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THE PHYSICS OF THE MISSISSIPPI RIVER.

A LETTER TO THE EDITOR OF VAN NOSTRAND'S MAGAZINE IN REPLY TO THE ARTICLE OF JAMES B. EADS IN THE SEPTEMBER NUMBER.

Sir: In the September number of your Magazine, a Review of the Report upon the Physics and Hydraulics of the Mississippi, by Humphreys and Abbot, appeared over the signature of James B. Eads, C.E. He states at the outset that he desires to reach the general intelligent public, rather than scientific readers. Although the Mississippi report was prepared for professional engineers, and its conclusions will stand or fall by their verdict, I ask space in your Magazine to expose some of the errors of this popular review.

Capt. Eads raises two principal issues with our report, each of which will be considered by itself.

RELATION BETWEEN VELOCITY AND SUSPENDED EARTHY MATTER.

The first question at issue is, whether the water of the Mississippi always holds in suspension the maximum amount of earthy matter, which water flowing with that precise velocity is able to support. If yea, then any diminution in the velocity must cause a deposit; and any increase may cause an excavation, provided the bed or banks are of such a nature as to supply suitable material. If nay, then no such results can be rationally predicated.

This question of fact—for theory has nothing to do with it—was conclusively decided in the negative by the long series of observations recorded in the Physics and Hydraulics of the Mississippi. Capt. Eads, with various adjectives which add nothing to his argument, denies the truth of this deduction, and asserts that it is based upon fallacies and blunders made by us in discussing our observations. The accuracy of the latter he does not dispute.

Our analysis of these observations, stated in brief, consisted in plotting two curves, of which the common abscissas were times; and the ordinates, respectively the mean velocity of the river in feet per second, and the corresponding number of grains of earthy matter held in suspension by one cubic foot representing the average for the whole river. If the latter quantity were a function of the former, the forms of the two curves would exhibit a certain symmetry; This was not the case, the amount of sediment per cubic foot when the river was flowing most rapidly being often no more than at the lowest stage.

These curves happen to be very unlucky for Capt. Eads' professional projects; for they overturn the theory upon which he proposes to dispense with levees, to improve the navigation of the upper river, and, in one word, to control

the appropriations which Congress may grant for the lower Mississippi. The world is not wide enough for them and him, and they must be removed from his path. Invoking, therefore, the divinities of Dynamics, Force, Matter, Space and Time to aid in raising a cloud of confusion over a very simple question of mathematics, he makes a desperate assault upon our method of reasoning. Not to follow him through several pages of this style of writing, which, although perhaps effective with the popular audience to which he appeals, will be passed over by professional men, let us examine his final conclusion. It is, that the ordinates of the sediment curve must be multiplied by the corresponding discharges of the river, before they can be compared with those of the mean velocity curve.

The absurdity of this proposition has been so clearly pointed out by Mr. R. E. McMath, C.E., in the *Engineering News* for February 28, 1878, that it is needless to repeat the elaborate algebraic demonstration by which he arrived at the conclusion that "mathematically this step is a blunder which would disgrace a boy in the junior class of our High School."

Suffice it to say, that the quantity under discussion is not, as Captain Eads argues, the total *mechanical work* performed by the river, which of course depends upon the number, as well as upon the supporting power, of all the cubic feet of water in motion; but *the force* which neutralizes the gravity of the earthy matter held in suspension. If this force be less than gravity, a deposit must occur; if it be equal to gravity, the water will be charged to its maximum supporting capacity with earthy matter; if it be greater than gravity, an additional amount of earthy matter may be added without causing a deposit. Now, Capt. Eads' theory confessedly requires that the water be always charged to the maximum capacity due to its velocity. The point, then, for observation to determine, is, whether there be any fixed relation between the earthy matter suspended in a unit of volume, and the horizontal velocity of that unit of volume. If any fixed relation existed between these quantities for the Mississippi, our curves would reveal it; but as they show the reverse, Capt. Eads' theory falls to the ground.

That the figures plotted by us correctly represent the quantities in question, is evident when we consider what they are. The mean velocity is the mean of the horizontal velocities of every cubic foot of the river. The amount of sediment was measured by collecting samples of water from three stations—one, 300 feet from the East bank, where the high water depth was 100 feet; another in the middle, where the depth was 100 feet; and the third, 400 feet from the West bank, where the depth was 40 feet. The total width was about 2400 feet. One hundred grammes of river water were collected daily, Sundays excepted, at surface, mid-depth and bottom, at the two deep stations; and at surface and bottom at the other. The figures plotted in our curves represent the mean weight of sediment per cubic foot for the entire river, computed by averaging the results from the eight stations. This averaging is legitimate, because the distribution of earthy matter held in suspension is remarkably uniform throughout the river, as is proved by the following yearly totals:—

DISTRIBUTION OF EARTHY MATTER.

Station.	High Water Depth	Total grammes collected in one year.		
		Surface.	Mid-depth.	Bottom.
Near East Bank.	100	15.302	17.552	17.880
Near Middle....	100	15.156	18.977	19.538
Near West Bank	40	13.845	—	20.070

The bearing of these figures upon Captain Eads' theory as "modified by depth" is too plain to need comment.

To recapitulate the foregoing views in more concise and mathematical language, the problem to be studied experimentally is, whether:

$$S = f(V).$$

In which S denotes the force which, opposed to gravity, maintains the earthy matter in suspension; and V is the horizontal velocity of the volume of water supporting said earthy matter. Now let us see what our critic proposes to do. If

he were not so bitterly in earnest about the problem, his blunder would be funny. He actually multiplies one member of the equation of which the truth or falsity is to be experimentally decided, by the *discharge*, which is nothing but the product of the area of cross section (a) by the mean velocity, (V); and leaves the other member unchanged, giving:

$$SaV = f(V).$$

And he then proceeds to congratulate himself, and to apply adjectives to us, in honor of the surprising discovery that, after he has introduced V into the first member, that member thus modified can be proved to be a function of V!

It would be easy to point out many other minor errors in this division of Capt. Eads' review, but after this exposure of the fundamental fallacy upon which his whole argument rests, it would be wasting time to do so.

Our position in the matter simply is, that the earthy matter which the river holds in suspension is chiefly that brought to it in suspension by its tributaries; and that if the amount they supply at any time be less than the velocity would support—as is usually the case in floods from the comparatively clear tributaries, like the Ohio—the water remains under-charged. Capt. Eads' claims:—"If the reader will bear in mind that the water is charged with sediment according to its velocity, and that it flows through a bed of precisely the same kind of material it is carrying in suspension, and that if its velocity is increased it will take up a greater charge from its own bed, or if its current be slackened it will drop some of its charge in the channel, and add to its bed, he will understand the important part which the speed of the current performs in the problem." Perhaps in thus committing himself, Capt. Eads' did not know that at Columbus, Ky., 20 miles below the mouth of the Ohio, the waters on the East side of the channel, which have issued from that river, although moving side by side, and with equal velocity with those from the Missouri on the West side, contain only about three quarters as much earthy matter in suspension. This fact, stated in the Mississippi Report, is proved by eight months' daily observations at Columbus. It would be ill-natured to wonder whether, if Capt. Eads' had noticed this ugly fact, he would have announced that, according to his investigations, these data "bear excellent testimony to the care with which Messrs. Webster and Fillebrown conducted the experiments at Columbus."

BED OF THE MISSISSIPPI.

The second main issue which Capt. Eads raises with our report, is in respect to the nature of the bed of the Mississippi. In this matter, he sets up a man of straw, and after overturning him, claims by so doing to have refuted our views.

As an example of the reckless misstatements of facts which characterize the whole review, I quote the following paragraph:

"By reference to pages 135 and 137 [Mississippi Report] it will be seen that this extract contains an astonishing exaggeration. Instead of *three years*, the current and sediment observations only occupied *eight months* at Columbus, and *one* year at Carrollton.

"When we remember that the junior author of the report on the Mississippi River was a prominent member of the Levee Commission, and that the senior author, as Chief of Engineers, warmly endorsed its report, it is difficult to reconcile this careless statement with the unusual scientific exactness which required four decimals to record their measurements of the current."

Turning to the page next preceding the one mentioned, we find a table giving in detail the results of the *two* years of sediment observations made by a party of the survey at Carrollton; and on page 142, another, giving the details of a continuous series of nearly three months' observations, made by Prof. Riddell. These with the eight months' record at Columbus, sufficiently sustain the accuracy of our language.

After such an exposure, it is sufficient without devoting space to other instances of similar inaccuracy in this Review, to ask the reader to apply the old maxim, "ex uno disce omnes," and take the precaution to refer to the report itself, before accepting as trustworthy Capt. Eads' statements respecting its contents.

This precaution is particularly necessary in order to understand our real

position in relation to the bed of the river. Capt. Eads endeavors to convey to the reader that our views rest solely upon a limited number of soundings in the river itself, and that all the information we possessed on the subject is reported in full in Appendix C. Neither of these ideas is correct. It is nowhere stated, nor is it true, that Appendix C describes every sample brought from the bottom. In the work done by me, personally, in which I used one of the transits on the bank, it frequently happened that when the boat brought the specimens for inspection, some uncertainty would exist as to the exact soundings in which individual specimens had been secured. In such cases, the samples were not bottled and no specific entry was made, but the general nature of the bottom was carefully noted, and this information was used in the final report. Capt. Eads might as well demand that a botanical collector should exhibit specimens of a plant gathered from every locality in which he discovered it, as to apply such a criterion to our work.

But if this kind of criticism is unfair, for a still stronger reason is the unfounded assumption that our views are solely based upon soundings in the river itself. In the report and appendices will be found stated at length many facts bearing directly upon *the geology of the region traversed by the river*, and which, from their wide range and perfect accordance with each other, will, it is believed, have great weight with any unprejudiced mind. Capt. Eads' whole argument on this point is that of a lawyer endeavoring to raise quibbles, rather than that of a judge stating the truth in an impartial manner. Readers who desire to study the subject should refer to the report itself, where the arguments are stated in as concise language as can well be used. They proved convincing to every member of the Commission of Engineers to which the subject was officially committed by Congress; and that they do not harmonize with Capt. Eads' wishes and theories, is our misfortune, not our fault.

One other point. Lest Capt. Eads' assertion that the shoaling of the bed of the river below Cubitt's Gap proves that crevasses cause a deposit in the channel, be accepted as true reasoning, I will simply say, that the location of this break, just above the head of the Passes, introduces anomalous features, which prevent occurrences there from having any legitimate bearing when applied to the river above. The great Bonnet Carré Crevasse, above New Orleans, has been open since April, 1874, discharging an immense volume of water at every high stage of the river; and yet no well-marked shoaling has occurred below its site. Capt. Eads may perhaps venture to deny this statement, which is fatal to his theories; but the facts set forth in the Report of the Commission of Engineers, ordered by Congress to investigate and report a plan for the reclamation of the alluvial basin, fully confirmed by later surveys, speak for themselves, and establish my position.

THE ST. LOUIS BRIDGE.

I must now refer to the single paragraph which has called forth this reply to Capt. Eads—his other statements, I think, might have been safely left unanswered, to be judged by professional men.

It must be apparent to every reader, that the personal animosity constantly exhibited by Capt. Eads toward the Chief of Engineers, and which marks every page of this review, unfits him from taking a fair view of the subjects discussed. This has led him not only to misrepresent the real issues, but also in the following paragraph, to introduce one entirely irrelevant:

"A few years ago the Chief of Engineers of the United States Army, being equally as well convinced that the steamboat smoke pipes were, like the bed of the river, unyielding in their nature, and that they were too high to pass under the bridge which spans the Mississippi at St. Louis, accordingly recommended that a canal with a drawbridge, through the bridge approach, to accommodate these unyielding smoke pipes, should be dug round the end of the bridge in the ancient geologic blue clay in Illinois, at a cost of over three million dollars! The fact that the river was proved by 'a glance at the two diagrams' to be always undercharged with sediment, was an assurance that the canal would be a success and would not silt up. But Congress did not look with favor on this plan. Doubts as to the unyielding nature of

the smoke pipes were openly expressed, and while the canal plans and estimates were being prepared the lucky discovery was made that the whole difficulty could be avoided by putting hinges in the pipes; and so three millions of public treasure were saved, and the commerce of the river now flows under the bridge without let or hindrance."

This paragraph is full of errors of fact, which might naturally mislead any one not familiar with the subject. The truth is the following:

On account of the strong complaints of navigators of the Mississippi River against the bridge which Captain Eads was building at St. Louis, a Board of Engineers was convened by the Secretary of War, to examine and report upon the matter.

At one of the public meetings of the Board after they had become satisfied that the complaints of navigators were well founded, and while they were discussing a remedy, a member stated that the case of the Louisville bridge was somewhat similar, and that he had provided for high water navigation by a channel cut into the canal bank at the South end of the bridge. He suggested that a similar plan might obviate the trouble at the St. Louis bridge, and proposed a cut into the shore behind the East abutment pier. The Board provisionally adopted this plan, and recommended that the local engineer officer be charged with the duty of working up the details and ascertaining the cost.

At a subsequent meeting of the Board, the local engineer officer reported as the result of his investigations, that the cut with its accessories would cost $922,436, and that the annual expense of operating the draw and keeping the cut open would be $15,000, which, capitalized at six per cent. would be $250,000, making a total cost of $1,172,436, (not three millions).

There can be no question as to Capt. Eads' knowledge of these facts, as he published a pamphlet in which he criticized this report with great bitterness.

From his statement above quoted, it would be inferred that engineer officers had never heard that steamboat chimneys could be lowered, and that as soon as the knowledge that such a thing was possible reached the ears of Congress, it at once decided not to interfere with the bridge.

In the first place, every boat engaged in commerce on the Ohio River, is provided with hinges, and this has been the case for many years. Even the little surveying steamer, the Major Sanders, in charge of one of the members of the Board, was provided with hinged chimneys. Evidently, therefore, they were no novelty to him, and the same remark can be made with regard to the other members, all of whom had had much experience in Western river navigation.

In the second place, the Board in their report stated that one of the objections to the St. Louis bridge was the following:

"The height under the lower arch is so small that a large proportion of the boats which will have occasion to pass under it must lower their smoke stacks at all, or nearly all, stages of the river; while many of the larger boats will not be able to pass under it during the higher stages, even with their smoke stacks down."

And yet Capt. Eads says that the discovery that steamboat pipes could be hinged, was made "while the canal plans and estimates were being prepared!"

The report of the Board contains no reference to "ancient geologic blue clay in Illinois;" indeed, there was no allusion whatever to the nature of the material of the East St. Louis wharf, unless an item in the estimate calling for "earth excavation (dredging)" at thirty cents per cubic yard can be considered as such. Nor was there the slightest reference to the question whether the river water was over-charged or under-charged with sediment; nor was there any "glance at the two diagrams." On the contrary, it was well known that the cut would probably silt up if left alone; and an estimate of $10,000 per annum was included for the specific purpose of keeping this cut open and the walls in repair.

Finally, *the only individual connection of the Chief of Engineers with these matters was, that the reports of the Board were submitted to the Secretary of War through him,* as required by the Army Regulations, and were formally approved with the recommendation "that the matter be submitted to Congress at its next

session for such action as in their judgment may seem to be necessary."

The above facts furnish such a commentary on the paragraph I have quoted, that their simple presentation is more severe than any language which could be employed. I leave the reader to draw his own inferences—mine being that any future attack of this nature from Capt. Eads, cannot be held to require notice.

HENRY L. ABBOT,
Major of Engineers and
Brevet Brig. General.

WILLETS POINT, NEW YORK HARBOR,
November 25, 1878.

SANITARY SCIENCE IN THE UNITED STATES—ITS PRESENT AND ITS FUTURE.

BY ALBERT R. LEEDS, PH.D.

Written for VAN NOSTRAND'S ENGINEERING MAGAZINE.

IT is generally conceded, I believe, by scientific laborers in this country, that we have been more fertile in invention than discovery. We owe to older nations a constantly increasing debt of obligation for those initial germs of thought which have fructified into new sciences, while we may, at the same time, ask a generous acknowledgment of the merits of many inventions which have opened up new fields of employment to thousands. Sciences which promise much for the improvement of the daily condition of mankind, and have in them a side largely practical are sure of welcome in our midst. Such a science is pre-eminently the one under consideration. It gathers into one the teachings of all other sciences, so far as they bear upon private and public health, and makes these teachings practically operative in the promotion of human welfare in this country. It grew into prominence during the war of the Rebellion, when the work of the Sanitary Commission was made co-extensive with every army camp and army hospital. Its principles have been expounded in Sanitary Associations formed in many States and in smaller communities. These have led to the formation of State and City boards of health, clothed to a greater or less degree with executive functions.

Every epidemic has fastened popular attention upon the subject, and what before was taught in book or lecture-room has been rehearsed in a thousand forms in the newspaper. In this present yellow fever plague more than twelve thousand people have perished, probably not less than sixty thousand have convalesced, and two hundred millions of dollars would not represent the aggregate pecuniary loss. During its awful course universal interest has been felt in the cause and prevention of this and similar diseases, a homily on private or public hygiene has formed a prominent feature of the daily paper, and this interest has culminated in the offer made by a lady, already widely known by her munificence in the cause of science, to defray the expenses of a commission of inquiry composed of sanitary experts. We believe that this is as it should be, and that in sanitary science this country is taking a foremost place, because popular sympathy and popular knowledge are running almost abreast of the science itself. The proper execution of sanitary laws demands the free and intelligent co-operation of the individuals; a system of enactments, however skillfully framed, and however supported by a strong central authority, alone will not suffice. Not only would it appear alien to the genius of our institutions, but also a mode ill-suited to attain its object, if a Health Department were added to the other departments of State at Washington. No one would deem it possible for such a department to legislate pure air, pure water and pure food into use throughout the nation. On the contrary, it would appear far wiser to leave such legislation to each State, and in the State as far as possible to each community, recognizing that the popular agitation and knowledge

requisite to obtain health laws is the best guarantee that they shall not afterwards remain dead letters. What I have to say concerning the present and future of Sanitary Science in this country is mostly the record of what such communities have already done, and of what ideas are now growing up in their midst, and which, ere long, will bear fruition in new laws and movements.

I. VITAL STATISTICS.

We shall begin at the foundation stones of exact sanitary science—Vital Statistics. To appreciate the advance in this direction, we have only to compare the condition of our greatest city, New York, at the beginning of this century with its present. At that time it had no registration even of deaths. The first "Bill of Mortality," as it was called, extended from November 1, 1801, to January 1, 1803, fourteen months. So little accuracy in the nomenclature of diseases was thought of or expected, that in this report the people are said to have died of "flux," "hives," "putrid fever," "rash," "lingering illness," (which certainly was not a rash performance), "stoppage," "breaking out," "fits," etc. The first reliable report was that made in the year 1866, after the organization of the Metropolitan Board of Health. In the second annual report in 1867, the beneficent results of the institution of the Health Board, and of the sanitary reforms executed under its superintendence, were shown by the fact that there were 3,152 less lives lost during the first year of its administration as compared with the preceding year. The report, moreover, showed that this gain had been mainly in the chances of life at the adult ages, and in the districts where the greatest amount of sanitary work had been performed. The causes of insalubrity affecting infant mortality were not yet within control. In the year 1868, the work of registration was extended and specialized in such a way that comparisons could be made in the death-rate between portions of the city occupied by a degraded and overcrowded population, and those more favorably situated, whether in point of natural advantages or in the character of inhabitants.

This specialization enabled the sanitary inspectors to judge of the value of their labors, in the matter of cleansing and disinfection. In fact, they had lowered the death-rate in certain of the most wretched wards below that in some of the best, sanitation in these latter having been omitted. The registration extended also to the effect of modes of living upon the death-rate, and in this manner pointed out the necessity of controlling the excessive mortality in tenement houses. That health reports when promptly and intelligently used might be effectively employed in the prevention of disease, was shown by the use of the returns made during the last two weeks in July, 1868. The Registrar, apprehending that the infective quality of Asiatic cholera might prove to be present in the rapidly fatal diarrheas then prevalent, sent warning to the Surgeon-General of the United States Army, in consequence of which the General in charge of the recruiting and transportation of troops, ordered the immediate suspension of that branch of the army service in New York. Valuable illustrations of the relation existing between damp houses and pulmonary consumption were obtained by selecting certain wards of the city, and forming maps in which every death from phthisis for a number of years was marked on the chart opposite the locality of its occurrence. The evidence so obtained pointed to an excess of consumption at the lowest levels, and in two of the wards to a crowding of fatal cases of this disease in localities unusually damp and in rainy seasons flooded, although these dwellings differed in no other respect from the average of the ward. The results obtained in this manner were deemed so valuable that the registration was applied to each house in the city. In this way excessive mortality in any locality, or from any special class of diseases, become known at once to the sanitary inspectors.

With regard to the registration of marriages, improvement was more difficult. The system of registration, expecting a voluntary support from clergymen and civil officers concerned, could secure very partial returns, and it was only by diffusing information through the press and the lavish distribution of circulars that the accuracy and completeness of the returns in this respect could be improved. An inquiry into the number of births

registered, as compared with that which the Board had reason to believe occurred, revealed a deficiency in the registry of 65 per cent. It has been stated by an American writer on these topics that "it would be impossible for a large portion of the adult men and women born in the United States to prove by any public records or other legal documents, that they were legitimate offspring, with a natural right to the name they bear, or even that their parents were ever married."

The system of mortality registration was gradually improved until the returns made in the year 1871 were probably nearly perfect. When compared with the mortality in other cities, this accuracy told against New York, for while its death rate was 28.6 per thousand, that of St. Louis in the same year was reported at 17 in a thousand; of Rochester, 16; Buffalo, 14, and Jersey City only 7. In the matter of marriages and deaths, the increased knowledge among clergymen, physicians and others, on whose voluntary co-operation the registration largely depended, had resulted in an apparent increase in the annual marriage and birth rate, but still the number of births returned was probably less by 10,000 than the true. In the following year the board instituted suits against these parties, which had a beneficial effect, but it became evident that nothing short of important changes in the law would secure completeness.

I have been thus particular in narrating the history of vital registration in New York, because this city was the first to undertake a reform, and because its reports were the first which attempted to keep abreast with the developments of sanitary science, and to diffuse this knowledge broadcast. The course of legislation on these points is one which every city and State has gone, or is going, through. In reference to New Jersey the facts are so fresh that I scarcely need recall them. At each meeting of the New Jersey State Sanitary Association, since its origin three years ago, the inaccuracy and worthlessness of the State Vital Statistics were conclusively shown in the reports of the committees on this subject. The association formulated a protest, and appointed a special legislature to memorialize the Legislature. By these means, and by the efforts of the State Board of Health, public opinion on the subject was awakened, and so far educated that, during the winter just passed, a law was enacted which gives to New Jersey one of the best systems of registration as yet devised in this country. It has incorporated in it two features to which its peculiar excellence are due, and which should be universally copied; 1st. That of issuing *burial permits* only after registry has been made by a properly qualified person. 2d. The returns are made to an expert, who collates them in accordance with the views of the most eminent authorities, and draws from them their most important teachings for immediate and very practical application.

II. REGISTRATION OF DISEASE.

We must not rest content, however, with the returns of mortality—we should advance to the registration of disease. This is practicable, and if not in all, yet in that large class of diseases in their nature preventable, of universally acknowledged utility. We do not delay until a street brawl becomes a riot, before notifying the magistrate and securing police aid, neither should we wait until diphtheria, typhoid, etc., become epidemic before sending intelligence to the custodians of the public health. But this is not all to make their knowledge of public utility, these custodians must be invested with adequate powers. At present there is little more expended upon the whole work of the Board of Health of the State of New Jersey during an entire year than the pay of two policemen. Its members labor without remuneration for the Sanitas Publica. Their power is mainly the educational impetus of just ideas, forcibly expressed. There are many ways of promoting sanitary reforms, but none it appears to me so practical as that of giving to the Health Board the money, means and men to register diseases, to investigate their causes, to suggest and promote their remedies, and not unfrequently to bring offenders to suitable punishment.

III. THE SANITARY LEGISLATION.

There is a source of danger, as this last summer has strikingly shown, which cannot be warded off by sanitary legislation when limited to a few of the States. If those States, which are the seats of

yellow fever, year after year, do not provide efficient precautions to suppress or control the epidemic, it will annually invade other localities following the lines of travel, and spreading northward to the Mississippi basin. We have recently seen the alacrity with which more favored communities came to the relief of those afflicted with the epidemic. Help of every description was sent until the bountiful public was asked to hold its hand. While the terrible plague lasted, was not thought a time for good advice, but for good deeds. Now that the danger is over, the time has arrived to avert similar visitations in future. Does it appear unreasonable to ask for the most skillfully devised sanitary regulations in localities where such a pestilence may germinate? Recent events have elicited a vast deal of discussion as to the origin of these epidemics and the modes of combating them. There is want of harmony, however, in all points but this, that some of the factors, which are concerned in originating the disease, are within human control, and prevention therefore, is the duty of the authorities where the disease germinates. Those involved in the consequences of neglect of these duties, however remote their homes, have a right to ask reform. This agitation should not be allowed to die out with the pressure of the calamity which aroused it. It should be continued until every one of the States has an efficient health code. At present the majority have either none or very deficient health laws. Massachusetts has strikingly shown its general enlightenment by being the first State to have an efficient health board and a wisely-devised code of sanitary legislation. New York and Pennsylvania have neither, though strenuous efforts have been made by public-spirited individuals to do away with the stigma. In the West, Michigan has been distinguished by the excellence of its sanitary legislation, and the voluminous and valuable publications of its State Health Board.

But Arkansas and Missouri are sadly deficient, and the case is even worse in Iowa, Kentucky and Indiana. Some attempts to supply the most pressing wants have been made in Florida, and in North and South Carolina, and health laws are not entirely wanting in the statute books of New Hampshire, Maine and Rhode Island. The necessity of educating the people in each State, before the requisite legislation is secured, will require a considerable period to elapse before all the States have systems of laws in accordance with modern knowledge. In the meantime, in the name of all those good men who have perished, and as an acknowledgment of the nation's charity, let the plague-stricken States of the Gulf and the Mississippi basin lose not a day in adopting the wisest precautions, experience and investigation can offer. Struggling as we are in this country to have the importance of sanitary legislation generally recognized, the progress made in some directions is highly encouraging. It is probable that no community will take steps to learn what is essential to its health before it has suffered from lack of information. To the distress of London the world owes those great works of the Royal Commissioners on Water Supply and the Pollution of Rivers, which are the repertory of the best knowledge on these topics. The manufactories of England have made it necessary for the government to take cognizance of aerial impurities, and which has been done in that country towards establishing a chemical climatology. Similarly the pollution of the Passaic by the manufacturing towns above has caused inquiries to be set on foot, akin to those referring to the pollution of the Thames, and has given rise to extended inquiries into the methods and aims of water analysis.*

An attempt was made to deprive the inhabitants of New York of some of their public parks and occupy them with buildings devoted to military and other purposes. The more public-spirited citizens came to the rescue, and through the influence exerted, a Public Parks Association, and other means preserved the open squares as breathing places and pleasure grounds. The association recognized as its principle of action, that to preserve the parks they must be improved. The proposition was made and eloquently advocated by Dr. Seguin, that the physical

* Report to the Board of Public Works of Jersey City. Professors Hurtz and Leeds, Analytische Bertrage, Prof. Leeds, Zeitschi für Anal. Chem., 1878. Recent Progress in Sanitary Science, *ibid.* Annals of the Lyceum of Natural History, N. Y., vol. xi., 1878. Water Supply of the State of New Jersey, *ibid.* Journal Franklin Institute, March and April, 1878.

as well as the spiritual well-being of the citizens at large would be powerfully augmented by making the public gardens *out-door schools*, supplementing the in-door school system by that in which they are lamentably deficient, an education in the phenomena of plant and animal life. A beginning in this direction has been made in the Botanical and Zoological Museums of the Central Park of New York, and in the Fairmont Park in Philadelphia, but these are remote from the centers, and the objects of study should be placed where they could constantly appeal to the eye. The hygienic value of gratifying the sense of beauty as well as satisfying the requirements of use is more and more recognized. The first society on this side of the water organized with this object, was the so-called "Land Hill Association," of the village of Stockbridge, in Western Massachusetts. After twenty years of activity, the result has been to produce a village of exceeding loveliness. Thousands of trees have been planted out along the road-sides. The village cemetery, formerly neglected, has been surrounded by an exquisitely kept hedge. Monuments have been erected to the memory of villagers whose subsequent achievements have made the place of their birth illustrious. Prizes have been offered for those who labored most efficiently to improve the health and beauty of their native town, and for these prizes the poor as well as the opulent contend. In fact the neatly kept side-walks, the attractive gardens, the pretty cottages of the poor are a better indication of what healthy pride can do for a community than the trim lawns of the rich. I need not add, that in a community where these things, which add grace and beauty to the daily life have been done, the more important works of water-supply, drainage, sewerage, etc., have not been left undone. Similar associations have sprung up throughout New England. In Williamstown the villagers have thrown down every fence, and this most picturesque of country villages is a beautiful park, through which the houses of the inhabitants are scattered. Some of the towns of New Jersey have caught the spirit of improvement, and recently I heard that Boonton, for which nature has done so much, was about to turn this bounty to account, and fill the houses, which a reverse of industrial prosperity had left vacant, with tenants attracted by its added charms of nature and art.

IV. VENTILATION.

In the matter of ventilation, a considerable advance on the whole is to be noted—in other words, the percentage of failures to successes, in cases where methods of ventilation, for the time being in vogue, have been tried, is slowly growing smaller. The volume of scientific literature, founded on our increasing knowledge of the properties of materials, of gases and of heat, grows more rapidly than the generally accepted rules by which the art of ventilation is to be practised. It is noteworthy that there are few persons who do not regard themselves competent to arrange the ventilation of an ordinary building, and it has hitherto been left largely to the builder, the vestryman and the school-trustee. This should not be the case. What advance has been made is mainly due to the specialization of this kind of professional labor—the formation of a class of engineers who devote themselves exclusively to problems of this character, and who have fought their way into practice by successful work accomplished. The architect submits his plans to these specialists, who add to them the requisite details of heating and ventilation. It would be a great step in the interests of sanitary science if the school or hospital trustee would not think it devolved upon them, as a portion of their office, to become for the time being an authority upon ventilation, and if they were, as a proceeding of sound economy, to relegate this duty to persons properly qualified. As a matter of fact these qualifications are obtained at a considerable expense to the community, for in this stage of the art of ventilation, there being no authority universally recognized and but few generally conceded rules, every sanitary engineer goes through a similar series of experiments and failures before he arrives at a reasonably successful method in practice.

As far as I can learn there appear to have been great successes as well as great failures, whether the system of ventilation by aspiration has been resorted to or that by propulsion. At the present

time many authorities of note have declared in favor of mechanical ventilation. And yet in a number, I might say in most of the asylums and hospitals in this country where fans have been introduced, they are now standing still. The Roosevelt Hospital, for instance, in New York, where the fan, after having been put in operation, was run backwards and was run so for months. It is now stopped. This is one fact of many, which would make us chary of affirming positively that either system is the better. Probably both discreetly applied yield good results, and in their skillful application and not less the faithful supervision of the ventilating apparatus after introduction are good results to be sought.

V. PHYSICAL EDUCATION IN SCHOOLS AND COLLEGES.

Progress in this direction has been initiated at our highest seminaries of learning, and is slowly working its way downward through our educational system. I do not refer to so-called athletic sports, although these had not attained to much prominence in our colleges prior to the year 1850, but to the introduction of physical exercise and instruction on hygiene as a part of the college curriculum. This, so far as I am aware, was first successfully accomplished in Amherst College, and now, after a trial of nearly twenty years, is still regarded as an indispensable adjunct of the college course. The dignity of this department of instruction is emphatically recognized by appointing to it only distinguished members of the medical profession and including them in the college faculty on the same footing as the other professors. It is made their first duty to know the physical condition of every student, and to see that the laws of health are preserved by them. In case of sickness, the students apply to this officer for a suitable certificate, which excuses them from college duties, and are put in the way of obtaining suitable treatment. The statistics of the bodily condition of the students are regularly and frequently secured, and are supplying a collection of "physiolgical constants," which are of growing interest, and supply practical helps in determining whether the student's physical condition is within the bounds of health, and whether their development from time to time is normal or otherwise.

All the classes are required to attend the gymnasium exercises four times a week, and the regularity and faithfulness in this is made an element in collegiate standing. The performances are accompanied with music, and arranged to give full play to the animal spirits. This and the advantages personally experienced by the students, have conspired to make the gymnastic fully as popular and well-attended as the literary exercises. Finally, the intelligent co-operation of the students is secured by instruction upon the means of preserving health, physical and mental, with supplementary lectures upon human anatomy and physiology. Prof. Hitchcock writing* of the chances of life of the young men under this hygienic discipline as compared with men of the same age elsewhere, says "it is regarded as an established law that the chances of life grow less and less from about the fifteenth to the twenty-third year, and the rate of decrease is very rapid. But the tables of health, as kept at Amherst College, show that there is an improvement in health from year to year through the course, the ages being from nineteen to twenty-three. For taking the number of sick men in the freshman class as unity, the number in the sophomore year is 0.912, in the junior, 0.759, and in the senior but 0.578, the percentage of sickness during the college course diminishing to nearly one-half.

In the light of this successful experi ment continued for a period of twenty years, it is not premature to urge upon colleges generally the formation of a similar department; and for my own part I see no method of raising the character of public school instruction so effectually, than that of giving to the *physical training* of the children a very prominent place.

VI. HEALTH RESORTS.

The growing facilities each year for travel are steadily increasing the number of citizens who visit the country, the sea shore and the mountains. A salubrious village is frequently transformed into a center of pestilence merely by such in-

* Hygiene at Amherst College. Prof. Hitchcock, Amer. Public Health Assoc'n.

flux of strangers, the entirely natural means of disposal of refuse and excuta no longer being adequate, and the artificial methods not being provided until an outcry due to disease is raised.

In no way, however, is the growing intelligence on sanitary matters more strikingly shown than by the extreme sensitiveness of pleasure seekers to the salubrity of summer resorts, in respect to water, sewerage and drainage. Of the multitude of illustrations I need but speak of Bethlehem, in New Hampshire, a beautiful village situated 1,700 feet above sea level, and so renowned for the purity of its atmosphere; that of the 40,000 hay fever patients, whom Dr. Beard has calculated exist (and hay fever patients say that life to them is only a tolerable existence) in this country, several thousands annually spend part of the summer there. Its popularity increased in a few years so rapidly that a crowded village soon arose, and during the summer of 1877 an outcry was made concerning drainage. The towns people realizing that the reputation of salubrity being the wealth of the town, steps must be taken at once to preserve it. They did so and during the present summer the influx of visitors has been greater than ever.

These two stages in the growth of a summer resort—its sanitary degradation and subsequent rehabitation—cannot be witnessed in every phase of their development, along the entire coast-line of the State of New Jersey. This great sea-side resort, a hundred miles in length, stretching from Sandy Hook to Cape May, is rapidly growing into an almost continuous city. It harbors each summer a vast multitude from our two Metropolitan cities, and from the middle and middle Western States. Even as a Sanibarium in winter, the physicians of Philadelphia, during the past lustrum, have recognized the great advantages that were pointed out by Dr. John Toney nearly a half century ago, and are sending their patients in need of change of air to Atlantic City and neighboring points. The arid expanses of its sandy shore have become in this way one of the principal sources of income to the State. Would it not then be a highly remunerative policy for the State to maintain their attractiveness. As a fact, nothing of a preventive nature was done. The shallow pits, which provided surface water for drinking and other purposes when the population was sparse, were multiplied when the visitors came by many thousands. Malarial and typhoid fevers were rife in spots which the sea-breezes visited every day. Only with the consequent suspicion and public alarm which threatened to empty them of their patronage, did these places tardily move in the matter of adequate sanitation, and now the universal introduetion of cemented cisterns and the diurnal removal of garbage under the stringent regulations of local Boards of Health, attest the purpose of the great sea-side sanitarium to retain its highly profitable reputation.

VII. SANITARY ADVANTAGE OF DOING AWAY WITH ILLUMINATING GAS AS A MEANS OF ILLUMINATION.

Any process of illumination which returns to the confined atmosphere we breathe the products of combustion is theoretically open to objection. All methods of illumination up to the present time have depended on some process of rapid combustion, oxygen being withdrawn from the air, an equal bulk of carbonic acid returned to it, and oftentimes a large amount of heat as compared with the amount of light liberated in the process. If now we can illuminate without the subtraction of vital and the addition of mephitic air, and if, however, we can produce an intense light without a corresponding heating of the surrounding atmosphere, we have made two steps of great hygienic value so far as the illumination and ventilation of rooms is concerned.

There is much reason for supposing that this will be soon accomplished in the wholesale introduction of the electric light. By very many roads a crowd of inventors is pushing forward to this end. The rapid destruction of the terminals with the corresponding need of frequent adjustment is being obviated by a variety of devices. In some the length of the carbons is made invariable by a supply of carbon from a hydrocarbon atmosphere in which the electric arc is taken. In another, wasting is prevented by an entire exclusion of oxygen, and the terminals are surrounded by an atmosphere of pure nitrogen. Another experimenter

separates his carbons by an intervening material over which the arc is found, and parts of this electric candle are burnt away at the same rate. In one of these ways, or in some other, the problem of lighting by electricity, more perfectly and as cheaply as by illuminating gas, will be solved, and we shall have the attendant train of hygienic benefits. In this matter of street illumination, the contamination of the atmosphere by gaseous products of combustion is, of course, not a matter of great moment, but in the illumination of those places of public assembly, the church, the theater, the lecture-room, the improvement will be of much importance.

VIII. SANITARY SURVEYS.

The intimate relation between health and the configuration of the soil has been recognized from time immemorial. In truth there is reason to suppose that more practical weight was given to it in ancient than in modern states of society; for while in the former security from enemies and the possible exigencies of a protracted siege made it imperative to select high places capable of good drainage for city sites, the demands of commerce are now best met by towns of the lowest level, frequently at the estuaries of rivers or marshes formed at the confluence of great streams. While the demands of commerce are inevitable, the call upon sanitary science to avert as far as possible attendant evils is not the less urgent. For this reason the rapidly increasing bulk of statistical information upon this subject is a matter of much gratulation. The geological surveys prosecuted by the State governments, and latterly extended to the shores of the Pacific by the munificence of the National authorities, have supplied an admirable foundation. The hydrography of sea-coast and the seaboard estuaries has been executed on a basis so broad and solid that the topography and hypsometry of the whole country can be built upon it. In addition to these we have a number of studies of the relation of topographic and geologic features to one of all the various types of disease.

Even before the inquiry instituted by the medical staff of the Privy Council of Great Britain, the extended research by Dr. Bowditch* demonstrated the intimacy of the relation between wet and retentive soils, and the prevalence of consumption, these conditions of surface-structure being chargeable with a thousand deaths from consumption in Massachusetts alone. Subsequently the fluviate and pond basins of Massachusetts were surveyed and mapped out by Kirkwood.

Staten Island having long lain under the ban of insalubrity, a number of gentlemen, interested in its occupancy and improvement, instituted a sanitary survey of the Island. Dr. Elisha Harris and Mr. Frederic Law Olmstead examining into the more general questions involved, and Prof. Newbury and Trowbridge the structural conditions. The influence of the surface soil, and of the underlying rock, its porosity, its bedding and its joints, upon the local climate, the drainage and the attendant salubrity have served as models for the conduct of similar investigations, in various other portions of the United States. Prof. Cook has been engaged for several successive summers in running a series of levels over much of the densely populated water-sheds of the Hackensack and Passaic, with similar objects, and recently a most minute inquiry has been made into the soil, contour and drainage of Hudson County, by Mr. L. B. Ward, C.E. He has determined how much of its area, which includes the communities of Jersey City and Hoboken, and the smaller towns of Harrison, Kearney, North Bergen, Union, Weehawken and Bayonne, is upland, how much is marsh, what portion is rocky, what occupied by soil of various kinds and where possible, the nature of the substrata, with the population on each tract, and their condition in regard to sewerage and drainage. The ability of each variety of underlying rock, serpentine, sandstone and trap, to carry off surface water is considered, with the corresponding influence upon the surface temperature, dryness and salubrity. When we consider the abrupt changes of habitat over this crowded area, it will be seen that it offers a field peculiarly favorable for the study of the effect of surface condition upon the rates and causes of mortality. Most fortunately the vital statistics of this district have been tabu-

* Consumption in New England, or Locality one of its Chief Causes.

lated with exceptional fullness and accuracy under the superintendence of the President of the Health Board, Dr. L. H. Elder. These statistics have been investigated by M. E. H. Harrison C.E., of Jersey City, with extreme care. He has plotted upon a unking map every case of fatal illness arising from insalubrious environments, each disease being indicated by a distinctive character. Other maps are in course of preparation, showing the relations of surface, contour, and drainage of soil, of rock, of sewerage and relative density of population over the same area. So far as preventable diseases are concerned, their causes will be made so plain that no inscrutable Providence, but wilful ignorance and neglect will be blamable for falling victims to their inexorable operation.

VIII. UPON THE COMPOSITION OF THE ATMOSPHERE.

In conclusion, gentlemen, I hope I may be permitted to say a few words with regard to one topic of sanitary science which for a long time has more particularly interested me, and which at the present time is the subject of especial study—the composition and purity of the atmosphere. As communities grow more dense, and factories multiply, the sources of aerial impurity augment in a rapidly growing ratio. In England, the Government has been forced to appoint an inspector, the celebrated Dr. Angus Smith, who has made thousands of examinations of the air in all parts of the country, and directs Government interference where persons and property are too much imperilled by atmospheric contamination. In Glasgow, a city analyst has been recently appointed with this special duty.

New York is already showing the effect of the sulphurous and nitrous vapors sent out from its myriad chimneys. Recently the U. S. Signal Officer, in his room at the Equitatable Insurance Building, took out from a carefully wrapped package an instrument which he desired to show me, and it was hopelessly corroded by the acid vapors sent out from the tall chimney of the United States Assay Office near by. In Philadelphia there is scarcely a house front which is not disfigured by some stain of magnesia and lime salts, a result in part due to the acid vapors in the atmosphere. And when rains sweep down and carry with them in solution such agents, they are more powerful to corrode metal and even stone surfaces than would at first appear credible. When one of the normal constituents of the atmosphere is wanting—the ozone—it has largely lost its sweetening and disinfecting powers, and there is much reason for believing that the prevalence or severity of certain diseases is intimately connected with the varying amounts of ozone in the atmosphere. Unfortunately there is much difficulty in estimating the percentage of this constituent of the air, and in guarding against the disturbing influences upon our determinations of other bodies possibly present.

To overcome these difficulties experiments have been on foot in the laboratory of the Stevens Institute of Technology for many months past.

Numerous analyses of the atmosphere collected in various parts of the United States, have been made and recorded. They will serve as contributions towards a beginning of a chemical climatology of this country, and might with great profit to the physician, the agriculturist and meteorologist be vastly extended by Government aid in connection with the Signal Service and the Department of Agriculture. We cannot do, unaided, as much in researches of this character as can be done in the laboratories of the Old World by Government assistance, but we can at least labor in the hope that the time is not distant when the importance of research of this kind, even if it does not end in a profitable invention, will be generally understood and generously encouraged.

The committee of experts appointed by Secretary Schurz to select from the fifteen plans submitted for the remodeling of the portion of the United States Patent Office destroyed by fire has adopted the plan of Mr. Vrydagh, of Terre Haute, Indiana. The plan embraces the addition of an attic story. The upper portion of the building, which has been used as a museum for exhibition of models and curiosities, will be remodeled and made into office rooms, as more are necessary, and the new attic story will be used for a model room.

FURTHER APPLICATIONS IN THE FLOW OF SOLIDS.*

By HENRI TRESCA, President of the Société des Ingénieurs Civils, Paris.

From "Iron."

It has long been known that heat is developed by the forging of a metal; and in some operations connected with the *platinage* of steel, pieces of steel subjected to blows rapidly delivered, may be raised to a dark-red heat. This phenomena does not ordinarily take place, except in working thin sheets; and it will be shown that, in working thicker pieces, the precise situation of the greatest development of heat can be recognized. In a forging operation which the author has had to conduct on a large scale on an alloy of iridium with platinum, a phenomenon occurred incidentally which engrossed his whole attention, bearing intimately as it did on the deformation of solid bodies. He may be permitted to refer to it, though the experiments are not yet completed; and it will be a source of great satisfaction to him to make known the first results of these experiments to an assembly of English engineers before any publication of them elsewhere. On the 8th of June, 1874, the author simply announced the main fact at the Academy of Sciences, that when the bar of platinum, after having been forged, had cooled to a temperature below that of red heat, it happened several times that the blows of the steam-hammer which at the same time made a local depression in the bar and lengthened it, also reheated the bar in the direction of two lines inclined to each other, forming on the sides of the piece the two diagonals of the depressed part; and this reheating was such that the metal was in these lines fully restored to a red heat, so that the form of these luminous zones could be clearly distinguished. These lines of augmented heat remained luminous for some seconds, and presented the appearance of the two limbs of the letter X. Under certain conditions as many as six of these figures produced successively could be counted simultaneously, following one another according as the piece was lifted under the hammer so as to be gradually drawn down for a certain part of its length. The appearance of these luminous traces can be explained beyond all doubt. They were the lines of greatest sliding, and also the zones of the greatest development of heat—a perfectly definite manifestation of the principles of thermodynamics. That the fact had not been observed before was evidently owing to this, that the conditions necessary to be combined at the same moment had not been present under such favorable circumstances. Iridised platinum requires for its deformation a large quantity of work to be expended upon it. The surface takes no scale, and is almost translucid when the metal is brought up to a red heat. The metal is but an indifferent conductor of heat, and its specific heat is low. All these are conditions which are favorable for rendering the phenomena visible in the forging of this metal, whilst it has remained unobserved with all others. Although this explanation was what was to be expected, the author nevertheless proceeded to justify it by experiments of a more direct character, of which some account will now be given; and which constitute the chief motive, and it may be added the chief point of interest in this communication. Given a bar of metal at the ordinary temperature, if, after having coated it with wax or with tallow on two faces, it be subjected to a single blow of the steam-hammer, the wax melts where the depression is produced, and it is observed that the melted wax assumes in certain cases the form of the letter X, as was observed in the case of the platinum bar. In other cases the limbs of the cross are curved, presenting their convex sides to each other. The heat has then been more widely disseminated, and the wax melted over the whole of the interval by which the curves are separated.

The prism, which has this melted outline for base and for height the width of the bar, represents a certain volume and a certain weight; and if it be admitted that the whole piece has been raised to the temperature of the melted wax, the elevation of temperature represents a

* Read before the Institution of Mechanical Engineers.

certain quantity of heat, or, in the ratio of the mechanical equivalent, a certain quantity of internal work which is directly exhibited by the experiment. In comparing this work with the work done by the fall of the hammer, a coefficient of efficiency is obtained which amounts to not less than 70 per cent. This value cannot be taken as final; it depends upon the conductibility of the metal, on the stiffness of the apparatus, and on the clearness of outline of the melted surface. But what the author is desirous to impress upon the meeting is, that here there is a return to the first methods of Mr. Joule, and that the author's investigations of the flow of solids conduct him to certain thermodynamic demonstrations.

The following are the numerical data for some of these experiments, together with the illustrative figures:—

Name of metal.	Work of the ram.	Form of the impression.	Area of wax melted.	Thickness of the forging.
	kgm.		eq.	cm.
Iron....	80	Rectangular.	1.45	2.5
"	90	"	1.50	2.5
"	110	Wide-spreading.	2.20	2.5
Copper..	60	Rectangular.	1.75	2.0

Volume of the corresponding prism.	Corresponding number of heat-units (heating to 80° C).	Equivalent work, at the rate of 435 kgm. per caloric.	Proportion percentage of total work converted into heat.
cc.		kgm.	
3.63	0.1498	63.72	0.796
3.75	0.1547	69.79	0.731
5.50	0.2269	96.44	0.877
3.50	0.1329	56.48	0.942

In the last experiment, taking as melted the area of wax included between the hammer and the crosses, a useful effect of 94 per cent. is obtained.

Stamping.—The object of stamping is to dispose the relative displacements in given directions, in order to pass from the primitive form, supplied direct by the maker, to the definitive form which is desired to be accomplished. From this point of view, the die is a kind of channel designed to facilitate the flow of the material, and to guide in the most suitable direction or directions. When it is required to draw down, by stamping, a square bar of iron, each blow of the hammer causes transverse enlargement as well as elongation; and the useless enlargement is advantageously obviated if it be prevented by the presence of the sides of the canal. If it be well to employ the stamp in simply drawing down a bar, how much more indispensable is it when the variation of form is more complex? The simple idea of flow supplies material for forming a rational judgment on the successive dispositions of the stamps required for the intermediate operations; and also on the adjustment of the sections of rolls, which are but circular stamps or molds, by means of which iron is drawn out. That all these phenomena are but various forms of flow, of which in most cases the circumstances can be anticipated, may be shown by other experiments which will now be described.

The most characteristic of these experiments is, perhaps, the following:—Having completely effaced the reverse in relief of a piece of money, place the flat surface on a sheet of lead, and flatten the second face in the stamping press. The whole relief of this face will be produced on the face which had been reduced to flatness; and the design of this relief will be imprinted on the lead. This effect is explained by the circumstance that each vertical thread or fibre of molecules, being separately compressed in the direction of its length, flows, when struck, with greater facility into the lead than into the other parts of the piece. The saliencies, as reproduced, are less, no doubt, than in the original relief, whilst the more delicate features are partially obliterated, but the general effect is reproduced, and it is apparent that the flow takes place in the direction of the depth, which is also the direction of least resistance. On the reverse of the sheet of lead, which has necessarily been reduced in thickness by the effect of the imprint, the image will be found repeated in a more confused manner, and it may be distinguished by a peculiar tint, which indicates a well-defined geometrical transformation; the lead having flowed in a horizontal direction, as the

only way of escape when its surface was depressed. This amplification or enlargement takes place in the proportion of 22 to 13, when the plate of lead is $\frac{1}{8}$-inch thick.

An entirely different effect is produced when a metal is struck. The blank piece having been placed in the matrix, the portions which are not to be raised in relief by the action of the press are reduced in thickness, for the benefit of the neighboring portions, which are raised; the metal literally flowing in radial directions, from the hollows to the reliefs by which they are surrounded. If the metal has only an engraved face, it may be made up of several blanks of equal thickness superposed. The same mode of distribution of the molecules takes place, and is manifested by successive imprinting at each face, in which the final relief is more or less obliterated. It is so clearly a manifestation of flow that takes place under these conditions, that if the bottom of the matrix be hollowed out at the center, then, the material which converges from the circumference exciting a pressure towards the center, the central portion of the blank is driven towards the orifice, where it forms a very regularly shaped boss; admitting of the transformation of a relief executed on a plane, into a similar relief on a surface which has become very convex or very concave, according as the design pertains to the upper or the lower face of the blank. To an analogous cause the presence of scars sometimes observed on metals highly relieved, is to be attributed; these scars being produced simply by the junction, during the latter strokes of the edges of the bosses which are formed by the earlier strokes. When the medal is relieved on both faces, if it be made up several plates superposed, it is interesting to remark the successive developments and effacements of the images on both sides of the plates; mingling and merging in each other in a singular manner. Rules cannot yet be formulated for the best forms of the grooves of rolls; but it may be accepted that they should be shaped in such a manner as to utilize as far as possible the natural flow of the metal in the direction of the pressures applied to it. It has been shown that, when a bar has to be drawn out, it is best to prevent any enlargement of it laterally, and to facilitate the longitudinal flow; the die should therefore be carefully gauged, short, and opened out in the direction of the length. It has been seen, also, that in stamping a disc, it may be useful to make use of centripetal compression. Each mode of action has thus its own mode of deformation, of which it is necessary to know how to take advantage. The following is a very remarkable instance. Given a disc of lead 4 inches diameter $\frac{1}{8}$-inch thick; if it be pressed in the stamping machine for a diameter of $2\frac{1}{2}$ inches at the center, the thinning of this central portion is only effected by the flow of the material outwards; and this flow is exactly symmetrical, when the centering is perfect. The exterior border is developed in the form of a tulip. By such means, without the employment of a matrix, geometrical forms of a perfectly definite character may be produced, which may be useful in some cases.

This general disposition of material has been long since observed by MM. Piabert and Morin, in the course of their experiments in drawing out blocks of clay. Around the orifice of entry the clay was thrown out in the form of acanthus leaves; and the same development is to be observed in the displacements which take place when projectiles are discharged against armor-plates. The metal displaced by the projectile is driven forward in flakes or strata more or less involved and dislocated, which have, nevertheless, a striking family likeness to the dispositions previously noticed. The geometrical condition of the development in tulip-form of the plate of lead may be very simply explained. The border of the plate, which makes an effort to retain unaltered its diameter and its thickness, continues to be attached to the central portion, the gradual crushing of which throws out rings which are successively thinner and thinner. These rings have, therefore, at at each instant a given thickness, and by their succession they necessarily form a surface of revolution, which is accurately calculable, on the hypothesis, which is perfectly justifiable, that the volume is constant. The conditions of such development may be modified by the employment of casings of various forms,

but attention will be confined to the case of a concentric casing so disposed as to prevent any increase of diameter. Eight discs of lead $1\frac{1}{2}$ inches in diameter having been placed in a cylinder, a piston of 1.20 inches in diameter is placed upon the pile formed by these plates. Since the material can only escape from the compressive action by the annular space comprised between the piston and the cylinder, it ultimately assumes the form of a sort of tumbler, of which the height is extended to the length of the piston, even beyond the length of the cylinder. The thickness of the tumbler, 0.15 inches, would have been more regular if but one disc of lead or of tin had been employed. But the mode of distribution of the layers in the thickness of the tumbler is in itself a useful subject for consideration. The uppermost plate has been developed, almost in one piece, to the upper edge of the tumbler, being connected by a continuous supplementary part, which becomes gradually thinner until it reaches the foot of the tumbler. The other plates are also developed in a parallel direction, supported by the sides of the cylinder, for a length which may be submitted to the same kind of calculation as that of the plates of the concentric jets. It is the same mode of deformation applied, in the present case, to an annular jet; and the complete analogy between the formulas which give expression to their relations is not one of the least remarkable facts in these transformations.

This method has for several years been adopted in industrial operations, under conditions of precision which are truly astonishing, in which a vertical and cylindrical jet, twelve inches high, is manufactured from a sheet of tin perfectly smooth and of uniform thickness. In the finest specimens of that size, the ends of the tube, which are pared after having been struck, do not show any irregularity exceeding 1-12th inch in height, even though the cylindrical envelope has been suppressed for the whole height. The substance driven out in the form of a ring, the thickness of which is measured by the difference between the radius of the punch and that of the matrix, is naturally disposed to form a thin cylinder, the several elements of which slide with equal facility upon the perfectly polished surface of the punch. A thousand examples of similar surprises may be found in industrial processes; but this instance, amongst them all, definitively sanctions the expression by which the author believes he is authorized to designate the results of his researches. The flow of solids is now recognized in science; much more will be accepted by the members, who are witnesses every day of the processes which are based upon it, as the true expression of the best ascertained facts.

Planing.—Of the various operations which have been described above, that of punching is the only one which has had for its object the dividing of a solid body, and forming two entirely separate parts—the burr and the punched block. The block is augmented by compression of a portion of the matter which constituted the cylinder which would have been simply pushed out by the punch, supposing that the cylinder could have slipped out without giving rise to other phenomena. The burr is reduced by the same amount. Cutting or shearing does not really take place until the moment when the burr, in consequence of lateral flow, has been reduced to its final height. It has been proved that from this moment the resistance opposed to shearing is actually proportional to the area of the zone of shearing. The coefficient of resistance applicable to this separation is no other than the coefficient of resistance of fluidity; or what amounts to the same thing, the coefficient of resistance to rupture; so that we are now put in possession of a certain formula, applicable equally to circular shearing by the action of the punch, and to rectilinear shearing by the shear blade or by the turning tool. In each case one of the parts of the piece slides upon the other part, producing at the two sides in contact a drawing out of the successive layers, which are bent over in the direction of the length of the shorn surface, in thin threads, like those produced by the punch. The separation only really takes place at the moment when these shreds are drawn to their extreme limit of tenuity.

This characteristic of the separated surfaces is met with in planing, although

the principal circumstances may here be entirely different; not less remarkable, however. The principal difference consists in this, that the chief compression takes place, not in the solid mass as before, but in the cutting which is detached by the tool, which, as it forms the exterior portion, opposes to the flow the least resistance. If the cutting be compared with the space which it occupied in the block before separation, it is easily observed that it is at the same time considerably shortened, and that, consequently, its thickness has been augmented in the inverse ratio of the shortening. This leading fact in planing is very well exemplified in the turning from the wheel-tire of a locomotive, comprising a cutting from the rivets. These are represented as of an elliptical section, $1\frac{1}{8}$ inch by $\frac{4}{16}$ inch, showing that the reduction in length affected by the action of planing was in the ratio of 10 to 28, or 0.36. This coefficient of reduction is still much greater than it is in many other circumstances; for the thinnest cuttings the coefficient is occasionally as low as 0.10. In another instance, a cutting planed off transversely from a double-headed rail, the height has not been altered, but the width has been reduced nearly in the same proportion as in the first example.

Another characteristic of cuttings produced by planing is, that the surface of the cutting, which rises from contact with the cutting tool, is always smooth and is developed geometrically. That surface, in fact, is moulded on the tool during the process of deformation, and slides upon it in such a manner as to roll itself up in the form of a cone or of a cylinder. At this moment, above all others, the plasticity of the metal is brought into play; and if the original form of the cutting should interpose too serious obstacles to this development, it tears or splits according to the direction of the generating surfaces of contact, still responding to the geometrical condition first referred to. It is well to avoid such rents as much as possible, for evidently they cannot be produced without the expenditure of additional power. Such loss of power must take place, especially where it is required to reduce a curved surface at one cut, of great breadth. An example of such fissures is shown on about a third of the width of another cutting from a tire; but those of the opposite edge are attributable really to a greater reduction of the length of the thinner edge in the process of planing. The other face of the cuttings is always rugged and wrinkled with fissures, or with transverse ridges of very variable aspect, according as the metal is more ductile and the cutting is thicker. For the greater thicknesses both iron and steel present on that surface a multitude of inclined ridges partly covering one another; and of which the incline is still better defined where complete separation has been produced. These scales have been drawn just as they appear under the microscope, on a cutting of Bessemer steel. Nothing can show better than their general inclination the sliding that may be produced in planing, in consequence of the compression which is produced in front of the tool before the cutting is completely detached from the block. In the greater number of cases the turning, when long enough, winds up into a helicoidal form, as may be seen on the cutting, of which the rugged face has just been shown. The inclination of the spirals depends upon that of the cutting edge of the tool, and their diameter upon the thickness of the cutting; the diameter diminishing with the percentage of reduction. It is thus that, in turning in the lathe a piece which is very slightly eccentric, the result is a number of parts of which the diameters are alternately greater and less. The demonstration afforded by this single specimen is quite complete. Without seeking to draw any conclusions from the study of these deformations with respect to the best form of tools for each of them, it follows clearly, from the foregoing discussion, that the work required for any cutting action whatever is expended in friction and in deformation by compression. The work of friction should augment with the number of cuts, and, as the shortening is greater for the finer cuts the molecular work expended should be greater. It follows, therefore, that it is most advantageous to make deep cuts, but, of course this mode of action demands more powerful tools and better foundations. It is in this direction, it appears, that the most recent progress in the manufacture of tools has been effected. The

different modes of cutting, rectilinear or circular, are applicable chiefly to flat surfaces, and to cylindrical surfaces. Flat surfaces are cut in the planing machine or in the lathe, and under most circumstances the two kinds of cuttings are almost identical in appearance—that of a cylinder formed of spirals more or less close, sometimes even in juxtaposition; but for this combination it is necessary that the two edges of the cutting should have been equally reduced, that is, that they should be of the same thickness. If it were otherwise the spirals would become conical; and such of these as appear to be most characteristic will now be described. The cutting obtained in mortising by means of a straight tool is absolutely cylindrical. When the tool cuts out, in this manner, a rectangular groove the material is compressed without any lateral deviation. If the cutting is of great thickness it is triangular, and the smooth surface is formed by the combination of the three faces at which the separation takes place, the direction in which crumpling takes place being the same as in all ordinary cuttings. The triangular form is the result of the compression being greater towards the middle line. To aid in forming an opinion on this point, two blocks were placed side by side, which were planed at the same time in the line of junction of the pieces. Two distant horns were formed, which parted symmetrically from one another; each half-cutting following the law of shortening by which it was bound to assume a form concave towards the side which was held by its attachment to the block. Having made a similar experiment in lead, the parallel and equidistant lines that were drawn upon the block before it was cut could be traced on the cutting, and they afforded the means of measuring exactly the average percentage of reduction and the mode of contortion of these transverse lines, which assumed successively the same inclinations as they lay one upon another at intervals of which the percentage of reduction varied from 0.10 to 0.30. The cuttings from a lathe, when they were produced from the annular groove by means of a straight tool, assumed exactly the same forms. For example, a cutting from a groove in what is called the Swedish piston, is a continuous ribbon rolled up as on a bobbin with the greatest regularity and of great length, without a rent. When turnings take the form of a helix, the small lateral displacement of the piece is not large enough to give to the ribbon a different character to that from a planing machine, when, for instance, it is required to turn a shaft to a uniform diameter, and it is then easy, with good metal, to produce cuttings of great length. But when it is required to turn the end of the shaft or of any cylinder whatever, the cutting follows a special course. If the tool be large in proportion to the diameter of the rings or circles on which it is acting, the difference of diameter between the two edges of the cutting makes itself felt in the cutting, which assumes the form of a helicoidal surface with inclined generating lines, of which the two directrices are two helices of the same pitch but of different diameters. This universal geometrical character moreover, is manifested in special ways according to the width of the ribbon and the interior diameter of the ring. In this way, three horns may be obtained encased one in the other if the cutting of the tool be radial. Successive spirals foul each other when the direction of the cutting edge is a little inclined. The inner helix is replaced by a straight edge, when the tool cuts right to the center of the face. Notwithstanding these differences of detail, the same rules prevail; a greater or lesser reduction or shortening according to the thickness of the cutting; a less reduction of length at the thicker edge of the cutting; a smooth surface of separation, which always forms a developable surface; a rugged reverse face ridged as if waves of metal had been successively projected there; in fact all the circumstances of a transverse flow of material—setting apart the secondary circumstances of transformation of the prism of metal from which the cutting is produced by augmentation of thickness and corresponding reduction of length. The author endeavored to represent, by a diagram, the triangular cutting which would be formed by planing from the edge of a block of metal a square prism, by means of a tool having two cutting edges, and of which the flat front is itself placed symmetrically. The effect of the

diagram, constructed on the assumption of a percentage of 0.30, is exactly reproduced by the model in relief. In agreement with the foregoing discussion and with the facts, it may be observed how the prism which is on the point of being separated from the block swells up by compression, commencing at a certain zone of fluidity, of limited length, in advance of the tool; and how, when this compression has arrived geometrically at the maximum which could be sustained by the material, the cutting is detached from the mass, to be subjected to the action of the face of the tool, upon which it slides, and which forces it to assume its ultimate form. Considerable as these modifications may appear, they are absolutely in accordance with the facts. They have been produced by the author, on lead as well as on the hard metals, under conditions which were exactly proportional to those which are represented by the model. The finest specimens of this triangular transformation of cuttings that have come under the author's observation, are produced by a mortising tool. They are not less than $\frac{6}{10}$ inch thick, and the rolling up of the metal could only be effected with the accompaniment of deep fissures in the lateral edges. The upper edge, on the contrary, is much more minutely serrated; one of the lateral faces is plaited for its own length—evidence of the compression of the material; whilst the other face, with its oblique fissures, shows still better the sliding by means of which the compression takes effect. There is a still smaller cutting which presents exactly the same characteristics.

It is the author's opinion, that for the construction of the best machine tools, with the most suitable thickness of cuts, the minute examination of the cuttings is of the greatest importance; and that by the same means, the surest evidence may be derived with respect to the qualities and homogeneity of the metal. Time does not permit of more than a passing reference to certain deformations which recall to mind, with a surprising degree of exactness, the constitution of certain rocks with their dislocations. A few experiments of this kind were made by the author in conjunction with M. Daubrée, from which the latter gentleman quite recently derived an explanation of a number of geological phenomena. The results of these inquiries would, no doubt, possess some interest for the members, but the author was desirous chiefly to lay before them such results of his investigations, as followed in natural sequence upon the substance of the communication already made in 1867. The idea of the flow of solids is, of all the modes of regarding their deformation, perhaps the one which most truly interprets all the phenomena of molecular mechanics and of the internal constitution of bodies, which underlie the various industrial operations.

PATENTS AND TRADE MARKS.

By Capt. HENRY GERNER, C. E.

Written for Van Nostrand's Magazine.

It is the purpose of the present paper to give the public an insight into the uses and abuses of protecting that valuable class of the community known as inventors, and also to promote the interests of our merchants in adopting, recording, using and reaping the benefits of trademarks. I may be permitted to speak with some authority on this subject, having had an experience of some thirty-five years in acting as a patent solicitor for myself and others. Nobody will deny that the world has received incalculable benefits from the inventors' fertile brain, and nobody will deny the justice of securing to inventors their inventions, and giving them a monopoly for a certain number of years during which to reap a reward for their ingenuity and toil. How to effect this in a manner satisfactory to the inventor himself and the public at large has been, for all governments, a problem, and is still so to-day. In most of the European countries, anybody who

declares himself to have made an invention, is, upon application, and by due process of routine and practice, presented with Letters Patent without any search or investigation whatever, it being left to the courts to settle all questions arising thereafter and therefrom. Here in the United States it is different. When an inventor applies for a patent, a kind of search is instituted by a corps of examiners who may refuse or give a patent at will. In the majority of cases, a patent is refused on the most frivolous grounds, these becoming a source of much vexation, expense and trouble both to the inventor and his attorney. It is, however, only a trouble which can be easily overcome, like all other troubles in Washington, by aid of the Almighty Dollar! Of course, the officials are unapproachable; you pay your money to somebody on the inside track and you get what you want. I speak from experience. I can name, in my own practice, hundreds of such rejected cases obtained by paying somebody in Washington. Thus, as this system of examination is no guarantee whatever, and only leads to injury, injustice and corruption, it ought no more to be indulged in; and it is to be hoped that Congress, in its wisdom, will abolish this reprehensible system. It is, after all, the courts who will have to pass upon the validity of an inventor's claim; the Patent Office can neither give him anything that he has no right to, nor take away anything that he has a right to. The whole question of all this brain-property, called patents, lies in the question of the *priority* of an invention, and all that an inventor in reality has got, is the unimpeachable proof that he was the first to invent what he claims as his invention. A United States patent, granted to any inventor either by foul or fair means, is no proof whatever that he owns what he claims, and the patent given is not worth the paper it is printed on when anybody else claims and proves that he made the invention prior to the alleged patentee.

Inventors in particular, and the public in general, seem to be entirely ignorant of this fact. The prevailing belief seems to be that a patent granted an inventor is absolute proof that he has a lawful right to all the privileges and benefits the patent purports to give, and it seems to be entirely overlooked that the grant he obtained is only a conditional one. The U. S. Patent Laws, Revised Statutes, of 1870, Section 4886, wisely give an inventor two years' time to put his invention into *public use or on sale* before he is obliged to make application for a patent, or can forfeit his right to a patent. All that is required is that he can produce absolute proof that he was the first to invent what he claims as his, and every inventor should, therefore, as a matter of necessity, have his invention fully and intelligently described and illustrated by drawings in the same way as is customary in patent practice, and such priority of invention documents should be signed and sworn to before a notary public, both by the inventor himself and by witnesses who know that he conceived and perfected the invention on certain fixed dates. This is a legitimate business for patent attorneys, and they should advise inventors asking their aid accordingly, but this not seeming to be profitable enough, inventors are advised to secure a patent in hot haste. On this latter advice, extensive patent mills, both in New York and Washington, flourish. The poor misguided inventor is made to believe and understand that a patent, once allowed, will end all his troubles and open the gates to a Golconda for him. Thus we have to endure the sorry sight of seeing an army of 20,000, mostly abjectly poor, inventors yearly rush in and ask for patents for alleged inventions of which barely one in a hundred are honored by the duplication in any shape of the model left in the Patent Office. Only allowing $100, in the average, as spent by every inventor on these 20,000 alleged inventions, and the patents allowed thereon, it foots up to the round sum of $2,000,000 yearly, wrung from the poor and misguided inventors. If, instead of parting with their own or mostly borrowed money, they had, at a comparatively trifling expense—which might later on be applied to the patent application if advisable—taken pains to properly cover up their priority of invention and then tried to put their invention into "public use or on sale," as the patent laws provide, to ascertain if their invention was marketable, and worthy of the expense of a patent, I believe that everybody who knows anything about the philosophy of patents, will

agree with me that instead of 12,000 patents being issued yearly, not 1,200 would get so far as to be worthy of such a distinction. Patents numbered among these 1200, gone through a fire of an actual trial of merit and actual ownership of invention, would be patents not only in name, as they are now, but patents in fact, and would any day find a regular market and be sought as the best investment of all by capitalists. We would no more see the repeated and pitiful sight under the New York *Herald's* "Business Opportunities," in offers to invest capital, of: "No patent right humbug will be noticed."

Already I fancy hearing the protest of the Patent Office and the patent mills: of the former, because, on the basis of judgment that I have sought to establish, enjoyable sinecures and clerkships will necessarily be dissolved; of the latter because a source of revenue to them, for which no absolutely beneficial return is given, will cease. Let them protest; first comes right, public benefit, national enjoyment; then sinecures and monopolies may follow; I hold that what I have said is true, and I hope to live to see it have become a living fact; it will follow, as a consequence, that the bulk of $398,-024.54 now annually expended for salaries, according to the last annual report of the Commissioner of Patents, will flow into the public treasury. To be sure, it would reduce the number of officials in the magnificent Patent Office to a mere corporal's guard, but then this reduced force would do useful work, by which they would benefit the sovereign people instead of, as now, being a constant source of irritation, injustice, injury and trouble in pretending to do a great deal of work benefiting nobody but themselves, and that in so far as the just quoted salary goes. I hope, for the public's own good, that all these ignorant and pretentious officials may be made to do more useful work in the future than in the past, and learn to behave themselves. I will here not go into the details of my own grievances. I have them in common with most every inventor and patent solicitor. I do not, however, propose to try the suit I have instituted against the managing officials of the Patent Office in the newspapers. If, however, the said officials should provoke an explanation beforehand, I shall be glad to enter into the arena. At least a thousand old clients and others of my friends, in the community wherein I have spent so many years of my life, will know that what I say has its force. I wish for nothing better than that the whole truth might be made known.

Our patent laws, and more particularly their proper execution, need a thorough overhauling. The integrity of the present system was ventilated in Congress last winter, and it is sincerely to be hoped that Congress, during the coming winter, will pass new measures which will prove beneficial alike to the inventor, his attorney, and the public at large: the attorneys especially, among whom are many good and deserving men, at the mercy of unscrupulous and revengeful Patent-office officials. There are several bills now pending before Congress aiming in this direction; among them is that of Mr. Phelps, "To provide for the protection of attorneys doing business before the Patent Office and other bureaus and departments of the Government," presented in the House on May 6 last, read twice, referred to the Committee on Reform in the Civil Service, and ordered to be printed. The penalty it provides for assailing the standing and character of an attorney, or attempting his disbarment, &c., &c., without due process of law, is imprisonment at hard labor and heavy fine.

It should be evident to every thinking man that "an inventor's right to reward and protection exists solely by virtue of the inventor's act of creation." The establishment, by legal means, of this act of creation, should be the basis for a valid patent. If an inventor can establish that he has *priority of invention*, then, and only then, a patent should be granted to him. I would leave it entirely to the courts to settle all other questions, and only require of the Patent Office that it should be an office of record of the inventor's Priority-of-Invention-documents, and the proper executor of patents to such inventors as would claim them. I would have Priority-of-Invention- documents in proper form filed for a registry fee, say, of $5, at the Patent Office; and the first registrar of substantially the same invention should be entitled to a patent after two years had elapsed, not

before and not later. I would require such Priority-of-Invention-documents to be executed in duplicate, and one of them returned to the applicant with a certification of the office thereon that he was the first to register specification and drawing. All later applicants should be denied this certification of being the "first and original"; and it should actually be certified to that somebody has received certification of being the first inventor, much the same as foregoing patents are quoted now in rejected cases by the preliminary examiner. Such certified documents of the first registrar should serve as a "provisional protection," after the style of the British provision, and should as such receive proper recognition and protection from the courts. It should be salable and transferable, being the inventor's absolute property. This will enable the poor struggling inventor to find out if his invention will ever be worthy of a patent; if so, he should have absolute right to a patent at the expiration of two years, but, neither prior to nor later than that expiration. Such a patent would be one of intrinsic value and ought to be paid well for and taxed well. The fee for issuing such a patent which would mostly get into the hands of the capitalists, ought to be $100. I would give the patent a life-time of twenty years, under no circumstances to be extended; and every year a tax of $50 should be required to be paid into the United States Treasury by the patentee or his assignees, under penalty of annulment of the patent.

If this patent system should be adopted, United States patents would rank with United States securities, as the best and the safest in the world. It would give a large revenue to the Government, without any complicated and unsatisfactory work; the public would cease to consider patents a humbug, and the deserving inventor would receive a place in society as a worthy and honored member, instead of being regarded, as he is now, in the light of a bore, a lunatic and a fraud.

I recommend and court a thorough investigation and discussion, by competent gentlemen, of this my proposed plan, and shall be glad, in the best interest of all concerned, to give all the time I can spare, and all the intelligence I can command, to so highly important a subject of so paramount an interest to the welfare and steady progress of this immense and fruitful part of the world in which we have the good fortune to be the pioneers. I believe nothing would prove more beneficial to the United States than a good patent system, and its simple and incorruptible (!) execution. It would stimulate invention to an extraordinary degree, but only the wheat would grow and the chaff perish.

Having thus given my views on patents proper, we now come to a subject of great interest to the trading community. This is the subject of trade-marks, often misnamed patent trade marks, or trade mark patents. It has been well that "the increasing attention now devoted both in this country and in Europe, to the value of property in trade marks, and the very stringent laws that have been enacted both here and abroad, as well as the recent agreement between Great Britain and this country on the subject of trade marks, are among the most significant facts of the times; and open to honest and reputable manufacturers a means of establishing a valuable property right as the result of their labor, skill and patience, which is as tangible as houses, lands, or merchandise itself. It is a sort of crystallization of good will that may be transmitted from generation to generation.

The constitution of a lawful trade mark has been defined by the United States Supreme Court, in Del. & H. Canal Co., *vs.* Clark, as follows:

"A trade mark must be distinctive in its original signification, pointing to the origin of the article: or it must have become such by association; and these are two rules which are not to be overlooked. No one can claim protection for the exclusive use of a trade mark or trade name which would practically give him a monopoly in the sale of any goods other than those produced or made by himself. If he could, the public would be injured rather than protected, for competition would be destroyed. Nor can a generic name, or *a name merely descriptive of an article of trade, of its qualities, ingredients or characteristics* be employed as a legal trade mark, and the exclusive useof it be entitled to legal protection."

It should be known furthermore,—it is curious to note how few are aware of the fact,—that *it is not obligatory to record trade marks in the Patent Office.* This is confirmed in the decision of the Supreme Court of the District of Columbia in J. Rodgers and Sons, *vs.* Philp and Solomons, in these precise words:

"The provisions of the act of Congress of July 8th, 1870, as far as it relates to the subject of trade marks, presents no obstacle to the rights of these complainants to maintain a suit to prevent an infringement of their rights by an imitation of their trade mark, notwithstanding they have ommitted to have their trade mark recorded at the Patent Office. The act is not obligatory on them in this respect. It offers to manufacturers an opportunity to have their trade marks recorded, but imposes no penalty or forfeiture of right for neglect to so record them. Trade marks are the property of their owners independently of statute, and are not the subject of a patent."

Therefore, save your money; save the exorbitant Patent Office registry fee for trade marks. The question of paramount interest to a manufacturer is: whether or not he is in possession of, and is using, a *lawful* trade mark. If not lawful, no Court can sustain it, no matter whether recorded in the Patent Office or not. There are a great number of *unlawful* trade marks recorded in the Patent Office on which the applicant has wasted his money, has got nothing, and fancies himself secured. The several Commissioners of Patents have repeatedly acknowledged this and promised reform. Such promises are idle, as nobody is in want of their fulfillment. Nobody has any need of expending $25 fee now, since no benefit is derived from so doing. The word "*registered*" is believed to be a proof of the legality and ownership of a trade mark; nothing can be more erroneous. For your $25 to the Government you simply obtain a certificate that your own statements about a certain trade mark has been filed in the Patent Office.

It does not guarantee you the absolute right to use the same, nor have you any remedy whatever if the registered trade mark is not a trade mark in the sense of the law. A great number of such I shall be most happy to show conclusively to their owners are not trade marks at all. The patent officials sometimes refuse to register a trade mark after you have paid your $25, alleging that it is not a subject for a legal trade mark. I had many such experiences in my long practice, and almost invariably found that the almighty dollar would make them legal and registrable in the Patent Office. I do not say, nor wish to be so understood, that I paid such money to the officials; I paid it to men who could get what my clients desired, and as it did them no harm, nor any good either, for that matter, but simply satisfied a peculiar whim of theirs and seemed to make them contented and happy, I had no objection, and shall have none in the future, to let such pay for the pleasure of having their trade marks registered in the Patent Office; but my advice always is, was and will be, not to lose money and patience in the attempt to have trade marks registered in the Patent Office. It is like climbing a tree after an apple when just as good a one has droppped from it and lies before you.

Any merchant or trader desiring to adopt or use a trade mark as the means of identification of his goods by the trade and the public, should, in the first place, seek information through proper and reliable sources, of the legality of his trade mark. This satisfactorily ascertained, certain legal declarations, properly verified and *recorded*, are *prima facia* evidence of ownership, which the courts in case of litigation for infringement will, and do, recognize as such.

A GREAT deal of irrigation is effected in Illinois with well water. In Iroquois County, eighty-five miles south of Chicago, 53,500,000 gals. of water from artesian wells are daily, it is said, supplied for irrigating land. No well is over 75 ft. deep. There are 200 wells within a radius of twenty miles, all of small bore. The prairie is 90 ft. above Lake Michigan, and there is no high land for 200 miles which can furnish a fountain head to these wells. A correspondent of the Baltimore *Sun* says that engineers are confident that the subterranean river flowing under San Francisco, leading direct from the exhaustless lakes of the Sierra Nevada, is quite adequate to supply several cities of its size.

MENTAL LOGARITHMS.

By EDW. DAVID HEARN, M.A., Columbia College.

From "The Scientific and Literary Review."

In his introduction to the "Complete and Immaculate Tables of Logarithms," published about half a century ago, Sir Richard Phillips remarked that "Logarithms are the most useful discovery ever made in the principles of arithmetic and calculation, and are so essential to operations in mathematics that in all education the facile use of logarithmic tables should invariably follow the study of decimal fractions." Since that time the teaching of logarithms has become so general that there is now scarcely a schoolboy to be found who is unable to employ logarithmic calculations to abbreviate otherwise tedious processes. To take a very simple example, let it be required to extract the cube root of 45.499593; and we have merely to take either the immaculate or any other reliable table, and turning to the natural number 45.5, which is quite near enough for all ordinary purposes, take out the logarithm, which is 658011, supply the characteristic which, as the integral part of the number, has two digits will be 1, and divide by 3; thus:—

$$3)\frac{1.658011}{.552670}$$

This quotient is the logarithm of the root ought, and again turning to the tables we find that 552668 is the logarithm of 357, and the characteristic being 0 we must write the natural number 3.57; therefore 3.57 is the cube root of 45.499593. Let those who doubt the simplicity and utility of logarithms attempt to extract the required root of the given number by the best known arithmetical process of evolution, and they will find not only that it is complicated and dificult to remember, but that the actual working will take ten times as long as is required to obtain the result by logarithms. To ascertain of what number some other number is the fifth, the seventh, or other high power is often altogether impracticable to the ordinary arithmetician without the use of logarithms; but the example given will suffice for the general reader.

Now, it must be understood that all logarithmic calculation is approximate only, and that it is a very good set of tables that gives the logarithms of natural numbers, as far as 100,000 (that I am using extends to a 10,000. only), but the results obtained are very close to the truth. The larger the natural number for which we can find the logarithm, the greater will be the certainty of accuracy in the results obtained. What then shall we say of a process which enables us to write without the use of tables the logarithm of such a number as 987.654321, and conversely to determine, also without the use of tables, that a given logarithm exactly represents the natural number—

.99302096503497900699?

The processes by which this can be done were explained many years ago by Mr. Oliver Byrne, but probably from his having employed a new system of notation, their application has been extremely limited, and to a large number of young students especially, the entire subject of his treatises has been altogether unintelligible This endeavor will here be made to popularize the system by retaining as far as possible the ordinary Arabic notation.

Some brief references to a few of Byrne's introductory remarks will add to the interest of the present study, and a few words on the manipulation and reduction of decimals will facilitate the accurate comprehension of the subject. Mixed decimal fractions are calculated precisely as whole numbers, all the care required being as to the placing of the decimal point. In addition and subtraction the decimal points must be ranged under each other. In multiplication we must in the product point off as many places from the right hand as there are decimal places in the multiplicand and multiplier combined. And in division we put the decimal point in the quotient as soon as the integral portion of the dividend ceases to be divisible by the integral portion of the divisor. Any decimal fraction can be reduced to a mixed

quantity by using the same multipliers as would be employed in the usual process of reduction, observing the rule of decimal multiplication; and any mixed quantity can be expressed in decimals by dividing step by step as in the reduction process. Thus to show and prove that £3.81875=&3 16s. 4½d., we have merely to employ the following processes:

A.— 3.81875
20
16.37500
12
4.500
4
2.0

B.—

4	2
12	4.5
20	16.375
	.81875

Therefore
16s. 4½d. = .81875

In the first working the figure preceding the decimal points represents pounds, shillings, pence, and farthings respectively. In the second working we first write the farthings, and divide by 4, because there are 4 farthing in a penny; then prefix the 4 pence and divide by 12; and, lastly, prefix the 16 shillings and divide by 20: the resulting quotient is the decimal fraction sought. But decimal mixed quantities did not always possess the advantage of the simple notation now used; so that we may hope that something may hereafter be done to simplify Byrne's notation. The first notice of decimals appears to be that found in a tract at the end of Stevinus' *Arithmètique* in the collection of his works by his friend and pupil, Albert Girard; the tract is entitled *La Disme*. This collection was first published in Flemish, about the year 1590. At this early date decimals in the first palce are termed primes and marked (1); those in the second place are marked (2), and called seconds, and so on; whilst all integers are characterized by the sign (0), which is put after or above the last digit; so that taking an example in addition it would stand thus:

A.D. 1590.	A.D. 1878.
(0)(1)(2)(3)(4)	
3 4 6 1 2	3.4612
2 1 4 7 7 2	21.4772
1 3 0 0 6	1.3006
2 4 0 0 4 9	24.0049
5 0 1 1 3 9	50.1139
(0)(1)(2)(3)(4)	

At the present time most civilized nations not only recognize the importance and simplicity of decimals, but employ the metric system; but the English and Americans are not yet sufficiently enlightened to follow the example, although it may be hoped that hereafter they may become more wise. But all people are naturally averse to change, and hence it is that Arabic figures and notation were not introduced into Europe until about 900 years ago, and were but little used until after A.D. 1600. Leonardo Bonacci, a merchant of Pisa, introduced the Arabian system of digital arithmetic into Italy, and wrote the first treatise published in Europe about A.D. 1228.

In his *Philosophy of Mathematics*, Auguste Comte remarks upon the difficulty experienced in putting mathematical questions into equations, and says that "it is essentially because of the insufficiency of the very small number of analytical elements which we possess that the relation of the concrete to the abstract is usually so difficult to establish. Let us endeavor now to appreciate in a philosophical manner the general process by which the human mind has succeeded in so great a number of important cases in overcoming this fundamental obstacle." He first considers the creation of new functions, and observes that "in looking at this important question from the most general point of view, we are led at once to the conception of one means of facilitating the establishment of the equations of phenomena. Since the principal obstacle in this matter comes from the too small number of our analytical elements, the whole question would seem to be reduced to creating new ones. But this means, though natural, is really illusory; and though it might be useful it is certainly insufficient. In fact, the creation of an elementary abstract function which shall be veritably new, presents in itself the greatest difficulties. There is even something contradictory in such an idea; for a new analytical element would evidently not fullfil its essential and appropriate conditions if we could not immediately determine its value. Now, on the other hand, how are we to determine the value of a new function which is truly simple, that is, which is not formed by a combination of those already known? That appears almost impossible. The introduction into analysis of another elementary abstract function, or rather another couple of functions, for each

would be accompanied by its inverse, supposes then of necessity the simultaneous creation of a new arithmetical operation, which is certainly very difficult."

Let us see how far Oliver Byrne's system meets the requirements referred to by Comte, and how far it can be applied for the development of mental logarithms. Decimal arithmetic, as now taught in schools, has been less than 150 years in use, so that the introduction of mental logarithms can scarcely be objected to on the ground that arithmetical knowledge has already reached perfection. Mr. Byrne explained that because the system of arithmetic invented by him requires numbers to be viewed under two aspects, and to distinguish it from other systems of operating upon numbers, he has called it *dual arithmetic.* By this new art, a number representing any given magnitude, or the function of any given magnitude, may be made to assume a form composed of factors of whole numbers having a known relation to one another; and these derived whole numbers may be readily made to assume a variety of forms, each form always reducible to the given number or magnitude, and hence the derived numbers, by a peculiar arrangement, may be developed to suit different operations; and the factors produced, after such operations are performed, are easily converted into natural numbers, expressing the required results. An acquaintance with the operations of common arithmetic is all that is required to permit of the acquisition of a sufficient knowlege of dual arithmetic for the ready solution of almost innumerable intricate and difficult problems, and in its more complete development it enables the reasoning of the differential and other methods of analysis to be dispensed with. But, as it is not here proposed to treat of the whole subject of dual arithmetic (which cannot be better studied than by consulting Mr. Byrne's own explanations), but merely to show one particular application of it, the theory of the art will only be referred to so far as is necessary to permit of the process being used. All that will be necessary will be to show how the dual logarithm of a natural number can be found, and then how to discover what natural number any given dual logarithm represents; for in the solution of any given problem the dual logarithms (which, as they can readily be discovered without reference to tables, we call mental logarithms) are manipulated in precisely the same way as the common logarithms given in the usual tables.

We are accustomed to represent that imaginary quantity, which is greater than any that can be named by $+\infty$ ("plus infinity"), and that other imaginary quantity which is less than any that can be named by—∞ (minus infinity). The signs will be here used with the same signification. The bases of dual arithmetic are:—

Descending Branch.	Ascending Branch
$-\infty$	$+\infty$
⋮	⋮
-999999	1000001
⋮	⋮
-999	1001
-99	101
-9	11
$(1-1=)$ 0	2 $(=1+1)$
.9	1.1
.99	1.01
⋮	⋮
.9999	1.0001
⋮	⋮
.99999999	1.00000001
⋮	⋮
1 =	1

It will be observed that in the case of quantities belonging to the descending branch the power is smaller than its root; for example, the power of 0.9 is only 0.81. In the ascending branch the power is greater than the root. In the descending branch, the bases are greater and greater as they approach 1, but cannot be greater than 1; whilst in the ascending branch the bases are less and less as they approach 1, but cannot be less than 1. Numbers in the dual system of arithmetic are expressed by the continued product of the powers of one or more of the bases, which are seldom introduced into the figurate operations of the art. As, however, the difference between the decimal and the dual systems of notation will probably be best understood from an example we may observe that, according to the decimal or usual system of notation, 73.598 is merely an abbreviated method of expressing:—

$$7(10)+3+\tfrac{5}{10}+\tfrac{9}{100}+\tfrac{8}{1000}$$

which, if we use the commonly accepted method of expressing powers, will become $7(10)^1+3(10)^0+5(10)^{-1}+7(10)^{-2}+8(10)^{-3}$, and we call $1, 0, -1, -2, -3$, the powers of the base which, in the decimal system, is always 10. Now, in dual arithmetic it is only the powers of the dual bases which are registered, and each dual digit is the power of a different base; and in order to avoid any departure from the usual Arabic notation we will merely prefix DNA: or DND: to the figures, according as they belong to the ascending or descending branch, and suffix P^1, P^2, and so on, to indicate the power of 10 in the decimal system with which they are connected; thus:—

1.41421356 = DNA: 36094110 P.
141421.356 = DNA: 36094110 P^5.
.99923682 = DND: 00076343 P.

It will be obvious that there is no greater inconvenience in representing a dual number in this manner than in representing an ordinary Brigg's logarithm by prefixing *log.;* and it may here be mentioned that it is proposed to indicate the dual logarithm or mental logarithm by prefixing DL: to the ordinary Arabic numerals in the same way. When some of the dual digits belong to the ascending and some to the descending branch either prefix, DNA: or DND: may be used, care being taken to place a negative sign over the digits belonging to the other branch; thus:

DNA: $3\overline{501}4738P^4$ = DND: $35014\overline{738}P^4$.

For the sake of uniformity it is preferable in such cases always to use the first form. It should be well understood that, although we have given 141421.356 = DNA: $36094110P^5$, many dual numbers may be found to represent the same natural number; and it is a peculiarity of the system that, although a given natural number may be represented by a vast number of dual numbers, yet every dual number which represents such given natural number may be almost instantly reduced to one and the same dual logarithm. Dual digits are intimately connected with binominal coefficients, and reference to the arithmetical triangle, formed when the binomial coefficients for the several powers are written down consecutively, will assist the memory for converting dual numbers into natural numbers, and *vice versa.* A line of units is written horizontally, and a second line vertically, the remaining squares being filled by a number equal to those in the squares immediately above and to the left of it; thus:—

1	1*a*	1*b*	1*c*	1	1	1	1	1
1*a*	2*b*	3*c*	4	5	6	7	8	&c.
1*b*	3*c*	6	10	15	21	28	36	&c.
1*c*	4	10	20	35	56	84	120	
1	5	15	35	70	126	210		
1	6	21	56	126	252			
1	7	28	84	210				
1	8	36	120					
1	9	45						
1	10							
1								

Now, we can see at a glance that taking the diagonals consecutively—first, *a*, *a*; then *b*, *b*, *b*; then *c*, *c*, *c*, *c*; and so on—we have the coefficients of the simple quantity, of the square, of the cube, and so on respectively, and these are precisely the operative numbers which we shall hereafter find very useful for giving us our mental logarithms. In the dual system of arithmetic, these numbers are used to find the powers of the bases .9; .999; .99; and so on, as well as the powers of the bases 1.1; 1.01; 1.001; and so on; but it must be remembered that in the descending branch the second, fourth, sixth, and other *even* coefficients must be regarded as negative and used accordingly.

The position of the dual digits is reckoned from the left hand; so that in the quantity DNA: $36094110P^5$, the 3 is said to be in the first position, the 6 in the second, and so on. In the first place, then, we must know how to find the natural number answering to a single digit of either branch in any position. Assuming as before that the working of an example is the readiest means of elucidation, let us find the value of DNA: 9 in the first position. The arithmetical triangle gives us, if we read diagonally for 9 either up or down, 1, 9, 36, 84, 126, 126, 84, 36, 9, 1, and we write these in

such a manner that the units shall form one horizontal line, the tens and hundreds running diagonally to the *left* beneath them, thus:—

1	9	6	4	6	6	4	6	9	1
	3	8	2	2	8	3			
		1	1						
2	3	5	7	9	4	7	6	9	1

Consequently, as there is nothing to denote that the power of 10 is other than simple, we say that 2.357947691=DNA: 9 in the first position. When the dual digit stands in the second position, we proceed in the same way, except that we write 0 between each diagonal. Let us, for example, find the natural number equal to the dual digit 5 in the second position; that is, DNA: 05. The arithmetical triangle gives us the coefficients or operative numbers as they are called in dual arithmetic, and we write—

1	0	5	0	0	0	0	0	5	0	1
		0	1	0	1	0				
1	0	5	1	0	1	0	0	5	0	1

so that 1.0510100501=DNA: 05. Upon the same principle, two cyphers, 00, must be inserted between the diagonals when the dual digit is in the third position; three cyphers, 000, when the dual digit is in the fourth position, and so on. We shall presently see that this property of numbers can be very extensively utilized. In arithmetical calculations it is usually sufficient for all practical purposes if we can ensure accuracy to eight places of figures, and we shall observe, if we take any dual digit in the fifth, sixth, seventh, or eighth position, that in the corresponding natural number all the places except one in the decimal fraction portion are filled with cyphers, the exception being the fifth, sixth, seventh, or eighth place, as the case may be, which contains a digit identical with the dual digit. It is unnecessary to show the working, but it will be found that—

DNA : 00002000P=1.00002000
DNA : 00000070P=1.00000070
DNA : 00009358P=1.00009358

The advantage of utilizing this fact is, that we can obtain a dual number correct to eight places of figures corresponding to any given natural number by the calculation of the first four figures only; which calculation can be performed in a few seconds. The natural number corresponding to the last four dual digits is written down, and this is manipulated with the operative number furnished by the arithmetical triangle. The unfound dual digits may be worked off in any order whatever; but care must be taken to locate the results in periods corresponding to the position of the dual digit sought; that is to say, when operating for dual digits in the first position the results must be written down in single figure periods; when operating for dual digits in the second position in two-figure periods; and so on. To take an example, let it be required to reduce DNA: 35014738P[4] to a natural number. We neglect the decimal and proceed thus:

DNA : 00004738=	1	0	0	0	0	4	7	3	8	1	operatives for DNA : 3
		3	0	0	0	1	4	2	1	3	
			3	0	0	0	1	4	2	3	
				1	0	0	0	0	5	1	
DNA : 30004738=	1	3	3	1	0	6	3	0	6		
					1	3	3	1	1	..−1	
DNA : 30014738=	1	3	3	0	9	2	9	9	5	..+1	operatives for DND : 05
			6	6	5	4	6	4	9	..−5	
				1	3	3	0	9	3	..+10	
						1	3	3	1	..−10	
									7	..+5	
DNA : 35014738=	1	2	6	5	7	0	1	1	5		

∴ DNA : 35014738P[4]=12657.0115

It will be observed that the operative numbers are always used as ordinary multipliers, and that all positive products are added, whilst all negative products are subtracted, the final results obtained being the natural number sought. We have seen that DNA: 00004738=100004738; and hence, generally, DNA : 0000 $v\ x\ y\ z$= 1.0000 $vxyz$; when $v\ x\ y\ z$ are digits occupying positions like 4738 in the example. But this is not all, for it will be found that DND: 00004738 is equal to 1.00000000.– 00004738 = .99995262; and generally DND : 0000 $v\,x\,y\,z$ = 100000000 —.0000 $v\,x\,y\,z$; and the knowledge of these facts can frequently be utilized in practice.

The rule for the reduction of common to dual numbers is of course the converse of that which we have just been considering. To find the dual digits for the first, second, third and fourth positions, we first take the common number corre-

sponding to a dual digit of either branch, so that the leading figures of this number may approach the leading figures of the given number; then the dual digits which have to be applied to bring the number selected to the given one will be the other digits of the required dual number. Owing to the properties just referred to, it will be obvious that when the dual digits for the first four positions have been found, the remaining four may, when accuracy to eight places is sufficient, be run in at one operation. By way of example, let it be required to reduce 12657.0115 to a dual number. We will explain the steps of the process after working it.

Given number=126570115.

DNA : 3	133100000	....+1
	66550000	....−5
	1331000	....+10
	13331	....−10
DMD : 05	7	....+5
	126576776	
DND : 0001	12657	....−1
	126564119	
DNA : 00004	5062	4
DNA.; 000007	886	7
DNA : 0000003	38	3
DNA : 00000008	10	8
	126570115	

We have now only to collect the dual digits and represent the decimal point in the given power by the index of the power, and we have: 12657.0115=DNA: 35014738P⁴, which proves the correctness of both reductions. With regard to the steps in the process it could be instantly determined by inspection that the common number 133100000=DNA: 30000000 was the nearest number to 126570115. As the selected number was too large, we knew that the next dual digit must belong to the descending branch. The number 126576776 was also too large, so that the descending branch had again to be used, and we obtained DNA: 35010000=126564119, which is smaller than the given number; therefore, the dual digits in the fifth, sixth, seventh, and eighth position will belong to the ascending branch, and by using such multipliers as will enable us to make up the deficiency we obtain 4738 for the remaining four places.

As there should now be no difficulty in reducing a given natural number to a dual number or in finding the natural number corresponding to a given dual number of either branch, all that remains is to show the method of finding the logarithm of a given dual number; that is to say, the method of calculating a dual logarithm, and then to explain the method of reducing a dual logarithm to a natural number. The first thing to be considered is whether the dual number belongs to the ascending branch, to the descending branch, or is mixed; as the method of reduction differs in each case.

RULE 1.—To reduce a dual number of the ascending branch to a dual logarithm (DL:), regard the given dual number as a natural number, add 31018 times the first dual digit; *plus* 33.09 times the second dual digit; then subtract five times the first three dual digits, a cypher (0) being placed after each, and the remainder, *minus* half the fourth dual digit, will be the dual logarithm sought.

RULE 2.—To reduce a dual number of the descending branch (of eight consecutive dual digits) to a dual logarithm (DL:), regard the given dual number as a natural number, add five times the first three dual digits, supposing a cypher placed after each; 36052 times the first dual digit; and 34 times the second dual digit. The sum will be the dual logarithm sought, and less than one unit out of truth.

RULE 3.—To reduce a dual number consisting of eight consecutive dual digits, some of which belong to the ascending and some to the descending branch, treat those belonging to each branch separately; subtract one result from the other and the difference is the dual logarithm sought; it belongs to that branch which has the greater number.

REMARK.—Dual logarithms are always whole numbers; those of the ascending branch have DLA: prefixed to them; those of the descending branch have DLD: prefixed. It must be remembered that the dual logarithm of 2 is DL: 69314718; and that the dual logarithm of 10 is DL: 230258509, as these logarithms are so frequently required that it is unnecessary to re-calculate them each time.

RULE 4.—To prepare a dual logarithm for reduction to its corresponding dual number, find the difference between the given DL: and some multiple of DL: 230258509, or of DL: 69314718, and repeat the process until the remainder is

less than half of DL : 23ä258509, and ultimately less than half of DL : 69314718. When necessary, complete this final remainder to 8 places by prefixing cyphers on the left; then apply one or other of the following rules according as such remainder is a DLA : or a DLD :

RULE 5.—To reduce a prepared dual logarithm of the ascending branch to its corresponding dual number, write down the given DLA : and add thereto 500,000 times the first figure (increased by 1 when the second is 5 or upwards); subtract 31,018 times the first figure from the sum; add 5000 times the second figure of the remainder, subtract 33.09 times the same figure. And, then add 50 times the third figure, *plus* half the fourth figure. The sum will be the dual number sought.

RULE 6.—To reduce a prepared dual logarithm of the descending branch to its corresponding dual number, write down the given DLD : and subtract therefrom 536052 times the first figure; then subtract 5034 times the second figure of the remainder; and lastly subtract 50 times the third figure of the new remainder. The final remainder will be the required dual number.

REMARK.—In applying the fifth and sixth rules the figure multiplied must not alter in the operation, but must reappear in the remainder.

We have placed these rules together, in order to facilitate reference to them, and will now show their application in practice. To economize space we will take one example, which will necessitate the application of all the rules. Let it, therefore, be required to find the DL: of 12657.0115, and prove that the DL: found is the correct one. As we have already found that DNA: 35014738 P^4 is a DN: corresponding to the natural number 12657.0115, and since, if our thoughts did not run precisely in the same direction, the resulting DN: would be different we will not recalculate it; but proceed at once to reduce DNA: 35014738 P^4 to a DL: which is thus effected:—

Separating the branches in Rule 3, we have DNA: 30004738, and DND: 05010000 to operate upon. We neglect the P^4 for the present and applying Rule 1 to the DNA; and Rule 2 to the DND: the workings will stand thus :—

Firstly		30004738
31018×3 (the first dual digit)	=	93054
		30097792*
30 00 00×5		=1500000
DNA: 30001738		=DLA: 28597792
Secondly		.05010000
00 50 00×5		25000
36052×0 (the first dual digit)		0
34×5 (the second dual digit)		170
DND: 05010000		=DLD: 05035170
Lastly		DLA: 28597792
		DLD: 05035170
		DLA: 23562622
P^4=230258509×4		=DLA: 921034036
DNA: 35014738P^4		=DLA: 944596658

Consequently we find that the natural number 12657.0115 is equal to DLA: 944596658, and we have now to prove that this is correct. We first apply Rule 4 to eliminate the powers of 10.

	Given DLA:	944596658
230258509×4=P^4=		921034036
Apply Rule 5 to		23562622
500000×2 (first dual digit)	×	1000000
		24562622
31018×2 (first fig. of rem.)	−	62036
		24500586
5000×4 (second fig. of rem.)	+	20000
		24520586
33×4 (second fig. of rem.)	−	132
		24520454
50×5 (third fig. of rem.)	+	250
Half of fourth fig. of rem.)		1
DLA: 944596658		=DNA: 24520705P^4

The remainder of the proof is simple enough, since it is obvious that we have merely to reduce DNA: 24520705 P^4 to a natural number, by the process with which we are already familiar; thus:

(*See process on following page.*)

Now a very peculiar circumstance may be noticed in connection with this series of reductions—in obtaining the DL: from the natural number we passed by way of one DN: whilst, in calculating the natural number from the DL: we passed by way of an entirely different DN: yet both these dual numbers are alike reducible to the same natural number. Moreover, if we

* The second dual digit being 0, the constant 33.09 is not used.

DNA : 2 =	1 2 1 0 0 0 0 0 0 0	
	4 8 4 0 0 0 0 0	
	7 2 6 0 0	DNA . 04
	4 8 4	
	1	
DNA : 24 =	1 2 5 9 1 3 0 8 5	
	6 2 9 5 6 6	
	1 2 6 0	DNA : 005
	1	
DNA : 245 =	1 2 6 5 4 3 9 1 2	
	2 5 3 0 9	DNA : 0002
	1	
DNA : 2452 =	1 2 6 5 6 9 2 2 2	
	8 8 6	(7
	7	(5
DNA : 24520705 =	1 2 6 5 7 0 1 1 5	

∴ DNA : 24520705P⁴=12657.0115

reduce DNA: 24520705 P⁴ to a DL: we shall find that we again obtain DL: 944596658 correct to eight places of figures; which is certainly strong evidence in favor of the accuracy of Byrne's rules, and should suffice to give universal confidence in the importance of the art.

In conclusion it must be understood that it is not proposed to supersede ordinary logarithmic tables by mental logarithms, but to use the dual arithmetical system as an auxiliary to and extension of the usual system of logarithmic calculation; and it will be found when the dual system is once mastered, as it quickly can be with a little attention and perseverence, that a great variety of problems, the solution of which usually involves complicated and laborious processes, can be accurately and readily solved without tables, without mental labor, and by the application of simply and easily remembered rules. For example it can quickly be ascertained that if—

$$3.01416x^6-28.233x^4+923.7x^3+ \\ +1234x^2-1862x=1609149128,$$

then x is equal to 56.43657, which would usually require much labor; and the dual system equally facilitates the calculation of angular magnitudes and trigonometrical lines, the solution of plane triangles, without the use of tables, the determination of the numerical value of elliptic and hyperbolic functions, the extraction of the roots of equation of all degrees, and indeed the solution of almost innumerable problems which have hitherto been considered to require great mathematical skill; all this moreover can be done by any one acquainted with the ordinary rules of arithmetic. But inasmuch as the present object is not to demonstrate the value and scope of dual arithmetic, but merely to show by a practical application of it that its advantages may be thoroughly utilized without the use of a special notation, these few references will suffice, and it is hoped that after the examples which have been worked, and the explanations given, this will be fully acknowledged, and that hereafter the system will be far more generally studied by all classes

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CLEVELAND STEEL.

From "Iron."

CAN Cleveland iron be turned into steel, and the steel thus made be now sold at a profit? Speaking broadly of "steel," we mean steel of suitable quality for rails, boiler and ship plates, and the other heavy branches of its manufacture. The cheapest steel now made and used for the above purposes is manufactured fram hæmatite or other ores uncontaminated by the presence of phosphorus. The Bessemer converter is the agent most largely employed for ridding the iron, subsequent to its delivery from the blast-furnace, of the impurities it has gathered up in the operation of its reduction and separation by fusion from the matrix of the ore. But this great cheapener of steel, the Bessemer converter, has been so far regarded as useless in dealing with cheaper irons like those of Cleveland, because it failed to rid such brands of their most objectionable impurity—phosphorus.

The cheapest mode of manufacture has thus far only dealt with the dearest raw materials. When the cheapest mode of manufacture deals equally well with the cheapest raw materials, then the steel

trade will take another giant stride, and settle in new quarters.

When in 1874 Cleveland turned out about 400,000 tons of iron rails, its manufacturers scorned the name of steel, and none of them cared to spend time or money in investigating a point they thought would never affect them. But before then, sundry far-seeing men, not thoroughly contented with anything short of absolute perfection, essayed to solve the riddle, and, commercially speaking, failed, they therefore laid the matter down, and enriched themselves by easier means.

The easier means have now for some time almost failed, and necessity has become the mother of invention.

Cleveland iron always has been deprived of its phosphorus by the agency of oxide of iron. There are some points, however, about the question, which have only lately been completely realized, and one is the precise conditions under which phosphorus has the least affinity for iron and the greatest for oxygen. The determination with certainty of these conditions (in which temperature was for a long time held to play a most important part) appears to indicate that the best substance for purifying Cleveland iron from its phosphorus is the long-used agent oxide of iron, but that it is inconvenient to use this substance also at the same time for the removal of carbon, as has been the practice since the days of Cort.

To make Cleveland iron into steel, speaking roughly, all that is necessary is to reduce to certain very small percentages its impurities, phosphorus, carbon, silicon and sulphur. In the ordinary puddling furnaces, these have been reduced at one operation, as shown below, from forge pig to puddled bar, and after piling and rolling, a rail was produced from this iron, whose composition is also shown. (*See table on following column.*)

But in a good steel rail these elements must be proportioned within narrow variations as under:—

	Per cent.
Iron	98.49
Carbon	0.32
Silicon	0.05
Sulphur	0.06
Phosphorus	0.08
Manganese	1.00
	100.00

	Forge pig.	Puddled bar.	Rail.
Iron	92.74	98.800	99.00
Carbon	3.11	0.050	0.25
Manganese	0.37	0.052	0.05
Silicon	2.00	0.271	0.22
Sulphur	0.25	0.035	0.02
Phosphorus	1.53	0.036	0.35
Oxygen (in cinder)	—	0.405	0.31
Copper	—	0.027	—
	100.00	100.00	100.20

Thus we see that in the Cleveland iron rail the phosphorus and silicon must be reduced and the carbon increased before it assumes a composition similar to the steel rail; and we need scarcely add that, with the same composition, equal strength and like qualities will, in every respect, be obtained.

Two different means are now projected, by one of which Cleveland will doubtless ultimately become as large a seat for the manufacture of steel as it has been for the production of iron. From recent disclosures we shall not be surprised if the great cheapener, the Bessemer converter, is yet brought into requisition to work out this problem, and whilst writing on this matter we are reminded of another new process in which Mr. Bessemer's so original and invaluable apparatus has found a use—no less, in fact, than to form a factor in the probable future manufacture of copper, sulphur and sulphuric acid.

It has been a generally accepted theory recently, amongst metallurgical authorities, that temperature was the condition that most strongly influenced the affinity of the phosphorus for oxygen and iron respectively; but it is now pretty clearly demonstrated that this was a mistake, and that, provided the cinder is sufficiently basic, phosphorus will leave iron and take to the oxygen in oxide of iron at any furnace temperature.

The two different means above-referred to for making steel from phosphoric iron are, therefore, these:—(1) To blow Cleveland iron in a Bessemer converter provided with a non-silicious lining, the phosphorus being removed from the iron during the operation, either by the agency of oxides put into the converter, or by oxide produced by overblowing. (2) To dephosphorize Cleveland iron by

washing with an excess of oxide of iron, and decarbonize the liquid product in a Siemens-Martin furnace.

We see no grounds at present for declaring that either of these processes is likely to prove so much the cheaper as entirely to exclude the other. The chief item of cost in both cases is oxide of iron, and as much of this must be employed in one case as in the other to obtain a like result.

As regards the first plan, how far silicon may be dispensed with when blowing phosphoric iron in the Bessemer converter, is not yet finally determined. If the carbon is eliminated along with the phosphorus, the heat must be got up by the oxidation of some element, and it is well known that no English iron, low in manganese, can be readily blown with much less than two per cent. of silicon. Now, the more silicon present in the phosphoric iron blown in a converter the more basic oxide must be at hand to hold the phosphorus.

On the contrary, pig-iron low in silicon is just the thing for the washing process in which phosphorus is removed and carbon kept in. A high temperature is unnecessary, and the heat due to the oxidation of silicon is not required, therefore, by this process a non-silicious pig-iron may be employed capable of being dephosphorized by a minimum weight of oxides.

Reducing the operation to its simplest form, the molten iron would be taken from the blast-furnace either periodically or continuously into a suitably-constructed refining furnace, either capable of being mechanically revolved, or agitated, or stationary. In whatever way the Cleveland engineers might work this out, the metal would be therein subjected to the washing of liquid oxide of iron. The process is not one involving heat absorption, but, on the contrary, there is a calorific evolution due to the oxidation of silicon and phosphorus to an amount, we have little doubt, quite sufficient to compensate for radiation, especially if the process could be made continuous. Looking broadly at this process, there is nothing about it, except the cost of the oxide of iron, which would appear to involve much expense. Non-silicious linings and a basic cinder, of course, apply in this case as in a converter.

The Bessemer converter has held its place for making steel rails almost to the exclusion of the open-hearth system, and it may be so in Cleveland in the future. But for mild steel, in which the exact percentages of carbon and manganese are of great importance, there is clearly at present a leaning in favor of the Siemens process.

In comparing the cost of these two systems of making steel, the chief item which strikes one is the difference in the wages account; the amount paid in wages on each ton of Siemens steel being about double what is paid per ton of Bessemer steel. The great length of time, viz., nine to eleven hours, which has so far been required to get out a heat of, say, ten tons of Siemens steel, accounts for this difference. But Cleveland iron, dephosphorized by the washing process, may be transferred liquid to the open hearth, and in the absence of silicon decarbonized with three cwt. of ore to the ton, and run out as good steel in from three to four hours. This would make nearly all the difference in wages, not to mention the saving in coals.

The very ingenious combination at work in Belgium, known as "The Forno-Convertisseur Ponsard," bids fair, under like conditions, even to complete such an operation in a still more limited time, for we are assured that this apparatus is now making good steel in less than four hours out of cold pig-iron and rail ends.

Many minds are now concentrated on the problem of making steel in the cheapest possible way, and there is a deep, but, for Cleveland, a tantilizing interest, in awaiting and noting each successive step. Stagnant and wearily as its fortunes now trail on, we cannot but think that from amongst the means we have glanced at, there is even now a radiant glory just arising to regild and illuminate the iron towers of Middlesborough.

SWITZERLAND has gained a considerable accession of valuable and prospectively productive land, by the opening of the canal from Aarberg to the Lake of Bienne, which has been nearly ten years in construction. About 74,000 acres of marsh land are drained by it, and the banks of Lakes Morat, Neufchatel and Bienne are secured from inundations.

THE REGULATION OF RIVERS.

From "The Engineer."

CERTAIN well-meaning individuals have suddenly become alive to the fact that British rivers stand in need of regulation; that is to say, in other words, that the beds of these rivers want straightening and leveling, and that the construction of embankments is required to prevent low-lying districts from being flooded in winter, while sluices, locks, or *barrages*, are indispensable to keep the level of the water up in dry weather. Having made this discovery, the gentlemen in question immediately got up a species of mild agitation, and at the present moment little else is talked of in some circles. The subject has been in one sense discussed very fully, and an earnest request was made at the recent meeting of the British Association for papers on subjects connected with rivers and their management. The request was complied with, and a great many hastily-constructed productions were read, or taken as read, in Dublin. Also a committee was appointed for the purpose of conferring with the Council as to the advisability of urging upon Government to take immediate action to produce unity of control of each of our principal river basins. It is understood, moreover, that not content with what has been done in London, the gentlemen who have taken the bad condition of our rivers so much to heart are about to hold meetings in the provinces to talk about the whole question, and "see what can be done." The object had in view is laudable, and we have nothing to say against it. It would be a pity that so much energy exerted in a good cause should be wasted; and to prevent this as far as lies in our power, we propose here to show that the apostles of river regulation are beginning at the wrong end, and to explain what are the preliminary steps that must be taken before any good whatever can be done; and this is the more necessary because not a few of the leaders of the movement have apparently lost their heads, and regard as "mere matters of detail" questions of the most vital importance.

As far as the memory of man goes back, British and Irish rivers have proved a source of trouble and anxiety to those who have had the misfortune to dwell on or near their banks. The same statement will apply to rivers in France, Germany, India, America—to rivers all over the world, in fact. This circumstance seems not to be generally known, and it has been gravely argued that floods are now much worse in every respect, and do more damage than they used to do. This is only partly true. Good land drainage means the rapid delivery of storm waters into the nearest stream, and inasmuch as Great Britain is now better drained than it ever was before, it is very likely that the floods are, heavier, more sudden in their occurrence, and caused to occur with less rainfall than formerly. Rivers have always made themselves so obtrusive by their bad behavior, that few subjects are better understood by engineers than the laws which govern the formation of floods, and the means which should be taken to prevent them; and it is not too much to say that half a dozen men might be found at this moment in Westminster, who could each design and carry out a system of work which would effectually regulate British rivers. It is quite unnecessary that we should go into any details to prove this just now; yet a very hasty perusal of much that has been recently written, or spoken, on the subject, will suffice to show that the writers and speakers regard the subject as invested with enormous difficulties of a purely engineering character; and very remarkable schemes, to say the least of them, have been proposed for the drainage of flood districts, the construction of *barrages*, and a hundred other things, necessary or not necessary. The answer to all proposals and schemes of this kind is that when the work needed comes to be done, engineers will do it with great ease; and that amateurs will have to stand on one side. The hyetological conditions of Great Britain are sufficiently simple and fairly well understood. We advise those, therefore, who have the will and the energy to agitate for the reform of our rivers, to dismiss from their minds

all schemes for the actual execution of the work. In a word, the time has not yet arrived for the interference of the engineer. When he is wanted he will step on the stage and play his part to perfection, but we are a long way from this part of the drama at present.

The difficulty which must be combated and overcome before our rivers can be regulated, is much more serious than any which engineers will have have to encounter. It consists in reconciling or overruling the contending interests of river-side proprietors. One of the proposals recently made is that what may be termed district water governments should be appointed, each of which should have the sole control and management either of a given stream, or of a given district traversed by several streams. The Thames Conservancy Board affords an example of one of these governments, which, if not accurate, is at least near enough for our purpose. In Holland, local governments of this kind control all work done on the dykes. The idea is no doubt good; indeed it is very difficult to see how any river, such as the Severn for instance, could possibly be regulated unless some special authority was appointed to attend to it. The moment we have come to this point we are face to face with difficulty number one. How is the board of regulation, or conservancy, whichever or whatever it may be called, to be constituted? Who is to appoint it? and from whom can it derive its authority? There can be but one answer. The Board must be empowered by Act of Parliament to raise the necessary funds by taxing those whose property is to be benefited; to carry out the necessary works, exercising in perpetuity the sole control over the stream; to rectify the course not only of main rivers, but of dozens of tributaries; to alter boundaries which have existed from time immemorial, and interfere all round with the rights of property in a way actually without parallel. How is Parliament to be induced to grant such powers? This is difficulty number two. Let us suppose that it was decided to-morrow that the river Severn was to be regulated. The first question asked by those holding property on the stream would be—Who has decided this? In all probability it would turn out that the originators of the movement were a comparatively limited number of individuals whose property was injured by floods; and these gentlemen would at once have to encounter the opposition of all the other people who, living on or near the banks of the stream, would be liable to be taxed for its regulation, but who would benefit in no way by that regulation. The minority would have to go to Parliament for a bill, and it is not too much to say they would not get it. If they were more fortunate and did obtain it, it would be only after the expenditure of an immense sum in Parliamentary costs and fees of all kinds. Amiable theorists maintain that it is the interest of every one living on the banks of a river to contribute funds for its regulation. We need not dispute the soundness of the theory, but it has no practical value in the present connection, because it is not generally held to be true. Indeed, there are individuals who usually display much shrewd common sense, and yet maintain that in a great many instances the value of property destroyed each year by floods would not pay two per cent. on the capital which would have to be expended to prevent them. Arguments like these cannot be dismissed as "mere matters of detail," for until they are answered, and that to some purpose, rivers will not be regulated in Great Britain, and the movement to which we have directed attention will die a natural death.

The first thing to be done by those who advocate the regulation of our rivers is to ascertain whether those most interested are of the same opinion. If they are, the work may perhaps be done; if they are not, the rivers will not be regulated. Let a committee be appointed either by the Society of Arts or the British Association, or some other competent authority. Let this committee begin at the head or the mouth of some important stream liable to floods, and ascertain step by step as they move along the stream, what are the desires of those to whom the river in a sense belongs. Let it be clearly explained to the farmer that certain engineering works will have to be carried out in his riverside fields, and that he will have to pay considerable sums every year for them. In a word, let the whole truth be put before him, and then ask him what his wishes are.

In the same way, let the mill-owners be consulted, and the proprietors of fisheries. In the course of half-a-dozen such trips as this the committee would acquire the information which no one else possesses, and armed with this, they could either recommend the abandonment of the whole idea, or its active prosecution. To use a parliamentary phrase, the sense of the country should be taken in the first instance, and if this is against the interference of Parliament in riparian matters, nothing can be done. We have already indicated the nature of several questions that may be asked. We regret that the list is not full. Suppose that a committee of the Society of Intellectual Hydrologists, let us say, in the course of such a tour in search of information as we have suggested, were to come across a sturdy riparian proprietor who asked them what business it might be of the Intellectual Hydrologists to inquire into the condition of the river of which he was part proprietor, what reply would be given? We really cannot suggest a satisfactory answer. But the last and most important question of all remains to be put. Have those who are so earnest for the regulation of British rivers any money wherewith to even commence to carry out the work they have so much at heart? If they have not, what is to be the end of conferences, and papers and discussions?

We have asked a sufficient number of unpleasant questions. We may be asked in return what our own views are. We can reply in very few words. We have no hesitation in saying that much might be gained by the construction of suitable regulation works on many of our principal rivers; that, as regards others, they are best left alone, because it would not pay to improve them; that any scheme of a comprehensive nature for the regulation or control of rivers must originate with Parliament; and must be a national work, carried out by public funds in the first instance, to be repaid in a certain number of years out of the profits to be derived from the works; that local or isolated efforts, either of individuals or learned societies, are not likely to directly produce regulation work; and that the efforts of such individuals or societies as take an interest in the matter should be devoted to inducing Parliament to appoint a committee to inquire into the whole subject of the arterial drainage of Great Britain and Ireland. In the appointment of such a committee, and in it alone, lies the hope of the would-be regulators of rivers. The interests to be reconciled are too opposed and too great, and the work to be be done is too important to be dealt with by individuals or scientific bodies. Any regulation work which will deal with our great rivers must be undertaken by the nation, and regulation operations which do not deal with our great rivers must be too limited in their scope to possess much value or deserve much attention.

The Strength of Wrought Iron.—A series of experiments has been carried on at the Washington Navy Yard, by Commander L. A. Beardslee, of the United States Test Board, to ascertain the strength of iron used in chain cables. It had been suspected, with just reason, that the British Admiralty tables for the strength of wrought iron needed revision. Not less than 2000 tests of the tensile strength of iron have been made at the Navy Yard, to determine the elastic limit, elongation, and reduction of area of the various specimens; and 42 complete chemical analyses have also been performed. Some of the conclusions which have been reached are remarkable. The admiralty tables are declared unsafe, and new ones have been prepared. The Board finds that the tenacity of two-inch bar for chain cables should be between 48,000 and 52,000 pounds per square inch; one-inch bar, between 53,000 and 57,000; and that stronger irons than these make inferior cables, because they have less ductility and capacity for welding. The strength of wrought-iron and its welding power are influenced quite as much by the reduction it has undergone in rolling as by ordinary differences in its chemical composition. In general, the processes for making wrought-iron give an uncertain quality to the product, while the methods of making cheap steel confer certainty and uniformity. The ordinary practice of welding is capable of great improvement, by being performed in an atmosphere freed from oxygen. The importance of the subject will be conceded, since the safety of human lives must often depend upon the strength of a ship's cable, or of the links in a bridge chain.

THE STRENGTH OF MATERIALS.

By WILLIAM KENT, M.E., Pittsburgh, Pa.

Written for VAN NOSTRAND'S MAGAZINE.

I.

INTRODUCTION.

The literature upon the subject of Strength of Materials is very extensive. Every professional engineer has, or should have, access to a library of volumes containing records of experiments made for more than a century past, upon every known material of construction, together with mathematical and logical discussions of various theories of strength and resistance, sufficient to enable him to design and proportion structures with that rough approximation to accuracy and economy of material which is at present allowed in most branches of engineering.

In some departments of engineering construction, notably in our American bridge building, the greatest care is taken to thoroughly understand and apply the principles of strength of materials, and to use materials of known quality; so that in both the theory and the practice of bridge construction, the engineers in charge have gone even beyond the books, and have done better work than any that the books have yet recorded.

In the large majority of constructions, however, this care is not taken. In many cases engineers are not employed at all in designing structures, and, in a certain degree, every man is his own engineer. This is especially true in the construction of ordinary buildings. The results are, in most instances, a reckless waste of constructive material, and frequently, a want of correct proportioning; heavy pieces being placed where light ones should be, and *vice versa*. The waste of constructive materials annually in this country might be figured in millions of dollars. On the other hand, the cost of saving material where it should not have been saved has too often been the sacrifice of human life.

It is chiefly for the benefit of those users of materials of construction who are disposed to be their own engineers that this series of articles is written, but it is hoped that they will not on that account be without interest to members of the engineering profession.

It is intended to present some facts and figures which will show that because a metal is called by the name "iron" it does not therefore necessarily possess a definite strength, but that its strength should first be determined by test; that published records of tests are not always to be relied upon; that many tests themselves are not reliable; and that in using any material in construction not only its strength but its other properties should be considered.

The articles will consist principally of compilations from various authorities, American and foreign, to whom credit will be given as far as possible; but many facts and figures will be given obtained from the writer's own experiments, and from those of his friends, which have not heretofore been published.

THE STRENGTH OF MATERIALS.

The term "strength of materials," in its widest sense, as used by many authorities, does not include merely what is known as the absolute or ultimate strength—or the resistance, expressed in pounds per square inch or other unit, to final rupture—but also the resistance within certain limits of distortion short of final rupture, as the elastic limit and the point of permanent set; the safe load; the resistance to steady and to suddenly applied loads; and the resistance to repetitions of loads and to shocks and vibrations. It also includes the amount of distortion of the material before final rupture, or within any limit short of final rupture, commonly called ductility; and the property of returning towards its original form after temporary distortion, or elasticity.

DEFINITION OF TERMS.

The external forces applied to materials tending to cause their rupture or alteration of form are called *stresses*.

They are of different kinds, viz. tensile, compressive, transverse, torsional and shearing stresses.

A *tensile stress*, or *pull*, is a force tending to elongate a piece. A *compressive stress*, or *push*, is a force tending to shorten it. A *transverse stress* tends to bend it. A *torsional stress* tends to twist it. A *shearing stress* tends to force one part of it to slide over the adjacent part.

Tensile, compressive and shearing stresses are called simple stresses. Transverse stress is compounded of tensile and compressive stresses, and torsional of tensile and shearing stresses.

To these five varieties of stresses might be added *tearing stress*, which is either tensile or shearing, but in which the resistance of different portions of the material are brought into play in detail, or one after the other, instead of simultaneously, as in the simple stresses.

TENSILE STRESS.

Testing Machines.—The resistance of materials to tensile stress is the one which receives most attention, as it is called into play more frequently than any other, except compressive, and is considered to be in some measure an index of all the other resistances. It is usually determined by means of an apparatus known as a testing machine. The character of this machine may vary with the nature and strength of the pieces to be tested. To test the tensile strength of a piece of twine, for instance, a convenient apparatus would be a spring balance, the twine being fastened at one end to a firm support, and at the other to the hook of the balance. It might also be tested by fastening one end to a firm overhead support, and attaching to the other end such a vessel as a tin pail, in which shot or sand could be poured till the twine breaks, and then weighing the pail and its contents. For testing large sections of metal, requiring many tons to break them, the testing apparatus may be a machine of great size and strength, requiring a high degree of skill both in its construction and in its manipulation.

Such machines have been built in this country in considerable numbers. One of the best known was built several years ago by the late Major Wade, for the United States Government. It is described in his Reports of Experiments on Metals for Cannon (Phila., 1856.) Copies of this machine, as improved subsequently by Capt. Rodman, are now in use at the Washington Navy Yark and in the U. S. Army Building in New York. One of these machines has been used for several years in the Woolwich Arsenal, in England.

Fairbanks and Co., the well known scale makers, of St. Johnsbury, Vt., and more recently Riehle Brothers, of Philadelphia, have paid considerable attention to the building of testing machines, and machines of their makes are to be found in various iron-making and manufacturing establishments throughout the country. All of these machines weigh the amount of applied stress by means of a combination of levers and scale beams. In the Riehle machine the stress is applied by means of a hydraulic press. In the Wade and the Fairbanks machines the stress is applied through screws, levers and wheel gearing, or some combination of them, the particular combination varying in different machines.

For testing very large specimens, requiring hundreds of tons to break them, hydraulic presses have been used, in which the stress is registered by gauges showing the pressure of the liquid used in the press. Such a machine is in use at the works of the Keystone Bridge Co. in Pittsburgh. A machine has recently been built by Albert H. Emery of Chicopee, Mass., for the use of the United States Board appointed to test Iron, Steel, etc., but it has not yet been erected in position for use on account of the neglect of Congress to make an appropriation for the continuance of the work of the Board. It is designed for a capacity of 800,000 pounds, and is believed by engineers who have inspected it to be the most accurate machine of large capacity ever made.

John L. Gill Jr. of Pittsburgh, has recently built a testing machine of 100,000 pounds capacity, for general work, which, as he claims, remedies some of the defects possessed by all machines of this class which have heretofore been built.

It is frequently supposed that to obtain the tensile strength of a piece of iron, or other material, any testing machine will answer, no matter how roughly built or how badly used, that the specimen may be of any convenient size and shape, and

that any person of ordinary intelligence is capable of making an accurate test. A review of what has been written by various "authorities" will convince any one how erroneous is this idea.

Mr. Kirkaldy, of London, the eminent experimenter, who has written several works containing his observations, quotes Mr. S. Hughes as saying that writers on the strength of materials in the last century seldom assigned a less tensile strength than thirty tons (of 2,240 lbs.) as the weight which would tear asunder a bar of ordinary wrought iron one inch square. Thus Emerson quotes the tensile strength of bar iron at 34 tons; Telford, 29.29 tons; Drewry, 27 tons; while at the present day Templeton gives 25 tons; Beardmore, 26.8 tons; Brown, 25 tons; and Hodgkinson, probably from more careful experiments than any other, 23.817 tons. In reference to these figures Mr. Kirkaldy states that he does not think that there is any satisfactory evidence in the experiments adduced to show that the iron now produced is inferior to that made during the last century. The difference is rather due to experiments having been performed by so many persons, whilst the pieces tested by each were so few—to the different kinds of apparatus employed—to the results having been more carefully recorded by some than by others—to the extreme meagerness of details, and to the complete want, with a few exceptions, of the makers' names or brands—which renderered futile any attempt at comparison. The various means employed to tear the piece asunder were—applying weights directly to the specimen, single lever and weights, compound lever and weights, combined with a hydraulic press, hydraulic press alone, with either a loaded valve or a gauge to indicate the pressure.

When such wide variations are found in the figures of the strength of bar iron published by those who are supposed to be authorities on the subject, and from whom accurate statements ought to be expected, is it any wonder that figures reported by manufacturers should vary still more, when their tests are frequently made on inaccurate machines, by unskilled operators, and when the results are apt to be influenced by incorrect and irregular methods of test, and by variety of shapes and methods of preparing test specimens, as well as by self interest?

A striking example of the difference in the apparent strength of wrought iron as obtained from different testing machines is given in a paper in the *Metallurgical Review* for September, 1877, by Mr. John J. Williams. In testing the iron for the Point Bridge, Pittsburgh, three machines were used. One is said to have given the tensile strength from three thousand to five thousand pounds per square inch below what it should indicate, and to have "pulled crooked," causing flat specimens to tear at one edge before the iron was strained to the greatest load it would carry. The second machine was supposed to be "nearly correct." The third machine was found on making comparative tests to indicate, on an average, 10,000 pounds higher than the first, "thus proving conclusively that the ultimate strength of iron depends on where the testing is done, and what kind of a machine is used."

Shape of Specimen.—The shape of test specimen has sometimes an important influence upon the record of strength. It is very common to shape test pieces as

Fig. 1

shown in Fig. 1., in which A represents a piece of plate or flat bar, and B a piece of round iron or other material. In testing pieces of this shape, especially in wrought iron, brass, or other ductile material, the result will almost invariably be higher than the true result as determined by test pieces of the shape shown in Fig. 2. The following facts may be given in confirmation of this statement.

Mr. D. K. Clark, in his "Rules, Tables and Data for Mechanical Engineers" notes a test made by T. E. Vickers, of a piece of steel turned down to one inch in diameter and fourteen inches in length, which gave a tensile strength of sixty

tons per square inch, while a bar of the same steel turned down to $\frac{3}{4}$ inch in a circular notch in the middle, broke at $79\frac{1}{2}$ tons per square inch.

C. B. Richards, C.E., in the Transactions of the American Society of Civil Engineers, Vol. II, p. 339, gives the following results of tests of different-shaped specimens of the same material: For "Burden's best" bar iron the "short" specimens, (shaped as in Fig. 1), gave 62,000 pounds as the average value for the tensile strength per square inch of original cross section of the finished specimen, while the "long" specimens (shaped as in Fig. 2), gave only 49,600

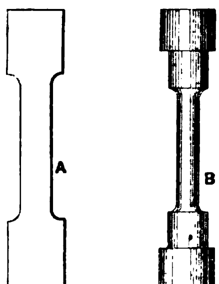


Fig. 2

pounds for that value, the difference in results corresponding to 25 per cent. of the smaller value. For "Bay State" boiler plate, the short specimens gave 52,100 pounds, and the long 47,450 pounds, the variation being 10 per cent. There was a great difference in the ductility of these irons, the Burden bar (long specimens) stretching on an average 30.1 per cent. before breaking, and the Bay State plate only 12.4 per cent., the length of the portion on which the stretch was measured being five inches in each case.

Col. Wilmot found, using proof bars of a form similar to Fig. 3, as a result of

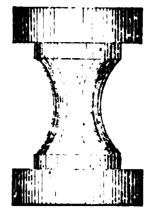


Fig. 3

eight trials of Bessemer steel, a mean of 153,677 pounds per square inch, and on the same steel turned to a cylindrical form three inches long and one square inch in cross section, the mean was only 114,460 pounds per square inch. (Levi and Kunzel's Report on Phosphor-Bronze).

Kirkaldy made some experiments to show the variations in results for tensile strength of Fagersta steel plates, arising from differences in the form and proportions of the specimens. One set was ten inches long and ten inches wide at the parallel middle portion, the second was $1\frac{1}{2}$ inches wide and $4\frac{1}{2}$ inches long, and the third was $2\frac{1}{4}$ inches wide and 100 inches long. The results were as follows (Clark's Rules, Tables and Data):

UNANNEALED.

	Elastic tensile strength tons per sq. in.	Breaking weight tons per sq. in.	Ratio of elastic strength to breaking weight per cent.	Permanent extension per cent.	Area of fractured section per cent. of original.
Length=breadth	16.05	26.39	60.0	29.7	48.8
Length=3 breadths..	15.56	25.56	60.3	35.0	43.2
Length=44 breadths.	13.71	23.00	59.2	14.0	43.1
ANNEALED.					
Length=breadth	13.53	23.65	57.0	33.1	39.1
Length=3 breadths..	12.98	23.16	56.0	39.3	36.5
Length=44 breadths.	12.04	21.31	56.5	16.5	34.4

Kirkaldy found that the influence of the shape of the specimens upon the results varied with the softness or ductility of the materials, and that this influence was important in the case of soft or ductile materials, but became hardly appreciable with hard and brittle materials.

Richards found that the breaking weight

per square inch of *fractured area* give nearly equal values with either shape. In the case of the "short" specimen the shape tends to prevent any contraction of cross section, and this would tend to make the apparent strength per square inch of fractured section approximate to that of a long piece which contracted to a greater degree.

The writer recently made some tests to determine the influence of shape of specimen upon some samples of charcoal-refined iron furnished by one of the mills in Pittsburgh which has a wide reputation for the superior quality of its product. The following table shows the results:

	Breaking Strength.		Elongation per cent.	Increased strength of short specimen per cent.
	Per square inch of original section.	Per square inch of practical section.		
	lbs.	lbs.		
No. 1.	52,261	97,567	16.	—
No. 2.	54,439	71,668	—	4.2
No. 3.	59,682	76,531	15.	—
No. 4.	63,268	75,601	—	6.0

No. 1 was a piece of flat bar, $2'' \times \frac{1}{2}''$, a "long" specimen, shaped as in Fig. 2, A, the distance between the shoulders being 5 inches and the original section of the portion stretched being $1'' \times \frac{1}{2}''$. No. 2 was a "short" specimen for the same bar, shaped as in Fig. 1 A, the minimum section being the same as No. 1, $1'' \times \frac{1}{2}''$.

No. 3 and No. 4 were a "long" and a "short" specimen from a piece of "C.H. No. 1" boiler plate $\frac{3}{8}$ inch thick, stamped "60,000 lbs.," shaped as in Fig. 2 and Fig. 1 respectively. All the tests were made in the direction of the fibre, all were made on the same testing machine, the same care was taken in fitting, and the same length of time taken in each test, so as to secure as far as possible a uniform set of conditions in each. It will be seen that the short specimen in both cases gave the highest results, although the variation is not so great as that given in the experiments of Richards. The strength per square inch of fractured section of No. 1 is unusually high, and much higher than that of No. 2, the short specimen of the same material. This does not accord with Richards' statement, given above, that the breaking weights per square inch of fractured area give nearly equal values with either shape. The strength per square inch of fractured section of No. 3 and No. 4, however, agree quite closely.

Commander L. A. Beardslee, U. S. N. has recently made a large number of such comparative tests, which confirm the general fact that the short specimens give the highest apparent strength; but the results have not yet been published.

The above results of experiments should be sufficient to show that tests of "short" specimens of ductile materials are of little or no value for giving their actual strength as used in construction. It is a well known fact, however, that in this city and elsewhere, tests are made of short specimens of soft and ductile irons, and the figures obtained published as the "strength" of the materials, when their actual strength as used in a structure is much lower. Such published figures are not only unreliable but unfair. The following reasons may be given why the use of short specimens should be abandoned in tests for commercial purposes.

1. In testing short specimens no accurate measurement can be made of their extension before rupture, as a means of comparison of ductility, a quality, the knowledge of which is quite as important as that of absolute strength.

2. In testing short specimens, two specimens of different materials may give the same results, while long specimens of these materials may give different results. Thus, two short specimens may each show a tensile strength of 60,000 pounds, while if the specimens were of the long shape, one might show 50,000 and the other 55,000 pounds, if one were more ductile than the other. The short specimens, therefore, fail to give not only correct absolute results, but also relative results.

3. There is no *standard* shape of short specimens, and as the difference in results due to various shapes of short specimens is unknown, no proper comparison can be made of the results of different experiments on such specimens.

4. As shown by the experiments given above, there is no standard relation between the strength of a short specimen

and that of a long specimen; hence, if a test is made of a short specimen, it furnishes no means of knowing what strain the material will bear in actual service.

5. As it is always desirable before using any material of construction to know its strength under the most unfavorable conditions, or to know its least, rather than its greatest, strength under any conditions, the test of a sample of the material should show its least strength, and not its greatest, as does the short specimen. Of what advantage would it be to know that a short specimen taken from a bridge-rod will show 50,000 pounds strength, when if a long piece, or the whole rod were tested it would show only 40,000 pounds?

Other Incorrect Tests.—As too high an apparent strength may be obtained in testing specimens of a certain shape, so a specimen of another incorrect shape, or one of a correct shape incorrectly placed in the testing machine, may give too low a result. This occurs when, on account either of the shape of the specimen or of the manner in which it is placed in the testing machine, the line of strain of the machine does not coincide with the axis or central line of the piece tested.

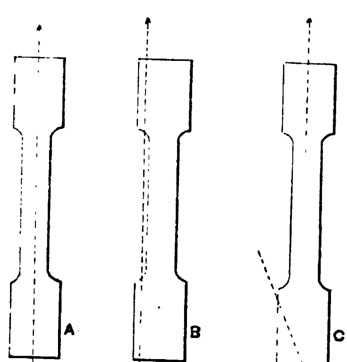


Fig.4.

In Fig. 4, A is a piece that is incorrectly shaped, the axis of the heads not being in a line with the axis of the middle portion. If placed in the testing machine in the manner that is ordinarily correct, viz.: with the axis of the heads in the line of strain, as shown by the arrows, the test will be an incorrect one and the result will be too low. The piece will break partly by *tearing;* it will *bend* as soon as the strain is applied, and the fibers will give way on one side before the other. B represents a correctly-shaped specimen, but being incorrectly placed in the testing machine, so that the line of strain is not through the axis of the specimen, as indicated by the arrows, it will also break partly by tearing and give too low a result. C is a correctly-shaped specimen, but the upper end is rigidly held in the clamps of the testing machine in the correct position, while from an inaccuracy of adjustment of the lower clamp, or of the testing machine itself, the lower end is pulled sidewise, causing a tearing strain. In all these cases the tendency is to bend the specimen before breaking it. The writer has sometimes seen tests made in which the specimen was so much bent before it was broken that the bend could be detected by the eye, and plainly shown on placing a short straight edge against the side of the specimen. More frequently he has seen tests in which the bend was imperceptible to the eye, but could be detected by an apparatus used for the purpose (which will be hereafter described) which magnified the visible appearance of bending. In many cases, by a slight re-adjustment of the clamps which held the specimen, the bend could be made to take place first on one side of the specimen and then on the other. It may be imagined how valueless the result of a test of cast iron, or of any brittle material would be in which the breaking was caused partly by pulling and partly by bending.

Mr. Hodgkinson states that the strength of a rectangular piece of cast iron drawn along its side is about one-third, or a little more, of its strength, to resist a central strain. It is evident that this ratio (one-third) would be given only by a few certain shapes pulled in a certain manner, and that it would vary with every shape and with every variation of the error of the test. The resistance of any material to tearing bears no relation whatever to its resistance to direct pull, as may be conclusively proven by the simple experiments of pulling and tearing a slip of paper or of tin-foil.

A gentleman in this city states that in making a great number of tests of special grades of cast iron, which have been frequently found by other experimenters to have a strength of about 30,000 pounds

per square inch, he has not been able to find a piece having a strength much over 20,000 pounds. The discrepancy is so large that it can scarcely be accounted for but by supposing that one of the testing machines is not correctly adjusted, or that in the second case the specimens are not correctly placed in the clamps.

The error which may arise in a test of this kind is usually much greater with cast iron or other brittle material than with a ductile material like wrought iron. The ductility of the latter allows it to be partially "drawn into a line," while the former will often break before the drawing into line can take place.

Influence of Time upon Tests.—A difference in results may be obtained in testing two specimens of the same material, if the *time* occupied in one test is longer than that occupied in the other. This difference due to time is in some cases inappreciable, perhaps too small to be measured; but in other cases it is enormous. *With wrought iron and soft rolled steel, the more gradual the test the higher will be the result. With tin, zinc and some other metals and some alloys, the more gradual the test, the lower the result.* These statements are not yet generally believed among unscientific men, probably because it has only been within the last few years that the facts have been brought into prominent notice by experimenters. For abundant confirmation of them, reference may be made to several papers by Prof. R. H. Thurston, published in the Transactions of the American Society of Civil Engineers, in the years 1874, 1875 and 1876. He divides the metals into two classes.

1. "Metals subject to internal strain by artificial manipulation, and which may exhibit an elevation of the elastic limit by strain, and *decreased power of resisting stress under increased rapidity of distortion.* The ordinary irons of commerce are typical of this class."

2. "Metals of an inelastic viscous character, not subject to internal strain, and not usually exhibiting an elevation of the elastic limit by strain, and which offer *increased resistance when the rapidity of distortion is increased.* Tin is a typical example of this class."

Among a great number of experiments by Prof. Thurston—in which the writer had the honor to be associated as assistant—upon the influence of time upon resistance, the following may be mentioned in confirmation of the above statements.

1. Two pieces of ordinary merchant iron, one inch square, taken from the same bar, were tested by bending stress. They were placed on supports twenty two inches apart, and the pressure applied by a screw and registered by a platform scale. One was tested as rapidly as possible, the pressure and deflection being read and recorded at every 20 or 40 pounds, about one hour being occupied in the whole test. A deflection of $5\frac{1}{2}$ inches was caused by a load of 2350 pounds. The other was tested very slowly, and was "rested," frequently under strain, for intervals of from 12 to 48 hours, the whole test requiring more than three weeks' time. A load of 2640 pounds caused a deflection of less than three inches.

2. Two pieces of cast tin, from the same bar, were tested by tensile stress. The test of one was made in eight minutes, and the highest resistance was 3,400 pounds per square inch. The test of the other occupied thirty minutes, and the highest resistance was only 2000 pounds per square inch. Tests by tensile, transverse and torsional stress on tin and on zinc, and on the soft white alloys of tin and copper, invariably gave similar results.

In tests of cast iron, hard steel, and brittle materials in general, the effect of time has not been determined, but it is supposed to be very slight.

Directions for Making Tests.—The object of a test of a material of construction is to learn all that should be known concerning the properties it possesses which make it valuable for the purpose for which it is to be used. No engineer should be satisfied to use in an important structure iron of which he knows only the tensile strength and that inaccurately. He should know its ductility, or amount of extension before breaking, which measures to some extent its resistance to rupture by shock, and its strength within the elastic limit, which measures approximately its resistance to distortion. For many purposes he should know its coefficient of elasticity, or stiffness, its uniformity of strength, and its possession of, or freedom from, internal strain. All

of these may be determined by a tensile test properly conducted, with a correct machine, and an accurate apparatus for measuring elongations. A few directions for making such a test may be of service.

In the first place the testing machine itself should be tested, to determine whether its weighing apparatus is accurate, and whether it is so made and adjusted that in the test of a properly made specimen the line of strain of the testing machine is absolutely in line with the axis of the specimen.

Secondly.—The specimen should be so shaped that it will not give an incorrect record of strength. Under no circumstances should the test of a piece shaped as was shown in Fig. 1 or Fig. 3 ("short" specimen) alone be relied upon to determine strength. The piece should be of uniform minimum section for several inches of its length. The writer recommends five inches in length between the extreme points between which measurements of extension are made, for the *standard* size of specimen, as being the most convenient length for the testing machines now most in use, for calculation of extension in per cent. of length, and for comparison of results with those of other experimenters. Considerable confusion exists in published records of the per cent. of extension of materials as given by different authorities, because they used different lengths in testing. A ductile material may extend 30 per cent. of its length if the specimen tested is five inches long, but if the piece tested is only 1 inch long it might show an extension of 100 per cent. Prof. Thurston gives the following formula for total extension

$$\text{Extension} = Al + fd,$$

where l is the length and d the diameter of the piece, A is a constant, and f is a variable function of the diameter. The length of a specimen, as well as the diameter or area of cross section should always be given in the record of a test.

Thirdly.—Regard must be had to the time occupied in making tests of certain materials. When wrought iron and soft steel can be made to show a higher apparent strength by keeping them under strain for a great length of time, it is well to test them as rapidly as possible to obtain their minimum strength; and in accepting results of tests of these metals from interested parties, it is well to know what length of time they have taken in testing them. It is fortunate that metals of the tin-class are not used in construction to resist heavy stresses. If they were, no test ought to be considered a reliable one which did not occupy a time as long as the material was expected to have "life" in actual service, for, as was shown in the case of the test of tin, above mentioned, the material might be nearly twice as strong under a rapid as under a slow test. In recording the tests of all such materials the time occupied in making each test should be given.

Fourthly—Accurate measures should be made of the extension under each successive increment of load in order to determine all the properties of the material, other than its mere absolute tensile strength, which make it valuable in construction.

Methods of Measuring Elongation.—The method of making measurements of elongation will depend upon the degree of accuracy which is desired. Measurements to $\frac{1}{100}$ of an inch may be made by taking distances by a pair of dividers between two fine lines drawn on the specimen, and reading this distance on a fine metal scale. In testing specimens five inches in length between these lines, this degree of accuracy will give approximately the elastic limit, and the percentage of stretch at different loads beyond the elastic limit, but it will not give the extensions corresponding to loads within the elastic limit, which are a measure of the stiffness, or co-efficient of elasticity, nor the position of the elastic limit with that approach to preciseness which is desirable in competitive tests or in scientific investigations. Reading of extensions to $\frac{1}{1000}$ of an inch may be made by means of two divided plates, clamped one on each end of the specimen and sliding on each other, one carrying a fine scale and the other a vernier; or by means of a micrometer screw, with scale attached, clamped to one end, the point of the screw abutting against a rod clamped to the other end—practically a modification of the micrometer calipers.

For still finer work, readings may be made to $\frac{1}{10,000}$ of an inch with accuracy, by the method first adopted at the test-

ing laboratory of the Stevens Institute of Technology, and now used there to the exclusion of all other methods. Two very fine micrometer screws are firmly clamped to one end of the test specimen, the ends of these screws abutting against two rods clamped to the other end. The screws have each fifty threads to the inch, and the head is divided into 200 parts.

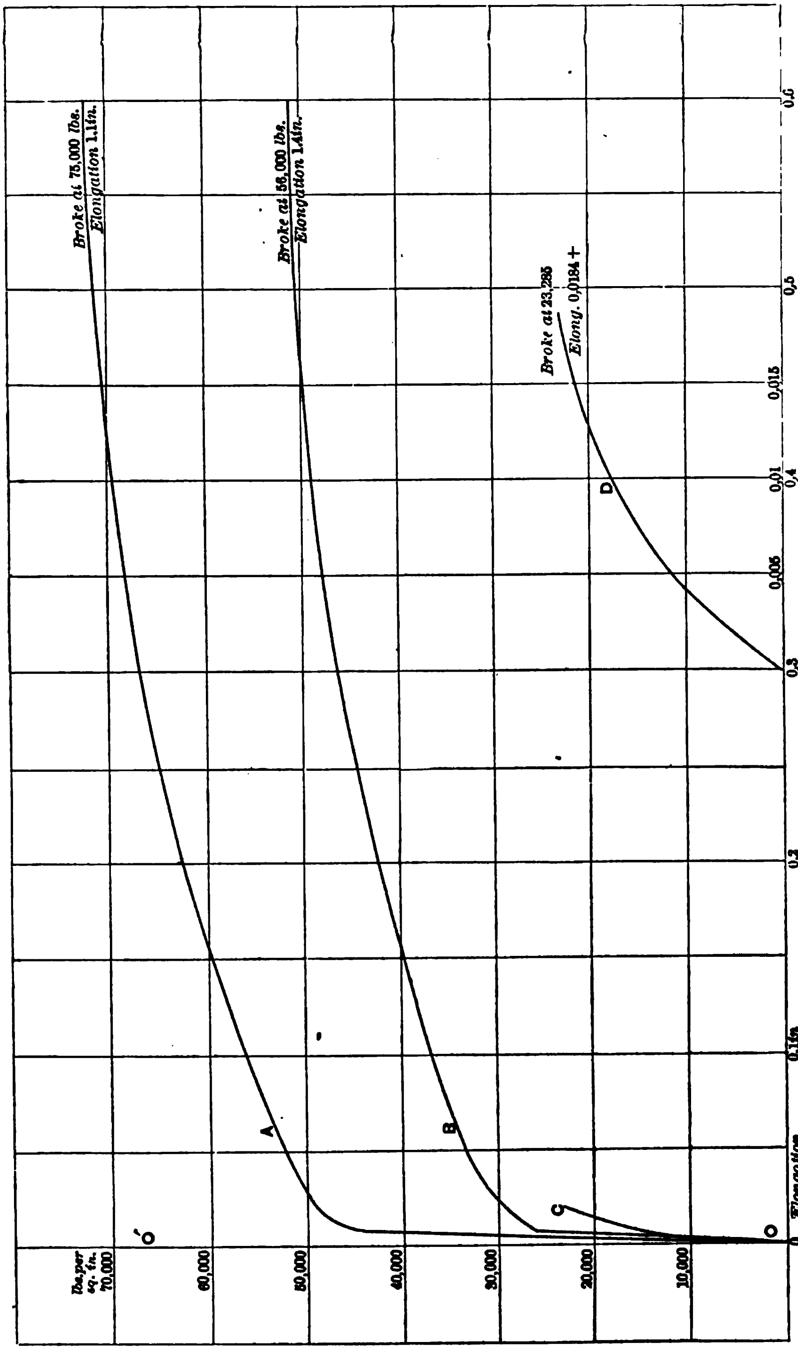
A sketch of this apparatus is shown above. A clamping piece, B, carries the two screws AA, and another clamping piece D carries the two rods CC. As the test-specimen S is extended in the testing machine, the points of the screws and rods recede from each other, and the distance each screw requires to be moved forward to make contact at each addition of load is the amount of extension due to such addition. It is impossible to obtain a reading of contact accurate to $\frac{1}{10,000}$ of an inch by the senses of sight or touch, but it is indicated with perfect accuracy by causing the touch of each screw on its opposite rod to close the circuit of a weak electric current and thereby ring a small bell. For this purpose the upper part of the rods CC require to be insulated from the rest of the apparatus. The writer made some experiments to determine the accuracy of the reading by electric contact, and found that there was no error as large as $\frac{1}{40,000}$ of an inch. In taking a reading, the screw A on the right hand is slowly turned till the bell rings, which indicates that contact has taken place. The reading on the divided head is then recorded; contact is broken with the right hand screw and made in like manner with the left hand screw. The difference between the mean of the two readings and the mean of two previous readings is the amount of change of length of the specimen. Two screws are necessary to eliminate the error which would occur if through any cause the piece bends during the pulling. In ordinary tests, there is, perhaps, not one test in a thousand in which the piece will not bend to some extent, and the bending will be plainly indicated by this instrument, although it would remain undiscovered without it.

It is frequently objected that measuring extensions of test pieces to $\frac{1}{10,000}$ of an inch is an unnecessary refinement; but by such measurements only is it possible to obtain the coefficient of elasticity or the elastic limit on specimens only five inches in length of portion stretched, and in many practical cases, as in bridge building, a knowledge of these is desirable. By the method of measurement above described, results have been obtained from five inch specimens of cast iron which are fully as accurate as those obtained by Hodgkinson on rods ten feet long, and with very much less trouble and expense. The following is the record of a test recently made by the writer, of a piece of cast iron, turned $1\frac{1}{8}$ inches in diameter and five inches in length between fillets.

(*See Table on page* 49.)

It will be seen that the greatest difference between the readings indicated by the screw on the right hand and those on the left, is only 0.0026 inches. As these screws were over six inches apart, the amount of bending of the specimen to cause this difference must have been almost infinitesimal. The accuracy of the test is further shown by the great regularity of the increase of the mean elongation and of the decrease of the coefficient of elasticity. The total error of any figure in the column of mean extension is not greater than 0.0002 inch.

Graphic Representation of Results.—The plate on the opposite page is a graphic representation of the results of tests of specimens of cast iron, wrought



STRAIN DIAGRAMS

A. *Bolt Steel, 0.35 Carbon.* B. *Wrought Iron.* C. *Cast Iron.* D. *same as C.*

A. B. and C. *actual elongation magnified 10 times.* D. *actual elongation magnified 100 times.*

iron and steel. The curves, or *strain diagrams* are made by "plotting" the figures of stress and elongation recorded during the test. The test of cast iron represented in the plate is the same as that recorded in the table above. Each curve is a complete record of all the properties of the material which can be

EXTENSION IN FIVE INCHES.

Load.	Right hand.	Left hand.	Mean.	Co-efficient of elasticity
lbs. per sq. in.	inch.	inch.	inch.	
500	0.0007	-0.0005	0.0001	25,000,000
1,000	0.0010	-0.0006	0.0002	25,000,000
1,400	0.0012	-0.0005	0.0003	23,333,333
2,000	0.0014	-0.0003	0.0006	16,666,667
2,500	0.0015	0.0000	0.0008	15,625,000
3,000	0.0017	0.0003	0.0010	15,000,000
4,000	0.0026	0.0000	0.0013	15,384,615
5,000	0.0024	0.0011	0.0018	13,888,889
6,000	0.0027	0.0016	0.0022	13,636,364
7,000	0.0032	0.0019	0.0026	13,076,923
8,000	0.0035	0.0028	0.0032	12,500,000
9,000	0.0039	0.0034	0.0037	12,162,162
10,000	0.0044	0.0038	0.0041	12,195,119
11,000	0.0050	0.0044	0.0047	11,702,128
12,000	0.0053	0.0055	0.0054	11,250,000
13,000	0.0057	0.0061	0.0059	11,016,949
14,000	0.0066	0.0066	0.0066	10,606,061
15,000	0.0074	0.0076	0.0075	10,000,000
16,000	0.0083	0.0086	0.0085	9,411,706
17,000	0.0090	0.0093	0.0092	9,239,130
18,000	0.0097	0.0104	0.0101	8,910,891
19,000	0.0108	0 0116	0.0112	8,482,143
20,000	0.0120	0.0130	0.0125	8,000,000
21,000	0.0134	0.0145	0.0140	7,500,000
22,000	0.0152	0.0168	0.0160	6,875,000
23,000	0.0171	0.0197	0.0184	6,140,218
23,285	broke.			

determined by test. The ordinates, or perpendicular distances of any point of each curve from the base line, represents the applied stress per square inch; and the abscissa, or horizontal distance from the line OO,′ represents the corresponding extension. That point of each curve at which it first bends towards the right hand indicates the elastic limit. The inclination of the initial portion of the curve to the vertical, OO′, measures the stiffness within the elastic limit, or co-efficient of elasticity. This method of representing results is now used by all scientific experimenters on strength of materials.

Elastic Limit and Coefficient of Elasticity.—These terms may here be defined. The elastic limit is that point at which the extensions cease to be proportional to the stresses, and begin to increase in a greater ratio than the extensions. In the diagrams, as stated above, it is the point at which the diagrams begin to curve away from the initial straight line. It will be seen from the diagrams that wrought iron and steel give a well defined elastic limit, while there is no elastic limit shown in the cast iron test, the elongations varying in a more rapidly ratio than the stresses, from the beginning of the test. The elastic limit is sometimes defined as the point at which the first "permanent set" takes place; the permanent set being the extension which remains after the load causing the extension has been removed. Within the elastic limit, according to this definition, a material that is extended by a load will, when the load is removed, return entirely to its original length; and beyond this limit it will only partly return, the amount of permanent increase of length being the set. This definition is not now considered, by the best authorities, as good as the first, as it is found that with some materials a set occurs with any load, no matter how small, and that with others a set which might be called permanent vanishes with lapse of time, and as it is impossible to get the point of first set without removing the whole load after each increase of load, which is frequently inconvenient. The elastic limit defined, however, as the point at which the extensions begin to increase at a higher ratio than the applied stresses, usually corresponds very nearly with the point of first measurable permanent set.

The co-efficient (or *modulus*) of elasticity is a term expressing the relation between the amount of extension or compression of a material and the load producing that extension or compression.

It may be defined as the load per unit of section divided by the extension per unit of length; or, the reciprocal of the fraction expressing the elongation in one inch of length, divided by the pounds per square inch of section producing that elongation.

Let P be the applied load, K the sectional area of the piece, L the length of the part extended, l the amount of the extension, and E the coefficient of elasticity. Then

$\frac{P}{K}$ = the load on a unit of section.

$\frac{l}{L}$ = the elongation of a unit of length.

$$E=\frac{P}{K}\div\frac{l}{L}=\frac{PL}{Kl}.$$

The coefficient of elasticity is sometimes defined as the figure expressing the load

which would be necessary to elongate a piece of one square inch section to double its original length, provided the piece would not break, and the ratio of extension to the force producing it remained constant. This definition follows from the formula above given, thus: If $K=$ one square inch, L and l each = one inch, then $E=P$.

In the diagrams the coefficient of elasticity within the elastic limit, is indicated by the inclination of the initial portions of the diagram from the vertical, the diagram whose initial portion deviates least from the vertical having the highest coefficient.

Strength Per Square Inch of Fractured Section.—The strength per square inch of fractured section, as determined by dividing the recorded breaking weight by the area measured after fracture, is stated by some writers to be a correct measure of the valuable properties of a material; thus of two samples of iron which show the same strength per square inch of original section, the one which shows the greater strength per square inch of fractured section is the more valuable. In many cases an iron which has a very high tensile strength per square inch of original section is an unsafe iron to use in construction, and one which has a much lower strength is safer, if the latter have the higher strength per square inch of fractured section. Kirkaldy states that for comparing the qualities of iron the breaking weight per square inch of the fractured area should be taken, and *not* the breaking weight per square inch of original section.

This method of comparing the qualities of materials, however, is not as good as that of comparing the strength per square inch of original section *together with the extension before breaking*, and with the uniformity of extension or of reduction of area in different portions of the length of the specimens, for several reasons:

1. The breaking strain per square inch of fractured area in ductile materials is frequently uncertain, as the piece may diminish in section very rapidly in the last few seconds before final rupture, and during those few seconds its resistance decreases rapidly, but apparently gradually, from the maximum recorded resistance of the specimen to perhaps less than half of the maximum. The writer has frequently found that the resistance of a piece recorded an instant before rupture was much less than half the recorded maximum resistance, and so rapidly did this resistance decrease that its final amount could not be measured. In such cases it was impossible to state what really was the resistance per square inch of fractured section. Commander L. A. Beardslee, U. S. N., in his tests at the Washington Navy Yard has found similar results.

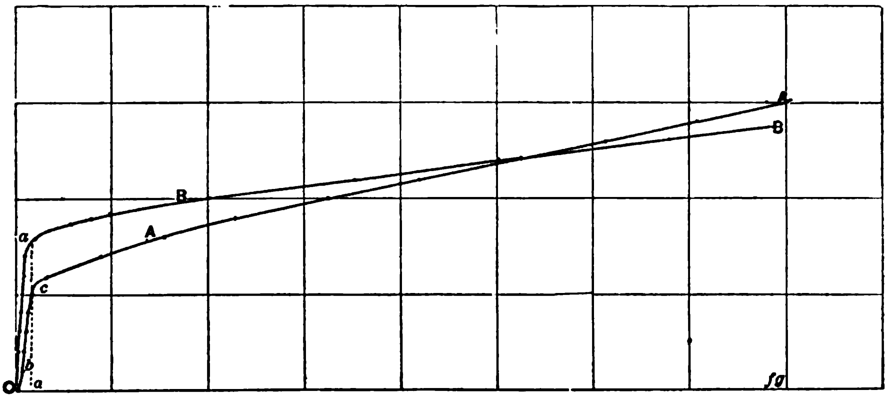
2. In ductile materials the fracture is frequently so much distorted that its area cannot be ascertained with any approach to accuracy.

3. Of two metals of the same class the one which has the greater tensile strength per sqare inch of fractured section *may not* be the better metal. For instance, of two pieces of a certain class of metal, say, soft steel, iron, or brass, of one square inch original section, suppose one has a breaking weight of 60,000 pounds, and the other of 50,000 pounds. The first elongates 20 per cent. and its fractured area is $\frac{3}{4}$ of a square inch. The second elongates 30 per cent., and has a fractured area of $\frac{1}{2}$ square inch. The first would have a strength per square inch of fractured section of 80,000 pounds, and the second of 100,000 pounds, yet it is easily seen that the first is the better metal. It is also seen by this example that the product of tensile strength per square inch of original section, multiplied by the extension, is not necessarily a measure of quality; for in the first case the product is 1,200,000 and in the second 1,500,000; yet the first may be superior, since any elongation greater than 20 per cent. is generally valueless.

4. In tests of metals, or of any materials, having no greater ductility than cast iron, the reduction of section is too small to be conveniently measured, while it is quite easy to measure its elongation. The ductility of such materials therefore cannot be compared by measuring their fractured area, while it can be compared by measuring their elongation with proper apparatus.

True Method of Comparing Qualities of Materials.—The only convenient method of comparing with accuracy all the

qualities of materials, so far as these qualities can be learned from tensile tests, is that of plotting the results of the tests and comparing the "strain diagrams," or the graphic method as above described. Such a comparison may best be illustrated by the two ideal strain diagrams in the cut below. The dots represent the observations made in each test. The specimens are supposed to be of the same shape and size and to be tested in exactly the same manner.



The following record of the two specimens here represented might be given by the operator of the testing machine:

	A	B
1 Tensile strength per square inch of original section....	60,000	54,000
2 Tensile strength per square inch of fractured section..	90,000	80,000
3 Elongation, per cent.........	25	24
4 Elastic limit, lbs. per sq. inch.	22,000	30,000

In a rough test items 1, 2 and 3 only would be given (by some item 1 only) and from these items any one would suppose that A was much the better material. Item 4, if given, might correct this opinion, but it might only lead to doubt as to the accuracy of the test.

An inspection of the curves, however, shows that B is a much better material than A. 1. In B the initial part of the diagram is a straight line to *a*, showing perfect homogeneity and freedom from internal strain; in A the initial line has a bend at *b* showing either want of homogeneity or the presence of internal strain. (This may, however, indicate only an inaccuracy in the record of the first part of the test.) 2. The inclination of the initial portion of B is much less than that of A, showing a higher coefficient of elasticity, usually a valuable property in constructive material. 3. The elastic limit of B is much higher than that of A. 4. The area of the triangle *O a d*, representing the *amount of work* done in extending the piece B within the elastic limit, or its "elastic resilience," is much greater than that of the corresponding triangle *O c d* of the piece A. The piece B, therefore, would be capable of enduring a much greater shock without permanent distortion than the piece A. 5. The total area included between the diagram of B and the base line *O a B B f* is greater than the corresponding area *O c A A g* of the piece A, showing that B has the greater total resilience, or requires a greater amount of *work* to be done on it to produce rupture. Here, then, are five independent points of superiority of the piece B over the piece A, while the bare record of items 1, 2, and 3, given above, would indicate the reverse.

The graphic method of recording results is now adopted by all scientific experimenters. The simplicity and accuracy of the method should commend its use also in ordinary tests for commercial purposes.

With a view to encourage improvements in agriculture in France it is announced that during the year 1879, one gold medal of 1,000f., one silver ditto of 700f., two ditto of 600f., two ditto of 500f., and one of 300f., will be awarded to such landowners or farmers in the Hautes or Basses Alpes as shall have utilized in the most intelligent manner the waters of the irrigating canals.

A PRACTICAL THEORY OF VOUSSOIR ARCHES. PART II.

By WM. CAIN, C.E., Carolina Military Institute, Charlotte, N. C.

Contributed to Van Nostrand's Magazine.

I.

1. The following is a continuation of a paper entitled "A Practical Theory of Voussoir Arches," which appeared in this Magazine for October and November, 1874, and afterwards reprinted as No. 12 of *Van Nostrand's Science Series.* In that treatise, the principles affecting the stability of arches, upon the hypothesis of incompressible voussoirs, were exposed and applied to the investigation of numerous experiments upon wooden arches, *at the limit of stability*, with which they were found to agree. A segmental stone viaduct was likewise examined; the theory for incompressible voussoirs being modified *empirically* for the elastic materials used in construction. It was mentioned in the former treatise, that we shall hereafter designate as Part I., that if the effects of the elasticity, causing the deformation of the compressible arch were known, that the investigation of its stability for a statical load could be effected.

The attempt is made, in what follows, to throw some light upon this effect of the compressibility of the voussoirs; and the empiricism, before mentioned, will be criticized in the light of the deductions, as well as from a further consideration of the experiments themselves.

Afterwards, the precise part played by the spandrels will be pointed out and certain theories concerning them discussed. The subject of the theory of arches will then be extended to *underground arches*, *groined* and *cloistered arches*, and *domes;* and practical examples will be given, worked out in detail, to illustrate the investigation—as far as it can be made—of the stability and strength of such structures.

EFFECT OF THE ELASTICITY OF THE MATERIALS.

2. We shall introduce the present subject with some comments on the fourth experiment given in Part I. Fig. 1 represents one-half of a wooden gothic arch and pier of fourteen inches span; the depth of voussoirs being two inches, the horizontal width of pier, 1.9; its height, ten; and the uniform thickness of arch and pier, 3.65 inches. The contour curves of each half arch are described from the opposite springing points. The voussoirs were constructed of equal weight, the pier weighing 2.3 voussoirs. The inner edge of top of pier coincides with the intrados at the springing. With no weight on the crown the arch and pier stood, but fell with a slight jarring, such as a person walking across the room. Now, as the crown joint and joint 5 opened on the intradosal, and joint 3 on the extradosal side, even when the arch stood; the voussoirs bearing at the very edges opposite the opening; it is evident that the arch, at the moment of rupture, was slightly deformed; *i. e.*, did not have exactly the figure above.

If that deformation had been noted and the figure drawn to correspond, then the resultants of the pressures on joints 1, 3 and 5 would have passed almost through the very edges; but assuming that the arch at the instant before rupture had the figure above, we find, if the horizontal thrust Q at the crown acts 0″.1 below the summit, that with the value of Q as given, the resultant pressure at joint 3 passes 0″.1 from the intrados, and at joint 5, 0″.2 from the extrados.

To pass a curve of pressure through the points noted on joints 0 and 3, lay off on the direction of Q prolonged, the distances $\overline{c1}$, $\overline{c2}$,... to the verticals through the centres of gravity of the loads from the crown resting on joints 1, 2,... respectively. In this case, as detailed in Part I., the distances $\overline{c1}$, $\overline{c2}$,... are respectively 1.7, 3.19, 4.39, 5.26 and 6.24 inches; the weights resting on joints 1, 2, 3, 4 and 5 being respectively, 1, 2, 3, 4 and 6.3 voussoirs.

These weights are laid off in order on the force line $\overline{05}$ on the right, and a line drawn from 3 parallel to the line 33 in the figure of the arch, cutting off, Q=0.63 voussoirs by scale. Then if from the

points 1, 2, 3 . . . on Q prolonged, we draw lines parallel to the oblique lines through 1, 2, 3, . . . of the force diagram, to the corresponding joints, we find the *centres of pressure* on the joints, the broken line connecting which is called the *line of pressures.* The magnitude and direction of the resultants are given by the oblique lines of the force diagram.

3. Now, if the material of which the arch is composed was absolutely incompressible, *i. e.*, able to sustain any finite effort on a mathematical line, then the centers of pressure at joints 0 and 3 would have passed through the very edges, and the arch would have balanced on a higher pier. We propose to show that this deformation of the arch was due entirely to the compressibility of the material. Again, it is a most important problem to ascertain the distribution of the molecular stresses on any joint, having given their resultant there in position, direction and magnitude. This latter subject is not restricted in its applications to arches alone, but applies to any plane joint or supposed section in a solid on which the resultant of all the external forces acting on the structure can be found; as on any joint of a chimney, retaining wall, abutment, arch, etc.

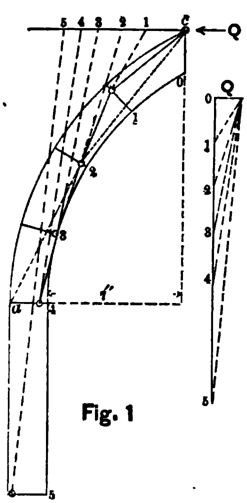


Fig. 1

Referring to Fig. 1, we see that the resultants on the joints are not generally perpendicular to them: resolve them into components I. and II. to the joints to which they correspond. The latter, or "shearing force," is resisted by the friction of the joint, resisting sliding, it is only the former component that we shall consider.

4. Conceive a slice DDA, Figs. 2 and 3, of width unity; the cross-section BD, perpendicular to the plane of the paper, being a *rectangle.*

Also suppose the resultant R of all outward forces on the cross-section BD, to lie in a plane parallel to the paper, through the center of the slice and perpendicular to the supposed cross-section. Let us suppose *first* that the cross-section can develop both *tensile and compressive* resistances as in a solid beam, and that R passes through *a*, *without* the cross-section.

If at some point, A in the medial plane, we conceive two opposed forces +R, −R, parallel and equal to R; the force R with the force −R forms a right-handed couple $\overline{RR}$, that can be replaced by the equal couple $\overline{pp}$ or the forces, *t*, *t′* . . . . *c′*, *c*, equal and opposed to the uniformly increasing tensile *resistances* from E to B and the compressive *resistances* from E to D, E lying in the center of gravity of the cross-section.

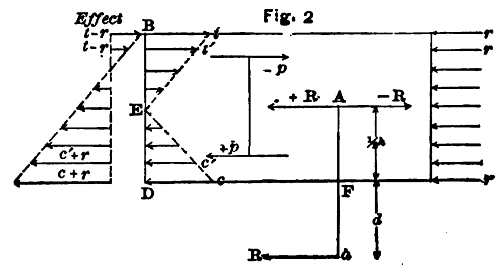


Fig. 2

5. In Fig. 3 A lies anywhere, in the medial plane, *within* the cross-section (a further limit will presently be indicated); in Fig. 2, A lies in the center of gravity of the cross-section, so that +R passes through E.

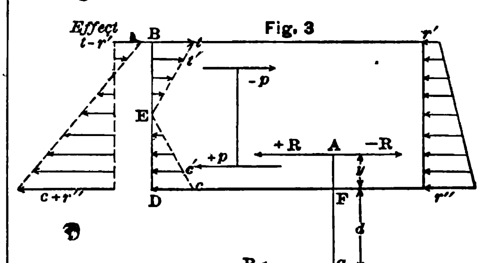


Fig. 3

The force +R at A, Fig. 2, is decom-

posed into forces, r, r . . . ., *supposed* uniformly distributed. In Fig. 3, $+$R at A is *supposed* decomposed into r', ..., r'', straight lines, limiting the arrows representing the forces.

Since the final "effect" is to give forces $t-r,'$ . . . , $c+r,''$ (Fig. 3) limited by *straight* lines, the result must be the same as in Fig. 2, since the forces $t-r$, . . . $c+r$, as limited by straight lines, can have but one disposition in order that R at *a* may be their resultant.

This would not be so, if some curved line limited the ordinates $r,'$ . . . . r''; which decomposition is thus incorrect if we assume, as is usual in the flexure of beams, that the forces exerted by the fibres and their consequent compressions or extensions, *within the limits of elasticity*, are directly as their distance from the neutral axis, or point of no strain, shown in the "effect" diagram of both figures.

6. Prop.—*If the joint* BD *is a plane joint;* i.e., *can offer compressive, but no tensile resistances, then if* R *falls inside of the joint, anywhere between* A *and* F *Fig.* 2, *it is decomposed into compressive forces only, which decrease regularly from the edge* D, *nearest* R, *towards the other edge, and thus, are proportional to the ordinates of a trapezoid or triangle.*

This is easily proved, if we assume, that *when* one edge of a plane joint is more compressed than the other, the actual shortening of the fibres and hence, (within the limits of elasticity) the forces acting on them, must be directly as their distance from the neutral axis.

There are only three suppositions:

1st. Suppose an *equal* shortening of the fibres on the whole joint. The stresses per square unit are thus the same throughout the whole extent of the joint; but then R, lying on one side of the center, cannot be their resultant. *One edge then must be compressed more than the other.*

2d. But the actual shortening of the fibres cannot be greatest at the edge farthest from R, for then, by hypothesis the stresses must regularly increase in going towards the edge farthest from R; but then R cannot be their resultant. This second supposition is then false.

3d. These two suppositions proving incorrect, the third as stated in the proposition is correct.

R can be, and is, the resultant of the forces distributed according to the law of the trapezoid; the most compressed edge lying nearest R.

7. When R lies so near the edge, that the limit of elasticity is passed in the case of some of the fibers, then although it looks probable that the actual shorten ing of the fibers is directly as their distance from the neutral axis, yet the corresponding resistances are no longer proportional to the compressions, for those fibers whose limit of elasticity is passed; hence R is no longer decomposed according to the ordinates of a trapezoid, except on a portion of the joint, the resistances being less than by this law as we approach the most compressed edge, for the balance of the joint.

8. Poncelet asserts that the "law of the decomposition of molecular forces at the exterior surface of a solid body" has not been solved hitherto. If we restrain ourselves to practical cases, such as that of one arch stone pressing upon another, or any single block pressing upon another block throughout its whole extent, the above *is* a solution; for in practice we should not allow the resultant to approach so near the edge that the limit of elasticity of any of the fibers is passed; and within this limit the solution is founded upon the same hypothesis as that used in discussing the laws of flexure.

It would seem as useless as impracticable to solve a problem, such as presented by Poncelet, of finding, the distribution of the pressure due to a heavy elastic prism, resting on a non-deformable horizontal plane; or as suggested by another author, of considering the stresses caused by an isolated weight resting on a common pedestal, &c., &c.

9. Where one block rests upon more than one, the decomposition becomes indeterminate. Thus, suppose at the foundation course of a pier, abutment, retaining wall, &c., the resultant passed outside of the middle third. We shall see presently, that if the course resting on the foundation is of one block, then the joint will open at the edge farthest from the resultant.

Where the stones are not cemented together firmly, it is doubtful if the resultant is decomposed according to the law of the trapezoid, where the courses are formed of many stones. The middle third is recommended as a good practical limit however. Still it must not be thought that it is a cure for all evils. Whenever the resultant on any course does not coincide with the center of fig-

ure, there will be settling on the side towards it; so that no pier etc., can be regarded as non-deformable; and the amount of this yielding to allow is simply a practical question; a slight opening of the joints, if not seen, being of itself of no matter, unless the pressures are thereby increased too much for safety, or water is permitted to enter, or some practical objection, other than want of required stability, is experienced.

10. Refering again to Fig. 2, and calling f the strain t or c, we know from the theory of flexure that

Moment of R at a, about $A=R(d+\frac{1}{2}h)$
$$=\tfrac{1}{6}fh^2,$$

whence,

$$f=\frac{6R(d+\frac{1}{2}h)}{h^2}=t=c \qquad (1)$$

The uniform compression $r=\dfrac{R}{h}$

$$\therefore (t-r)=\frac{R}{h^2}(6d+2h), \ldots\ldots \qquad (2)$$

$$c+r=\frac{R}{h^2}(6d+4h)\ldots\ldots\ldots \qquad (3)$$

11. Supposing tensile resistances at the joint, these formulæ give correct results for the solid beam; and likewise for a plane joint, when R is so near A, that $t-r$, $t'-r$, . . . , may all act as compressive forces; since, by this decomposition, the law of the trapezoid previously established, art. 6, holds; R being the resultant of the regularly increasing stresses.

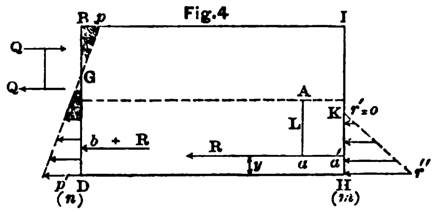
12. Now let $t-r=0$. From eq. 2, $d=-\frac{1}{3}h$ or $aA=\frac{1}{6}h$. As long as aA is less than $\frac{1}{6}h$, there are only compressive resistances on the joint; but if the resultant leaves "the middle third" of the joint BD, then $(t-r)$ is positive and we *assume tensile resistances* at B to oppose the forces.

13. *But* suppose the joint BD can offer no tensile resistances, then it is wrong to decompose R as before, if by another method of decomposition stability is assumed. Many writers seemed to have overlooked the method of decomposition proved in art. 6.

Thus, R at a, Fig. 4, is the resultant in position on the joint BD. We may decompose R according to the ordinates of a triangle as at (m), if we assume the law of the trapezoid as before, r' is now 0, hence $y=\dfrac{k}{3}$, where $k=KH$=height of trapezoid or triangle.

Placing

$$\overline{Aa}=L,\ HK=h=3\ \overline{Ha'}=3(\tfrac{1}{2}h-L) \quad . \ (4)$$



14. Let us now compare this result with the case shown at (n), where the beam is supposed to supply tensile resistances along BG. Call the resistances per square unit at B and D, p and p' respectively; *i.e.*, $p=t-r$ and $p'=c+r$.

From similar triangles, calling DG=x,

$$p' : x :: p : h-x \ \therefore\ x=\frac{p'h}{p+p'}$$

Now, replacing $(d+\frac{1}{2}h)$ in eq. (1) by L; deducing p and p' as in (2) and (3), and substituting, we have,

$$x=DG=\frac{\frac{R}{h}\left(\frac{6L}{h}+1\right)h}{\frac{R}{h}\ 2\frac{6L}{h}}=\frac{h^2}{12L}+\frac{h}{2} \quad . \ (5)$$

Now from eqs. (4) and (5)

For L=$\frac{1}{6}h$,	HK= h,	DG= h,
$\frac{1}{3}h$,	$\frac{1}{2}h$,	$\frac{3}{4}h$,
$\frac{5}{12}h$,	$\frac{1}{4}h$,	$\frac{7}{10}h$,
$\frac{1}{2}h$,	0,	$\frac{2}{3}h$,

That is HC<DG, when L lies between $\frac{1}{6}h$ to $\frac{1}{2}h$; or the point of no strain in the beam lies nearer the edge DH, where the joint can oppose no tensile resistances than when we suppose them exerted.

15. Let us now compare the strains p and r''. Representing R by the area of the triangle whose base is KH and altitude r'', we have,

$$R=\frac{kr''}{2}=\frac{3yr''}{2}$$

$$\therefore\ r''=\frac{2}{3}\frac{R}{y} \quad . \quad . \quad . \quad (6)$$

Again moment of R at a about $A=R(\frac{1}{2}h-y)$, &c. See art. 10. Hence,

$$p'=(c+r)=\frac{R}{h^2}(4h-6y) \quad . \quad . \quad (7)$$

Thus for any values of y between 0 and $\frac{1}{3}h$ (see table) we find that, $r''>p'$, always :

y	r''	p'
$\frac{1}{12}h$	8	$3\frac{1}{3}$
$\frac{1}{6}h$	4	3
$\frac{1}{4}h$	$\frac{8}{3}$	$\frac{5}{2}$
$\frac{1}{3}h$	2	2
	$\frac{R}{h}$	$\frac{R}{h}$

There is therefore greater compression at edge DH when the real forces are as at (m) than when the beam can oppose tensile resistances as at (n). Also since KH $<$ DG, within the same limit ($y=o$ and $\frac{1}{3}h$) the compression is more uniformly distributed at (n) than at (m); hence the beam will bend more when, as in the voussoir arch, a joint BD can only oppose compressive resistances, than when, as in a solid beam, tensile resistances can be exerted. As a consequence of this compression at lower edge, there being none (according to our hypothesis—the law of the trapezoid) at K in case (m), the joint above K must open, and could in fact be removed without interfering with the distribution of the forces at all. . . . Some writers have erroneously asserted that the opening was due to tensile forces. It may be observed, that if, as in the voussoir arch, these tensile forces are unbalanced rupture must ensue.

16. *The decomposition of* R *cannot be the same for an open joint as for a solid beam, when* L *lies between $\frac{1}{6}h$ and $\frac{1}{2}h$;* for if, for a solid beam, we assume the disposition (m), the edge nearest R is most compressed, and the joint above K would tend to open; but as there are tensile resistances there that prevent it, the disposition (m) is not correct for the solid beam; hence some other disposition as (n) is correct.

17. It is evident, from the reasoning in art. 6 that, *in a voussoir arch, whenever the resultant on a joint does not pass through its center, that the edge nearest the resultant is most compressed, and the arch is consequently deformed.*

If some external force, as a spandrel thrust, keeps the line of pressures in the middle of the arch ring, then there will be no deformation of the arch, save that due to the uniform compression at each joint.

Conversely, if the arch retains its shape, save that due to a uniform compression at each joint, the line of pressures must coincide with the center line of the arch ring; for otherwise there would necessarily be deformation of the arch, which is not supposed.

18. It is evident, that as the resultant pressures on the joints are farther removed from the center, the deformation is greater; increasing gradually until, when the line of pressures leaves the middle third of the arch ring, the joint just begins to open, this opening increasing, [see Fig. 4 (m)] until when the resultant is very near the edge, the strain r' on the most compressed fiber exceeds the ultimate strength of the material and crushing ensues, followed or accompanied by rotation. For very light arches this crushing may not be perceptible. Thus, in the experiment given Fig. 1, there was deformation of the arch, before rotating began, due to the compressibility of the material, joints 0, 1 and 5 compressing most at their outer, joints 2, 3 and 4 at their inner edges. The resultants at joints 0, 3 and 5, obeying the law of nature's economy of force, pass nearly through the very edges; so that $k=\overline{\mathrm{KH}}=3y$ (Fig. 4, m) is very small, and as the face of one voussoir can be supposed to rotate about K, the opening at I must be very appreciable, as it was at joints 0, 3 and 5 in the actual experiment.

19. It is likewise evident that the greater the compressibility of the material, the greater the deformation of the arch; hence, with weak materials, rotation will occur sooner, *i.e.*, with less loads or lower piers, than for less compressible materials. This principle is proved experimentally by exps. 13 and 14 of part 1, with cloth joints.

The above theory now perfectly explains, that apparent anamoly in the experiments, of the resultant on the base of a pier, made up of several bricks, approaching the center as the height of the pier was increased; the arch and pier being at the limit of stability. The true explanation is, that it is due to a compression of the edges, causing a deformation of the arch; so that if the exact figure of the arch at the instant of rotation could be obtained, we should find that the line of pressures approaches closely the very edges at the joints of rupture.

In fact this is necessarily so, for as these edges alone bore, just before rotating, the line of pressures must pass through them. The same remarks apply to all the experiments. It was not attempted to find the figure of the deformed arch just before rotating, for the figure in most cases was not constant and hence impossible to obtain. In truth a very limited time scarcely permitted of the experiments that were performed.

20. It is well to note that so long as R

remains within the *inner third* of the arch ring, that compressive forces act on the *whole extent* of the joint, and thus there will be no opening. Eq. (3) will give the unit strain on the fiber at the most compressed edge, noting that d is now minus. When $d=-\frac{1}{6}h$, $(c+r)=\frac{2R}{h}$, or double the compression $\frac{R}{h}$, if the resultant were uniformly distributed on the joint.

Hence at those joints where the resultant cuts the joint $\frac{1}{3}$ the depth of joint, from either edge the strain induced in the most compressed fiber, is double that due to a uniform distribution of the resultant on the joint.

21. *It is clear likewise that inversely if in any voussoir arch the joints are all closed, that the line of pressures keeps somewhere within the middle third;* for if it did not, then there would be compression, uniformly increasing, as at Fig. 4 (m) over *only a part* of the joint; so that while the fiber at K is unaltered, those fibers on the part KH are compressed, which necessitates the opening of the joint above K; but this is against the supposition of closed joints, hence the actual line of pressures keeps within the middle third. Eq. (6) gives the value, $r''=\frac{2}{3}\frac{R}{y}$, of the unit strain on the edge nearest R. When R lies in the *outer* third of the arch ring, the mortar being supposed to offer no appreciable tensile resistance.

Thus if R passes $\frac{1}{4}$ depth joint from an edge, $r^4=\frac{2}{3}\frac{4R}{h}=\frac{8}{3}\frac{R}{h}$, being slightly over double the unit strain $\frac{R}{h}$, if R was uniformly distributed.

22. For arches of medium spans, it can matter little, so far as strength and stability are concerned, whether, by the aid of a spandrel thrust or other device, or from inherent strength, the line of pressures is restrained to the middle third or to limits, $\frac{1}{4}$ depth joint from edges, or other limits. Thus in the stone viaduct of 50 feet span, given in Part I, the horizontal thrust at the crown, in a slice 1 foot wide, is nearly 25 tons, the depth of voussoirs being 2.5 feet. Now if the compression were uniform, the strain per square foot would be 10 tons.

If the resultant at the crown passed, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ depth joint from extrados or intrados, the strains per square foot at the most compressed edge would be 20, 26.7, and 33.3 tons respectively. Since the crushing weight of granite varies from 400 to 800, of limestone from 250 to 600, and of sandstone from 200 to 300 tons per square foot, it is evident there can be no danger of crushing in these cases for such materials. The crushing weight of best brick in cement, Trautwine gives as 50 to 70 tons per square foot; of brick alone, at from 50 to 300.

If the material, in the most compressed edge, is not to be subjected to more than $\frac{1}{8}$th of its breaking weight, then brick should not be used of the proportions above.

23. If an iron band is placed around that part of the extrados, where the joints would otherwise open, it may entirely or partially prevent this opening. The arch then ceases to be strictly a voussoir arch, so that this device will not be further noticed.

24. We have been careful to expose this law of the decomposition of the resultant on a cross section in detail, since so many writers have fallen into error on this point: some applying the principles affecting a solid beam to the open joint, and thus discovering a veritable case of rotation when the line of pressures passes outside the "middle third" of the arch ring, due to supposed tensile forces; and others again rejecting the law of the trapezoid entirely, and falling back upon the theory of incompressible voussoirs.

It is needless to give all the various theories, even in outline, that have been proposed by so many able writers. Suffice it to say, that it is probably agreed, that the true solution of the arch is intimately connected with the law of its compressibility; or more plainly, the law of its deformation due to its elasticity.

The aim is, therefore, whilst not attempting a thorough solution, to endeavor to present clearly some important points bearing on this subject.

25. It is assumed as proved, that if, *in any completed arch, no joints open, that the actual line of pressures keeps within the middle third of the arch ring.* In fact, it is well to design the arch ring so as to satisfy this condition.

It does not follow, by any means, that if this actual line is found outside the middle third, that the arch will fail, as Rankine's eq. asserts. We have just seen that both strength and stability may be satisfied when this line approaches the edges at certain joints quite closely, so that other limits might, in many cases, with safety be instituted.

But it seems advisable, entirely from a practical point of view, not to have any open joints; it gives an appearance of insecurity and besides may leave too small a margin for shocks.

This principle does not apply to culverts, buried out of sight and never sustaining any shocks or much variation of pressure; hence it is extravagance to give them the same depth of keystone as a stone bridge. It is for the same reason proper to increase the depth of arch stones for a railroad arch bridge over the sizes usually employed in road bridges, as the live loads are heavier and move faster. For tunnel arches the judgment of the engineer must be largely exercised in allowing for the different thrusts of the various materials found in tunneling; and here it is better to be on the safe side and use a deep arch ring to allow for variations in thrust, especially if the soil is treacherous.

26. Let us recur again to the gothic arch, Fig. 1; which, as remarked, spread outwards about the haunches when set up on the solid piers. If with the hands the tops of the piers are moved inwards until the span is just 14 inches, the joints are all closed as stated in Part I, p. 50. The line of pressures is, therefore, confined to the middle third. If the top of the pier is pushed still farther in, the crown joint opens at the top, leaving at the bottom edge; and this is so, whether the other joints that may open are wedged up just to close or not.

Now there are some intermediate positions for the top of the pier when the horizontal thrust at the crown acts at different points along the joint: for as the top of pier is moved gradually inwards from its limiting position the thrust at the crown must travel *gradually*, down the joint, until it reaches the lower edge, when, of course, the lower edge alone bears.

It cannot jump at once from its highest to its lowest position.

This granted, a very important proposition is established: *that by cutting or fitting the arch stones in a certain manner that the line of pressures at the crown may be made to pass through any desired point of the crown joint.* Thus, in this case, with a span less than 14 inches, the thrust at the crown may be confined to any point of the crown joint between the top of the middle third limit and the bottom of the joint; and the voussoirs may be so cut that no other joint opens, even if the crown joint opens.

The same remarks apply to any other joint. A line of pressures may thus be made to pass at will through any point at the crown joint, and through some point below the previous one, generally on any other joint. Thus the arch may be pivoted at the crown and at the haunches, as see exps. 3 and 10 of Part I; or the line of pressures may be compelled to take the same position by properly chipping away part of certain joints; so that the true line of pressures is, after all, dependent on the mason. If he so cuts and lays the stones that on completion the bridge shows no open joints, as is the rule, then the line of pressure is somewhere within the inner third.

27. Next, suppose the piers removed and that the arch stands upon a firm support, the joints being closed as stated: where is the actual line of pressures? The line as first drawn corresponds to the principle of least resistance; but this cannot be the true one, *since, for no other reason, the consequent rotation about the upper edge of the crown joint and the lower edge of joints* 3 *or* 2, *would necessitate a rotation about the extradosal edge of joint* 4. But this last cannot occur, unless the line of pressures passes nearly through *a*, the outer edge of joint 4; involving a new curve of pressures, as shown by the dotted lines, *cra*, similar to the *actual* one drawn in Fig. 11 of Part I, referring to the first experiment, in which the joints opened as just described. In fact when the crown is lowered, the haunches must spread, and consequently the springing joint be most compressed on the extrados side. Now, in practice, this spreading always occurs at the haunches, so that the line of pressures there is below the centre line, whilst at the top and springing it is above it or outside of it.

If no joints open, as was the case, the line must keep within the inner third besides. Now, if the law of this deformation of the arch was known, the curve could be located; as it is, we can only approximate to its true position, by noticing the manner in which arches settle, or fail as just shown. Now, it was proved by numerous experiments in Part I, *that when an arch failed by rotation, in every case the line of pressures corresponded to both the maximum and the minimum of the thrust.*

This is illustrated by figures 11, 12, 13, 23, and 24, of Part I, which figures refer to a few of the experiments made on the wooden arches. Now, it will probably be admitted that, with voussoirs that fit perfectly before decentering, and with incompressible abutments, that the deformation of the arch due to its compressibility, is in the same direction as that when the arch is at the limit of stability from a similar kind of loading; *i. e.*, the curve of pressures crosses the centre line the same number of times, und near the same places. Then it seems highly probable, under the conditions assumed, that *the actual line of pressures*, in such an arch, *is confined within such limiting curves, approximately equidistant from the center line of the arch ring, that only one curve of pressures can be drawn therein, corresponding, therefore, to the maximum and minimum of the thrust in the limits taken.*

Assuming this to be true, let us critisise the constructions given for the segmental bridge in Part I. Thus, in Figs. 9 and 10, the line of pressures is nearer the centre line than drawn. The curve for the limit of stability is represented in Fig. 13, when the weight is at the crown; and by Fig. 24, for a single weight on one haunch.

Now, according to the principles exposed above, a line of pressures for the bridge, Fig. 21, loaded eccentrically, should be drawn through the lower middle third limit, at joint 6, on the loaded side, the upper limit, at joint 6, on the unloaded side, and a point at the crown jotnt slightly lower than before. On a large scale drawing (3 feet to the inch), assuming the thrust at the crown joint to act 1.1 ft above the intrados, we find that the line of pressures passes above the middle third limits at joint 2, under the load 0.2 ft.—its max. departure—just touching the lower limit at joints 1 and 2, on the unloaded side. If preferred, the direction and amount of the thrust at the crown to pass through the given points on joints, 6, may be found by successive trials in place of computing them. Thus, assume the direction of the thrust: then find its amount to pass through the point on one joint 6, and use this amount in finding the center of pressure on the other springing joint, which should coincide with the point taken on that joint, otherwise, try again. Three trials only sufficed in this case.

From the above, we see that the depth of arch ring should be increased 0.2, in order that a line of pressure, giving the max. and min. thrust in the limits, may everywhere keep within the middle third. The recommendation, though, is repeated to increase the depth of the arch ring 0.5 foot, to allow for the influences mentioned in the next article.

If the object, therefore, is simply to investigate the stability of a proposed structure (which is indeed the real object of our investigation), it is evident that, *if any line of pressures can be inscribed within limits, so that no crushing occurs, that the arch is stable against rotation for statical loads.* For with less horizontal thrusts than that taken, the curve departs more and more from that which corresponds to the ultimate maximum and minimum, without which the arch cannot fail by rotation. As explained above, it is well to confine this line of pressures to the *middle third* of the arch ring, so that no joints open. If the line drawn corresponds to the minimum, but not to the maximum of the thrust within the limits taken, it is not the actual line of pressures (if the abutments are fit in, etc.), since this is probably contained within still narrower limits. In fact there will be an excess of stability in this case. In arts. 55, *et seq.*, the characteristics of the maximum and minimum thrusts will both be investigated in the most general manner.

Remark.—It should be observed that *if*, in a voussoir arch, there are no mortar joints, and the stonef are cut so perfectly that the compression is the same next the joints, as in the body of the stones, then *when* the pressure line keeps within the inner third, the conditions are exactly similar to the case of the solid or rigid arch "fixed at the ends." For the graphical treatment of this case, the reader is referred to Professor Greene's articles on this subject in *Engineering News* for 1877, in connection with Bell's article in VAN NOSTRANDS' MAGAZINE, Vol. 8; also to Professor Eddy's "New Constructions in Graphical Statics." The analytical treatment is given in full in Du Bois' "Graphical Statics." On testing the curve of pressures corresponding to the max. and min. of the thrust, if the segmental bridge with the eccentric load examined in Part I, by the three conditions, $\Sigma M=o, \Sigma My=o$, and $\Sigma Mx=o$, the center line of arch ring having been divided into 16 equal parts, it is found that the curve of pressures should be raised slightly, to satisfy these conditions, which are simply, that

the tangents at the springing are fixed, the span is invariable, and the vertical displacement of one springing above the other, equals zero. It seems, however, useless to enter into this refinement for actual bridges with mortar joints, rough beds, etc.; therefore, it is not mentioned further.

28. The abutments or piers have previously been considered as unyielding. If their tops lean outward, from a yielding of the foundation on the outer side where the greatest pressure is ordinarily thrown, or if the span lengthens from the compression caused, we see from the remarks on Fig. 1, that the actual line of pressures corresponds to a less horizontal thrust, so that it approaches the intrados at the springing, and, of course, retreats farther within the abutment at its base. If the abutment continues to yield, it approaches still more the contour curves at the crown and haunches, so that a sufficient yielding may cause the line of pressures to approach very nearly the very edges at those joints.

If the abutment is narrow, and especially if each course is built of a single block, like the voussoirs, for the thickness taken, it becomes really a continuation of the arch, so that all the principles of art. 27 apply; the base of the abutment becoming the actual springing joint.

Now, in practice, the abutments do yield somewhat; so that in most arches the actual line of pressures corresponds to a less thrust than as suggested in the previous article. In view of the fact that the arch-stones may not fit perfectly (for openings of the joints do occur sometimes, slate rock being driven in them), in addition to the foregoing, it seems impossible to say exactly where the true line of pressures is to be found. In most bridges the joints do not open, so that in them, it is confined to the middle third.

To be on the side of safety in designing piers or abutments, the maximum thrust in the limits may be used. If, however, the top of the pier leans, the thrust forcing it over becomes less, whilst the opposing thrust of the arch on the other side increases. When there are a series of arches resting on piers, the resultant on them due to dead load is vertical if the arches are all alike, or have the same horizontal thrust. In all cases, the pier should be investigated when one span is loaded and the adjacent one unloaded, exactly as shown in Part I., Fig. 22.

When the piers are very high, a series of arches are often placed below the first, which must, therefore, supply sufficient horizontal thrust to keep the line of pressures within the middle third of the horizontal joints of the piers from the lower arches to the base. The part of the piers between the series of arches are to be examined as before. Some magnificent structures have thus been built with tiers of arches. Instance of modern construction, the Morlaix Viaduct; also, the bridge over the Seine at the Pont du Jour, Paris. Often the piers of the upper series rest upon the arches beneath, which are generally of greater span than the upper series.

29. In addition to the influences exerted on the line of pressures by the cutting of the stones, a similar one producing similar effects is *Temperature.* Stoney says, "With increased temperature the crown rises and joints in the parapets open over the crown, while others over the springing close up. The reverse takes place in cold weather; the crown descends, joints over the springing open and those over the crown close. When stone or iron arches are of large span, these movements, from changes of temperature, will generally dislocate to a certain degree the flagging and pavement of the roadway above. This is very conspicuous in Southwark Bridge."

These combined influences, like the shocks due to rolling loads, can only be guarded against by empirically increasing the depth of the arch ring over that due to the statical loads, as recommended in art. 27.

Practical Conclusions.—In view of the combined influences of misfits, shocks, temperature, and yielding of abutments the true curve of pressures may be found in wider limits than those in which it has the characteristics of the max. and min. of the thrust in the arch ring.

If we design an arch ring then, in which a line of pressures corresponding to the max. and min. of the thrust can *just* be drawn in the middle third, some joints may still open from some of the influences mentioned above; therefore it is recommended to use somewhat narrower

limits than the middle third. Or better, require that a line of pressures may always be inscribed in the inner third, and increase the depth of arch ring empirically to allow for the other influences; since they are different for each bridge; dependent as they are upon the thrust, height of pier or abutment, kind of foundation, accuracy with which the stones are cut and other considerations detailed above.

INFLUENCE OF THE SPANDRELS.

30. The part played by the spandrels has not previously been enquired into. When the centers of an arch are struck, it is found, particularly for full center arches, that the crown descends and the haunches spread outwards. This spreading is *resisted* by the spandrels, which if solidly built, exert horizontal forces sufficient to prevent much lateral motion of the arch and thus help to keep the joints closed. If there is no tendency to spread in the arch, the spandrel exerts no horizontal resistance; it being simply a wall resting on a rigid arch ring, which thus cannot differ in its action from any other fixed foundation of the same shape. As generally built, with vertical joints next the arch, and even with inclined joints there, in most cases, there is no tendency to *slide*, which alone can cause an *active* horizontal thrust.

One author, M. Y. Von Villarceau, assumes that the spandrels exert an *active* horizontal thrust, proportioned, like liquids, to the depth below the surface! He thus obtains, after hundreds of pages of intricate calculations, "*the hydrostatic arch*"—one of the worst forms of arch a constructor could well choose. What a regret that such industry could not have been devoted to a more practical end.

Rankine's theory will be noted further on.

If in any of the experiments given in Part I, especially with the gothic arch where the spreading was very appreciable, a sufficient horizontal thrust was exerted by the hands or spandrel walls against the extrados, the arches, even under much greater loads would have been stable. Similarly in a stone bridge.

If the spandrel thrust, at different points was given, the investigation of the stability of the arch ring would be as simple as that for a culvert or tunnel arch given further on.

But it seems impossible to estimate it. If each spandrel course was composed of one stone, and the yielding of these stones noted, then by the law, "*ut tensio sic vis*," we could deduce the horizontal resistance offered by each course. But if each course was made up of several stones, the forces corresponding to the compressions of the mortar joints would be too uncertain to rely upon, and thus the horizontal resistances could not be computed.

31. As, however, arches, such as the semi-circular, semi-elliptical, etc., that require a very effective spandrel thrust to maintain stability, are often built, it is necessary that the engineer comprehends all the reactions experienced in such arches, although he may not be able to precisely estimate them in tons, etc.

To this end, let us consider the semi-circular arch, Fig. 5, of 100 feet span, and 3 feet depth of keystone; the solid spandrel extending to 3 feet above the top of the keystone.

Divide up the spandrel by vertical lines, 5′ apart for 40′ from the crown, than 2′ apart for the next, 10′ and 1′ apart for the remaining 3 feet. The joints 1, 2, 3, ... are then drawn as in the figure.

Our object is first to ascertain the weight from the crown resting on any joint, and the position of the centre of gravity of this weight. The line of pressures, disregarding the spandrel thrust is then drawn as explained for Fig. 1.

In the following table the first column indicates the joint. In the next four columns, the upper numbers, opposite any joint number, refer to the trapezoidal figures; the lower numbers to the voussoirs on which the trapezoids rest:

(*See Table on following page.*)

We assume that the surface s of a *trapezoid* is equal to its horizontal width, $w \times$ mean height v; the latter being measured approximately from the top of the spandrel to the extrados along the medial vertical line.

For the *voussoirs*, surface$=s=w\times v=$ length measured along center line$\times$ depth (3′ in this case), see Part I., p. 78. Column c gives the horizontal distances from the crown to the medial vertical of the trapezoid (assumed to pass through its center of gravity), and to the

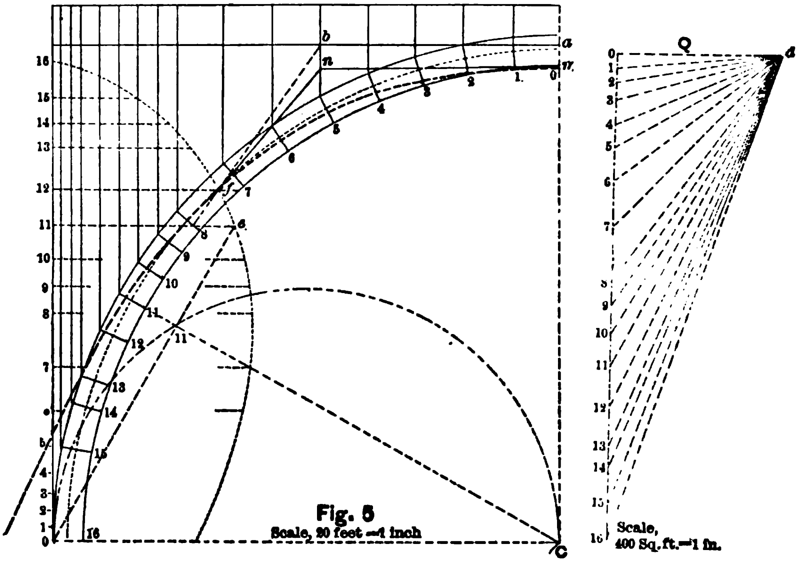


Fig. 5
Scale, 20 feet = 1 inch

Scale, 400 Sq. ft. = 1 in.

Joint.	w	v	s	c	m	S	M	C
	5.	3.1	15.5	2.5	39.			
1	4.9	3.	14.7	2.5	37.	30.2	76	2.5
	5.	3.5	17.5	7.5	131.			
2	4.9	3.	14.7	7.4	109.	62.4	316	5.1
	5.	4.5	22.5	12.5	281.			
3	5.	3.	15.	12.3	184.	99.9	781	7.8
	5.	6.	30.	17.5	525.			
4	5.2	3.	15.6	17.1	267.	145.5	1573	10.8
	5.	8.	40.	22.5	900.			
5	5.3	3.	15.9	22.	350.	201.4	2823	14.
	5.	10.7	53.5	27.5	1471.			
6	5.75	3.	17.2	26.8	461.	272.1	4755	17.4
	5.	14.2	71.	32.5	2307.			
7	6.2	3.	18.6	31.7	589.	361.7	7651	21.1
	5.	18.6	93.	37 5	3488.			
8	6.87	3.	20.6	36.6	754.	475.3	11893	25.
	2.	22.3	44.6	41.	1829.			
9	3.1	3.	9.3	39.9	371.	529.2	14093	26.6
	2.	25.	50.	43.	2150.			
10	3.3	3.	9.9	41.9	415.	589.1	16658	28.3
	2.	27.9	55.8	45.	2511.			
11	3.7	3.	11.1	43.8	486.	656.	19655	30.
	2.	31.4	62.8	47.	2952.			
12	4.3	3.	12.9	45.8	591.	731.7	23198	31.7
	2.	35.9	71 8	49.	3518.			
13	5.13	3.	15.4	47.7	735.	818.9	27451	33.5
	1.	39.9	39.9	50.5	2015.			
14	3.23	3.	9.7	49.1	476.	868.5	29942	34.5
	1.	43.5	43.5	51.5	2240.			
15	4.2	3.	12.6	50.2	632.	924.6	32814	35.5
	1.	49.	49.	52.5	2572.			
16	10.	3.	30.	51.3	1539.	1003.6	36925	36.8
			1003.6		36925			

center of gravity of the voussoir respectively. The product, $s.\ c = m =$ moment of s about the crown. Column s now gives the area of the trapezoids and voussoirs from the crown to any particular joint, and is formed by the successive

addition of the numbers in column s. Similarly M is formed from m. The quotient $\frac{M}{S}=C=$horizontal distance from the crown to the center of gravity of the surface S, corresponding to the same joint. Thus, for joint 8, the sum of the moments of each trapezoid and voussoir from the crown to joint $8=M=11893$; the sum of their area$=S=475.3$; hence by mechanics $C=\frac{M}{S}=25$. If we take a slice of the arch of the width unity, then S will represent the corresponding volumes,and is proportional to their weights.

The whole surface of the arch and spandrel = rectangle − quadrant $=56\times 53-\pi\frac{2500}{4}=1004.5$, differing 0.9 square foot from the value found approximately in the table.

32. Let us now pass a curve of pressures through a point a, 1′ below the extrados at the crown, and a point 1′ from the intrados along joint 8.

To do this we lay off on the horizontal through the upper point, $\overline{ab}=C=25'$ to the left of the crown. From the point b so determined, draw a straight line to the lower point at joint 8. This line gives the resultant on joint 8 in position and direction. Next, laying off to scale in the force diagram to the right, the surfaces S in order, and drawing through 8 a line ∥ the direction of the resultant just found, it will cut off on a horizontal through o the value $\overline{od}=Q$ of the horizontal thrust at the crown.

Drawing lines through 1, 2, . . . (force diagram) to d so found, we have the directions and magnitudes of the resultant on joints 1, 2, . . . represented by their lengths. To find the center of pressure on any joint as *e. g.*, 5, we lay off C=14 on the horizontal $\overline{ab}$. From the point thus found draw a line ∥ $\overline{ds}$ of force diagram; where it intersects joint 5 is the center of pressure on that joint: for the thrust Q at a, combined with the weight resting on joint 5, acting at its center of gravity must give the resultant on joint 5 in position, magnitude and direction. Similarly for other joints.

The broken line traced through these centers of pressure—not drawn in the figure to avoid confusion—is the "*line of pressures*." In this case it everywhere keeps within the middle third of the arch ring, except at joints 15 and 16. At joint 15 it passes in the arch ring 0.6 feet from the extrados; at joint 16, 1.9 feet outside of the extrados. [If there was no spandrel, the true line of pressures must lie in the arch ring, if possible, and satisfy the conditions of art. 27.]

33. At joint 7, the center of pressure is at the middle of the joint; the line of pressures then goes below the center line of the arch ring, approaches nearest the intrados at joint 8; at joint 13 it again crosses the center line, and keeps above it to the abutment. Joints 8, 9 and 10 are the most compressed on the intrados side. Joint 8 is often called the joint of rupture.

34. Now if this were the true curve of pressures, the effect of the compression, not being uniform on the joints, would be to lower the crown and spread outwards the haunches (see art. 28). So that if the spandrels were solidly built up to joint 7, they would offer horizontal resistances, in addition to their vertically acting weights to partially prevent the deformation of the arch ring.

Let us suppose, for an instant, that the spandrels are absolutely incompressible; the arch, then, cannot change shape where the tendency to spread occurs; therefore, the compression must be uniform on such joints; *whence the actual line of pressures must pass through the centers of those joints.*

This compression along the arch ring shortens it slightly, but we shall neglect this shortening. Our untrue hypothesis conducts us to the following construction, similar to the one given in Rankine's Civil Engineering, article 138: lay off, on the extreme left vertical, the loads from o (Fig. 6) upwards. Now, *assume* that the curve drawn tangent to the resultants on the joints coincides with the "line of pressures" (which hypothesis is sensibly incorrect as we approach the springing). Draw $\overline{oe}$ parallel to the center line at joint 11—the construction shown effects this easily, by drawing $\overline{o,11}$ through the inter-section of $\overline{c,11}$ with the semi-circle. Next draw a horizontal line through 11 on the scale of loads, to the intersection e with

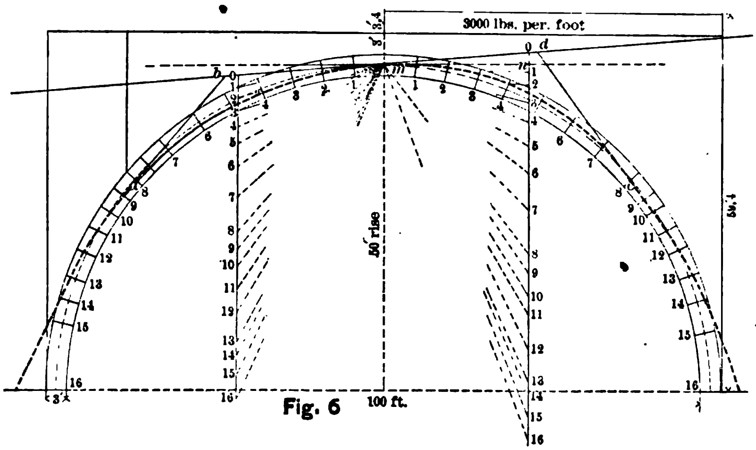


Fig. 6

the line just drawn; then $\overline{oe}$ represents the magnitude and direction of the resultant at joint 11, whose two components $\overline{o,11}$ and $\overline{e,11}$ are respectively, the load from the crown to joint 11, and the *total horizontal thrust exerted below* joint 11.

On repeating this construction for each joint, we find the horizontal thrust exerted below each joint; the horizontal thrust, then exerted upon a single voussoir, as that between joints 11 and 12, by the spandrels, is thus the difference between the line $e, \overline{11}$ and the horizontal $f, \overline{12}$. At joint 8, the horizontal thrust obtains its maximum; and above this point, in this case, the spandrel would have to exert tensile forces to cause the center line to become the true line of pressures; but as it cannot do this, the horizontal thrust from joint 8 to the crown is constant.

35. If an accurate construction should be desired, to ascertain the horizontal force supplied by the spandrel at the extrados of each voussoir, in order that the center line may be the curve of pressures, from the joint where spreading first occurs to the abutment, we may proceed as follows: assume the thrust on that joint to pass through its center; then having assumed the position at the crown of the horizontal thrust, we find, as in art. 32, the magnitude and direction of the resultant on the joint considered. Combine this resultant with the weight of next voussoir and load, acting through their common center of gravity, and the resultant so found with such a horizontal force, acting through the middle, approximately of the extrados of the voussoir, as to cause the final resultant to pass through the center of the next joint. This construction may be continued to the abutment.

If a force polygon is drawn to one side, the amounts of the horizontal thrusts supplied to each voussoir by the spandrel becomes evident.

36. Continuing the construction of art. 35, we draw a tangent to the center line at joint 8, to intersection n with the vertical through the center of gravity of the load from the crown to joint 8, and from this point draw $\overline{mn}$ horizontal, to intersection m with the crown joint. From m the center of pressure on the crown joint, the curve of pressures to joint 8 is drawn as before explained in art. 32. It is shown by the dotted line through m which point is 0.2 feet below the crown. The curve is continued below joint 8, supposing *no* spandrel thrust, and cuts the springing joint 4.5 feet to the left of the extrados.

On the supposition of incompressible spandrels however, the true curve is that drawn through m to joint 8; then it follows the center line. Rankine indeed, asserts that a linear arch parallel to the intrados and drawn within the middle third is the true curve, whence n and m may be slightly raisedor lowered. (See Rankine's Civ. Eng., art. 285.)

This cannot be if the arch is to perfectly preserve its figure (art. 34). Again, for compressible spandrels, the line of pressures about joint 8 *must* lie below the center line, *never* above it (art. 28).

37. But is this the true curve? Decidedly not. If there were no spandrels, and the abutments yielded sufficiently, the curve would be somewhat as drawn in art. 32, tending to flatten the arch from the crown to joint 7; then to render it more convex; and below this

point causing the greatest compression to occur at the extrados.

Now, *theoretically*, with spandrels built in the usual manner, when the centers are struck, the tendency of the arch to spread at the haunches *causes* the compression of the spandrels, which thus, in partially resisting this spreading, put forth horizontal forces. The true line of pressures then between joints 7 and 13, about, must pass *below* the center line; below joint 13 approximately, it keeps to the left of the center line. The latter follows from the fact that any spreading at the haunches is accompanied with a diminished compression of the intrados at joint 16, (see art. 27).

Again, the spandrel thrust must be greatest where the spreading is greatest; whence from joint 8 the spandrel thrust necessarily diminishes down to the springing or near it, where it ceases. No spandrel thrust is experienced above joint 8 about, if the spandrel moves as one mass.

In practice, the spandrel is not solidly built above joint 8, except on the faces of the arch, so that on that account, the thrust above joint 8 will be small if any.

38. *Practically*, the first solution above conducts to this absurdity; the arch should be unstable because m lies below the arch ring, but the depth of keystone was obtained from a comparison of examples in actual practice, so that no engineer would believe that a full center arch of 100 feet span and 3 feet depth of keystone should be unstable. Thus, from Rankine's formula, founded entirely on practice, the depth of keystone should be, for a single arch $\sqrt{.12 \times r}=2.45$ feet, or for an arch of a series $\sqrt{.17 \times r}=2.9$ feet; r being the radius at the crown, 50 feet in this case.

Again, *in practice*, the crown falls on decentering; hence it seems probable that the true line of pressures there is *above* the center line, not below it.

39. *Now if the joints keep closed, the actual line of pressures probably passes above the center line at the crown; between the center line and the lower middle third limit at joint* 8 *about; from this point it again approaches the center line, crossing it about joint* 13 *and passing near the outer middle third limit at the springing joint, keeping throughout within the middle third of the arch ring. The spandrel then, from the point of greatest spreading to, or near, the abutment, must exert the least horizontal resistances that will effect this object. Some idea of their magnitudes could be gained from the construction of art.* 35, *if we knew two points, say at the crown and joint* 8, *through which to pass the curve of pressures, assuming its position below joint* 8.

If this be true, then the thrusts exerted by the spandrels are evidently much less than as given by Rankine's construction, and the spandrel in a semi-circular arch does not sustain the whole of the horizontal thrust. Rankine's theory is the only one, except Y. Von Villerceaus, that has yet been proposed to evaluate the spandrels' influence.

40. If the top of the backing of an arch is sloped downwards from the arch, it may not be capable, near the top, of exerting much horizontal thrust.

If we *knew* the total spandrel thrust down to a certain horizontal joint, the weight of the spandrel above this joint multiplied by the coefficient of friction of stone on stone is the force that resists the sliding tendency; so that the height of spandrel, on this supposition, is easily computed.

As Rankine's construction gives an excess of spandrel thrust, it may be used to evaluate the least height of backing, both loose and solid, to be used.

41. Whatever doubt may exist as to the precise *measure* of the forces exerted by the spandrel, its important action in preventing deformation of the arch ring from a theoretical stand point is rendered plain by the above discussion; and the locus of the true curve of pressures is more precisely ascertained than hitherto; which was the object to be accomplished in the present instance. It is usual with constructors to strike the centers, after the keystone is driven in and the backing carried up such a distance above the "joint of rupture" (as joint 8 is often called) that the arch ring will be stable when the supports are removed. This would seem to be a matter to be determined from practice; though a curve of pressures can be used in approximately testing the stability of the unfinished arch.

It may be observed that, if the resultant at joint 8 maintains its position and

direction approximately, that the center of pressure at the crown for the completed arch will be lower than for the unfinished arch, *if* the vertical through b moves to the left as the arch is completed; otherwise the reverse happens.

42. *Width of Piers and Abutments.*—For full security, in these arches where the spandrels exert a marked influence, the horizontal thrust at the crown may be taken as acting at the lower middle third limit, whence the center of pressure at the base of an abutment is determined exactly as in art. 2, Fig. 1, (also, see art. 28). The right half of the arch being supposed removed and Q being applied at the crown to produce the same effect; its combination with the weight of the semi-arch and abutment must give the resultant acting on the base of the latter, irrespective of internal actions, such as the real distribution of the spandrel, thrust, &c. For the piers, suppose again, for safety, that Q acts at the lower middle third limit at the crown; then find the resultants acting at the level of the springing, due to the arches on both sides of a pier; on combining them with the weight of pier, acting at its center of gravity, the center of pressure at the base can be found. It is a good practical rule to limit this center of pressure in both abutments and piers to the middle third of the base (see art. 9); and to cause it to approach the center as the foundation becomes more insecure.

43. Let us next suppose the full center, Fig. 6, loaded with cars weighing 3000 pounds per lineal foot from the crown to the right abutment. If this load bears upon a width of six feet, it is equivalent to a layer of stone of same density as the bridge (150 lbs. to the cubic foot), 3.4 feet high, as represented in the figure.

The following table for the right half of the arch is made out exactly as explained in art. 31, only the voussoir numbers, for each joint, are placed above the corresponding spandrel numbers. Now let us pass a curve of pressures $\frac{1}{12}$th depth of arch ring below the center line

	s	c	m	S	M	C
1	14.7 32.5	2.5	37 81	47.2	118	2.5
2	14.7 34 5	7.5	109 259	96.4	486	5.
3	15. 39.5	12.5	184 494	150.9	1164	7.7
4	15.6 47.	17.5	267 822	213.5	2253	10.5
5	15.9 57.	22.5	350 1283	286.4	3886	13.6
6	17.2 70.5	27.5	461 1939	374.1	6286	16.8
7	18.6 88.	32.5	589 2860	480.7	9735	20.2
8	20.6 110.	37.5	754 4125	611.3	14614	23.9
9	9.3 51.4	41.	371 2107	672.	17092	25.4
10	9.9 56.8	43.	415 2442	738.7	19949	27.
11	11.1 62.6	45.	486 2817	812.4	23252	28.6
12	12.9 69.6	47.	591 3271	894.9	27114	30.3
13	15.4 78.6	49.	735 3851	988.9	31700	32.
14	9.7 43 3	50.5	476 2187	1041.9	34363	33.
15	12.6 46.9	51.5	632 2415	1101.4	37410	33.9
16	30. 52.4	52.5	1539 2751	1183.8	41700	35.2
	1183.8		41700			

at the crown joint and joints 8, through m, a and c. The thrust at the crown is generally inclined. Let us deduce some general formulæ to enable us to find it.

44. Call P_1 = weight from crown to joint 8 on left,

a_1 = its lever arm about a,

P_2 = weight from crown to joint 8 on right,

a_2 = its lever arm about c,

Q = horizontal component of thrust at m,

b_1 = lever arm about a

b_2 = " " " c

P = vertical component of thrust at m, considered positive when it acts downwards as regards pressure from the right half of the arch upon the left.

g_1 and g_2 are the horizontals from a and c respectively to the vertical through the crown.

Now suppose the right half of the arch removed and its effect replaced by P and Q acting at m; we have taking moments about a.

$$a_1P_1+g_1P=b_1Q. \quad . \quad . \quad . \quad (5)$$

Next, conceive the left half removed, &c., and take moments about c.

$$a_2P_2-g_2P=b_2Q. \quad . \quad . \quad . \quad (6).$$

Eliminating Q, we have

$$P=\frac{a_2b_1P_2-a_1b_2P_1}{g_1b_2+b_1g_2} \quad . \quad . \quad . \quad . \quad (7)$$

From (5)

$$Q=\frac{a_1P_1+g_1P}{b_1} \quad . \quad . \quad . \quad . \quad (8)$$

See more general formulæ in Part I, art. 12, and in art. 63 following.

45. From the tables of arts. 31 and 43, we have $P_1=475.3$, $P_2=611.3$; and their centers of gravity are distant from the crown, respectively, 25. and 23.9 feet.

From the drawing, we have thus, $a_1=13.7$, $a_2=14.8$, $b_1=b_2=17.5$, $g_1=g_2=38.7$, whence,

$$P=\frac{-a_1P_1+a_2P_2}{2g_1}$$

$$=\frac{-13.7\times475.3=14.8\times611.3}{2\times38.7}=32.8$$

$$Q\frac{=a_1P_1+g_1P}{b_1}$$

$$=\frac{13.7\times475.3+38.7\times32.8}{17.5}=444.6$$

From m lay off to the right, horizontally, $Q=444.6=\overline{mn}$; then vertically upwards, $P=32.8=\overline{no}$: $\overline{om}$ represents the resultant at the crown joint. Now, lay off the force lines $o, \ldots 16$ from colums S in the tables; so that $\overline{m1}$, $\overline{m2}$, ... now represent the directions and magnitudes of the resultants on joints 1, 2, ... right and left of the crown. Their positions are found as follows: draw a horizontal through m, and lay off on it, the numbers in column C; the first table referring to the left half of the arch, the last table to the right half.

From the points so found, draw vertical lines to intersection with $\overline{mo}$, produced if necessary; which thus give the points where the inclined thrust at m is to be combined with the weight from the crown to any joint, to find the resultant on that joint; whose intersection with it is thus the center of pressure for that joint.

Thus, P, acts 25′ to left of m: lay off 25′ on the horizontal through m, then drop a vertical to intersection b with mo; then draw $\overline{ba} \parallel m8$ of force line for left of arch, to find a the center of pressure for joint 8. Similarly d and dc are found for joint 8 on the right. These should be the first constructions made to test the values of P and L found, which correspond to the line of pressures passing through a, m and c.

The line of pressures thus drawn passes below the middle third of the arch ring, on the unloaded side, the following amounts in feet: at joints 2, 3, 4, 5 and 6, .3, .4, .3, .2 and .1 respectively; it then crosses the arch ring, passes above the middle third about joint 12, and cuts the springing joint 4.5 feet outside of the arch ring.

On the loaded side it passes above the middle third 0.1 at joints 4 and 5; then across the center line and is just tangent to the lower middle third limit at joint 10, below which it again crosses the arch ring and passes into the abutment, cutting joint 16 about 3 feet outside of the arch ring.

46. On the unloaded side this curve below joint 8 follows very closely the curve drawn in fig. 6; so that if horizontal forces, as large as the ones supposed supplied by the spandrels by Rankine's construction are applied here, the line of pressures will coincide nearly with the center line. On the loaded side the line of pressures touching the lower

middle third limits at joints 9 and 10 would be forced too far in by the supposed horizontal thrusts. As mentioned in art. 39, this spandrel thrust is probably much less than given by the above construction; thus if c is at the center about, and a at the lower limit, the thrust at the crown being raised slightly above the center, the spandrel thrusts, necessary to keep this line of pressures within the middle third say, are nearer the true ones than as given by the previous construction. It seems, at present, impossible to locate exactly the true curve of pressures when spandrel thrusts are exerted.

47. If, however, the abutments are large enough and unyielding, and the spandrels sufficiently strong to resist the thrust, the arch *cannot fail* by the sinking of the crown, which is the usual method of failure of full center arches. Thus Gauthey says (see extract in Haupt's Bridges, p. 126), "let *cvc'*, Fig. 7,

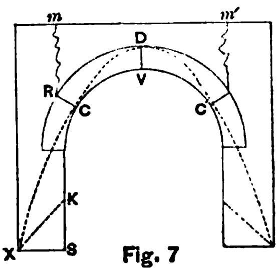


Fig. 7

be the intrados of any arch, whether semicircular, elliptical, gothic or composite. Let D be the crown of the extrados, or back of the arch, which is supposed to be filled up level with the haunches at m and m'. If a weight be placed upon the crown too great for it to bear, it yields, and the arch stones open beneath at the crown, while the extrados is found to open at some point on each side; either at the spring, if it be a flat arc of a circle, or about 30 degrees of a semicircle, or at various other points if it be composed of arcs of circles, tangent to each other, and of various rises, whether $\frac{1}{2}$ or $\frac{1}{3}$ or $\frac{1}{6}$ of the span; and the arch *only falls*" (italics our own) "by pushing aside the abutments at C and C', the opening at R extending itself up to the top at m and m'. It has, moreover, been observed that when the abutment gives way, it leaves a portion of itself standing, viz., XKS, the line XK being at an angle of 45° with the horizon, which only adheres by the strength of the mortar or cement made use of."

This last fact should be observed in designing pillars of any kind. The line of pressures for the kind of rotation just mentioned is shown by the dotted line.

If the arch ring has a very small depth some of the voussoirs may crush; or again, the arch may fail by rotation by the *crown rising* and the haunches falling in, as in the bridge over the Tâfe (see art. 51). Eccentric loads, as we saw in Part 1, cause the haunch under them to fall, and the opposite one to rise. In these cases the rising of the crown—we repeat, the only way in which a bridge can fail with solid, immovable abutments, and spandrels sufficient to resist the thrust against them—tends again to relieve the spandrels of a certain amount of strain, thus causing the curve of pressure to *rise* at the crown, with a less horizontal thrust, and thus partially counteract the tendency towards rotation about the intrados at the crown.

The depth of voussoir, in this case, can only be determined by an empirical rule founded on practice, as given below. A rolling load evidently causes different spandrel resistances from the dead load, though Rankine's construction does not make any difference for the unloaded side; another proof of the incompleteness of his theory.

48. Let us investigate the part *amc* of the bridge as though it was a *segmental bridge*, resting upon fixed abutments at a and c. It is only needful to examine it for an eccentric load (see Part I.) Pass a curve of pressures through the upper middle third limit at joint 8 on the left, the lower limit at joint 8 on the right, and 1.25 ft. above the intrados at the crown joint. We find on a drawing of 3 ft. to the inch, $g_1=39.25$, $g_2=38.48$, $a_1=14.25$, $a_2=14.58$, $b_1=17.16$ and $b_2=17.81$. As before, $P_1=475.3$, $P_2=611.3$: whence, by eq. 7,

$$P=23.8,\ Q=449.1.$$

The line of pressures drawn with these values keeps everywhere within the middle third, barely touching the lower limit at joint 2 on the left, and passing 0.16 ft. inside of upper limit at joint 3 on the right, and corresponding (art. 27)

nearly to the maximum and minimum of the thrust in the limits chosen.

The span of this arch is 75.45 feet, its rise being 17.2 feet, between $\frac{1}{4}$ and $\frac{1}{5}$ of the span. The deformation is small for this segmental bridge, so that the spandrel resistance may be neglected, or rather regarded as simply adding to the stability of the bridge, already safe, unless from the dynamical effects of moving loads. Concentrated loads on one side (as in art. 12, Part I.) should next be tried, in positions that cause the most hurtful effects. This influence, together with the influences of art. 28, may cause an increase of depth of arch ring of half a foot over the three feet.

49. Recurring to the semi-circular arch, Fig. 6, it is evident that for an unsymmetrical load, the spandrel on the opposite side to the load will exercise a thrust for a greater height above the springing than for a uniform load, as compare the line of pressures in the two cases.

It may be asked, why does not the part of the arch below the joint of rupture, a, Fig. 6, act as a part of the abutment simply; so that if the part amc satisfies the conditions of stability, when it is treated as a segmental bridge, why should not the whole bridge be stable?

Rankine's empirical formulae for the depth of keystone, $\sqrt{cr}$, r being the radius of the arch, and c a constant, seems to be founded on such a hypothesis; for by it, the depth of keystone is the same for spans of any length, provided the radius is the same. Thus, if $r=50$ feet, as in Fig. 6, this depth is the same for any span between 100 and 0 feet!

The above query may be answered thus: the actual line of pressures in an arch bridge like Fig. 6, is dependent upon the form of the arch below the points a and c, since the deformation of this part induces the spandrel thrusts (art. 37) which influences the position of the true line of pressures.

In fact, suppose that the line found in Fig. 6, on the supposition of no spandrel thrusts, to be the true one for a segmental bridge amc of span ac: by the reasoning of art. 46, the actual spandrel thrusts exerted below c would force this line out of the middle third, certainly, and most probably out of the arch ring, whereas, the spandrels are supposed to allow no joints to open at least. Trautwine's empirical formulæ for depth of key in feet, d, is,

$$d=\frac{\sqrt{r+\text{half span}}}{4}+.2;$$

and is more agreeable to theory than Rankine's; although for railroad bridges it gives too small values; at least for bridges of 50′ to 75′ span, and rises $\frac{1}{4}$ to $\frac{1}{5}$ span, as for the two segmental bridges examined in art. 48, and in Part I., art. 12. It is evident again, from the reasoning of art. 28, that it would be advisable to increase the depth of arch ring, the smaller the ratio of width to height of abutments; and by the same rule, an arch in a series should have a greater depth of voussoirs, as recognized in Rankine's rule for that case.

50. Having resigned the above arch, that requires a spandrel thrust to keep it from falling, to the domain of empiricism, it may be asked, if by some device of construction, whether it may not be brought within the limits of a strict investigation? Plainly, if the depth of arch stones be increased towards the abutments, so that a line of pressures, with a constant horizontal thrust, can always be inscribed within the middle third, the arch will be stable; and the spreading will be so much diminished that the arch will require but little spandrel thrust to cause stability, so that its influence may be neglected in constructing the line of pressures.

This increase in the depth of arch stones is earnestly recommended; as well as the continuation of the arch ring into the abutment, when, as in segmental bridges, there is some danger of sliding at the springing.

If necessary, the increase in depth of the arch ring may be made up of several stones. They should, of course, break joint with stones above and below them, and be well bonded with the spandrels.

The last pier of the first fixed bridge across the Lim Fjord, an arm of the sea stretching right across the Danish continent from east to west, has, according to recent news, now been completed. The new bridge will connect Aalborg on the south side of the Fjord, with Norresundby on the north, and it is thought that it will be opened for traffic during the autumn.

ELECTRIC LIGHTING.

Compiled from various sources.

When Davy, in 1813, first produced the electric light between the charcoal points of two conducting wires, it was at once proposed to use it for all the various purposes of illumination, simply because in volume of light it excelled all other known sources of light. But it became soon evident that great obstacles presented themselves to the general application of this light. Davy had used for its production a gigantic galvanic battery of 2,000 elements; the lengths of charcoal were quickly consumed in the enormous heat of the arc of light, and had to be moved towards each other, if the light was not to be extinguished, in the same degree as they were consumed; and, finally, the light was concentrated upon a small space, the attempt of dividing it into several lights having also failed. A step in advance was made when, using stronger acids, new elements were constructed which gave more powerful currents; it was possible to produce with from forty to fifty of such elements a beautiful electric light; and the efforts were successful of maintaining, by means of the current itself and the clockwork suitably connected with it, the charcoal points at the distance required for the supply of a continuous light for hours without manual intervention. But notwithstanding the great improvement of the galvanic elements, the cost of the light produced by them was still too great, and the management of the batteries too troublesome, for its general practical application for the purposes of illumination to be thought of. Its use was confined to some scientific purposes, and the occasional production of great effects of light in theatres, &c.

Faraday, in 1831, found a new way of generating electrical currents, by moving magnetic poles towards closed wire coils, or *vice versa*, and since then opened out to physicists and engineers a new, wide field to produce by the suitable construction of magnetic machines powerful electrical currents, and with these the electric light without batteries, consequently in a purely mechanical way. Step by step progress was made in this direction, while on the other hand it was tried to improve the electric lamps, with a view of keeping the charcoal rods at the same distance during combustion. Already in 1850, we find both, the magnetic machine and the electric lamp, improved to such a degree as to provoke serious thought of applying the electric light to lighthouses, for illuminating large spaces of ground for war purposes, &c.; and yet it was not until the end of 1863 that the first electric light was shining in the lighthouse of Cape la Hêve, near Havre.

In the meantime, Dr. W. Siemens, of Berlin, already then well known for his eminent services rendered to telegraph engineering, had invented his induction cylinder, and had employed it both for the working of the telegraph and in the construction of a new magnetic machine, small but powerful; and Wilde (1866) undertook to make use of the Siemens machine in combination with a steam-engine of fifteen horse-power in such a manner as to be able to produce electrical currents and the electric light of a power until then never thought of. But Wilde's machine was very expensive both as regards construction and working, and found but little favor in consequence.*

In the same year, Dr. Siemens discovered the dynamo-electrical principle, or the manner of changing any description of mechanical work without steel magnets into electrical currents. It is well known that there are no different descriptions of forces in nature, but only various forms of one and the same fundamental force, the essence of which is movement, either the movement of larger bodies (mechanical work) or the movement of the smallest particles and the ethereal body of the universe (heat, light, electricity). For that reason it is possible to transform any of these forms of force—for instance, the muscular pow-

* Mr. Wilde has obtained on interim injunction, restraining Messrs. Wells, of Shoreditch, from using the light which they have adopted at their premises, on the ground that it involves an infringement of his patent.

er of living beings, steam-power, water-power—into heat or light or electricity. With this invention of Siemens, galvanic batteries were doomed, and a means offered to engineers of producing electrical currents of a power hitherto unknown, and on proportionately small machines.

Of the various magneto-electric machines that have been brought forward, that of M. Gramme is the most generally used. As soon as his machine became practicable, an English company purchased the English and American patents, the Société d' Encouragement awarded a gold medal to the inventor, and a large number of manufacturers ordered the apparatus. The exhibitor has been awarded prizes at the exhibitions of Lyons, Vienna, Moscow, Linz, and Philadelphia; and now about 500 machines of his construction, with magnets or electro-magnets, have been delivered, and the demand for them is still increasing. Electric lighting, which before M. Gramme's invention did not exist, speaking industrially, is at the present day within the domain of things practical. It is not within our scope to give a detailed description of his machine; it is sufficient for our purpose to state that it furnishes continuous electric currents, the direction and intensity of which may be changed. The electric current having been generated is conducted through an insulated wire to a perpendicular rod of retort carbon, the point of which is placed exactly above the point of a similar rod, the distance between the points being less than a quarter of an inch. This intervening space is the electric arc, and the current passing through that interval from the rod above to that below heats the points to a state of intense incandescence, producing the electric light. The current passing only in one direction—from the upper or positive pole to the lower or negative pole—consumes the positive at double the rate of the negative, and consequently the distance between the points has to be continually readjusted by clockwork and a magnet, constituting the regulator or electric lamp.

Davy made use of rods of wood carbon extinguished in water or mercury. They burned with great brilliancy and very regularly, but they were consumed very rapidly, so as to be useless for practical purposes. Now retort carbon, the deposit collected from the walls of gas retorts, is employed. It is much more dense than wood carbon, and resists for a long time the destructive action of the voltaic focus. Here, also, there is room for improvement, for retort carbon is not all that could be desired, but it gives satisfactory results in most of its appli cations.

The best known apparatus for regulating the consumption of the carbons are those of Archereau, Lacassagne and Thiers, Gaiffe, Foucault and Duboscq, Hefner-Alteneck, and Serrin. The first two are no longer in use.

By the side of the electric lamp with regulating apparatus for the carbon rods as they become consumed, Jablochkoff's candle has also become the material for electric lighting. M. Jablochkoff's light completely suppresses the regulator. His invention was presented to the Academy of Sciences in October, 1876. The carbons, instead of being opposed, are placed side by side, and are separated by an insulating fusible substance. When the current begins to pass, the voltaic arc plays between the ends of the carbons. The layer of insulating matter melts, volatilises, and the double rod of carbon slowly consumes, exactly as the wax of a candle progressively exposes its wick. M. Jablochkoff now burns in his candles, as they are called, powdered asbestos. It seems as if the interesting labors of M. Jablochkoff will have practical result, and that they will increase the domain of electric lighting, for his lamps are now largely used. In Paris, besides numerous larger electric lamps, at the present time, there are burning many Jablochkoff candles, of which we may mention eight on the Place de l' Opera, twenty-four in the Avenue de l' Opera, eight on the Place du Theatre Francais, six at the Palais Bourbon along the front facing the Place de la Concorde, seventy in the Grands Magasins du Louvre, eight in the shops of the Belle Jardiniere, sixty in the Concert de l' Orangerie des Tuileries, thirty-two in the interior of the Hippodrome.

Returning to the Gramme machine, it appears that the first light machine constructed by M. Gramme fed a regulator of 900 Carcel burners; its total weight

was over 2,000 lbs. This machine served for a long time for the experiments on the clock-tower of the Houses of Parliament at Westminster, The fault found with this machine was that it became heated, aad gave sparks between the metallic brushes and the bundle of conductors on which the current was collected. This, however, has not given rise to any serious inconvenience during five years. M. Gramme's next machine was less powerful, of a power of only 500 burners, and consequently of smaller dimensions. When a current is sent into two regulators, each will give 150 Carcel burners. This apparatus has been introduced on board the *Suffren* and the *Richelieu*, of the French navy; on the *Livadia* and the *Peter the Great*, of the Russian Navy; it is employed by several Governments for service in fortified places. This machine is described as excellent, but its luminous intensity is slightly feeble when the atmosphere is foggy; its price, however, is somewhat high. The inventor has improved upon this machine, and constructed one which, when coupled in tension, gives a luminous intensity of 800 Carcel burners at 700 revolutions per minute, and, if coupled in quantity, 2,000 Carcel burners, with 1,350 revolutions per minute. It has been adopted by the French Ministry of War, by the Austrian navy and artillery, by the Norwegian, Turkish, and other Governments. By further simplification, M. Gramme has been able to introduce a machine most suited for industrial purposes, large workshops, and large covered spaces.

As has already been remarked, the electric light may be advantageously employed in a large variety of works, for it admits of obtaining a great quantity of light at a small expense. By its means, the loading and unloading of cargoes, the mounting of machinery, carpentry, weaving, dyeing, and similar trades may be carried on by night just as well as in broad daylight. It is necessary, however, to employ two machines, in order that the light of the one should counteract the shades thrown by the other. It has been found by experience that the naked light may be employed, the workmen themselves having asked for the removal of the opal globes which it was thought at first necessary to usa. The electric light preserving the tints of colors, this property has been utilized with success by several dyers for standardizing their colors by night.

The electric light is most effective for high rooms; when ceilings are of a less height than twelve feet its introduction becomes more difficult. As a rule, there may be conveniently lighted with a single apparatus about 5,000 square feet of fitters' shops, lathe-shops, tool-shops, and modeling-rooms; half that space in spinning-mills, weaving establishments, and printing-rooms; and about 20,000 square feet of yard, court-yard, dock-yard, quay, and open air works.

In a country like the British Isles, where the safety of the mercantile marine and the navy depends so very much upon the amount of security with which ships may enter ports, and the care bestowed upon keeping up an effective system of lighthouses to warn the navigator against approaching dangerous coasts, the electric light would be sure to prove a welcome auxiliary in effecting those objects; and so in reality it has, being now employed at many of the stations. It renders visible at night, at distances varying from 2,000 to 6,000 yards, objects such as buoys, ships, coasts, etc. The electric light was first applied to lighthouses in 1863. In that year trial was made with an Alliance machine at the lighthouse of La Hève, near Havre, the results being so satisfactory that no doubt all lighthouses would have been provided with the new light if the question of expense had not stood in the way. It has been stated that the electric light is seen at least five miles farther than the oil-light, and that in hazy or foggy weather the range of the light is twice as great with the former as with the latter.

In England, official opinion was at first against the introduction of the electric light in lighthouses, on account of the peril of interruption; but this has been overcome. There are now electric lighthouses, besides those of England and France, in Russia, Austria, Sweden, and Egypt. Everywhere their action is pronounced satisfactory. Hitherto machines of only 200 Carcel burners have been tried; but it is stated on good authority, that the French Administration of Lighthouses are about to experi-

ment with a Gramme machine of 2,000 burners. This machine, probably, will greatly enhance the advantages, already recognized, of electricity over oil, and will, perhaps, determine a radical change in the existing illumination of lighthouses.

The lighting of works by night is highly interesting. The Spanish Northern Railway, after trial, used the light as early as 1862 in the works proceeding in the Guadarama mountains. The expense per hour for material consumed was 2.90 francs per lamp; the saving effected upon the use of torches was 60 per cent. The light has also rendered important services in the mines of Guadarama. The air became so vitiated in the workings by the explosions of charges and the combustion of the miners' lamps that the ordinary lamp would not burn. When a Serrin's regulator was sent down, a complete change took place, respiration "becoming as easy as in the open air," the lamps remaining alight. Amongst open-air works may also be cited, as executed by the aid of the electric light, those of Fort Chavagnac at Cherbourg, of the Chemin de Fer du Midi, the reservoirs of Ménilmontant, the building for the *Moniteur Universel*, and, more recently, those of Havre harbor and docks, the Exhibition of 1878 in the Trocadéro, the Avenue de l'Opéra, the Grands Magasins du Louvre, and other establishments.

With respect to the introduction of the electric light on board ships, the experiments carried on board *L'Amérique*, one of the steamers of the Compagnie Générale Transatlantique, were so conclusive that it seemed as if nothing could oppose the immediate adoption of the electric light on all vessels, for it is satisfactorily proved that the chief number of collisions result from the difficulty which captains experience in estimating the exact position of an approaching vessel. Yet it will be only gradually that the electric light will take possession also of the ocean. Quite recently, the electric light has been introduced into the British Navy. The electric lighting apparatus of Messrs. Wilde & Co., of Manchester, which has been tried on board the Alexandria, Temeraire, and other vessels, having proved efficient, the order has been given by the Admiralty to supply similar apparatus to the ironclads Northumberland, Warrior, Repulse, Superb, Nelson, the steel aviso Iris, and the despatch boat Lively. For at least two years the introduction of the Gramme machine has been progressing on board the war vessels of the French, Danish, Russian, and Spanish navies. As far as we know, up to the present time, no disappointment has been experienced, and increasing labor will doubtless improve the first arrangements and lead to still better means.

As far as the application of the electric light to military operations is concerned, several Governments have ordered powerful Gramme machines for the defence of fortified places. An apparatus specially constructed for this purpose has been adopted in France, Russia, and Norway. From trials made at Mont Valérien with the apparatus, and a special projector, it resulted that an observer at the side of the apparatus could see objects 6,500 yards distant, and clearly distinguish details of construction at 5,500 yards. M. Gramme has also designed a machine for war-signaling of very small dimensions that may be turned by hand. The French army possesses at present two of these machines.

Very little attention has hitherto been bestowed upon the great service electricity will ultimately render in lighting up theatres and similar places of places resort. Besides the comparative cheapness of the electric light, its use will do away with the expensive fitting-up necessitated by gas. The great drawback to thorough enjoyment caused by the flare and heat of hundreds of gas-flames will be entirely removed. No longer broiled and heated up to almost fever point, we shall be able to sit in comfort and, more than that, perfect safety against that most awful of all calamities, a fire in a theater, or even a panic such as quite recently occurred at Liverpool. Panics will be avoided; for people will soon come to know that fire from electric light is impossible. We were forcibly reminded of the great danger to which audiences are now exposed during a recent visit, on a Saturday evening, to the Covent Garden promenade concerts. We pictured to ourselves the scene that would ensue if, during one of those

crowded performances, the mass of inflammable material which has been piled up, in addition to what is already stored there—with the evident endeavor to "decorate" the place—were to catch fire; it required but a little fancy to conjure up a picture of an Inferno to which nothing was wanting. With the electric light, on the contrary, we should have, instead of a sweltering, gasping multitude, an audience able to enjoy the musical or dramatic fare set before it. As yet, however, little progress has been made in the employment of the electric light inside theaters.

What is said of theaters applies with even greater force to places of public worship. Let any one who doubts this attend on a sultry evening at such popular churches as the Metropolitan Tabernacle, or at Mr. Newman Hall's church, or even the City Temple, where the ventilation is said to have been recently improved. The heat in all those places is simply unbearable; yet people are expected to worship under such conditions.

For these reasons, it seems to us, that the reform pointed out will not be long before it is introduced, especially as more perfect machines for producing the light are to be had. In Paris, experiments have recently been made for lighting up the interior of the Opera, with a view of preserving the paintings with which the walls of the *foyer* are covered. The general opinion expressed was favorable to its employment.—*The Builder.*

Of the present state of progress in this country, the *Tribune*, of Nov. 16th, says:

The situation in regard to the electric light at this time resembles in a marked manner the state of things when engineers were trying to make a ship go by steam power. For forty years after the close of the Revolutionary War in America, hundreds of ingenious minds were busily at work trying to solve the problem of the application of steam power to propulsion. The United States, England and France were foremost in the study of the matter, as they are at present in the field of electric lighting. It is not extravagant to say that a thousand forms of banks of oars, paddle-wheels, screw propellers, power pumps, and other devices for making a boat go, were invented, cast aside, taken up again and again, re-invented by men who were not aware of what had been already thought of and pondered over, during that period of forty years, until out of the whole vast throng of crude notions the world finally decided upon two forms of propelling apparatus as practical, and put them into general use throughout the field of marine engineering. Exactly the same thing is going on now with reference to electric lighting. Since 1845, when the voltaic arc emerged from the laboratory as a thing which could possibly be put to practical use, the problem of the electric light has enlisted the attention of a large number of able men in Europe and America. Experiments have been making in hundreds of laboratories. Valuable ideas have been occasionally hit upon and patented, and every ten years or so, the announcement has been made that the means for subdividing the electric light has been found, and the attention of the world has been riveted for a while upon some new lamp which has run a brief course of popularity, and then has dropped out of sight never to be heard of again, until some new inventor has come upon the same idea and brought it out afresh, believing it to be new. No doubt, out of this busy investigation will come eventually a practicable method of sub-dividing the electric light for domestic uses, just as success attended the study of paddle-wheel and screw propulsion. Practical modes of electric lighting on a large scale have already been found. In that direction further time seems to be necessary only for perfecting the details. Lighting on a small scale is yet a thing of the future, unless Edison and a few others have attained to it, but it can hardly be doubted that success will eventually be reached.

Rumor asserts that Mr. Edison's plan is the revival and improvement of a very old one; that of rendering platinum wires incandescent on each of the many branches of his divided circuit. His invention it would seem, however, has not yet passed the experimental stage.

Another light which claims attention is, the so called Sawyer-Mann light. It is produced by the incandescence of a tiny perpendicular bar of carbon, an inch long and $\frac{1}{16}$ inch in diameter, which is sustained in place by two large thick bars, arranged one above the other hori-

zontally. The current, in passing from one large carbon bar to the other, through the small one, encounters great resistance. It heats the small bar to whiteness, and produces a light of the most admirable character. It is white, mellow, and pleasing to the eye, and floods a room with a radiance resembling that of daylight. The blaze in the Sawyer lamp can be turned up and down just like a gaslight. In order to prevent the combustion of the carbon, which would be almost instantaneous in the open air, the lamp is enclosed in a sealed glass tube, two inches in diameter and six or eight high, from which the atmospheric air has been expelled, and into which nitrogen or some similar gas has been introduced. Mr. Sawyer refuses to state at present exactly what the composition of this gas is. He claims simply that it will preserve the carbon from combustion for an indefinite length of time. The lamps are very handsome. Five of these lights are now on exhibition at the shop of the inventor. Three of them are operated each by a branch from the main wire. The two others are worked by one branch.

The Rapieff light now undergoing trial in London, is of another kind. The voltaic arc is produced in this lamp between four carbon rods, arranged in pairs, each pair forming the letter V. The apices of the Vs meet in a common center. A regulator is attached, which maintains the carbons at an invariable distance, and results in a light as regular as that of any other similar lamp. One of the carbons of this patent can be removed and replaced without interrupting the current, which is a new feature. Three lights are maintained in one circuit.

The light in the Brush lamp is produced by the voltaic arc between two carbon points, which are kept at a proper distance from each other by a special device of the inventor. When more than one lamp is used in a shop, the light is practically as steady as that of gas. The lamp is hidden in an opalesque globe which softens and diffuses its radiance. The carbons last for fourteen hours. The light costs one-fifth the expense of gas. A dispatch from Cleveland says, that a new Brush machine now maintains seventeen strong lights.

In the Wallace lamp the light is produced between the edges of two carbon plates, which will last one hundred hours. The passage of the current consumes the edges of the two plates at the point where they are the nearest together, and then flies off along the crack that separates them to some other point, always appearing at the place where the plates are the closest together. The flitting of the voltaic are back and forth causes the light to flicker, and seems to exclude the lamp from domestic use. The furnishing of a shop with a number of them results in a partial correction of the evil of flickering. As far as the light which falls upon the work which is going on below is concerned, it seems sufficiently steady for practical use.

The Arnoux & Hochhausen light, and the Weston light, which is on exhibition at the Equitable building, both employ the voltaic arc between carbon points.

In the Lontin lamp a slender rod of carbon is kept in contact with a slowly revolving wheel, touching it on the outer rim; the small rod is kept for about an inch of its length incandescent.

Of the Werdermann lamp we take the following account from English sources.*

This gentleman uses carbon electrodes, with a peculiarity in their use, which is of some importance. A long slender rod of carbon is secured in a vertical position, and is pointed at the upper extremity, which impinges against the under side of a block of carbon made in the form of a thick disc. Thus one electrode is a pointed rod, while the other presents a flat surface. In regard to the vertical rod the upper part only is in the circuit, and the length of this portion can be increased or diminished by shifting the collar which transmits the current. The rod is drawn up so as to press its point against the under surface of the disc by means of a fine cord and a counter weight. As the rod burns or wastes away, it continues to be drawn up so as to remain in contact with the disc. When the current passes, that portion of the carbon rod which is above the connecting collar becomes incandescent, and an infinitesimally small electric arc is also formed at the spot where the point of the rod touches the flat surface above it.

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* *Iron.*

By this arrangement the waste of the upper electrode is exceedingly slow, and is imperceptible unless extending over a period of at least some few days. The rod retains its pointed form during the whole period of its combustion, and may be several feet in length. At a recent trial the current was derived from a small Gramme electro-plating machine, requiring a steam-engine of only 2 horse-power to put it in full work. It may, therefore, be assumed that this was about the limit of the power at work to produce the light. At the commencement of the proceedings two lights were maintained, each stated to be equal to three hundred and twenty sperm candles. At this rate the two lights would be equal to six hundred and forty candles, or forty full power gas lights, each consuming 5 cubic feet of sixteen-candle gas per hour. The two lights burned with extreme steadiness, there being no undulation or flickering whatever, although there was no glass globe to tone down any variations of luster. The lights were perfectly bare and unprotected, and the place where the trial was made was a workshop of moderate size. Later in the evening, one light was exhibited outside the building, in an open thoroughfare, and the same perfect steadiness was observable. After the two lights had been burning for a time they were extinguished, and the current was sent through a row of ten lamps. The light per lamp was of course reduced, but there was the remarkable fact that ten lights were main tained by a comparatively weak machine, driven by an engine with a power of only two horses. The light of each of these ten lamps was stated to be that of forty candles, making, therefore, a total of 400. A reduction of light, consequent on the further division of the current, is thus apparent; but for this loss there may be ample compensation in the economy of a distributed light as compared with one that is concentrated. In the case of the ten lamps, the light is equal to that of twenty-five full power gas lights, consuming altogether one hundred and twenty-five cubic feet of gas per hour. The extremely small arc due to the peculiar arrangement of the carbons in the Werderman light has the advantage of offering the least possible resistance to the passage of the current. Hence the electric power is economized, and it becomes possible to make use of an electric current large in quantity but of low intensity. The tension being small, there is the less difficulty with regard to insulation.

Concerning the transmission of the current, it may be remarked that there are two main wires connected with cross wires which pass through the lamps. If one lamp or more should be accidentally extinguished, the rest will continue to burn. The whole of the lamps can also be extinguished and relit by merely stopping the current and then sending it on again. No nice and troublesome adjustment with reference to the length of the electric arc is requisite. The lower carbon, drawn by the line attached to the weight, travels up through the collar or ring which connects it with the circuit, and simple contact between the point of the rod and the surface of the disc is sufficient for the manifestation of the light. Mr. Werdermann asserts his ability to distribute the current from the small machine so as to divide it among sixty lights. In that case, the light would inevitably be small; but enough is apparent to prove that so far as a current can bear division, Mr. Werdermann will be able to utilize it. In respect to duration, a carbon rod four millimeters in diameter and a yard long, obtained from Paris, costs a franc. This, placed in the large lamp, having an estimated lighting power of 320 candles, will last from twelve to fifteen hours. The smaller lamps take a carbon of three millimeters in diameter.

Of the success from an economical point of view of electric lighting the accounts are somewhat conflicting. The following is the abstract of a report of Mr. Stayton the surveyor for Chelsea, who inspected the Paris system:

The Municipality of Paris have contracted with the general Electricity Company to light up certain streets and places, of which the principal are the Avenue de l'Opera and the Place de l' Opera, a street 900 yards long and 30 yards wide. To do this 46 columns have been set up at an average distance of 38 yards apart. Jablochkoff "candles" are used, and Gramme machines. There are three machines, each supplying sixteen lamps, and each driven by a 16-horse

power steam-engine, set up for the purpose, so that every lamp takes one-horse power. The conducting wires are laid in the subways below the road. The "candles" last an hour and a half, and cost 7½d. each. The number of candles required for the evening are placed in what is termed a "chandelier," within a ground glass globe, a fresh candle being automically switched in as the previous one is consumed. Each light is equivalent to 700 wax candles, but the globes take off one-third of this light. The ordinary London street lamp is equal to 12 or 15 candles.

The contract is at present merely an experimental one. The company undertook to light the lamps for a period of six months, ending in November next, from dusk till shortly after midnight, and to provide the whole of the apparatus, for 1f. 45c. (1s. 2¼d,) per light per hour. Shortly before the electric light is extinguished, about one-third of the gas lamps are lighted, and continue till sunrise, the former light being unnecessarily powerful, and too expensive to be maintained all night.

The Avenue and Place de l'Opera are usually lighted by the large number of 400 gas lamps, set three or five together on columns, with short intervals between them. In spite of the amount of gas burnt, the City Engineer says that "the cost of the electric light is four times that of gas, but a greater amount of light is obtained." On the other hand, in the lighting of the courtyard of the Louvre, it is asserted that a saving of 29⅜ per cent. is effected by replacing 201 gas lamps by 16 electric lights, although 3½ times the amount of light is given.

Besides the place above mentioned, the electric light has also been adopted for lighting the Place du Théâtre Françaíse, the Madeleine, the Arc de Triomphe, the Orangerie des Tuilleries, the Magasins du Louvre, and about thirteen other places in Paris. It is also in operation in the principal places in Brussels, Madrid and St. Petersburg.

Mr. Stayton then proceeds to consider the cost of lighting various parts of Chelsea by means of electricity, as compared with that of gas lighting. To begin with, there is less gas used here than in Paris. The distance between the lamps varies a good deal, for instance, it averages 55 yards in Sloane street, 70 yards in King's road, 35 yards in Lowndes square, 35 yards in Cadogan place, 28 yards on the Chelsea Embankment. In Piccadilly the distance is 30 yards, and in Cromwell road, South Kensington, 27 yards.

To light Sloane street, which is 1,100 yards long and 20 yards wide, with two electric stations, each supplying sixteen lamps, would cost for plant and alterations £3,200, and 16s. per hour for 3,250 hours, or £2,600 per annum. The present cost of a gas lamp in Chelsea burning 3,850 hours per annum is £3 6s. 7d., therefore the expense of the 40 lamps in Sloane street is 8¼d. per hour, the total per annum for the street being £133 3s. 4d. The outlay for lighting the Chelsea Embankment with 48 lights in place of the 109 gas lamps is estimated at £4,800, and the hourly cost at £1 4s. for 3,250 hours per annum. The present cost of the gas lamps is 2s. 1½d. per hour for 3,850 hours per annum. In Sloane street the light would be 31 times as great as at present, and it is believed that half the above number of lights would be sufficient. These, however, could not be worked from a single station, so that the chief cost, that of the machines, &c., could not be thus saved.

The main conclusions Mr. Stayton draws are: That the present arrangements for electric lighting are unsuitable for long distances, especially in London, where the lamps are so much farther than in Paris. The close proximity of the electric stations is a great drawback to the system, and their establishment in business streets would be a matter of considerable difficulty. These are the disadvantages of the system. The following are the advantages:

About 1½ hours' daily consumption is saved in consequence of instantaneous lighting and extinguishing; the light is vastly superior to gas, and is not injurious; there is an absence of noxious smells both in the production and combustion; the heat in a room, so often unbearable in the case of gas, is scarcely felt; the most delicate colors are preserved; air is not consumed as in the case of gas; there is no chance whatever of explosion; and although thè light is so powerful in the streets, no accidents to horses have occurred.

On these grounds, he says that after a careful consideration of the whole question, he is of opinion "that at present the electric light is not suitable for street lighting in the metropolis; that it is suitable and can be utilized with splendid effect in large squares and places, such as Trafalgar square or Parliament square; but although in each of these places at the present time the lamps are numerous, the cost would be greater than gas." He also makes some remarks on the improvements and modifications required before electric lighting can come widely into use, such as the necessity for further subdivision, and for a means of working over greater distance, points which are of course familiar to those who have paid any attention to the subject.—*Journal of the Society of Arts.*

Regarding the matter of efficiency and economy of the dynamo-electric machines, Mr. Sprague,* the electrician, says:—These machines are manifold in name and in appearance, but they all are different modes of obtaining the desired transfer of energy by the same fundamental principles; the whole object of the different inventors is to discover how to arrange the parts so as to do the work at the least first cost and at the smallest working expense. The whole question among them is, *For a given horse-power of engine which machine will produce the largest electric current in a given external resistance?* That question is much too large a one to even enter upon, and it is a good way from being finally settled as yet.

Batteries produce a constant current always in one direction; machines generally produce a constantly breaking and reversing current; but the Gramme machine succeeded in producing a current exactly resembling that of batteries. It is a very debated question which kind of current is best adapted to electric lighting. The positive carbon, which is usually the uppermost, consumes much more rapidly than the lower, and also forms a hollow crater which encloses a large part of the light; the intermittent alternating current generates a resistance in the arc which diminishes the light. However, the Jablochkoff candle, in its most advanced form, requires this alternating current in which the two carbons burn away equally, and at the same time the insulating material is rendered incandescent and contributes to the light; it being, however, not a little doubtful whether this contribution is fairly equivalent to the share of the electric current thus appropriated and taken from the true arc. I merely indicate these points as practical questions by no means solved as yet, but which cannot be discussed here.

As one consequence of this property of the candle, to enable the Gramme machine to be used a second machine has had to be devised, in which the continuous direct current of the Gramme proper is converted into an alternating one. This second machine, however, at the same time raises the power of the current in the same manner as was done in the machines of Wilde, the origin of all the later forms.

The other machine in use in Paris is that of Lontin, which is the one employed also at the Gaiety Theatre in London, and to which public exhibition of the electric light in a supposed practical application is due much of the excitement on the subject. This machine, likewise, is in two separate parts, so arranged as to provide several distinct circuits to work separate lights. Serrin's regulator is used, but M. Lontin claims to have effected some improvements therein. We may now proceed to the applications made of these two systems, and no doubt some of my remarks may excite surprise, because there has been an immense deal of exaggeration and nonsense published by people who, led away by first impressions, and having only partial knowledge of the subject, have, as is usually the case with unskilled observers, seen imperfectly and recorded impressions rather than judgments.

There are at present displayed in Paris a very great number of electric lights, but they may be studied in a very simple set of groups based upon the purposes for which they are used; this also happens to properly distribute the different systems in operation.

1. *Railway Stations.*—At the station of the Western Railway (Gare St. Lazare), Lontin's system is in operation.

Two lamps are placed at the principal entrance; five lamps are placed in the

* Electric Lighting, its State and Progress, by John T. Sprague.

principal hall, where they replace thirty-four gaslights, and will no doubt do so to very great advantage as far as light is concerned, as the hall is so large that the gas has little power over it. They will all be worked by one apparatus, and were to be in action in a few days; when I saw them the engine was not ready.

The goods department is, however, lighted by six naked electric lights, derived from one machine driven by a steam engine. The result is extremely effective, as the men loading and unloading can see the addresses on the packages with ease, and the lights being distributed around the area of operations (which is, however, only a small one), they are able to work easily and safely.

2. *Hotels and Shops.*—In the large buildings opposite the Louvre, which include a great hotel and an immense series of rooms, forming the "Magasins du Louvre," where an endless variety of goods are displayed, there are a large number of electric lights employed which belong to the system of the Jablochkoff candle, as do all the others I shall notice. The courtyard of the hotel was formerly lighted by 14 two-light gas lanterns, and by 24 single globes upon the staircase which forms one side of the roofed-in quadrangle. It is now lighted by 2 electric lanterns on one side, and 6 globes spread over the staircase; 2 other lights in the entrance also assist the effect. In the show-rooms there are a number of electric lights, and also a number of 6-light gas chandeliers, which latter quite equal the electric candle, except as to the inherent difference of the lights as to color, one of the special advantages of the electric light being its possession of all the colors forming perfect light—so that, unlike gas, it shows up clearly the blues and greens, and all the various shades of color; of course, also, the electric light is less heating, does not consume the air, and does not distribute vapors which can condense among the goods.

It will be seen that in this courtyard 52 gas burners, giving, at say 10 candles, a light of 520 standard candles, did formerly the work upon which at least 8 electric lights are now employed, giving, at 300 candles, a light which should be 2,400 candles, and this being nearly five times the quantity of light, the effect ought to be great. In fact, this courtyard is really one of the most beautiful results of the electric lighting that I have seen. I took two friends to see it, and they expressed the same opinion, and thence arose a somewhat curious circumstance. Before leaving Paris they said, "Let us go and have another look at the Hôtel du Louvre." They went, and said as before, "Well, it is really beautiful." "It is beuutiful," repeated one, "but *look again.*" They looked again—the *electric light was not in action; it was the gas light they were admiring.* This shows how large a part imagination plays in many observations.

In this instance I propose to make some remarks here upon the question of cost.

It is stated that here "201 gas burners, costing 90.4 francs per day, have been replaced by 16 electric lights, costing only 63.6 francs, being an economy of 30 per cent., with three-and-a-half times the light." This statement is made by those interested in the light, and I understand that the electric system is worked by an already existing engine, and do not know if a proper charge is included for the steam engine, and for depreciation and interest on the outlay. But *in Paris the gas costs 6s. 9d. per 1,000 feet*—more than double the London price—and its illuminating power is lower than ours—a statement which will surprise those who, seeing the brilliant lighting of Paris, overlook the reckless consumption of gas to which this result is due. The most favorable result, according to English prices, is that, in this case, the actual electric lighting cost 35 per cent. more than the previous gas lighting.

3. *The Streets.*—The Jablochkoff candle is used in many places, chiefly in front of fine public buildings, which they illuminate very effectively, but it will be sufficient to deal with the principal exhibition in the Place and Avenue de l'Opéra, a roadway which extends some 900 yards by 30 wide, and offers from the present aspect one of the most beautiful and interesting sights of Paris.

In front of the Opéra there are two columns, each carrying 3 candles in globes. In the Place are four columns, carrying lanterns, each containing 2 lighted candles. In the Avenue there

are 36 candles irregularly distributed, but averaging nearly 40 yards apart. In the square in front of the Théâtre Français there are two circles of 6 lights each and two single lamps. There are thus in one view 64 electric lights. Besides this there are innumerable gas-lights in the shops, and although by the effects of contrast the gas lights seem to be nearly extinguished by the electric ones, they none the less supply their quota of light.

It is needless to say that, regarded simply as a sight, the effect is very striking; it is not at all surprising that those who look at it with the eye, unguided by the judgment, and compare it with our London streets, exclaim about its perfection, and abuse our London gas. By-and-bye I will count the cost; at present I will only indicate some facts which no one has mentioned as yet, and which will somewhat surprise both the casual observer who goes away exclaiming that such a light must be introduced everywhere, and also those who think only of the vivid character of the electric light itself.

(1) It will naturally be supposed that two electric lights placed only forty yards apart will illuminate *perfectly* the intervening space; on the contrary, a distinct gradation of light approaching to a comparative shade, is observable on the roadway, just as with our gaslights. This is, however, a natural consequence of the fundamental principle of light and of all radiant forces that their quantitative effects diminish in the ratio of the squares of the distances; for this reason, where equally diffused light is required, a number of small lights are necessarily more effective than a much more powerful light concentrated in one or few centers. In this case, with the two lights at forty yards, the combined lights give at the middle point of twenty yards a light somewhat less than half of what they give at the ten yards distance from either post; the exact relative values being as $\frac{1}{2}$ is to $1\frac{1}{16}$, calling the value of each light singly 1 at ten yards distance.

(2) People talk about the gaslights in the streets superseding the shop lights. On the contrary, not one single shop light has been put out, nor has the light the least effect upon the shops. I spoke to one jeweler (besides several other shopkeepers) whose front was directly under a candle, distant only five yards. He still used the ordinary outside lamps, and when I asked if he found any advantage from the electric light, he replied, with the usual French shrug, "Why, sir, it is simply moonlight." This is, in fact, the exact description of the illumination; it is that of a moderately clear moonlight with the eye caught by points of brilliant light, and, so far as the shops are concerned, this light is absolutely ineffective. We often complain that our gaslights are largely wasted upon the heavens; most of the beauty of this Paris light is really due to that cause, the long rows of lofty white houses catch the eye and also reflect part of the light back again, and the air itself is filled with a luminous haze due to the reflections from the innumerable minute particles with which it is always loaded.

(3) The light is variable. Each candle is continuously flashing and frequently changing to a rosy tint. This might not be noticed by a casual observer, being lost in the general effect, but on examining the light by writing directly under a lamp, I estimated the fluctuations as amounting to fully one-fourth of the full light.

(4) The light is rapidly lost in the distance. There is a row of lights in face of the house of the Legislative Assembly which light up the façade very effectively. In the Place de la Concorde these lights stand vividly out from the surrounding gas lamps; at the front of the Madeleine, double the distance, they are only distinguishable from the ordinary street lamps by their superior whiteness of color, the Paris gas being very yellow. This observation will no doubt seem incredible to many; it appeared so remarkable to me that I have worked out some calculations which prove that my sight and judgment told me the strict truth, as will be seen in a subsequent paragraph.

4. *Theatres.*—The Hippodrome, a very large circus, is now lighted with the Jablochkoff candles. There are twenty in globes along the line which divides the audience from the arena, besides three gaslights (or sixty burners turned towards the audience) on the intermediate columns. Above these again are sixteen naked candles with reflectors throwing the light on the middle of the arena.

There are thus thirty-six electric lights in full action, and in addition, there are four naked lights suspended from the roof to throw light upon the Trapeze performances when required. But with all these powerful lights the result was poor compared with the rich radiance we are accustomed to in theatres. As in the streets, it was an effect similar to moonlight, and the fluctuations were very perceptible. I was myself greatly surprised at this, as I was prepared to find the light especially well suited to this purpose, and I can only explain the actual fact by supposing that the eyes are dazzled by the number of cross lights directly influencing them, instead of illuminating only the objects intended to be seen.

II. The Cost of the Light.—As to this, it is nearly impossible to attain any reliable information. We can only accept at a suitable discount the statements furnished by the interested parties.

The Gramme machines, to maintain 16 lights, cost £400 in addition to a steam engine or other motor of at least 20 horse-power. The fittings of the lamp cost £8, besides columns, &c., and the conducting cables 3*s.* 4*d.* per yard of the circuit. These are Paris prices, and would be higher in England. Smaller machines for fewer lights would cost more in proportion.

The cost of working per hour for the carbon candles, firing of engine, and attendance, is put at 8*s.* 8*d.* per machine, or 6½*d.* per light: to this must be added a due charge for repairs, depreciation, and interest upon outlay, so that the hourly cost cannot be put at less than 8*d.* per light. But any calculation on this basis, without taking into consideration the circumstances of each case, would be so uncertain that I shall here confine myself to the one salient fact agreed to by all the interested parties, Gramme and Lontin in France, and Siemens in England, that *each separate light consumes one horse-power of an engine*, besides all the incidental expenses.

I have not the prices of the Lontin system, but I believe they are somewhat less, but I know that the Lontin light, as displayed at the Gaiety, tries to the utmost the powers of a 12-horse engine. I would not propose omnibus drivers as judges upon a scientific problem, but I may mention that one of them explained to his passengers a few evenings ago that these lights brought great crowds to look at them, and that it was found they were too strong, so that two of the lamps had to be put out. The man was quite right, they were too strong—*for the engine.*

The quantity of light produced in return for this cost is somewhat uncertain. It is claimed that the regulator with Lontin's system gives a light equal to 100 Carcel lamps, and the Jablochkoff candle about 80. I very much doubt this. I found no difficulty whatever in examining the light steadily at a distance of 12 or 15 feet, nor did it produce any subsequent unpleasant impression of the spectral nature; and I cannot think that this would be the case with a light of the intensity stated, proceeding from an illuminating area not larger than a sixpence. It is, however, claimed as a merit of the Jablochkoff system, that, "instead of a luminous point emitting divergent rays of a most disagreeable character, it bathes the brightness of the carbon points in a white flame which gives a diffusive light," but my remark applies to both lights.

There is some doubt as to the value of the Carcel lamp in our standard candles, but Mr. Sugg, who is a good authority, valued it some years ago at 9.5 candles, and I will take this value as the highest. This gives the light of the "candle" as 760 standard candles: Mr. Sugg, however, states that on a trial before the jury the light of each as tested was 500 candles. These lights for most purposes have to be enclosed in opal glasses about 16 inches diameter, which are said to absorb one-third of the light. It is, however, well known from the best experiments that

Plain glass	absorbs	10	per	cent.
Ground "	"	30	"	"
Opal "	"	60	"	"

This reduces the effective lighting power of each light to 300 candles, assuming its original value to be, as claimed, 760. With these globes the light no longer issues from the radiant point, but from the whole surface of the globe, its inherent brightness being therefore proportionately reduced. An ordinary gas

burner giving, say 10 candles light, has an illuminating area of about 5 square inches, so that its inherent light is about 2 candles per square inch. The area of the diameter of a globe of the size used is 220 square inches; and if its light is 300 candles, the inherent light is only about 1.4 per square inch, less than that of the gas burner. Hence it is that when we increase our distance so far that the size of the flame no longer influences the eye, and that its rays are practically parallel, the electric lamp is, as I before stated, distinguishable from a gas lamp only by its different color.

Ordinary gas, burnt properly, will give the light of 14 candles for 5 feet per hour; that is, 1,000 feet give a light equal to 2,800 candles for an hour at a cost of, say 3*s.* 4*d.* Assuming that an electric light gives a net effect of 300 candles, and that its cost is only 8*d.* per hour, the cost of the same 2,800 candles is 6*s.* 2½*d.* With naked lights, however, the cost of gas would be somewhat greater for equal amounts of light. But all such calculations are at present only vague approximations. Improvements also must be looked for, and, indeed, I have seen in operation machines, not yet before the public, which will, I believe, lower the cost. On the other hand, there is abundant room for improvement in the use of gas, as it is well known that even our best burners do not give anything like the light the gas is capable of, while those in common use burn the gas most wastefully. In fact, we burn too little gas in single flames, so that the surrounding air floods and cools the flame; the more gas we can burn perfectly in a single light the greater the light the gas will give for the quantity burnt. A simple experiment will prove this: Fit two burners on pieces of elastic tube so that they burn side by side, and note the light given. Now bend them to each other so that the two flames cross and blend in one; it is evident there is no increased consumption, but the increased illuminating power is something surprising. If powerful lights, such as the electric ones, are required, there is little doubt they can be produced economically from gas, and all the other objections can readily be overcome, for the color of gas light will whiten as the quantity properly burnt is increased, until a pure white light is obtained, superior in quality to that of the electric arc, because the intense temperature of this involves the production of an excess of the violet rays of light and of those chemical rays called actinic, which, while consuming energy, do not give light, and from their other properties are likely enough to have injurious actions upon health. The removal of the products of combustion also is a very simple matter, already effected in some forms of gas burners, and involving nothing more than an outlet to a chimney or to the open air. In fact, all the defects attributed to gas are simply matters of management and economy, and if people really wish to correct them, and are prepared to spend only a small portion of the amount needed for the fitting up of electric lights, they can easily have what they desire, and yet retain that great advantage in simplicity, the mere turning on of a tap, as compared with the difficulties of special and elaborate mechanism.

At present it is found that the current for producing light cannot be sent to any great distance with advantage. Leakage has been the great trouble of gas engineers, but it is a mere trifle in their case as compared with the transmission of large electric currents. What is called *resistance* in electricity is of two orders: there is the conductive resistance, the specific capacity of each substance to permit electricity to pass, and which may to some extent be compared to friction in mechanics. This is a simple enough matter, and its law is that with a given kind of conductor, say copper wire, to maintain a fixed resistance the section of the conductor must increase in the same ratio as the length, or, what is the same thing, the total weight of wire must increase as the square of the length. This alone is simply a question of first cost; but then also the tendency to leakage or loss increases in a greater ratio than the length, and the insulation becomes more and more difficult as the size of the conductor increases. But this is insignificant compared with what is called inductive resistance, that which in submarine cables limits the speed of transmission. This comes into action only with intermittent currents, and it will increase in some ratio exceeding the

square of the distance, if the conductor is enlarged as required to increase the conducting power. We have as yet no experience in this subject with large currents, but it is so serious an obstacle in the minute currents used in telegraphy that it is not unlikely to prove an absolute barrier to the sending of large alternating currents to any considerable distance. I was told that arrangements are making to send a light two miles here in England, and when I pointed out this probability, it seemed that it had not been anticipated, but I learnt at once that it had arisen in some experimental trials. Whether it will be overcome remains to be seen; if not, systems depending on alternating currents, as with the Jablochkoff candle, will be limited to short distance; at present they work at a distance not exceeding 300 yards.

III. Influence upon Gas Interests.—From what has been stated it is evident that the electric light can only be applicable in some circumstances, and that the necessity of cumbersome machinery and qualified attendance must limit its use. But also in many cases where it would be suitable, it by no means follows that its use would involve diminished consumption of gas. An American experimentalist states that with a petroleum engine and electric apparatus he has obtained more light than from the direct combustion of the petroleum. Now gas engines have received of late very great improvements, and these improvements are not final: it is quite certain that from their great simplicity, their freedom from danger, and from the great fact that they require little or no attendance, they will gradually supersede steam for many uses, and among others they would often be employed to furnish the motive power for electric lighting, and more gas would be employed than was required for the former gas illumination, because no one will be contented with merely the same quantity of light.

For street lighting there is small probability of its extensive use. Paris is a great bazaar; abundant light, however great the cost, pays there, just as it does in our public-houses. The people there live in the streets, which are filled with fine public buildings and endless sources of attraction. Neither our climate, nor our cities, nor habits of life, offer us the same inducements; and what would the English ratepayers say if asked to maintain even the ordinary gas lighting system of Paris, where in many streets the public lights are equal to one burner every 4 or 5 yards; such was in fact the light now displaced by the electric light, and still employed in the Rue de la Paix, close at hand and easily comparable, where every 12 or 14 yards there is a post with three lamps on it, giving a light not so white and brilliant, but quite as useful practically, as the electric light itself.

There are a few places such as Charing Cross or Oxford Circus, where a full electric system might be advantageously employed, but its general introduction into English streets is a probability so distant as hardly to call for consideration.

To make this evident I have carefully gone over Trafalgar Square and estimated the electric lights required to illuminate it *exactly as the Place de l'Opèra in Paris is now lighted.* It would require 16 single and 7 double lights, that is 30 electric candles with engines of some 40 horse-power. It is at present lighted by 73 gas burners, not counting private lights.

In lighting railway stations, in large factories where the vitiation of the air by gas is an important consideration, probably in picture galleries and libraries, and in some of the larger hotels, the electric light must be expected to come into use, but its introduction will be gradual, and its utmost effect on gas consumption can only be to somewhat diminish the regular annual increase of consumption. This *might* diminish the need for increasing powers of production, but as the dividends are earned by actual production, these would not be affected, even by a complete stoppage of the increase; it is however much more likely that by the use of gas engines and other employments as yet undeveloped, this increase will grow rather than diminish.

Finally, it needs no prophetic power to foresee that ere long a dead set will be made in the markets both upon gas shares and upon the pockets of that confiding public who are always ready to give their money to any one who asks boldly and promises freely. It is but a few years since gas shareholders went

through a similar scare over new gas, air gas, and other inventions which were to ruin them and make the fortunes of people who gave £20 for £1 shares in the new scheme. The day some bold financier asks for a million of money to light England with electricity, no doubt timid shareholders will rush about to sell their shares, and it will be useless to hint to the speculative investors that the new lighting, when introduced, will probably not be powerful enough to show them their money back again. Gas shares would no doubt go down, and that would be an excellent opportunity for prudent men to invest in gas shares. In fact, after a thorough examination of the matter, so far as it has yet progressed, and so far as its further unquestionable progress can at present be foreseen, I feel no hesitation in expressing the conviction that the gas companies, at all events of London, have nothing whatever to fear, and that whatever owing to market operations may happen to gas *shares*, gas *dividends* are in little danger from any competition with the electric light."

In the meantime each and all of the various systems are on trial in this country. The American machines are the Wallace, the Brush, the Weston, the Arnoud-Hockhausen, and the Fuller—all dynamo-electric machines, converting by slightly different arrangements of permanent and electro-magnets, mechanical power into electric currents.

The lights are of two classes: 1st, those which consist of a luminous arc between two slightly separated carbon points; and 2d, those which consist of a slender rod of carbon or platinum, rendered incandescent by the constant passage of a strong current through it.

In the former class, if the carbon points approach each other from opposite directions, an automatic regulator is necessary to maintain the light; for such purpose are the Serrin, the Browning and the Focault regulators; the Jablochkoff light, although depending on the arc, requires no regulator, because the carbons are in fixed relative positions parallel to each other and at proper distance.

Of the second class, the "incandescent" lights; the Sawyer-Man; the Lontin; the Regnier; the Werderman and rumor says the Edison are the present examples.

It would seem that we are yet without sufficient knowledge of the relative economy of either the machines or the lights, but the public may await with patience the results of the very brisk competition which will certainly be induced by the present urgent demand for improved systems of lighting.

CONDITIONS OF MAXIMUM MAGNETIZATION OF ELECTRO-MAGNETS.

By T. DU MONCEL.

From "Comptes Rendus de l'Académie des Sciences," Abstracts published by the Institution of Civil Engineers.

In former papers the maxima conditions of electro-magnets have been deduced from formulæ in which it has been supposed that the attractive forces were proportional to the squares of the intensities of the currents, to the squares of the number of turns or convolutions of the magnetizing helix, to the diameter of the iron core, and to the square roots of their lengths. The Author doubts that these laws are applicable under all degrees of magnetization, and he has already pointed out* considerable departures within certain limits as regards the diameter and length of core. He now finds this also the case with the law that represents the electro-magnetic forces as proportional to the squares of the intensities of the current. It has long been known that Joule, de Haldat, Müller, and Robinson have found that at the commencement of a current, and when the magnetic state of the iron is far removed from the point of saturation, the attractive force, instead of increasing as the square of the intensity of the current, increases in a much more rapid ratio, and that this ratio again diminishes near the point of saturation,

* Comtes Rendus, vol. lxxxv., pp. 877, 466, 481, 497 and 652.

remains for a short period stationary at this limit, and subsequently diminishes to that of simple proportionality to the current intensities. The question then arises, what should be the resistance of the magnetizing helix with regard to the exterior circuit? The helices, the author concludes, should always have less resistance than the exterior circuit, in the amount of half where g is taken as variable, and in the ratio of $2a+\frac{c}{a}$ to $a+c$ in the case where the variable is a. It is then to be concluded that on circuits where the interruptions of the current are multiplied, the resistance of the electro-magnets should be proportionally greater as the completions of the circuit are of shorter duration; and it is for this reason, as well as for faulty insulation, that Mr. Hughes has considerably reduced the resistance of electro-magnets employed on long circuits. On a line of 310½ miles, Mr. Hughes has found that the electro-magnets of his instrument should not have a resistance above that of 74½ miles of the line. Of course the imperfections of insulation conduce also to this diminution.

THE PROGRAMME OF ENGINEERING AND OF ARCHITECTURAL STUDENTS.

From "The Builder."

We have regarded the main divisions of the professional study of the engineer as far as concerns, first, survey; secondly, public works proper; and thirdly, the application of these motor-powers which nature herself offers to the service of man. The point thus reached may, indeed, be considered as a sort of landmark. The discovery, by Watt, of a means of applying for the production of motion the expansive power of steam, forms an epoch in the history of engineering. The mechanical engineer is sometimes regarded as if he occupied a lower position in his profession than his civil brother. No view can be more inaccurate. The civil engineer must be, to attain any eminence, a mechanist and a workman also. Our first engineers have been mechanics. The minute details of the locomotive have received as much and as enlightened attention from Robert Stephenson and from Isambard Kingdom Brunel as they have done from Fairbairn, or from any exclusively mechanical man.

The use of steam-power, first for the drainage of mines, then for the general movement of machinery, and thirdly for propulsion by movable engines, both by lnnd and by sea, had attained a very important development before the theory of mechanical motor-power was at all properly understood. It is forty-five years since Ericcson, by his successful construction of a hot-air, or caloric, engine, gave a practical proof that the value of the vapor of water for mechanical purposes was due to its capacity for heat; and that heat itself was the source of motion. The determination of the thermal unit was a step of extreme importance in the advance of mechanical science, and the researches which are now being daily pressed further and further into the study of electricity, point to a possible improvement in our methods of applying the force of nature, as much in advance of our present mechanical knowledge as that is superior to the science of a hundred years since. It is true that the cost at which a certain quantity of heat can be produced by the combustion of coal is very far less than that attainable by any other known process of chemical decomposition or recomposition. That fact has been a barrier to the use of electro-mortar machinery. But the discovery of the conversion of motion into the electric current, as in the case of the magneto-electric machines of Gramme, is one of which it is impossible to foresee the ultimate results. We can command, in the tidal movement of the sea, a source of motive power that is practically unlimited, as well as gratuitous. The main obstacle to the future

application of this power to the service of civilized life is the difficulty of transmission. The present means of hydraulic and of pneumatic transmission of power are limited, costly, and clumsy, as compared to the use of the insulated wire for the transmission of the electric current. Up to the present time only currents of low tension have been required for telegraphic purposes. But the extraordinary results of the application of a steam-engine of only seven-horse power to the production of the electric light by the Gramme machine, on the one hand, and the discovery of the quadruple or multiple divisibility of the electric current on the other, seem to intimate that we are possibly on the threshold of brilliant discoveries, as to the means of availing ourselves of an unlimited source of motive power now very little employed or regarded.

It is thus, first to the production, and, secondly, to the transmission, of mechanical motor-power, that we hold that the chief attention of the engineer is invited by the actual state of scientific discovery. The study of the accumulator, and of all the appliances of hydraulic transmission of motion, is already assuming a practical importance hardly less that of the study of the steam-engine itself, whether fixed, marine, locomotive; or movable, as in the case of agricultural steam-engines. As to the latter, the chief attention at the moment, especially on the Continent, is turned to the improvement of what are called secondary locomotives, which are required either for the ascent of steep inclined planes, or for the propulsion of vehicles or tramways at comparatively low speeds. It is probable that the advantages thus to be obtained are, at present, rather overrated than otherwise. The economy which is attainable for locomotion over an approximate level, and at a speed not less than twenty miles per hour, is so great, as compared with the former methods of transport, that a saving of four-fifths in the cost is estimated to have been obtained. But with the introduction of the counteracting power of gravity, in steep ascents, and with the reduction of speed, the great economical advantages tend to disappear. We may find a steam-engine cheaper than a pair of bullocks to take a given load up a given hill. But it will be the persons who have to ascend the hill who will have, as a rule, to provide and apply the engine-power. There will not be even that return for the expenditure of the capital of independent persons, who merely seek a profit for their money, that is afforded by our railways; and even that is now only 4.32 per cent.

After the general study of the engineer with regard to the apparatus for the production, and for the transmission, of mechanical motor-power, comes the wide field for his skill in the application of such power. Here we enter on mechanism proper—industrial mechanism, the substitution of mechanical contrivances, exact in their operation, for human skill, which is less exact. The line may here be drawn with unusual precision between mechanism and art—between the mechanic and the artist. It may not be too much to say that we anticipate that almost anything, for the production of which skill may be acquired as matter of routine, will sooner or latter, be better, as well as more cheaply, effected by machinery than by human fingers. On the other hand, in all that into which the real soul and genius of the artist enters—all that which no toil or labor will enable an unsympathetic scholar to acquire—will remain the province of man as distinguished from machinery. In the vast factories now erected for the production of textile fabrics, the operatives are becoming, more and more, the mere attendants on the machines. The saving of labor is now being made in the direction of increasing the number of spindles, of looms, or of other machines, which can be tended by one man. But while a faultless and accurate piece of machine-made lace, for example, is turned out at an incredibly low price per superficial inch, the manufacturer of this fabric does not lower the price of Rose Point or of Honiton. The individuality of the artist, even in so humble a matter as the execution of a delicate pattern in thread, can never be attained by the machine. When what is wanted are great numbers of indistinguishable copies, they will be produced, from year to year, cheaper (and it may be hoped better) by the mechanic. When what is required is to gratify the taste, or to elevate the tone, it will always remain the province of the author.

It is, of course, far beyond our present

limits to attempt to sketch an outline of the application of mechanical power. An instance which may be cited as one of the most extraordinary triumphs of this branch of mechanics, of recent times, is the sewing machine. While for many purposes the work done by this instrument is inferior to hand-work, there is no doubt that the burden of an enormous amount of purely mechanical oil is thus taken off mankind, or rather womankind. The only question is, how far the workwoman has been raised in the social scale by the invention. We very much fear that it is precisely where the heaviest burden formerly fell that it is least diminished, even if changed in the mode of its incidence, by the change. Amongst the most striking of the modern mechanical applications of scientific principles, of comparatively late discovery, may be named the bicycle; and the beautiful American drilling machine, in which one coil of flexible steel wire revolves within another, and the instrument while rapidly working may be applied in any direction. The more wide becomes our acquaintance with the field of mechanical invention, the more profound will be our conviction that the engineer is, in this branch of his calling, but at the very commencement of an immeasurable progress.

Among those special mechanical industries, which at the present moment demand so much of the attention of the engineer, may be mentioned the transmission of signals to a distance by electricity, whether they are designed to attract the eye or the ear; the application of the same power to purposes of illumination, whether by way of instant ignition and extinction of coal gas, or as a direct source of light; the labor-saving machinery of which the farmer is daily making more and more use, and the application of steam locomotive power to what are called secondary purposes—that is to say, to the ascent of steep hills, or to the propulsion of vehicles through the streets of populous towns. In these matters, we see on the one hand the successful application of patient care, and even the flashes of brilliant genius. On the other hand, we have to lament the adoption of suggestions, any one of which may in itself be good, but which, taken together, are mutually exclusive. Such, for example, is to be found in a recent report on local railways, or, as we should call them, tramways, in France, in which the combination of gradient, engine-power, weight of rails, and possible weight of train, is so impracticably proportioned that no train could exceed a gross weight of 7½ tons, exclusive of the engine; while a rate of charge equal to that at present made for horse haulage by the common road would be insufficient to make any return on the capital expended. Here is a case in which the schoolmaster is signally abroad, and in which the need of sound scientific programme is abundantly illustrated.

When we turn from the rough programme of the study proper for the education of the engineer to that appropriate to the architect, we shall see that there is at the same time a close resemblance, and a sensible difference, in the points which it is needful to block out, as compared with those already indicated. As the study of the engineer commences with survey, so does that of the architect. But as the aim of the former is to work in accordance with mechanical law and routine, while that of the latter is to produce the work of the artist, so from the very commencement of their studies should a certain difference of bias be impressed on the respective pursuits of two students. We have regarded survey, as conducted by the engineer, as composed of two branches—that of the making of maps, plans, and diagrams, and that of the inspection of materials and workmanship. This second branch is common to the two professions—the differences, when they exist, being only those of detail. But while the work of the engineer, in the wider branch of science, is mathematical, that of the architect is artistic. Not that the importance of exact and actual measurement of details is to be undervalued. Not that the drawing-board, the T and set squares, the dividers, and the compasses, ought to be less familiar implements for the one pupil than for the other. Not even that we undervalue for the engineer the use of the sketch-book. But his sketch-book, if what it ought to be, will be something very different from that of the architect. A certain degree of proficiency in free-hand drawing, desirable for every one, is indispensable for the student of architecture. Above all, he must be gifted with

that power—rare as it may be—which enables a man to see a picture where it exists; in fact, to appreciate the pictorial in nature. The survey of the architect, in this sense, consists in the filling his mind with the noblest and best specimens of structural beauty and excellence. His memory, no less than his note-book, will be enriched, his taste will be formed, and his judgment enlightened by travel and observation. Not only will he have to view a building in the grand, to appreciate the proportions of its most striking features, and to apprehend the law of architectural subordination, but he will at the same time become aware how far climate and situation are elements of style. He will understand how Egyptian architects burrowed in the live rock for a grateful shade from the burning sun of Africa; how Greek and Italian architects were contented with the artificial shade of peri-style or of vault for the brief heat of the Mediterranean summer; how Teutonic architects throw up pointed and gabled roofs to throw off the burden of the winter snows. As the survey of the engineer gives him exact measurements of space, so the survey of the architect should give him truthful ideas as to situation, adaptation, and proportion.

Distinct from this preparatory schooling and enrichment of the mind is scientific knowledge of structure, both as to mathematical construction and as to strength of materials. Here the architect, for the most part, has to deal with problems of less magnitude than the engineer; and tables and examples will be readily forthcoming to guide his ordinary practice. But the higher and more scientific forms of structure; the groined arch, the flying buttress, the frozen cobwebs of such ceilings as those of King's College Chapel, Henry VII.'s Chapel, St. George's Chapel, with pendentives that seem to mock the unsleeping power of gravity; the heavy dome of a Michelangelo or of a Wren; must contain no secret hidden from his research. Perfect structural skill, founded on mathematical knowledge, is an essential part of the furniture of the mind of the architect.

Hand in hand with this structural skill must rank what, if we retain the original meaning of the Greek word, may be called the economical skill of the architect—his ability in the distribution of the house. This part of his qualification may be described as the education and organization of common sense. It is the portion of education which is most usually and most cruelly neglected, not in one pursuit alone, but in the general upbringing of the young men and boys of England. By the service of this faculty the architect learns, as if instinctively, to make the best of a given site, to proportion means to object, to subordinate parts to the whole, as a matter of distribution of space. All questions of sanitary provision here come in; not only as to drainage, ventilation, illumination, and removal of smoke, but as to due proportion of open area to building, of vestibule and staircase to rooms, of offices and stabling to residences. An architect possessed of this kind of ability will never run a line of street so as to cross a line of thoroughfare and cut off a main street ventilation, as we so often see done in some of the ever-growing suburbs of London. He would as soon make his drains run up hill (as in fact has been done with some of the main drainage of London). Nothing is more easy than to undervalue, or to neglect, this important part of the education of the architect. Nothing is more fatal to the use and the value of his advice.

To these, the mechanical, although far from being the self-moving parts of his art, has to be added yet another element,—the æsthetic. Taste, it is true, cannot be given to him who has it not; but it may be cultivated in him who has it, even if the portion with which he is naturally gifted be but meagre. And even taste may be aided by intelligent perception of structural fitness. For all those features of building which give the finest contrasts of light and shade there is a structural reason. The man who attempts to beautify a building by *appliqué* ornament, stuck on without reason arising from structure, produces only costly abominations. It requires a noble and a cultured taste to make structural lines assume the most pictorial forms, as is the case in so much of our pointed ecclesiastical work. But the man who is unaware of his primary principle of decoration is like one who builds sham ruins to decorate a newly-planted shrubbery. The lamp of truth can never be left unlighted by any one who aspires to the name or

the name of an architect, except at the cost of his reputation.

Brief and imperfect as the above sketch admittedly is, it may yet not be altogether without use. The traveler who is encumbered with parcels soon becomes aware of the comfort and advantage that he derives from a list of his *impedimenta*. He knows each package, no doubt, well enough. Yet if they are not numbered or catalogued he will often feel puzzled to tell which is missing in the hurry of a railway station. The value of order, even of the most simple and artificial kind of order, is often incalculable. Were those who direct architectural education to set before themselves the task of blocking out the duties of the profession—of watching against the danger of directing any one, instead of giving attention in proper proportion to all—the buildings now going on all over the country would be of a higher stamp than is too often the case. It is no doubt true, when we look back to the work of architects of a former generation, that much ignorance and neglect of detail often characterize their work. We may see many a building which, viewed at a certain distance, is imposing from its just or even noble proportions, but which yet pains the educated eye, on a closer approach, by its rudeness or even vulgarity of detail. It is quite intelligible that as architects have become aware of this deficiency their efforts to obviate it may have led them to err in the opposite direction. In the careful study of the best details of the old masters, the students have but too naturally lost what is more important than any detail, namely, grasp of the dominant idea. With men of real taste, and real desire to excel, it ought to be sufficient to give this hint. When the architect takes breath—steps back from his drawing-board, as the painter does from the easel—asks himself, "What, after all, was the original motive of this moulding, or of that feature of detail?" he may often start to see how he is forgetting the whole in a too careful study of the parts. The essential value of an exhaustive programme may thus be held to be twofold, both to the architect and to the engineer. First, it should insure exhaustive study, by enumerating every branch of skill and learning proper to the profession. Secondly, it should insure, by the order of its own arrangement, a corresponding order in the mind of the student, and a fit subordination of parts to a complete and perfect whole.

REPORTS OF ENGINEERING SOCIETIES.

American Society of Civil Engineers.—The last issue of Transactions contains: No. 143, On a Newly Discovered Relation Between the Tenacity of Metals, and their Resistance to Torsion, by Robt. H. Thurston.

No. 144, Observations on the Stresses Developed in Metallic Bars by Applied Forces, by Theodore Cooper.

No. 145, Cushioning the Unprocting Parts of Steam Engines, by John W. Hill.

Discussions of previous papers by Prof. Wood, Edward Searles, J. Foster Flagg, and others.

Edinburgh and Leith Engineers' Society.—A meeting of this Society was held on Wednesday evening, November 20th, at No. 5 St. Andrew square, Mr. Robert C. Reid, C.E., president, in the chair. A paper was read by Mr. Robert H. Smith, formerly Professor of Engineering in the Imperial University of Tokio, Japan, "On the Calculation of the Strains and Stresses in Redundant Structures." He described a redundant structure as one in which the number of links was greater than $2n-3$, where n means the number of flexible joints, the above-number $2n-3$ being just sufficient to make the structure stiff. He pointed out that what is usually considered a non-redundant structure becomes redundant when laid upon and fixed to its abutments, and that it was on this account that the horizontal components of the abutment reactions were usually said to be indeterminate. He then described in detail the graphic and other methods of calculating these horizontal components from the *moduli* of elasticity of the different members and of the abutments, treating separately, link-work structures, and massive arches, neither of which, he believed, had hitherto been completely investigated. After explaining the application of the general method to structures of any degree of redundancy, he concluded with some remarks upon the impossibility of profitably avoiding all redundancy in engineering structures.

Institution of Civil Engineers.—The Council of the Institution of Civil Engineers are inviting communications on the subjects named below, and for such as are approved they are prepared to award premiums arising out of the special funds bequeathed for the purpose:—(1) The triangulation survey, and mapping of countries and districts, including the astronomical observations required for latitude and longitude, and the measurement of bases, with a description of the instruments employed, the reduction of the observations, and degree of accuracy of the results; (2) The leveling of countries either by spirit-leveling, vertical angles, barometers, or the boiling point

of water; with a description of the instruments employed, the reduction of the observations, and degree of accuracy of results; (3) The effect of the lapse of time on the strength of materials strained beyond the supposed limit of elasticity, but within the ultimate strength; (4) The stresses inducing the failure of iron ships, as bearing on the probable endurance of large iron structures under high strains and repeated bendings; (5) The causes of slips in rocks and earths of different kinds, and the conditions that induce treacherous ground in railway cuttings, tunnels, and the sides of valleys near reservoir banks; (6) The best combined system of warming, ventilating, and lighting large buildings; (7) The most suitable materials for, and the different systems of, road-making for large towns, where the traffic is heavy, including a comparison of first cost, maintenance, and durability; (8) The construction of iron piers for viaducts, with practical examples of French, American and other systems; (9) The design and construction of a steel bridge, with particulars of the weight and cost, and of the tests to which it has been subjected compared with an iron bridge of the same span; (10) The works carried out on the Continent of Europe for the improvement of rivers, and of inland navigation generally; (11) The design and construction of movable weirs across rivers; (12) The treatment of estuaries, with special reference to tidal capacity; (13) The design and construction of dock gates and caissons, including the requisite external and internal arrangements, illustrated by recent practical examples; (14) The design and construction of building slips for large vessels; (15) The construction of tide gauges, and the usual method of carrying out a systematic series of tidal observations; (16) The storage and filtration of water, both natural and artificial, and the arrangements for the distribution of water in towns; (17) The systems of domestic water supply suitable for rainless districts; (18) The benefits and expedients of irrigation in India and in other warm climates, and the proper construction of irrigating canals, so as to avoid erosion or silting, and to prevent the growth of weeds; (19) The bearing of recent experiments on the resistance of vessels, and on skin friction, upon other hydraulic problems; (20) The most recent types of iron sleepers for railways, with practical results and statistics; (21) The best practical use of steam in steam engines, and the effects of the various modes of producing condensation, and of various grades of expansion; (22) Compressed air as a motive power, particularly as applied to machinery in mines, and for traction on tramways and in tunnels; (23) The relative advantages of steam, heated air, gas, water, and electricity as the motive power in small engines; (24) Wind and water as motive powers, compared with steam power, and the motors most suitable for utilizing them; (25) The differences in design of British and foreign locomotive engines; showing the benefits derived from increase in weight, and the relation that ought to exist between the diameter of the wheel and the load it has to carry; (26) The various descriptions of pumps employed for raising water or sewage, and their relative efficiency; (27) The different systems of lifts in use in warehouses and in dwellings; (28) The relative loss of power due to friction in various parts of machinery; (29) The "output" of coal in the United Kingdom, as compared with that of other countries, illustrated by statistics, showing where coal is produced, where and how it is consumed, and the relative quantities exported; (30) The appliances and methods used in different countries for tunnel-driving, rock-boring, and blasting, with details of the cost and of the results attained; (31) The methods and machinery employed in sinking and in working deep coal mines; (32) Coal depots for ocean steamers, the various points involved in their management, and the methods of preserving large quantities of coal from deterioration; (33) The metalliferous or other mining districts in different countries, and the mode adopted in working them; (34) The methods employed in securing the excavations in mining large and irregular-shaped mineral deposits, for example, the Almaden mines, the Great Comstock lode, etc.; (35) The appliances used in different countries for dressing the ores of lead, copper, zinc, and tin, and the smelting of such ores, with details of the results and cost by various methods; (36) The disposal and utilization of slags from various smelting processes; (37) The management of underground waters in mining districts, and the relative economy of distributed or trunk pumping engines, adits, etc., in particular cases; (38) Recent progress in telegraphy, with a notice of the theoretical and practical data on which that progress has been based; (39) The application of electricity to lighting purposes, contrasted with the best systems of lighting at present in use; (40) Torpedoes, and their influence on naval construction.

IRON AND STEEL NOTES.

The Cooling of Steel During Hardening.—One of the most serious losses common to our tool and implement manufactories is that of the cracking and splitting of steel during the hardening process. Not only is the article or piece lost after having incurred the cost of its manufacture, but, in many cases, the completion of the machine of which it forms a part is arrested until the lost piece is replaced. In many cases this is done at increased expense, because the piece has to be made singly instead of with a number of others, involving as much setting of machine and adjustment of tools as would be required for a large number of pieces. Successful hardening and tempering is, indeed, even under ordinary and unvarying conditions, considered and kept as a trade secret. Visitors are excluded from the hardening and tempering room. In some cases the method of heating, in other cases the material used for heating, in yet others the cooling mixtures form the supposed secret. As a matter of fact, however, some of the very best tool manufacturers employ the simple open fire or furnace and water, and it is probable that with these two simple agents good cast-steel can be as successfully and properly hardened for any purpose as it can be

under any other process, and the advantage gained by heating in fluxes consists in increased expedition and the necessity for a less expert manipulation.

The splitting or cracking of steel occurs during the cooling part of the hardening process, and is to be easily avoided even with the most unfavorable of steels, if the conditions of cooling are made to conform to the form and size of the article. The cooling is, in a large majority of cases, performed by dipping the heated steel in water; and the manner in which the dipping is performed may be made at will to crack, warp, or straighten the article.

The instant the surface of a piece of red-hot steel enters the water a rapid contraction of the submerged portion takes place, and unless this contraction is kept equalized to suit the shape of the article, the side or part most contracted will bend hollow, causing the diametrically opposite metal to bend to accommodate the inner curve. Suppose, for example, we heat a piece of steel, an inch square and twelve inches long, to a red heat, and dip it slowly in water, so that one side of the square will strike the surface flat and evenly; then that surface will contract while the diametrically opposite or upper surface will remain expanded; the lower face will curve to a concave, the upper one to a convex. If, then, such a bar were curved during the heating process, we may help to straighten it by dipping it slowly in the water, with its convex side downward. If it was bent at one end only, we may dip it at that end first, diagonally, and with the convex side downward. If, however, we dip it with its length lying either diagonally or horizontally, we are apt to warp it, no matter how quickly it may be dipped, and the reason is, in addition to the above, as follows: Experiments have demonstrated that the greater part of the hardness of steel depends upon the quickness with which its temperature is reduced from about 500° to a few degrees below 500°, and metal heated to 500° must be surrounded by a temperature which renders the existence of water under atmospheric pressure impossible; hence, so long as this temperature exists, the steel can not be in contact with the water, or, in other words, the heat from the steel vaporizes the immediately surrounding water. The vapor thus formed penetrates the surrounding water and is condensed, and from this action there is, surrounding the steel, a film of vapor separating the water from the steel, which continues so long as the heat from the steel is sufficiently great to maintain that film against the pressure of the water and the power of the water which rushes toward the steel to fill the spaces left vacant by the condensation of the vapor as it meets a cooler temperature and condenses. The thickness of the vapor film depends mainly upon the temperature of the steel, but here another consideratien claims attention; as the heated steel enters the water, the underneath side is constantly meeting water at its normal temperature, while the upper side is surrounded by water that the steel has passed by, and, to a certain extent, raised the temperature of. Hence the vapor on the underneath side is the thinnest, because it is attacked with colder water and with greater force because of the motion of the steel in dippings. Suppose, now, we were to plunge a piece of heated steel into water, and then slowly move it laterally, the side meeting the water would become the hardest, and would be apt to become concave in its length.

From these considerations we may perceive how important a matter the dipping is, especially when it is remembered that the expansion which accompanies the heating is a slow process compared to the contraction which accompanies the cooling (although their amounts are of course precisely equal), and that while unequal expansion can only warp the article, unequal contraction will in a great many, or, indeed, in most cases, cause it to crack or split.

After an article is dipped to the required depth, it should, if straightness is of importance, be held quite still until reduced to the temperature of the water, because, if taken out before so reduced in temperature, it is especially apt to crack; and it is better to have a deep tank of water, if the body of the metal is great, so that the steel may be dipped slowly downward and become cooled sufficiently rapidly to harden without any lateral movement, except it be after the steel has lost its redness.

When a piece of steel requires to be hardened at one end only, the dipping must be performed with a view to make the graduation from the soft to the hard metal extend over a broad section of metal; for if the junction of the hardened with the soft metal is abrupt, the hardened end is apt to break short off. The method of dipping, therefore, is, in this case, to plunge the end of the steel vertically into the water, to a depth a little more than equal to the depth it requires hardening, and, after holding it still there until it is black hot (that is, as soon as its redness is gone) dip it slowly a little deeper, and then raise it up to the amount of the increased dipping, and slowly immerse again.

When a piece of metal requires hardening and tempering at one part only, we may heat the steel back of the part to be tempered to redness, and dip the article so as to harden the required part, and leave sufficient heat in the contiguous metal to raise the temperature of the hardened part enough to temper it. This plan is always followed in the tempering of the lathe and planer tools, flat drills, &c. If, however, the method of dipping is to hold the steel in the water at an even depth after the immersion, the temper-color will be very narrow, while, if the steel is raised and lowered in the water, the color-band will be broad.—*Polytechnic Review.*

RAILWAY NOTES.

THE fact that some steel rails wear much longer than others has attracted much attention from engineers and metallurgists. The question is one of considerable importance, and for this reason different rails, which had been in use on the Pennsylvania railway, have been carefully analyzed, and their chemical composition and other qualities ascertained and compared with their wear. Dr. Dudley, chemist in the testing department of that railway, read a paper at a late meeting of the American Institute of

Mining Engineers, in which some of the results of his investigations were given. These confirm the observation often made, that the hardest rails do not wear the longest. The wear to which steel rails are subjected is that of rolling friction, which is in reality a succession of blows. The effect of blows on a hard substance is to crumble it, while on the softer materials a permanent distortion or change of form is effected, which is in reality the "flow" of the metal under the pressure of the blows. A material, then, which is so soft that it will not crumble, and so hard that it will not flow, will probably offer the greatest resistance to wear under a succession of blows like those produced by rolling friction. The question is of great importance, as uniformity in quality of steel rails would secure a great annual money saving.

It is stated that chilled wheels were used on the Emperor Ferdinand Northern railroad, of Austria, as early as 1855, and have ever since increased in number, so that now, with 10,000 luggage trucks, 23,140 such wheels—21,696 from Ganz and Co., Buda, and 1,144 from Count Andrassy's works, at Dernoe—are in use. The following figures are deduced from a table giving some data concerning the life of chilled wheels on the Northern railroad. The time is given and not the mileage; the latter can be, however, approximately found by multiplying the years by the average mileage per year, namely, every wheel is guaranteed to last five years, and in case of failure is exchanged by the manufacturer. Between the years 1855 and 1872, 4,309 chilled wheels were bought; of these, 25,497 or 74 31 per cent. were exchanged before reaching five years of service, and up to 1878, 6,505 more, or 19.2. per cent., had been disabled, leaving but 2,307 still in use. Of the latter number, 10 have been running since 1862, 41 since 1864, 129 since 1865 and 1867, 530 since 1868 and 1869, 709 since 1870, and 888 since 1871 and 1872. In the last five years an additional number of 24,854 was bought, and of these, 5,465 have failed already. The average life of those wheels that failed before reaching the guaranteed time was 3·1 years, and the average life of wheels which exceeded five years but failed before 1878 was seven years, giving a total average life of 3.9 years for the wheels that failed. The 2,307 wheels still remaining will probably not exceed an average of 4.5 years. From 1865 to the end of 1877 only thirteen wheels have been broken.

ENGINEERING STRUCTURES.

The English Channel Tunnel.—The Channel Tunnel is still only a possibility. Nothing is being done on the English side, but on the French coast the borings are continued, and the information obtained confirms the geological evidence on the strength of which the undertaking was proposed. The scheme, however, does not meet with favor in influential circles here.—*English Mechanic.*

The Grand Canal of China.—This canal is likely to share the fate of the Great Wall. This water-way was constructed by Kublai-Khan and his successors of the Yuen race, and is 600 miles in length. There are 10,000 flat-bottomed boats on this canal, and these are used in the transportation of grain. The *Echo* states that this great water-way is an enormous "white elephant," as it costs an enormous amount every year for repairs, the appropriations there, as elsewhere, not being entirely devoted to the purpose for which they are meant. Junks are delayed every month while channels are dug for their passage. This year, for the first time since the construction of the canal, the grain from Nanking, with the consent of the government, has been forwarded by sea, and this fact has impelled the Peking authorities to consider the expediency of abandoning the canal as a commercial highway.

ORDNANCE AND NAVAL.

Determination of the Resistance Offered to Ships.—In an article contributed to the *Rivista marittima*, Signor A. Lettieri has described an apparatus for determining the resistance offered to ships by experiments on their models. The inventor considers that the determination of the resistance encountered by a vessel moving at different velocities in still water is a most important question, which has been solved by Mr. Froude. The law which this gentleman has formulated, by which to deduce the resistance met by a vessel from those encountered by its model, Signor Lettieri considers to have been fully verified by the experiments made by Mr. Froude on the *Greyhound* and its model. The further prosecution of similar experiments Signor Lettieri thinks useful, or even necessary, with the view of ascertaining, before the launch of a vessel, the curve of the resistance that it will encounter with different loads and displacements. Being unacquainted with the apparatus used by Mr. Froude, Signor Lettieri has invented one of his own, the description of which he illustrates with a drawing. In experiments of this nature the elements to be determined are two: the uniform velocity, and the resistance encountered at that velocity. The first of these is obtained by the measure of the space passed through in a unit of time. It is, therefore, desirable to have an apparatus which shall graphically denote this velocity by a curve, and refer it to a measure of the resistance. To effect this, Signor Lettieri has designed a vertical cylinder (the drawing shows the length to be fourteen times the diameter, but neither scale or dimensions are given), which revolves on a fixed axis. The upper part of this axis sustains a pulley, and a second pulley is fixed beneath the cylinder, with a small drum on its axis. A line attached to the drum passes over the upper pulley, and sustains a scale pan, to which is fixed a pencil, the point of which presses against the cylinder. The model is attached by a line to the lower pulley, so that the descent of the weight corresponds to the movement of the model through the water; while the weight itself is a measure of the resistance. Movement is given to the vertical cylinder by means of a pair of conically-

toothed wheels, one of which is attached to the cylinder itself. The motion of the latter being made thus uniform, and its velocity known, the curve traced on it by the pencil will indicate the relation between the movement of the model and that of the cylinder, and will form a regular spiral when both movements are uniform. The remainder of the paper is occupied by an algebraical investigation of the curves thus to be obtained, and by the relation between the weight placed in the scale pan, and the resistance encountered by the model in its passage through the water.

BOOK NOTICES.

OUTLINES OF MODERN ORGANIC CHEMISTRY. By C. GILBERT WHEELER. Professor of Chemistry in the University of Chicago. Chicago: Jansen, McClurg & Co.

The selection of materials for this compact little volume, from the great mass of materials at hand, has been judiciously done.

The student is supposed to be familiar with modern Inorganic Chemistry; he will then find this the best supplement to his course that he could well get, unless he intends to become a professional chemist. For all other needs of a student this work is sufficient.

The typography is excellent, the chemical formulas being exceptionally neat

THE HEMPSTEAD STORAGE RESERVOIR OF BROOKLYN: ITS ENGINEERING THEORY AND RESULTS. By SAML. MCELROY, C.E. New York: D. Van Nostrand. Price 50 cts.

This pamphlet discusses certain important questions of hydraulic engineering upon which the author has taken issue with the engineers by whom the late construction of the Brooklyn works has been directed.

As presented by Mr. McElroy, engineers will find the discussion very interesting. The estimates and promises of the projectors of the work as completed are carefully compared with the performance down to very recent date.

HYDRAULICS AND PNEUMATICS. By PHILIP MAGNUS. London: Longmans & Co. 1878. Price 75 cts.

This volume of the London Elementary Science Classbooks, by one of the editors of the series, is intended for upper-form pupils about to commence the study of a branch of physics which has an important connection with practical mechanics. The method of treatment employed is in some respects novel, the more popular course of illustration by experiment being used as much as possible; while, when it is necessary to deduce results by the help of mathematics from more elementary principles. the experimental proof is likewise superadded. The subject-matter is divided into short sections in a convenient way for the classroom, and to most of these a set of exercises is appended. At the end of the volume a series of miscellaneous problems is given. The information supplied is well brought up to date, and the diagrams and other illustrations are abundant and suitable.

RESEARCHES IN GRAPHICAL STATICS. By H. T. EDDY, C.E., Ph.D. New York: D. Van Nostrand. Price $1.50.

These "Researches" include a considerably amplified edition of two papers entitled respectively "Certain New Constructions in Graphical Statics" and "A New Fundamental Method in Graphical Statics," originally read by the author at Buffalo, in 1876, and of a third paper containing considerable new matter in problems relating to the combination of states of stress, and entitled "The Theory of Internal Stress in Graphical Statics."

In the first paper the author, who is Professor of Mathematics and Civil Engineering in the University of Cincinnati, deals largely with the various forms of the elastic arch, and has aimed by presenting, so to speak, a pictorial representation of the thrusts, moments and shears, to make clear those relations which in the analytic method are obscured by the intricacy of the required formulæ. He also gives a new investigation of the continuous girder with various sectional areas, as well as graphical solutions of many problems relating to domes of iron and masonry, the stability of retaining walls and other matters.

Professor Eddy discusses the stability of arch ribs, with and without hinge joints, by means of the "Equilibrium Polygon." In arches a special polygon appertains to a given arch and load, in which the horizontal stress is the actual horizontal thrust of the arch. The methods used to obtain the latter depend upon the separation of the stresses induced by the loading into two parts; one part being sustained by the arch in the same manner as an inverted suspension cable, that is, as an equilibrated linear arch, and the remaining in virtue of its reaction as a girder. When this thrust has been obtained, the equilibrium polygon permits of an immediate determination of all other questions respecting stresses, whether induced by load, change of temperature, or otherwise, because, as demonstrated in two places by Professor Eddy, if the equilibrium polygon and the arch itself regarded as another polygon, are so placed that their "closing lines" coincide, and their areas partially cover each other, the ordinates intercepted between these two polygons are proportional to the real bending moments acting in the arch.

As a practical example of the arch rib with fixed ends, the author takes the St. Louis steel bridge, and confirms the original calculations, and as an illustration of his graphical method applied to the elucidation of the really indeterminate strains on the stays, and stiffening girder of a suspension bridge of the Roebling type, he takes the Cincinnati and Covington Bridge of 1075 ft. span. The author appears to prefer a flexible arch rib with stiffening girder to the rigid rib universally adopted, and would anchor the girder to the piers in such a state of bending tension as to exert considerable pressure upon the arch. This in effect is an adaptation of the American system of "counterbracing" in "quadrangular" trusses to the special case of an arch, and may be advantageous in pin-jointed structures. He makes the somewhat surprising statement that the St.

Louis Bridge (525 ft. span) is wanting in "initial stiffness" to such an extent that the weight of a single person is sufficient to cause a considerable tremor over an entire span. This may appear incredible to engineers who have neither been intrusted with the building of a wrought-iron or steel arch bridge nor have watched with intelligence the proceedings of others, but will be looked upon rather as an interesting confirmation of their own experience by those responsible for the design of such structures. We know of no wrought-iron arch bridge in this country which has not exhibited this sensitiveness to vibration, and so long ago as 1868, in criticising the proposed design for the St. Louis Bridge, we predicted that it would prove unusually "lively," unless the arch rib or spandril filling was made considerably stiffer than was at that time proposed. In execution, the arch rib was made twelve feet instead of eight feet deep, but even then, owing to the intentional absence of all rigidity in the spandril filling the bridge, though enormously strong, is still to the inexperienced eye rather too sensitive to tremors. This, after all, should be no matter for surprise or alarm, since we need not go further than Westminster to find conditions of strata such that the passage of a spring van down the roadway will set thousands of tons of soil and houses into very perceptible vibration.

In the paper entitled "A New General Method in Graphical Statics," a fundamental process or method is established of the same generality as the "Equilibrium Polygon." The new method is designated as that of the Frame Pencil, and both the methods are discussed side by side in order that their reciprocal relationship may be made the more apparent.

Finally, the author, by a presentation of the subject of internal stress from a graphical point of view has, he believes, added a branch to the science of graphical statics which has not heretofore been recognized as susceptible of graphical treatment, and by which the entire investigation is brought within the reach of any one who might wish to understand it.—*Engineering*.

URE'S DICTIONARY OF ARTS, MANUFACTURE, AND MINES. By ROBERT HUNT, F. R. S., assisted by numerous contributors. Vol. IV. Supplement. Longmans, Green and Co. 1878. Price $12.00,

Scientific discoveries follow one another in these days so closely, and the application to the industrial arts takes place so rapidly, that scientific and technical dictionaries must almost immediately after their appearance be followed by the publication of supplemental parts. The volume before us is well up to date; it contains notices of the liquefaction of the so-called permanent gases, and of the telephone. In glancing over the articles devoted to the more important chemical industries, we find them, with few exceptions only, replete with the best information.

The refining of pig-iron, and in particular the elimination of the phosphorus, is a subject of much interest. Jacobi proposes to separate the phosphorus from the ore itself by breaking the latter into small pieces, calcining in vertical kilns, heating the calcined mass in tanks with a solution of sulphurous acid—obtained by burning iron pyrites and condensing the fumes in water—heating the acid solution in a coil of cast-iron pipes, and allowing the liquor to settle in tanks, where the phosphate of alumina deposits in the form of a white powder.

The application of the spectroscope for determining the precise moment, when the charge of pig-iron in the Bessemer converter has been burned into steel, induces the editor to give an epitome of the theory of the spectrum, which will prove acceptable to practical men, though we must warn them against some of his notions. The use of spectral analysis for quantitative determinations, as suggested by Lockyer and Roberts, is mentioned, though we doubt whether it is good for anything.

The articles on water and wine disclose a very dismal state of things. It appears that the so-called filtration of the Thames water by the Metropolitan companies is absolutely worthless. Prof. Frankland has demonstrated that in the river water supplied to London the soluble organic matters and some of the suspended matters of sewage and manure reach the water-drinker in a few hours, and in substantially the same condition in which they leave the sewers and the fields. Turning to the paper on wine, we learn that the manufacture of wines is now carried on as openly as any legitimate trade, sherry and champagne in particular being subject to a complicated artificial treatment. Can the adulteration act not be applied to the vendors of "plastered" and "fortified" sherry and "scented" champagne? The production of aniline without the use of arsenic must be considered a great improvement. The use of salicylic acid for antiseptic purposes seems to spread; not only beer and wine, but bread and meat may be kept by it for several weeks in good condition. In the account of the application of dynamite in the clearing of land, we miss the information supplied by Trautzel in the well-known little book of his. In sugar-refining, evaporation of the syrup by means of blowing air through the mass, has been advantageously used by E. Moride at Nantes. He succeeds in bringing the syrup to 35 and even 40 deg., without changing its color or inverting the crystalline sugar. The saving in time and fuel is considerable. The final evaporation has, however, to be effected in vacuum.

Although the larger and more complete articles in this dictionary are devoted to manufactures and mines, considerable space is devoted to mechanical arts. Among others of these noticed in this division of the supplement before us, is "Agricultural Mechanics," a title much more comprehensive than the articles that have commonly been written under it. In a dictionary we cannot expect to find a treatise on each subject, and must, therefore, overlook incompleteness in the consideration of machinery in detail, and, perhaps, to some extent, number of implements referred to under a title such as the above. But it should be complete as far as it goes. We have in this article reference to steam ploughing apparatus, mowing and reaping machinery, thrashing machines, corn elevators, chaff cutters, and pulpers.

Under the head of electric light, a clear de-

scription is given of the Siemens dynamo-electric machine, and of the Gramme machine, and the principle upon which it is constructed. The description of the former is rendered less clear by the incorrect use of capital letters, instead of italics, in the bottom line of page 339.

In dealing with rock-boring machinery, a very useful and satisfactory sketch of recent advances in this direction is given. Of wood-working machinery some excellent examples are given, but it seems hardly fair that all these should bear the name of one maker, however good. —*Abstract from Engineer.*

THE ELEMENTS OF GRAPHICAL STATICS AND THEIR APPLICATIONS TO FRAMED STRUCTURES, WITH NUMEROUS PRACTICAL EXAMPLES OF CRANES, BRIDGE, ROOF AND SUSPENSION TRUSSES, ETC. By A. JAY DUBOIS, C. E., Ph. D. New York. 1875 John Wiley & Son.

In the course of a review of DuBois' "Graphical Statics," published in the *Zeitschrift des Ver. Deutsch Ing*, the writer says:

"This surprisingly long title is followed by a preface of ten closely-printed pages, which contains notices valuable to the student while using the book. The table of contents, of twelve pages of fine print, is preceded by a four-page note. 'Elements of Graphic Statics,' intended especially for student and teacher. Then follows, under the title 'Introduction,' an excellent and exact translation (!), including references of the capital work of our German colleague, Dr. J Weyrauch, '*Ueber die Graphische Statik,' Leipsig: Verlag von Teubner.* The title of the first chapter, 'Historical and Critical,' is accompanied by an asterisk with the reference 'Weyrauch, U. S. W.; and in his preface DuBois says: 'For the historical and critical introduction we are indebted, a few alterations excepted, to the pen of Weyrauch. It will be useful, &c.. &c.' As regards the 'few alterations' of DuBois, we have not been able to discover them, except in the omission of several scientific references of Weyrauch. The American reader is led to infer from DuBois' method of reference that only one page of his 'Introduction' is taken from Weyrauch; when, in fact, as I find after a thorough examination, there are twenty-seven pages of close translation.

"What particular use was made of Culmann, Mohr, Ritter, Winkler and Reuleaux, and how much Cremona, Favaro and others were studied, after the entire literature had been collated by Weyrauch's diligence for the benefit of the translator, we shall not determine; but to DuBois belongs the credit of industry in collecting, and of the introduction of practical examples."

Such a work will find recognition among German scholars; and because of the want of such a work, long felt, his American colleagues will gladly avail themselves of its benefits; especially in view of the fact that he has made use of the very abundant literature on the subject in German, French, Italian and English, which has appeared since the first edition of Culmann's Graphic Statics, in such a way that one can obtain a comprehensive view of all the works which the subject comprises. Teacher and student are under obligations to him for having made it possible to get a clear apprehension of the controversy concerning the necessity of the use of the methods of the so-called Modern Geometry, as maintained by Culmann, in opposition to the methods of Bauschinger, who dispenses with the Modern Geometry. The author has avoided partial treatment by the introduction of the elegant methods of Von Standt, adapted by Reye to Graphic Statics, into his translation of Weyrauch's work, making its use possible to those who, already engaged in professional life, have never studied the Modern Geometry; and who have, therefore, been obliged to give preferences to the graphic methods deduced by analysis. Though the author inclines to Bauschinger's methods, which prevail in Germany, we must object to his selection of details merely from the diligent work of the German scholar, when he should have presented his elaborated material entire. For example: Bauschinger undertakes the laborious task of determining the center of gravity of a rail-section; DuBois merely gave Bauschinger's diagram—which is a masterpiece of drawing—to the lithographer, who copied it correctly, but not with the delicacy and precision of the original. Besides this, "entire plates show a lack of the care in delineation which is required in a work like this."

We would not convey the impression, by our severe criticism, that this work is not of extraordinary advantage for American students and teachers. On the contrary, we think it will be of great use to practical men, because of the numerous examples; and to the scholar, because of Weyrauch's diligent collation of authorities.

The book is recommended to German readers, especially to students, as a good exercise book in English, on account of its clearness of exposition To the American student it is especially useful, because of the careful directions given in the introduction as to the order of reading and the arrangement of material.

[NOTE.—An abstract of the above review was given in our November issue. As the author expressed some dissatisfaction with the incompleteness of the translation, in his note in the December number, we have given above the entire review.—ED.]

A TEXT-BOOK ON THE STEAM ENGINE. By T. M. GOODEVE, M.A. New York: D. Van Nostrand. (*In preparation.*)

THOSE best acquainted with the subject are aware that the English literature of the steam engine is exceedingly imperfect. A considerable number of treatises on the theory and practice of steam engineering has been written and published, but not more than half a dozen of these treatises deserve a place in a well selected library. The little volume before us may be regarded as one of the best text-books of the steam engine of its size that have yet been produced; and we say this advisedly, and giving due consideration to the writings of Lardner, Bourne, Rankine, Cotterell and Rigg.

Professor Goodeve has given us a treatise on the steam engine which will bear comparison

with anything written by Huxley or Maxwell, and we can award it no higher praise. The author's brief preface so plainly sets forth the plan of the book that we reproduce it just as it stands:—

"The first chapter contains a sketch of the steam engine as it existed in the time of Watt, together with an account of the ideas then prevalent, as to the nature of heat, and concludes with a summary of some physical properties of steam. The second and third chapters are occupied by an investigation of the principles of the modern theory of heat in its applicaton to the steam engine. Then comes a chapter on the conversion of motion, which deals with certain salient points in the mechanism of an engine, The fifth chapter is mainly devoted to the expansion of steam, to the action of valves, and to the application of Watt's indicator. The sixth chapter treats of boilers and the consumption of fuel. The seventh chapter is on compound cylinder engines, and is illustrated by some drawings of the engines constructed by Messrs. Maudslay, Sons & Field, for the White Star line of Mail steamers, making the voyage between Liverpool and New York. Finally, there is a chapter on miscellaneous details, such as steam engine governors, Giffard' s injector, and the link motion The work concludes with a series of examination questions. The author's chief object has been to point out the influence which the change in our views as to the nature of heat has excercised on the practical construction of the steam engine, and he has further endeavored to show the manner in which Watt's diagram of energy has enabled us to accomplish a scientific analysis of the action of heat engines generally, and in particular of the steam engine, under all its varied forms."

Space would fail us did we attempt to follow our author through his volume, and indicate step by step the way in which he has carried out the programme which we have reproduced above. The book is an octavo of 269 pages, and its price is so moderate that no one interested in the steam engine need be without it.

A large portion of the volume is devoted to a consideration of the laws of heat as affecting the steam and other engines, and he has actually succeeded, or we are very much mistaken, in making the theory of heat engines perfectly intelligible without having recourse to any but very simple and easily manipulated formulæ. This is in itself a considerable achievement.

As an example of this useful quality, let us take an instance at haphazard. Our author is dealing with the properties of a vapor, and he points out that a vapor, being a gas, has that property of indefinite expansion which characterizes gases. It follows that if a small quantity of vapor be formed in a closed vessel, it will expand and fill the whole of it. There are, he goes on to say, two cases to be considered: (1) when the vapor is in contact with the generating fluid, (2) when it is entirely separated therefrom. It would be very easy to write many pages considering these two cases, but our author does nothing of the kind; he cites an experiment which places the facts which the student has to learn at once before his mind's eye. We have never seen any allusion to this experiment in any other book, and we therefore reproduce our author's account of the way in which it is to be performed:—

"Take a barometer tube, say, about 33 inches long, and closed at one end. Fill it with clean mercury, which may be done by pouring in mercury nearly to the level of the open end, closing the end with the finger, and then passing the large bubble of air two or three times up and down the tube. This removes all the minute bubbles of air which adhere to the glass, and mercury may be added up to about ½ inch from the open end; then fill this empty space with bisulphide of carbon, a very volatile liquid, and insert the tube in a deep well of clean mercury. The bisulphide of carbon will rise to the top of the tube, vapor will form in the empty space above the mercury, and will, by its pressure, drive down the column of mercury so as to shorten it considerably as compared with the column in an ordinary mercurial barometer. We have accordingly a small layer of liquid lying on the top of the mercury, and several inches of apparently empty space above the liquid. A singular result may now be exhibited. Depress the tube by the finger so as to sink it in the well or cause it to rise higher, when it will be found that the height of the column within the tube—measured from the surface of that in the well—remains absolutely constant. If the tube be raised quickly the liquid begins to boil, fresh vapor is formed instantly, and the pressure is kept at a constant intensity; on the other hand, a portion of the vapor passes into the liquid state, when the space which it fills is contracted, and nothing will alter permanently the height of the mercurial column except a permanent change in the temperature of the liquid and the tube.

No illustration of the operation of a great natural law could, we think, be happier. Hypercritics may take exception to the use made by the author of the word vapor, yet it would perhaps be impossible to find any other which would answer the required purpose equally well.

Not the least valuable portion of Mr. Goodeve's work is that in which he handles Carnot's principle, which affords a remarkable example of a great truth evolved from a wholly erroneous premise. Carnot believed that heat was a material fluid, and yet he evolved the truth that as the whole work done by a heat engine is traceable to the disappearance of heat, and to that alone, it follows that a heat engine is entirely independent of the nature of the substance with which it performs its functions. Whether the apparatus is a steam, air, or ether engine, the result is the same. The amount of work done by a reversible heat engine depends only on the constant temperatures at which heat is received and at which it is rejected, and is uncontrolled by the nature of the intermediary agent, such as steam or air. It is impossible to attach too much importance to this law. Because it has been overlooked, much money has been wasted in vain endeavors to attain an economy which cannot be had

—*Abstract of a Review from Engineer.*

VAN NOSTRAND'S

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NO. CXXII.—FEBRUARY, 1879.—VOL. XX.

A PRACTICAL THEORY OF VOUSSOIR ARCHES. PART II.

By WM. CAIN, C.E., Carolina Military Institute, Charlotte, N. C.

Contributed to VAN NOSTRAND'S MAGAZINE.

II.

HEIGHT OF SURCHARGE, IN ORDER THAT THE CENTER LINE OF ARCH RING, MAY BE A POSSIBLE CURVE OF PRESSURES.

51. Let it be required to find the proper height from the soffit to the top of roadway, in order that the center line of the arch may be a possible line of pressures.

The black line at the top, Fig 8 shows the line of roadway for the segmental arch of 100′ radius and 4′ depth of keystone, found by the following approximate construction:

Divide the semi arch into nine portions aa_1, a_1a_2, . . . , each of the same horizontal length; the weight of each portion is nearly proportional to its medial vertical line, limited by the soffit and roadway (yet to be found), drawn through the centers c_1, c_2, ... The horizontal $\overline{aA}$ and the line Aa_9 drawn tangent to the center line at a_9 represent the directions of the resultant pressures at a and a_9, assuming the pressure at a_9 to be tangent to the center line. From the point A draw $\overline{A_2}$, $\overline{A_3}$, . . . parallel to $c_1, c_2, c_2c_3, \ldots$; the points of the type c lying in the center of the arch ring; also draw the vertical $\overline{1, 10}$, at such a distance $\overline{A1}$, that $\overline{12}$ is equal to the distance from the soffit at c_1 to the roadway. Then $\overline{23}$, $\overline{34}$, ... are the heights from the soffit to the roadway at c_2, c_3, ... , which may now be laid off as in the figure.

The accurate construction is as follows: the horizontal thrust acting at a must be combined, at the intersection of $\overline{aA}$ with the vertical through c_1, with such a force $\overline{12}$ (force diagram) that the resultant will pass through the center line at a_1. This resultant is, in turn, produced to intersection with the vertical through c_2, where it must be combined with such a force $\overline{23}$ that the resultant $\overline{A3}$ will pass through a_2, and so on; the forces 1 . . . . . 10 being thus found and laid off as before. Now, since the resultants at a_1, a_2, ... are tangent, or very nearly tangent for a segmental arch, to the center line, their directions are evidently parallel to the chords $\overline{c_1, c_2}$, $\overline{c_2, c_3}$, . . . as assumed in the first construction.

It is seen from Fig. 8, that a lightening of the spandrel walls, about from a_3 to a_8, conduces to stability. This is often done in large bridges. By this means the ignorant mason who built the Pont-y-Tu-Prydd arch of 140 feet span, 35 feet rise and only 1 foot 6 inches depth of rubble arch ring in the body of the arch, managed to cause the bridge to stand; which when first built fell, by the weight of the haunches forcing up the crown.

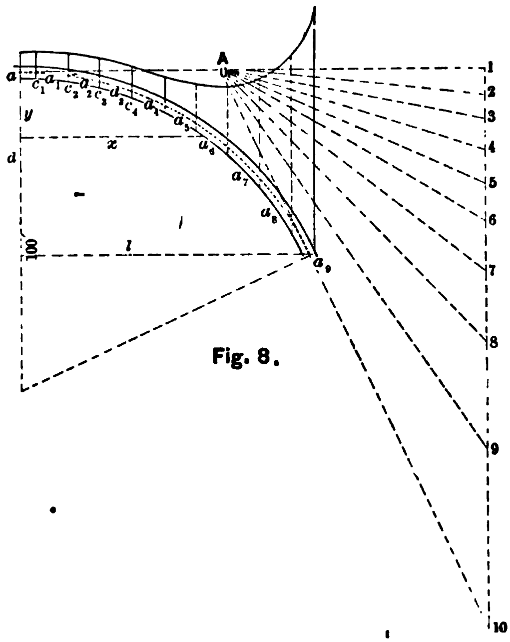


Fig. 8.

On being rebuilt, the spandrels were lightened by cylindrical openings, the spaces between, being filled with charcoal (see VAN NOSTRAND'S MAGAZINE for March 1873, p. 193) and the bridge stands to this day. Though it evidently does not admit of heavy rolling loads, may suffice for a light traffic. As a precedent however, in construction it is to be avoided.

52. On testing a parabolic arch of 200′ span and 100′ rise in the same way, the line limiting the roadway will be found to be everywhere the same vertical distance from the soffit; so that where the surcharge is very high the parabola is the best form of arch ring.

This may be shown analytically, in an easy manner. Thus conceive $a_1, a_2 \ldots a_9$ consist of a thin metallic ring, that is to sustain a uniform horizontal load, w per foot, without bending; required the form of the curve $a \ldots a_9$. It is necessary that the line of pressures coincides throughout with the rib, for if it departs from it at any point, the resultant on that point multiplied by its lever arm to that point, gives a bending moment, which the thin rib is supposed incapable of resisting. (See art. 4 Fig. 2.)

Let $\overline{aA}$ be the axis of X, the vertical down from a the axis of Y. The resultant at a is the horizontal thrust Q. Now take moments of this force, and the downward acting weight on the part aa_6, about a_6, whose coördinates are y and x,

$$\therefore M = Qy - \frac{Wx^2}{2}.$$

Now M must equal zero for every point of the arch, in which case *the line of pressures will coincide with the figure of the rib.*

Placing $M = o$, we deduce,

$$x^2 = \frac{2Q}{W} y$$

the equation of a parabola, Q.E.D.

53. We see from the foregoing that for a simple arch ring, or for a uniform horizontal load on the ring, and approximately for a very deep surcharge, level at top, the parabola is the best form for the arch ring.

For bridges level at top, at least for bridges whose rise is not over $\frac{1}{8}$ span, the circular is a better form than the parabolic. It is needless to speak of the superiority of the segmental arch over the elliptical and allied forms, whose inherent weakness at the haunches is generally remedied by a greater depth of arch ring, a sufficient reason for choosing

"The rainbow's lovely form"

as the best figure for an arch, however beautiful the elliptical or oval curve may be considered in itself.

54. Fig. 8 will suffice to illustrate the common method, as given by many authors, of binding the curve of pressures. Extend $\overline{aA}$ to c_1, then draw $c_1c_2 \parallel A_2$, which is thus the resultant on joint a_1. Extend this resultant to intersection with vertical through c_2, from which

point draw $c_2c_2 \parallel A_2$ and so on. This method, so simple in theory, does not work well in practice, owing to the fact that any error made in finding any point of the curve of pressures is carried on; whereas by the method given in art. 2, any error made is confined to the joint where it is made. It is evident that by the latter method, using *straight* edged rulers and triangles (steel are the best), feathered edged scales, prickers, hard pencils and smooth paper, that the centers of pressure should be found almost to a very needle point. Such accuracy is moreover essential to properly testing an arch ring. It is well to observe that the constructions can be made by drawing only a very few lines. In fact too many lines only serve to confuse the drawing and should be avoided.

CURVE OF PRESSURES CORRESPONDING TO THE MINIMUM AND ALSO TO THE MAXIMUM HORIZONTAL THRUST IN AN UNSYMMETRICAL ARCH.

55. These cases were not demonstrated for an unsymmetrical arch in Part 1, though they offer no difficulty, and are essential to a complete investigation of the stability of an arch. In Fig. 9, *eca* represents an unsymmetrical arch, or an arch acted on by forces, not symmetrical —vertical or inclined.

Let P = resultant of the external forces, acting on the arch between a and c, not including the reaction R at a. Then on combining $R = \overline{ak}$, with P, we get the center of pressure c on the joint cc_1. Similarly we could proceed for other points b, d, e, of the curve of pressures,

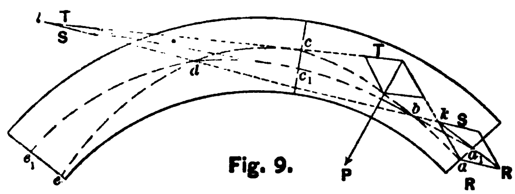


Fig. 9.

corresponding to the resultant $R = \overline{ah}$, acting through a in the direction $\overline{ak}$.

Let a_1 b c_1 d e_1 be a second curve, corresponding to the reaction R′ at a_1. Now *if* S *is such a force, acting towards the left, that when combined with* R, *it gives* R′ *as a resultant*, we can find a point c_1, on joint cc_1, of the new curve of pressures, either by combining R′ with P as before, or by combining its components with P: thus call the resultant of R and P, T; this combined at l with S, gives a resultant which cuts joint cc_1 at c_1, *a point lying between* $\overline{kl}$ and c, $\overline{kl}$ being in the direction of s produced.

56. By this construction, it is seen that the new curve of pressures, corresponding to the reaction R′ at a', *passes through b and d* the points where kl intersects the first curve of pressures; for other joints, as ee_1, the new curve lies nearer $\overline{kl}$ than the first curve; since when s acts to the left, the combination of T, for any joint, with S gives a resultant acting *between* T and S, which therefore cuts the joint nearer $\overline{kl}$ than the first curve; since when S acts to the left, the combination of T, for any joint, with S, gives a resultant acting *between* T and S, which therefore cuts the joint nearer $\overline{kl}$ than the first center of pressure.

The above supposes that neither R nor S are vertical, but that both act to the left, whence the horizontal component of R′ exceeds that of R. The joints are, moreover, not supposed inclined more than 90° from the vertical counting from the top.

57. *Prop. If two curves of pressure cut each other, the curve which lies nearest the straight line, which joins their common points, corresponds to the greatest horizontal thrust.*

We have seen in the preceding article that the two curves *can only* intersect on the straight line kl (Fig. 10) as implied in the proposition.

Now if, at any joint cc_1, the center of pressure c_1, corresponding to the curve $a_1bc_1de_1$, lies nearer $\overline{kl}$, the straight line joining b and d, than the curve $abcde$, then we may suppose a force S, acting in the direction $\overline{kl}$, to be combined with T at l, to effect it. The force S, thus found, must therefore when combined with R at

a give R′; since R and S produce the some effect as R′; so that all points of the first curve can be found by combining R with the resultant of the forces P, up to the joint, and afterwards combining their resultant with S.

The force S, acting to the left, increases the horizontal component of the resultants on each joint; hence the curve $a_1bc_1de_1$ corresponds to greater horizontal thrusts than the curve *abcde*, as stated in the proposition.

If the arch is symmetrical, the curves of pressure are symmetrical with respect to the crown, whence $\overline{kl}$ must be horizontal, whence follows the conclusions of art. 4. Part 1 demonstrated these in another manner.

58. 1/. *If a curve of pressures has two points common to the intrados and an intermediate point common to the extrados, it corresponds to the minimum horizontal thrust.*

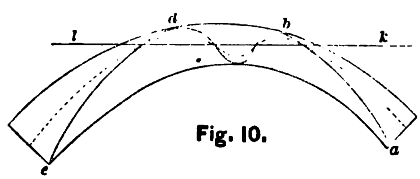


Fig. 10.

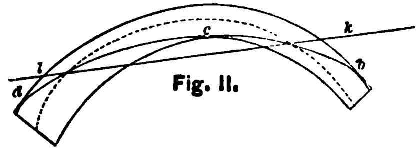


Fig. 11.

For, suppose the curve *abcde*, Fig. 10, touches the extrados near *c*, the intrados on both sides nearer the abutments.

Then any other curve of pressures, a_1 bc_1 de_1 that remain in the arch ring, must cut the first, only in points on the straight line $\overline{kl}$, joining any two points of intersection.

Now the new curve, near the points of contact of the first curve with the contour curves of the arch ring, must, if it remains in the arch ring, pass nearer $\overline{kl}$ than the first curve; whence, by *Prop.* art. 57, the first curve corresponds to a less horizontal thrust. Q.E.D.

2/. *If a curve of pressures abcde, Fig.* 11, *has two points of contact b and d with the extrados, and an intermediate point of contact c with the intrados, it corresponds to a minimum horizontal thrust, if bcd, in the vicinity of c, lies between the intrados and the straight line* $\overline{bd}$.

For any other curve, lying in the arch ring, as the dotted curve, *must* lie nearer the straight line *kl*, joining their points of intersection, than the first, in the vicinity of *b c* and *d*, and thus corresponds to a greater horizontal thrust.

3/. *If however, the intrados, in the vicinity of c lies between the curve bcd, Fig.* 12, *and the straight line* $\overline{bd}$, *the curve corresponds to a maximum horizontal thrust;* since this curve lies nearer $\overline{kl}$ than any other as the dotted curve of pressures.

It is seen that $\overline{kl}$ in Fig. 11 lies above $\overline{bd}$, whereas the reverse occurs in Fig. 10.

59. When a curve of pressures possesses both the properties of the maximum and minimum of the thrust, the arch is at the limit of stability; as see all the figures relating to the experiments in Part I. The above principles were first stated by Dr. Hermann Scheffler.

60. When the arch is symmetrical, the curves of pressure are so, whence follow the conclusions of arts. 5 and 6 of Part I, including the cases not there demonstrated. The second case of the minimum is but rarely, if ever, found in practice; and of course does not occur when the first case obtains.

61. For the mature hypothesis of incompressible voussoirs, that curve of pressures which corresponds to the minimum of the thrust is, by the principle of the least resistance, the true one.

We have previously explained in art. 27 how this principle is modified for materials used in practice, both by their compressibility and by the cutting of the stones.

We infer, from the reasoning in art. 27, that with well fitting stones and unyielding abutments, that the actual line of pressures in an arch is found within limits, approximately equidistant from the center line of the arch ring, and having the characteristics of both the max. and the min. of the thrust within those limits. Thus in Fig. 12, the dotted line represents the actual line of press-

ures within the limits drawn, since the part *abc* corresponds to the max. and *bcd*

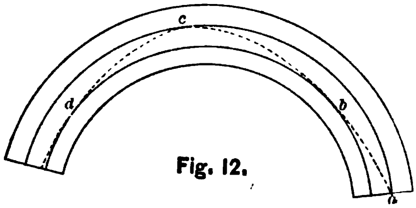


Fig. 12.

to the min. of the thrust within those limits; for *b* lies above a straight line drawn from *a* to *c* (art. 58, /3), and *c* lies between *b* and *d* (art. 58, /1). If the abutments yielded however, the true curve is found within wider limits, and simply corresponds to the min. of the thrust within those limits (see art. 28).

In view however of the influences of temperature, not fitting the stones perfectly, yielding of piers and abutments and shocks, it is recommended to require *in designing an arch*, that for any position of the rolling load, the max. and min. line of pressures may be drawn within somewhat narrower limits than the middle third, otherwise increase the depth an amount, to be left to the judgment of the engineer, as suggested in art. 29.

PRESSURES PER SQUARE UNIT IN EXISTING BRIDGES.

62. By computation, it is found, that in existing bridges, the pressure per square foot on the voussoirs at the crown, *supposing this pressure uniformly distributed*, varies from less than one ton per square foot, for the smallest arches, to 20 tons per square foot and over, for arches of 150 to 200 feet span; the strains being estimated for dead load only.

The normal pressure per square foot at the abutment is greater, often three or four times the above.

It is thus the practice to increase the unit strains with the span.

The material should not be strained at the most compressed edge more than one fifth the crushing weight. Therefore, at the joints of rupture, where the resultant is supposed to pass one-third depth joint from an edge, and the pressure per square foot at the most compressed edge is *double the mean*, the material should not be strained to more than one-tenth the crushing weight, *if* the pressure is supposed uniformly distributed. On this supposition, if we take the crushing weights of sandstone, limestone and granite (which vary between wide limits) as 300, 400 and 500 tons per square foot, respectively, the allowable strains at the most compressed edges will be 30, 40 and 50 tons. If weaker materials are used the unit strains must be less, and we should increase the depth of voussoirs.

If the curved courses of bricks are in concentric rolls, without bond, the determination of the line of pressures, as well as the distribution of the pressure on each course, becomes uncertain and indeterminate. As a rough guess, if there are n rolls, each roll may be supposed to be $\frac{1}{n}$ of the pressure. As the outer roll has the greatest span, it is only necessary to test its stability under $\frac{1}{n}$ the total load. It is not recommended, though, to trust to any such rule, but to bond the rolls, using strong cement; so as to approximate the structure to a "rigid arch."

ARCHES WITH VERTICAL AND HORIZONTAL LOADS.

63. In the arch ADB, Fig. 13, suppose it required to pass a curve of pressures through the points A, E and B.

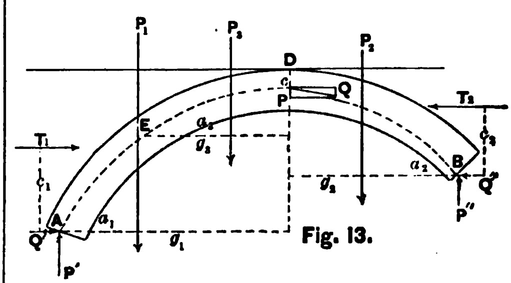


Fig. 13.

Let C be another point of this curve, at the crown, where the horizontal component of the pressure is Q, the vertical component P. Call the vertical components of the loads on the segments AD, DB and ED, P_1, P_2, P_3, respectively; their *horizontal* components T_1, T_2, T_3, respectively.

Call the perpendicular distances from P_1 and T_1 to A, a_1 and c_1; from P_2 and T_2 to B, a_2 and c_2; and from P_3 and T_3 to E, a_3 and c_3, respectively.

Also call the vertical distances of C' the point of application of the inclined thrust at the crown, above A, B and E, b_1, b_2 and b_3 respectively; and the horizontal distances of the same points A, B, E, from the crown, g_1, g_2 and g_3.

64. We now take moments in turn about A, B and E. In eqs. (9) and (11), we suppose the arch to the right of the crown removed, and its effect replaced by the resultant of P and Q acting to the left, P being + when acting upwards; in eq. (10), the part left of the crown is supposed removed and a force equal and directly opposed to the resultant of P and Q acting to the right.

We thus find:

$$a_1P_1 - g_1P + c_1T_1 = b_1Q \quad . \; . \; . \; . \quad (9)$$
$$a_2P_2 + g_2P + c_2T_2 = b_2Q \quad . \; . \; . \; . \quad (10)$$
$$a_3P_3 - g_3P + c_3T_3 = b_3Q \quad . \; . \; . \; . \quad (11)$$

Equating the values of Q in (9) and (10), we find,

$$P = \frac{b_2(a_1P_1 + c_1T_1) - b_1(a_2P_2 + c_2T_2)}{b_2g_1 + b_1g_2} \quad . \; . \quad (12)$$

From (9) we obtain,

$$Q = \frac{a_1P_1 - g_1P + c_1T_1}{b_1} \quad . \; . \; . \; . \quad (13)$$

These equations suffice to determine P and Q, when the position of C is known. When, however, we can only locate the points A, E and B, the values of P and Q and the position of C is found as follows.

For convenience let us make the following abbreviations:

$$g_2 + g_3 = d_1, \quad g_1 - g_3 = d_2, \quad g_1 + g_2 = d_3,$$
$$b_2 - b_3 = e_1, \; b_1 - b_3 = e_2, \; b_1 - b_2 = r.$$

Now subtract (10) from (9),

$$a_1P_1 - a_2P_2 - Pd_3 + c_1T_1 - c_2T_2 = Qe_3 \,. \quad (13a)$$

also, subtract (11) from (9)

$$a_1P_1 - a_3P_3 - Pd_2 + c_1T_1 - c_3T_3 = Qe_2.$$

Equating the values of Q drawn from these last two equations, and noting that

$$a_1P_1(e_2 - e_3) = a_1P_1e_1; \; c_1T_1\,(e_2 - e_3) = c_1T_1e_1$$

we have,

$$P = \frac{e_1(a_1P_1 + c_1T_1) - e_3(a_2P_2 + c_2T_2) + e_2(a_3P_3 + c_3T_3)}{e_2d_3 - e_3d_2} \quad . \; . \; . \quad (14)$$

Substituting in eq. (13*a*) the value of P just found, reducing the terms of one member to the same denominator, collecting like terms, whose coefficients are of the type *ed*, and noting that, $e_2 - e_1 = e_3$ and $d_3 - d_2 = d_1$, we have,

$$Q = \frac{d_1(a_1P_1 + c_1T_1) + d_3(a_2P_2 + c_2T_2) - d_2(a_3P_3 + c_3T_3)}{e_2d_3 - e_3d_2} \quad . \; . \; . \quad (15)$$

From (9), we have,

$$b_1 = \frac{a_1P_1 - g_1P + c_1T_1}{Q} \quad . \; . \; . \quad (16)$$

to fix the position of C at the crown.

We have always for the reactions at A and B, $P' = P_1 - P$, $P'' = P_2 + P$, $Q' = Q - T_1$, $Q'' = Q - T_2$.

65. The above equations apply directly to unsymmetrical arches, solicited only by vertical forces by making T_1, T_2 and T_3 zero. Compare Part 1 art. 12.

When the arch and load is symmetrical, $P = O$. If the point of application at the crown is known, we have from (13),

$$Q = \frac{a_1P_1 + c_1T_1}{b_1} \quad . \; . \; . \quad (17)$$

If two points A and E are given, we have then $g_1 = g_2$, $h_1 = h_2$, $d_3 = 2g_1$, $e_3 = o$, $P_1 = P_2$, $T_1 = T_2$, $a_1 = a_2$, $c_1 = c_2$; whence from (15),

$$Q = \frac{a_1P_1 + c_1T_1 - (a_3P_3 + c_3T_3)}{e_2} \quad . \; . \; . \quad (18)$$

The position of Q is then found from (16) by making $P = o$.

Eq. (18) is very easily deduced independently.

66. *Application to Underground Arches.*

Let Fig. 14 represent a culvert, with the embankment above it partially completed; so that when the material of the embankment is reduced to the same specific gravity as that of the arch, a line $\overline{ai}$ will limit its top; the earth being level to the left of *a* and to the right of *i*.

If the surcharge is of the same specific gravity up to *bh*, as the voussoirs, then if the earth has a natural slope, the line $\overline{ai}$ will be straight, as drawn; otherwise it may be curved.

The tables for the *vertical forces* are made out as usual. The mean heights of the trapezoids are represented by the dotted lines and the sum of the first

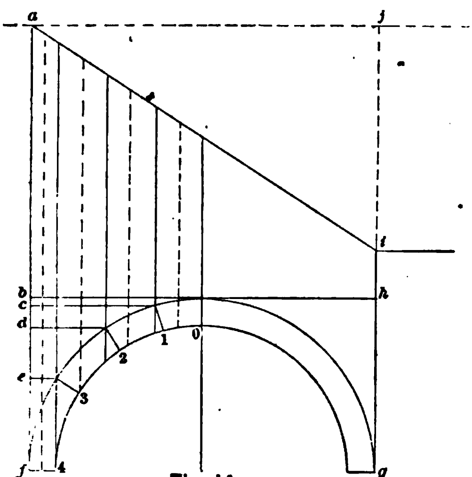


Fig. 14.

three trapezoids will be considered as the surface from the crown to the third joint; similarly for other joints.

This is a sufficiently near approximation for a deep surcharge. For greater accuracy the method detailed in art. 31 may be used.

The horizontal forces are due to the earth pressure and are very difficult to estimate exactly. In a mass of earth with an *unlimited level surface*, the horizontal pressure per square unit at a depth x*

$$p = wx\frac{1-\sin.\ \Phi}{1+\sin.\ \Phi} = wx\ tan.^2\ (45^\circ - \tfrac{1}{2}\Phi).$$

When the upper surface is at the angle of repose Φ, the pressure per squre unit, parallel to the slope, is,

$$p' = wx\ \cos.\ \Phi.$$

w represents the weight per cubic unit of the earth.

These formulæ are modified, when the earth is not of unlimited extent, the friction of the abutting surfaces causing a change in the direction of the pressure.

Again, the surface above is sloping from a to i, and level elsewhere.

Cohesion, likewise plays an important *role* in earth pressure; its influence becoming much more marked as the embankment grows older. For new embankments it is well to neglect it.

Let us assume, as an approximation, that the horizontal pressures, due to the earth, on voussoirs 1, 2, 3 and 4, are due to the heights x measured along the dotted lines from the extrados of each voussoir to the surface of level topped earth.

The surfaces against which these pressures act for voussoirs 1, 2, 3, 4, are, $\overline{bc}$, $\overline{cd}$, $\overline{de}$, $\overline{ef}$, respectively; so that the horizontal pressure acting on the third voussoir, for instance, is equal to the product of the height $\overline{de}$, by the height of the surcharge from the extrados to the surface, by $tan^2\ (45 - \frac{1}{2}\Phi)$, ($w$ being taken as unity). In the following examples let $\Phi = 30^\circ$, so that, $tan^2\ (45 - \frac{1}{2}\Phi) = \frac{1}{3}$.

The horizontal pressure then upon the third voussoir is, $\overline{de} \times x \times \frac{1}{3}$. It may be written $\frac{yx}{3}$ for any voussoir. The lever arms of these forces, about the top of the arch, are the vertical distances from the line $\overline{bh}$ to the middle of the segments $\overline{bc}$, $\overline{cd}$, $\overline{de}$ and $\overline{ef}$. The moment of these forces, down to any joint, divided by the sum of the same forces, gives the vertical distance from the line $\overline{bh}$ to the resultant of the forces taken, as given in the last column of the following tables concerning horizontal forces.

67. *Example.*—Let the span of the semi-circular culvert be, 11.30 units of length, the depth of voussoir 0.94, the height $\overline{ab}$ of the reduced surcharge 25.12, and the height $\overline{hi}$, 12.56. The billing up to $\overline{bh}$ is taken of the same density as the voussoirs. If the backing was solid up to $\overline{bh}$, the horizontal forces would be due more nearly to the depth from a to the voussoirs on the left, and from i on the right.

Each of the semi-arcs with its load is divided, as shown in Fig. 15, into eight parts (approximate trapezoids), of which the first six have a width of 0.94, the two last a width of 0.47.

In the following table for vertical forces, column (1) gives the joint, columns (2) and (3) the force from the crown to the joint and its lever arm, respectively, for the left semi-arch, columns (4) and (5), giving the same quantities for the right semi-arch:

* See Rankine's Civil Engineering, p. 322.

	Left Side.		Right Side.	
Joint.	Force.	Lever Arm.	Force.	Lever Arm.
(1)	(2)	(3)	(4)	(5)
1	18.98	0.47	18.25	0.47
2	38.99	0.95	34.07	0.93
3	60.24	1.45	50.98	1.39
4	82.79	1.95	67.57	1.85
5	106.87	2.47	84.16	2.31
6	133.23	3.01	101.28	2.79
7	147.84	3.29	110.58	3.05
8	162.63	3.57	119.68	3.29

The next table refers to the horizontal forces; column (1) referring to the joint, column (2) gives the product $\frac{1}{2}yx$ (see art. 66), column (3), its lever arm about the summit, column (4), the moment, columns (5), (6) and (7) the sum of the forces down to any joint, their moment and the distance of their resultant below the line $\overline{bh}$, respectively.

	Left Side.			Horizontal Forces.		
Joint.	Force.	Lever Arm.	Moment.	Force.	Moment.	Lever arm.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	0.58	0.05	0.03	0.58	0.03	0.05
2	1.29	0.19	0.25	1.87	0.28	0.14
3	2.73	0.47	1.28	4.60	1.56	0.34
4	4.35	0.94	4.09	8.95	5.65	0.63
5	6.37	1.62	10.33	15.32	15.98	1.04
6	11.09	2.64	29.28	26.41	45.26	1.71
7	9.40	3.77	35.44	35.81	80.70	2.25
8	23.46	5.43	127.39	59.27	208.09	3.51
	59.27		208.09			

Columns (1), (5) and (7) are next given for the right side.

Right Side.			Horizontal Forces.		
Joint	Force.	Lever Arm.	Joint	Force.	Lever Arm.
(1)	(5)	(7)	(1)	(5)	(7)
1	0.59	0.05	5	11.56	0.99
2	1.64	0.14	6	18.51	1.62
3	3.80	0.33	7	24.17	2.12
4	6.96	0.60	8	38.25	3.34

Now let it be required to pass a curve of pressures, 0.41, below the top of the crown joint, and through the lower middle third limits, at joints six on either side.

Now the vertical loads from the crown to joints six on left and right are (see table) $P_1=133.23$, $P_2=101.28$; the distances of their resultants from the vertical through the crown are 3.01 and 2.79 respectively; whence by measurement on the drawing, $g_1=g_2=5.1$ and $a_1=5.1-3=2.1$, $a_2=5.1-2.8=2.3$.

Similarly,

$$T_1=26.41,\ c_1=3.5-1.7=1.8$$
$$T_2=18.51,\ c_2=3.5-1.6=1.9$$

whence by eq. 12, art. 64,

$$P=\frac{a_1P_1+c_1T_1-(a_2P_2+c_2T_2)}{2g_1}=5.8$$

Also by eq. (13), Q=96.

Now lay off on vertical lines, $\overline{08}$, to left and right of the center, the numbers in columns 2 and 4 respectively, being the vertical loads from the crown to the joints in order. From columns (3) and (5) of the same table lay off the distances, on the horizontal through the summit, from the crown to the centers of gravity of the vertical loads in order. Thus $\overline{S6}=3.01$ corresponding to $P_1=133.23$.

Next, from the tables referring to horizontal forces, lay off on the horizontals $\overline{08'}$, the forces given in columns 5, for the left and right side respectively. Also lay off on vertical lines the numbers in columns (7), measuring from the line $\overline{gS}$. Thus the total horizontal earth thrust from the crown to joint 8 on the left is $T_1=26.41$; and its point of application is $\overline{g6}=1.71$ below the summit. To find the thrust at the crown, lay off $\overline{mn}$ =Q horizontally, and $\overline{no}$=P vertically downwards: $\overline{mo}$ is then the resultant at the crown joint in position and magnitude. Draw the lines $\overline{44''}$, $\overline{55''}$... parallel and equal to $\overline{mo}$. Now, to find the center of pressure on a joint, as the 6th on the left, draw vertical and horizontal lines $\overline{6b}$, $\overline{6b}$, through the points of application of P_1 and T_1, to the intersection b; which is thus the point of application of the resultant of P_1 and T_1, represented by the line 66′ in the force polygon on the left. From b draw $ba \parallel \overline{66'}$ to inter-

section a with $\overline{mo}$ produced; from a draw $\overline{ac} \parallel \overline{6'6''}$ to intersection with joint 6 at its center of pressure. It is evident that the resultant there is represented by the line $\overline{6'6''}$, the resultant of $\overline{06}$, $\overline{06'}$ and $\overline{66''}$, or of P_1, T_1 and the inclined thrust at the crown; similarly on the right side, to find the position of the resultant on joint 8, we find d, 3.29 to the right of S and 3.34 below it; thence draw $\overline{de} \parallel \overline{88'}$ to intersection e with $\overline{mo}$ produced; thence draw $\overline{ef} \parallel \overline{8'8''}$ to f the required point; the magnitude and direction of the resultant being represented there by the line $\overline{8'8''}$.

The line of pressures thus found, represented by the dotted line, leaves the middle third at joints 4,5 and 8 on the right, and at joint 8 on the left.

By raising the point m nearly to the upper middle third limit, and the center of pressure at joint 5 on the right to the lower middle third limit, the curve of pressures will remain within the inner third except near the abutment. The arch stones should be increased in depth there, up to about joints 6.

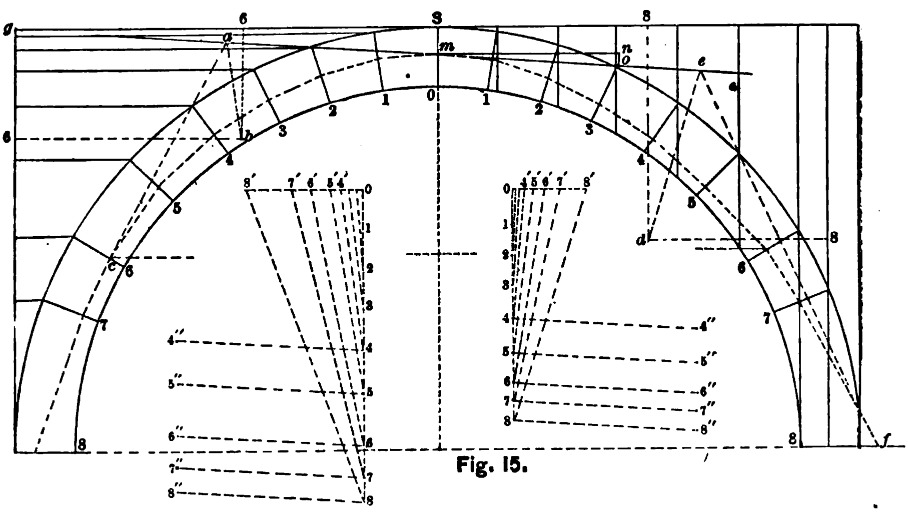


Fig. 15.

The earth is next supposed *level at top*, the distance $ba=hj$, fig. 14, being 25.12 units. On making out a table of vertical and horizontal forces as before, for one side only, we find from eq. (17), art. 65, $Q=122.2$ cubic units of stone, on passing a curve of pressures through the upper middle third limit at the crown and the lower middle third limit at joint 6.

The curve thus found keeps everywhere in the middle third except at joints 8, where it nearly reaches the extrados.

68. If the arch stones are not increased in depth near the abutment, joint 8 will tend to open at the intrados; but this it cannot do unless the haunches spread; which is in turn resisted by the spandrels; or if there are none, by an increased horizontal thrust which the earth is capable of putting forth, thus keeping the line of pressures within the arch ring, *e.g.*, within the middle third if the deformation that the earth permits is small.

Experience shows that very thin arch rings, built in rubble, often can fulfill the conditions of stability when embanked over carefully; the centers being struck after the embankment is mostly completed.

In such cases the earth must exert larger horizontal forces, than given above; so that it is well to be guided mainly by experience in designing underground arches as before remarked.

By increasing the depth of arch stones near the abutment, as suggested, we are safe in presuming on stability without the aid of extra horizontal forces over the ordinary active earth thrusts (see art. 50).

69. The dimensions of the preceding culvert and surcharge may be taken in any unit as feet, meters, etc.

If the unit taken is the meter it corre-

sponds to a railroad culvert at Schwelm, the top of the embankment being $31.^m40$ above the top of the arch, corresponding to a weight $25.^m12$ high of materials as dense as the voussoirs, as given by Scheffler. From the diagram for the earth level at top, we find that the normal components of the pressure on joints 6 and 8 are about 180 cubic meters of stone; so that, if uniformly distributed, the pressure per square meter would be $\frac{180}{0.94}=192$ cubic meters of stone.

If we take the weight of a cubic foot of stone at 140 pounds (very low), since there is about 35 cubic feet in one cubic meter, 192 cubic meter$=140\times35\times192$ pounds$=448$ tons. This pressure being on one square meters$=10.8$ square feet, the pressure per square foot$=41.5$ tons. If the line of pressures, at joints 6 and 8, is $\frac{1}{3}$, $\frac{1}{4}$ depth joint from edge, the pressure per square foot at the most compressed edges is 83, 111 tons respectively (see arts. 21, 22).

If the material is nowhere to be subjected to more than $\frac{1}{5}$ the crushing weight, it is evident that the best granite or limestone should have been employed in this bridge, having a crushing weight of 400 to 500 tons per square foot. It is stated that the material was not of a good character, and that a large number of voussoirs in all parts of the arch were crushed in consequence.

If the unit of length taken is the foot in the preceding example, we see, that a semi-circular arch of 11.3 feet span, and 1 foot depth of voussoir, is stable against rotation, when the top of the embankment is about 30 to 40 feet above the crown, provided the voussoirs are increased in depth towards the springing, say to 2 feet. The normal pressure now on joint 6 is 180 cubic feet stone$=11.3$ tons per square foot. At the most compressed edge the pressure is 22.6, if no joint opens, which even sandstone can very well stand.

Any multiple of these dimensions, as 22′.6 span, 2′ depth voussoir and 60′ to 70′ height of surcharge, will show equal stability against rotation. The thrust is now 180 cubic units of stone$=1440$ cubic feet of stone$=90.4$ tons. This acts on a surface of 4 square feet; so that the uniform compression is 22.6 tons and the compression at the edge of joint 6 is 45.2 tons about; which sandstone can again bear.

70. It is evident that the height of surcharge should be considered in designing culverts; though it is neglected in the practical formulæ proposed by some authors.

Trautwine (Engineers' Pocket Book p. 347) says:

We have known nearly semicircular arches of 30 to 40 feet span, to be thus built successfully (*i.e.*, when the centers are left standing until the earth-fillling is completed above the culvert) with scarcely a particle of masonary above the springs to back them." He recommends *not to do less*, and that only in small spans, than make the height of backing above the springing over the abutment, $\frac{1}{2}$ the total height of the arch from the springing to the top of keystone; and from the point so found to draw a tangent to the arch to limit the backing. As suggested above, it is still better to increase the depth of voussoirs towards the abutment.

Rankine well suggests that "over the arches of culverts, the earth rammed in thin layers should rise to at least half the height of the proposed embankment; the remainder may be tipped in the usual way."

71. It is more than probable that in culverts, where the loose earth is deposited after the culvert is built, especially if the centers are not struck until the embankment is completed over the arch, that the *entire height* of surcharge *does not* press upon the culvert. Cohesion and friction both influence the result. Thus if the grains of earth had the cohesion of stone, the *slight sinking of the culvert*, due to its compressibility, would relieve it of part of the load above it, especially in small culverts. The same sinking would cause a portion of the weight above the arch to be transmitted by friction to the sides:

Thus let s=span of culvert, h=height of surcharge above its top. Now the horizontal pressure of the earth at a depth x on a surface $\triangle x$ being, wx tan.^{2} $(45°-\frac{1}{2}\,\Phi)\triangle x$ nearly, the limit of the sum of quantities of this type between the limits, $x=h$ and $x=o$, $=\frac{Wh^2}{2}$tan.$(45°-\frac{1}{2})\Phi$, is the total horizontal earth thrust exerted on a vertical plane 1 foot wide and h deep below the surface.

If the culvert were suddenly removed, the mass above, if it had no friction or cohesion, would, like a perfect fluid fall, the top surface changing. to a lower level. Now consider friction alone, its coefficient being called f=tan. Φ. The weight of the mass above the arch is, wsh; the friction of the vertical paral-

lel walls on either side is $2f\frac{wh^2}{2}\tan.^2(45°-\frac{1}{2}\Phi)$

Now if the arch yields somewhat, the weight still sustained by it is the difference between these expressions.

There is no weight sustained by the arch for,

$$h=\frac{s}{f\tan.^2(45°-\frac{1}{2}\Phi)}$$

and for one half this height, the difference above gives the maximum weight sustained by the lowered arch.

Thus if $s=15$ feet and $\Phi=34$, there is no weight on the arch for $h=75$ feet; the maximum load obtains when $h=37\frac{1}{2}$ feet. In the first case, the weight above the arch is entirely spread to one side.

In reality the lower particles descend, as in a beam, so that a wedge-shaped mass would probably fall down, the material above forming a natural arch. This often happens in brick walls with arched openings in them; the arch yielding so that a natural arch is formed above it, and as a consequence the arch does not sustain the whole weight of surcharge. This happens over every lintel or other compressible support at the top of windows doors etc. in the walls of houses, churches, etc.

The analysis above is given only to illustrate partially the principle enunciated, and not for use in practice.

72. The weight resting on a culvert is not that due to the total height of surcharge for another reason. Draw the trapezoidal cross section (perpendicular to the roadway) of the embankment, and divide it by a vertical line into two equal parts. In consequence of the symmetry the earth thrust of one-half of the slice shown by the cross section (supposed to have any width) against the other half, is horizontal. On combining this thrust with the weight of one-half of the slice acting at its center of gravity, the resultant of course strikes the base farther from the center than if there were no horizontal thrust. Its effect is evidently to increase the vertical pressure towards the slopes and diminish it near the center of the cross section, Q.E.D.

It is not considered advisable to proportion large culverts for a less weight than that due to the whole surcharge; but small drains may be made smaller than such theory requires.

73. *The abutments* of culverts are treated in the graphical construction exactly as though they were a part of the arch. The horizontal thrust of the earth on their outside is not always that due to the height of surcharge, as the ground generally rises abruptly from the foundation of the abutment walls.

It is best, then, to estimate for both cases: the earth pressing and not pressing against the back of the culvert. Again, as part of the weight above the arch is transmitted to the sides, the weight sustained by the abutments may be increased; also the horizontal pressures acting on them and the arch. It is best to err on the same side in designing these structures.

74. *Tunnel arches.*—In treating tunnel arches, only a part of the weight above them is supposed to press on the arch, the balance being transmitted to the sides. Cohesion now plays the most important part. Tunnels have been executed, even in clay, that have stood for some time without support. In such tunnels large masses often fall in, completely choking up the tunnel; so that if an arch of wood, stone, or other material was built before the fallen mass lost its cohesion it would eventually have to support all, or a part of its weight. What weight presses on a tunnel arch cannot be estimated; we can only resort to experience here.

Rankine gives the following formula, founded on practice, for the minimum thickness, t of tunnel arches,

$$f=\sqrt{.12\,r},\ r=\frac{a^2}{b^2};$$

where a=rise and b=half span.

"This is applicable where the ground is of the firmest and safest kind. In soft and slippery materials, the thickness ranges from $\sqrt{.27r}$ to $\sqrt{.48r}$."

75. The *arch* is peculiarly adapted for a tunnel support; for it is the great advantage of the arch, that it *will not be forced in* at one place *without it is forced out* at another. The latter, the enveloping mass generally prevents, if stones and earth are packed in tight back of the arch; so that the arch so constructed should generally stand unless crushed from a too heavy load.

As in practice, tunnel arches are not thus crushed, we may infer, as stated before on theoretical grounds, that only a part of the superincumbent material presses on them. In every deep tunnel, the thickness of the arch ring is not increased over that due to a compara-

tively small height, as is inferred from the preceding formula.

If a quicksand is encountered on one side or the other, the curvature of the arch must be sharply increased there, or the arch may be forced in, as has happened in certain treacherous clays.

76. Assuming the preceding formulæ for earth thrust and the depth of surcharge that is supposed to press, in accordance with this theory, the stability of a tunnel arch is investigated as previously explained in the case of culverts.

Thus take the tunnel arch under the Thames, Fig. 16; whose dimensions in meters are as follows: the thickness of the arch ring is about 0.94, the radius of the upper part $\overline{0..6}$ is 2.16, and of the inferior part $\overline{6..10}$, 8.61 meters; the upper part being formed of three concentric rolls without bond.

The earth and water above the tunnel is supposed to exercise upon the arch a pressure corresponding to a load $7^{m}.54$ high of material like that of the voussoirs; the reduced surcharge being supposed level at top for simplicity.

The upper part of the arch is divided into six parts, having widths of 0.90, 0.63, 0.63, 0.31, 0.31, 0.31, respectively. The lower part is divided into four parts, having lengths, along the center line, 0.47, 0.76, 0.75 and 0.94 respectively and whose lever arms are the distances of their centers of gravity from the vertical axis of the tunnel, or from the horizontal through the top of the arch, for the vertical or horizontal forces respectively. The tables are made out as in art. 67. If preferred, the voussoirs and surcharge may be considered separately as in art. 31, to attain greater accuracy; but the usual method elsewhere followed is sufficiently near for the object in view.

The method followed is moreover as correct for voussoirs 7 to 10 as the one followed in art. 31, since the particular division of the arch preceding any voussoir is immaterial as concerning that voussoir.

In the following condensed table the first column gives the joint, the next the vertical force on that joint counting from the crown, the third column its lever arm counting from the vertical through the crown; columns 4 and 5 give the horizontal force (acting on the extrados of the arch from the crown to the joint considered) and its lever arm about the top of the arch respectively.

Joint.	Vertical Forces.		Horizontal Forces.	
	Force.	Lever arm.	Force.	Lever arm.
1	7.67	0.45	0.38	0.075
2	13.25	0.77	1.11	0.22
3	19.24	1.11	2.45	0.47
4	22.54	1.28	3.54	0.66
5	25.84	1.46	5.40	0.99
6	29.14	1.60	9.38	1.63
7	29.58	1.64	11.08	1.89
8	30.29	1.66	14.23	2.35
9	30.99	1.68	17.39	2.82
10	31.87	1.70	22.49	3.49

To pass a curve of pressures through the upper middle third limit at the crown and the lower middle third limit at joint 5, we have by measurement and from the tables,

$$Q = \frac{aP_5 + cT_5}{b} = \frac{.79 \times 25.84 + 1.02 \times 5.40}{1.69} = 15.34.$$

Now lay off the line of vertical loads, $\overline{0\ .\ .\ 10}$, from column 2, and the line of horizontal forces, $\overline{0\ .\ .\ 10'}$ from column 4.

Next, on the horizontal, through the summit, lay off the lever arms in column 3; and on the vertical, tangent to the extrados, the lever arms in column 5; also from each point, 3′, 4′, 5′, . . . lay off to the right $\overline{3'\,3''}$, $\overline{4'4''}$, $\overline{5'5''}$, . . . , each equal to Q.

To find the resultant of the presssure on any joint, as the 7th, we draw $\overline{7a}$, $\overline{7a}$, representing the positions of the vertical and horizontal forces ($\overline{07}$, $\overline{07'}$, force diagram) to intersection a; from a draw $\overline{ab} \parallel \overline{77'}$ to intersection b with Q prolonged; at b draw $\overline{bc} \parallel \overline{77''}$ to intersection c with joint 7, which is thus the center of pressure on that jont. The resultant on joint 7 is represented by the line 77″ of the force diagram, being the resultant of $\overline{07}$, $\overline{07'}$ and Q.

The line of pressures thus found is represented by the dotted line, and leaves the inner third of the arch ring at joints 9 and 10. It does not follow that the

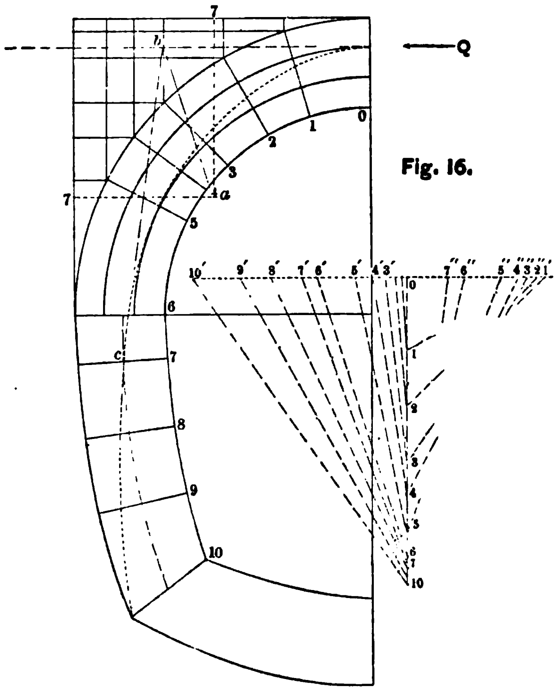


Fig. 16.

arch is unstable. The active forces may not be, and probably are not, as estimated. Whether this be so or not, the joint 10 cannot open inside without the arch being forced out at the haunches; but this, if the earth is packed tight around the arch, is resisted by the earth outside, which there exerts a larger horizontal force than estimated (partly passive), and thereby restricts the line of pressures to narrow limits.

It is thus evident that the arch is stable unless the surrounding earth itself gives way, which will generally not happen. The surrounding mass thus plays the part of spandrel, besides exerting an active thrust, being the least thrust it is capable of. It is even more effective than a spandrel, since it can prevent the crown from rising, which, in stone viaducts, is one method of rupture which the spandrel actually aids in producing.

77. The above construction applies when the rolls of the arch are bonded together. As this is not true in the present case, the problem of determining the true line of pressures becomes impossible of solution.

If we conceive one-third the vertical and horizontal forces, given in the previous table to act on each roll, we shall find that for the outer roll that a curve of pressures cannot be drawn within the middle third, though one can be drawn within the arch ring from the crown to joint 6.

However, the passive earth thrust in good earth will again prevent deformation and thus cause stability as before; but the arch cannot be considered as strong as if it were bonded throughout so as to act as one mass.

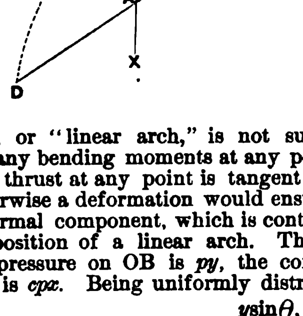
78. The reversed arch at bottom is not supposed to exert any horizontal force, so that the sides of the arch simply rest on it, as though it was an abutment. Otherwise it is used to prevent a forcing in of the sides; and the sum of its horizontal component and that at the crown equal the total horizontal thrust of the earth or fluid on one side.

79. The curve of pressures shown in fig. 16 resembles somewhat a *quarter ellipse;* so that if such a curve was taken for the center line of a tunnel arch, a curve of pressures might be drawn very nearly following this center line; so that if the arch was really acted on by the forces supposed, the arch designed would offer a very great stability.

Similarly if the centre of the ellipse is taken at $\frac{1}{8}$ height of arch above the inverted arch, it will be found that a curve of pressures may be drawn very near the center line, especially if the surcharge has a less specific gravity than the arch. In fact, the ellipse is recognized as the proper form for tunnel arches and any part of the curve may be used that will best subserve the purpose in view—the axis of the tunnel always coinciding with that of the ellipse.

From fig. 16 we see that if a semi-circle is used for the curve of the upper part, that the lower part had best be made vertical on the sides. This form of tunnel arch is quite common in practice and is commended when good material is used.

REMARK.—We can easily prove analytically that the ellipse is the proper form for a tunnel arch, when the depth of surcharge is so great that the thrust of the earth at any part of the arch is practically the same. To take the most general case, let the top surface of the earth be ∥ OY in the adjoining figure. Call the pressure, per unit of inclined plane OY, in a vertical derection, p; the conjugate pressure in a direction ∥ OY, per unit of a vertical plane OA, can be represented, according to the theory of earth pressure, by cp, c being a constant. Let $OA=x$, $AB=y$, $OAB=\theta$, and call the thrust at O in the direction OY, tangent to the rib BOD at O, pT.



This rib, or "linear arch," is not supposed to have any bending moments at any point, so that the thrust at any point is tangent to the rib; otherwise a deformation would ensue, due to the normal component, which is contrary to our supposition of a linear arch. The total vertical pressure on OB is py, the conjugate pressure is cpx. Being uniformly distributed, their lever arms about B are, $\frac{y\sin\theta}{2}$, $\frac{x\sin\theta}{2}$, respectively.

Now if any point, as B, of the arc is to be a point in the line of pressures, we must have, taking moments about B,

$$pTx\sin\theta=\left(\frac{py^2}{2}+\frac{cpx^2}{2}\right)\sin\theta$$

$$\therefore y^2=2Tx-cx^2$$

the equation of an ellipse,

Q.E.D.

The equation of ellipse referred to a diameter and the tangent at its vertex, a and b being the semi conjugate diameters is, $y^2=\frac{b^2}{a^2}(2ax-x^2)$.

Comparing with the above, we have,

$$T=\frac{b^2}{a},c=\frac{cp}{p}=\frac{b^2}{a^2}.$$

Or *the intensities of the conjugate pressures are as the squares of the diameters to which they are parallel.*

If in the eq. above we make, $x=OA=a$, we find, $y=AB=b$; whence from the last eq., $\frac{a.\ cp}{b.\ p}=\frac{b}{a}$. Now the thrust at $O=pT=acp$, whilst that at the ends of the conjugate diameter DB, acting ∥ OX, is bp; hence *these forces are proportional to the diameters to which they are parallel.*

To construct the arch, c and a or b must be given to find the other semi diameter from the eq., $c=b^2\div a^2$.

WHEN THE TOP SURFACE OF EARTH IS LEVEL, OY becomes level and $\theta=90°$. a and b are now the semi axes of the ellipse. From the theory of earth pressure, $\frac{cp}{p}=\tan.^2(45°-\frac{1}{2}\Phi)$ whence, for a tunnel arch,

$$\frac{\text{horizontal semi axis}}{\text{vertical semi axis}}=\frac{b}{a}=\tan.(45°-\tfrac{1}{2}\Phi):$$

Φ being the angle of repose.

Next, make $c=1$ (θ being 90°) and the ellipse becomes a *circle*. At any point of the circle consider the two equal forces p_1 p_1 at right angles and acting on a unit of area of planes ⊥ to them. Their resultant acts normally to the circle, and its intensity is easily found to be p, the intensity of the vertical and horizontal components.

Calling $r=a=b$, the radius of the circle, we have the thrust at $O=pT=pr$, or *the product of the intensity on a unit of circumference by the radius.* This thrust is the same all around the ring.

The above are the principal deductions given by Rankine for the arches given above. The reader is referred to his Civil Engineer for the "geostatic" and other forms of linear arches.

A SILVER medal has been awarded by the Royal Cornwall Polytechnic Society to West's six-cylinder engine, exhibited by Messrs. Plambeck and Darkin, of Queen Victoria Street, London.

SEWER VENTILATION.

From "The Engineer."

While the attention of house builders and the public is daily directed to the necessity for carefully excluding sewer gas from dwelling-houses by the use of improved traps, and plenty of them, it is to be feared that a very important thing is being wholly lost sight of. We allude to the ventilation of main sewers. In the metropolis this has in a few instances been properly provided for; but in the suburbs of London, at all events, the arrangements employed are of the most crude and unsatisfactory description. In theory, sewers ought not to be ventilated at all; this is to say, they should be hermetically sealed by effective water-traps. The scheme has been tried very fully, and with disastrous results, and no more need be said about it. There is reason to believe that the gases generated in sewers are lighter than the air, as a rule, to which there are certain exceptions to which we shall come presently. Be this as it may, it seems to be certain that when drains are laid on anything like a steep inclination, the sewer gas will rise to the highest point in the drain; and it will, under special atmospheric conditions, then force the seals in any traps not of the best kind, and will rush into and flood the houses. To prevent this, street ventilators are fitted which are simply gratings communicating directly with the main sewer below, and it is assumed that these will suffice to prevent any accumulation of pressure in the sewer, great enough to force a seal of say, 1in. of water. But besides these street gratings, the rain-water stack pipes of all suburban houses, with the exception of a few built under special supervision, communicate directly with the main sewer, and it follows that throughout the length of a street the main sewer is tapped at every 20 feet or 30 feet by house rain-water drains, and that an accumulation of pressure in the sewer is apparently impossible, as the sewer gas can rise freely in the stack pipes to the level of the eaves. This being the case, it is not easy to see what good purpose the street grating ventilator can possibly serve.

In theory these arrangements are by no means defective. The sewer gases are provided with means of escaping freely, and it is apparently impossible that the seal of any fairly well-made trap can be forced; but in practice the system works very badly, and it is because this circumstance seems to have been overlooked by nearly all writers on sanitary subjects that we now call our readers' attention to it. In calm weather, with a steady barometer, all goes well, but a sudden fall in the barometer will at once let loose much gas previously imprisoned in the liquid or semi-liquid contents of the sewer. Next we have rain, which stirs up the said contents, and a very foul-smelling and noxious emanation at once begins to try to issue from the street grating and the open tops of the stack pipes. A stack pipe is very like a chimney in the sense that it may or may not "draw," and it is worse off than a chimney, because the force with which the current of sewer gas rises within it is very feeble unless the weather be warm. The position assumed by the open mouth of the stack pipe, just under the eve, is about the worst possible so far as the performance by the pipe of the functions of a ventilator is concerned. In certain states of the weather, the wind deflected from the rood down the mouth of the pipe, not only stops all upward currents, but creates down currents, and causes a plenum in the main sewer. We have seen this effect so strongly manifested, that in a winter's gale 1 inch water seals in a dwelling-house were broken with ease at every gust of wind. Prudent people will trust their lives to nothing less secure than a 2-inch seal, while three inches are easily to be had, and with these the sewer gas no longer finds its way into the houses through the sinks, &c. But it gets in none the less certainly. At times, while a down draught is going on in one set of stack pipes, a sluggish up-draught may, as a conse-

quence, be taking place in others at the opposite side of the street. The sewer gas is often positively heavier than the air, and, under the conditions just named, overflowing from the stack pipes, it descends to the ground, and then, as there is always an in draught to every inhabited house, for obvious reasons, the sewer gas will find its way into the basement, and render it for the time absolutely uninhabitable. If at such periods the intelligent householder will take the trouble to run the stench to earth, he will find from the evidence supplied by his sense of smell, a layer of sewer gas, probably not more than 1-feet thick, spreading itself along the street and overflowing into areas and down steps, and that this gas is lazily welling up from the street grating ventilators. He will also find the sewer gas descending in streams outside certain of his stack pipes, and he will also find that a gentle breeze is blowing, but that the offensive ventilator and stack pipes are sheltered from it. At such times he may rest assured that at some other point in the sewer there is a direct influx of air caused by a down draught in stack pipes, due in its turn to the breeze. We exaggerate in no wise when we say that hundreds of houses in the suburbs of London are liable at any moment to be flooded in this way with sewer gas from the outside, and this with the most perfect traps known to science indoors.

It will be urged that the remedy consists in cutting off the stack pipes from the main sewer by special traps; that is to say, there ought to be a trap in the pipe which unites all the stack pipes, sinks, and closets with the main sewer. This is right, so far as it goes in theory, but in practice the arrangement is not found to work well. In most cases there is not room for such a trap. We are not dealing now, be it remembered, with what are known to builders as "mansions," but with the houses letting at from £40 to £60 a year, which may be counted by the thousand all round London, east, west, north and south. Such traps, to be efficient, must be so placed that they can be examined from time to time. They must be of good proportions, and they are rather expensive affairs. We happen to know that in many instances where they have been fitted they have had to be removed, because they caused more trouble and nuisance than they were worth. If care were taken to provide 3-inch seals to the sinks and closets of a dwelling-house, with proper ventilating pipes, then would the external trap be wholly unnecessary. But let us suppose that the system were universally adopted; we should then have the main sewer cut off from the stack pipes. Would matters be improved? We much doubt it. The gas would then rise through the street ventilators in greater volumes than ever, and flow over the roadway and into the houses. That these ventilators are dangerous nuisances is known to most sanitary engineers; and various devices, in the shape of charcoal baskets, have been adopted to render them harmless. In many cases these have proved useful, but Sir Joseph Bazalgette at all events has pronounced them worse than useless, and they are not now fitted in London. When a complaint is brought before the local authorities that a street ventilator is causing a nuisance, a man is sent with a bucket of disinfectant, which is emptied down the ventilator, and the authorities rejoice that they have done all that is needful for the well-being of the community.

We may be asked, what would we have? Ought street ventilators to be done away, and if so how are drains to be ventilated? To this we reply that some years ago the ventilation of town drains constituted a subject of constant discussion among sanitary engineers, while now hardly anything is said about it. Are we to assume that the difficulties to be encountered are too great to be overcome? We think not, and we write in the hope that the subject will once more receive the attention it really deserves. May we venture to suggest that the best way of ventilating a sewer would consist in taking a lesson from the performance in this connection of stack pipes, and developing the idea in a practical shape? As, for example, let it be made compulsory on every builder to carry up a flue through one of the walls of his house, the top of which flue may be made to assume the form of an ordinary blind chimney cap; this flue should freely communicate at its base with the main sewer, either by means of the common drain from the house, or by means of a subsidiary drain. The best position for the flue would be alongside the kitchen

chimney, from which it would constantly derive heat. The result would be at all times a strong—for a sewer ventilator—upward current through the drain flue, which would discharge the sewer gas through an aperture in its side, where it ought to do no harm—at an elevation, at least, far safer than that to which any stack pipe reaches. The cost of the arrangement would be very trifling, and it could be adapted under every conceivable circumstance. When small houses are built in a group, one or two ventilating flues only would suffice, instead of one for each house. Of course, all street ventilators would be closed up.

ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION.

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I.

THE following paper contains the mathematical investigation of some of the cases of the motion of incompressible, frictionless fluids. The results obtained are to be considered as applying only to this class of fluids, unless the contrary is expressly stated. The paper is intended to be introductory to a treatise which I hope before long to be able to publish.

In a subject so difficult as Hydrodynamics, there is but little chance for the discovery of hitherto unheard of properties of the quantities dealt with, so, in what follows, the reader will not look for much that is absolutely new in the way of fact, although the arrangement of the work and in many cases the methods employed are my own.

The references to the original sources from which information has been drawn, are given in every case, and I trust that these references, together with the matter contained in this paper, will prove of value to any one interested in the most difficult but beautiful problem of fluid motion.

Of late years, much has appeared in different places upon the subject of Hydrodynamics, but, so far as I am aware, there is no general work either in the English, French or German languages. The aim of this paper and the treatise which will follow will be to combine in one work, all of importance that has been written upon the subject, and so enable the student to forego the immense amount of research necessary in order thoroughly to inform himself upon any one branch of the subject.

The short section which appears upon the theory of the Potential, is principally taken from Clausius's work upon that subject. The references to theoretical mechanics are, unless otherwise stated, to Thomson and Tait's Natural Philosophy. Kirchhoff's Mathematische Physik and Clifford's Elements of Dynamic, have also been consulted.

§ I.

GENERAL EQUATIONS OF FLUID MOTION.

Let X, Y, Z denote as usual the component forces acting at the point (x, y, z) of the fluid reckoned per unit of its mass —then denoting by ρ the density of the fluid we have for the forces acting upon the elementary mass $\rho dadydz$ the expressions

$$X\rho dxdydz,\ Y\rho dxdydz,\ Z\rho dxdydz;$$

Now for the fluid pressure acting upon one face of the elementary parallelopiped, say, $\delta y\delta z$ we have $p\ \delta y\delta z$, p denoting the pressure on unit of area; upon the opposite face it is, neglecting powers of dx higher than the first,

$$-\delta y\delta z\left(p+\frac{dp}{dx}\delta x\right)$$

Consequently the resultant force due to fluid pressure acting in the direction of the axis x is,

$$-\delta y\delta z\frac{dp}{dx}\delta x.$$

The equilibrium of this portion of the fluid therefore requires that

$$\delta x\delta y\delta z\frac{dp}{dx}-\rho X\delta x\delta y\delta z=o$$

with similar expressions for the other pairs of faces. We have thus for the equations of fluid equilibrium,

$$\frac{dp}{dx}=\rho X$$

$$\frac{dp}{dy}=\rho Y$$

$$\frac{dp}{dz}=\rho Z$$

These three equations can, of course, be replaced by the single equation of equilibrium.

$$dp=\rho(Xdx+Ydy+Zdz)$$

when dp denotes the variation of pressure corresponding to the changes dx, dy, dz, in the co-ordinates of the point at which the pressure is estimated. We see from this equation that the expression,

$$Xdx+Ydy+Zdz,$$

is either an exact differential or capable of being made so by a factor. If the forces X, Y, Z belong to a conservative system, that is, a system possessing a potential, or for which the absent expression is an exact differential, we know that

$$\frac{dX}{dy}-\frac{dY}{dx}=o \text{ \&c.}$$

But when these conditions are satisfied the quantity $Xdx+Ydy+Zdz$ is an exact differential, or in the case of a system of conservative forces we have without the assistance of any interpreting factor

$$Xdx+Ydy+Zdz=dR$$

when R is the potential of the forces at the point (x,y,z). It follows from this that $dp=-\rho R$, or that p is a function of R, then for all surfaces for which R is constant p is also constant, *i.e.* the pressure is constant over all equi-potential surfaces. From these equations of equilibrium we can pass directly to the equations of motion, by means of D'Alembert's principle. Call u, v, w, the velocities of a particle of the fluid whose co-ordinates at the time t are x,y,z, thus,

$$u=\frac{dx}{dt},\ v=\frac{dy}{dt},\ w=\frac{dz}{dt};$$

let u', v', w', denote the accelerations to which these velocities give rise, then in our equations of equilibrium replacing X, Y, Z by,

$$X-u',\ Y-v',\ Z-w'$$

we have for the equations of motion

$$\frac{dp}{dx}=\rho(X-u'),$$

$$\frac{dp}{dy}=\rho(Y-v'),$$

$$\frac{dp}{xz}=\rho(Z-w')$$

when we have of course,

$$u'=\frac{du}{dt}+\frac{du}{dx}\frac{dx}{dt}+\frac{du}{dy}\frac{dy}{dt}+\frac{du}{dz}\frac{dz}{dt}$$

$$=\left(\frac{d}{dt}+u\frac{d}{dx}+v\frac{d}{dy}+w\frac{d}{dz}\right)u.$$

We may for brevity replace this operator by $\frac{D}{Dt}$, and we have thus for the equations of fluid motion the following:

$$\frac{dp}{dx}=\rho\left(X-\frac{Du}{Dt}\right),$$

$$\frac{dp}{dy}=\rho\left(Y-\frac{Dv}{Dt}\right),$$

$$\frac{dp}{dz}=\rho\left(Z-\frac{Dw}{Dt}\right).$$

Concerning the operator which we have denoted by $\frac{D}{Dt}$, it is important to observe that it relates to a particular particle and not to a particular point in space; the velocities u, v, w, are functions of x, y, z and t, and denote the velocity which *any* particle has when it occupies the position denoted by x, y, z *and* $\frac{du}{dt}$ dt denotes the increase of velocity of a second particle over the one originally in this position, which arrives at this point after the lapse of time dt, while on the other hand, $\frac{Du}{Dt}dt$ denotes the change in velocity of the original particle during this time. When the motion is very small, the terms $u\frac{d}{dx}$, &c. may be neglected, and we would have,

$$\frac{D}{Dt}=\frac{d}{dt}.$$

To our equations of motion it is necessary to add one more, expressing the continuity of the fluid. This equation simply expresses the fact, that during any natural motion there can be neither annihilation or germination of matter, or, referring to our problem, that the amount of fluid in any space at any time must be equal to the amount originally contained in that space, increased by the amount which has entered it during the time which has been allowed to pass, diminished by the amount which has left it during that time.

Let a, b, c, denote the co-ordinates of any particle of the fluid at an initial instant, x, y, z, denote the values of these co-ordinates at the time t; now in order completely to specify the motion it is necessary to express these latter quantities as functions of initial co-ordinates and the time. Suppose further, that δa, δb, δc, are the edges of a small parallelopiped of the fluid, as these are assumed to be infinetisimal, the figure will remain a parallelopiped during the motion.

We have now for the co-ordinates of the extremities of the edges meeting in the point a, b c,

$$a+\delta a,\ b,\ c,\quad a,\ b+\delta b,\ c,\quad a,\ b,\ c+\delta c$$

at the time t the co-ordinates of these points will be,

$$\begin{array}{ccc} x, & y, & z, \\ x+\frac{dx}{da}\delta a, & y+\frac{dy}{da}\delta a, & z+\frac{dz}{da}\delta a \\ \vdots & \vdots & \vdots \end{array}$$

From these we arrive, by a smple geometrical process, at the volume of the parallelopiped, which is then at the time t.

$$\begin{vmatrix} \frac{dx}{da}, & \frac{dy}{da}, & \frac{dz}{da} \\ \frac{dx}{db}, & \frac{dy}{db}, & \frac{dz}{db} \\ \frac{dx}{dc}, & \frac{dy}{dc}, & \frac{dz}{dc} \end{vmatrix} \delta a\delta b\delta c$$

or representing the determinant by Δ, $\Delta\,\delta a\delta b\delta c$; hence by our definition of continuity we must have

$$\Delta=1,$$

or in general if ρ_0 and ρ denote the initial and final densities of the fluid contained in this portion of space

$$\rho\,\Delta=\rho_0 -$$

which simply expresses the fact that the density of the fluid contained in this portion of space must vary inversely as the volume of the space.

This equation is known as the integral equation of continuity. The form of the equation most generally employed, however, is that which expresses the fact that the rate of diminution of density bears to the density at any instant the same ratio that the rate of increase of the volume of an infinitely small portion of the fluid bears to the same infinitely small volume at the same instant. The symbolical expression of this fact constitutes the *differential equation of continuity* of the fluid.

Let the flow towards the inside of an elementary parallelopiped of the fluid be considered as positive, then the flow towards the outside will be negative. Representing as before the edges of this elementary parallelopiped by δx, δy, δz we have for the flow through the face δy δz in the direction of x and during the time dt

$$\rho\delta y\delta z u dt$$

through the opposite face the flow will be during the same time

$$-\delta y\,\delta z\left(\rho u+\frac{d\rho u}{dx}.\delta x\right)dt.$$

These together give rise to an increase of mass

$$-\delta x\delta y\delta z\frac{d\rho u}{dx}dt$$

with similar expressions for the other pairs of faces respectively, perpendicular to the axis of y and z. Then the total increase of mass is

$$-\delta x\delta y\delta z\left(\frac{d.\rho u}{dx}+\frac{d.\rho v}{dy}+\frac{d.\rho w}{dz}\right);$$

but this increase of mass is also given by

$$\delta x\delta y\delta z\frac{d\rho}{dt}dt;$$

equating these values and we have for the equation of continuity

$$\frac{d\rho}{dt}+\frac{d.\rho u}{dx}+\frac{d.\rho v}{dy}+\frac{d.\rho w}{dz}=o$$

or for incompressible fluids simply,

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=o$$

It may not be uninteresting to show how this differential equation of continuity can be derived from the integral equation. We have, denoting the mass of this elementary portion of fluid by m,

$$m=\rho\begin{vmatrix}\frac{dx}{da}, & \frac{dx}{db}, & \frac{dx}{dc}\\ \frac{dy}{da}, & \frac{dy}{db}, & \frac{dy}{dc}\\ \frac{dz}{\delta a}, & \frac{dz}{\delta b}, & \frac{dz}{\delta c}\end{vmatrix}\delta a\ \delta b\ \delta c$$

Differentiating this with respect to t we have

$$o=\Delta\frac{D\rho}{Dt}+\rho\frac{d\Delta}{dt}$$

or

$$o=\frac{D\rho}{Dt}+\frac{\rho}{\Delta}\cdot\frac{d\Delta}{dt}$$

the quantity $\frac{1}{\Delta}\frac{d\Delta}{dt}$ will be found by easy reductions to be equal to

$$\frac{du}{dx}+\frac{dv}{dy}\frac{dw}{dz}$$

and the equation thus becomes

$$o=\frac{D\rho}{Dt}+\rho\left(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\right)$$

from which we obviously obtain the forms given above. Particular forms of this equation for special cases are often quite simple—as, for example—suppose the motion of the fluid to be wholly parallel to the plane xy; in this case we have simply

$$\frac{du}{dx}+\frac{dv}{dy}=o$$

but $\frac{du}{dx}=-\frac{dv}{dy}$ is the condition that the expression

$$udy-vdx$$

be an exact differential; calling it $d\Psi$ we have

$$u=\frac{d\Psi}{dy},\quad v=-\frac{d\Psi}{dx}.$$

The quantity Ψ is called the stream function, and all motion takes place in the direction of the curves Ψ=const. If the motion be steady, the lines Ψ=const. will form a system of tubes in the fluid, which may be called the tubes of flow. A much more general simplification of the equations of hydrodynamics exists however for certain classes of motion. It is a fact, the discovery of which is due to Lagrange that if at any time the expression

$$udx+vdy+wdz$$

is an exact differential, it will remain so throughout the motion; that is, if at any time we have

$$\frac{du}{dx}-\frac{dv}{dy},\quad \frac{dw}{dx}-\frac{du}{dz},\quad \frac{du}{dy}-\frac{dv}{dx},\ =o$$

these quantities will remain so throughout the motion. Representing these quantities by ξ, η, δ, we may express this fact in another manner, viz. if at any time the motion of the fluid be irrotational, it will remain so during the entire motion. In particular, if the fluid originally at rest be set in motion, by a system of conservative forces or pressures, there will be no motion of rotation throughout the entire motion. The following proof of this theorem is that given by Sir Wm. Thomson in his paper on "Vortex Motion," Edin. Trans. 1869. As this proof does not depend upon the quantity ρ, we can give it in general representing by ω the integral $\int\frac{dp}{\rho}$. We have then from our equations of motion

$$d\pi=Xdx+Ydy+Zdz-\left(\frac{Du}{Dt}dx+\frac{Dv}{Dt}dy+\frac{Dw}{Dt}dz\right)$$

Now, according to hypothesis,

$$Xdx+Ydy+Zdz=dR,$$

and obviously,

$$\frac{Du}{Dt}dx+\frac{Dv}{Dt}dy+\frac{Dw}{Dt}dz=\frac{D}{Dt}(udx+vdy+wdz)-\left(u\frac{Ddx}{Dt}+v\frac{Ddy}{Dt}+w\frac{Ddz}{Dt}\right)$$

or since

$$\frac{Ddx}{Dt}=\frac{dDx}{Dt}=u,\ \&c.$$

$$d\pi = dR - \frac{D}{Dt}(udx + vdy + wdz) + (udu + vdv + wdw)$$

or representing $u^2 + v^2 + w^2 by V^2$,

$$\frac{D}{Dt}udx + vdy + wdz) = d(R + \tfrac{1}{2}V^2 - \pi).$$

Integrating this along any arc (12) moving with the fluid we have

$$\frac{D}{Dt}\int(udx + vdy + wdz) = (R + \tfrac{1}{2}V^2 - \pi)_2 - (R + \tfrac{1}{2}V^2 - \pi).$$

If the arc be a closed circuit the second member of this equation vanishes and we have

$$\frac{D}{Dt}\int(udx + vdy + wdz) = o,$$

or this may be expressed by saying that *the line integral of the tangential component velocity around any closed curve of a moving fluid remains constant throughout all time.* The line integral is called the *circulation*, and the proposition may be stated. *The circulation in any closed line moving with the fluid remains constant.* In a state of rest the circulation is, of course, zero; therefore, for the assumed case of motion generated by pressures or conservative forces we have that the circulation is always zero, so that $udx + vdy + wdz$ is an exact differential. Representing this quantity by $d\varphi$ we have

$$u = \frac{d\varphi}{dx}, v = \frac{d\varphi}{dy}, w = \frac{d\varphi}{dz}$$

from which we have for the differential equation of continuity

$$\frac{d^2\varphi}{dx^2} + \frac{d^2\varphi}{dy^2} + \frac{d^2\varphi}{dz^2} = o$$

or simply

$$\Delta^2\varphi = o.$$

The quantity φ is appropriately called the velocity function, and the velocity in *any* direction is expressed by the corresponding rate of change of φ. We may just here observe one fact concerning φ. If φ is a minimum at any point, *i.e.* if it increases as we go away from that point there must evidently be a positive expansion of the fluid from this point in all directions. Similarly if φ be a maximum at any point. Then the motion is in all directions towards this point and there is compression of the fluid.

If there be neither expansion nor compression of the fluid within the region bounded by a closed surface, the greatest and least values of the velocity potential in that region must be on the surface; for since there is no expansion or contraction, there can be no maximum or minimum value within this surface. If, therefore, the velocity potential is constant over the surface, it must be constant throughout the enclosed region, since its greatest and least values are now equal. In particular, if it is zero over the surface, it must be zero throughout the enclosed region. When the velocity potential exists, the equation for determining the pressure can be put into a very simple form, viz.

$$d\pi = dR - \frac{D}{Dt}d\varphi + \tfrac{1}{2}dV^2$$

integrating

$$\pi = \int\frac{dp}{\rho} = R - \frac{D\varphi}{Dt} + \tfrac{1}{2}V^2$$

but

$$\frac{D\varphi}{Dt} = \frac{d\varphi}{dt} + u^2 + v^2 + w^2 = \frac{d\varphi}{dt} + V^2$$

so that

$$\pi = \int\frac{dp}{\rho} = R - \left(\frac{d\varphi}{dt} + \tfrac{1}{2}V^2\right)$$

or for our assumed case of incompressible fluids

$$\frac{p}{\rho} = R - \left(\frac{d\varphi}{dt} + \tfrac{1}{2}V^2\right)$$

Another form of the equations of fluid motion due to Lagrange is worthy of notice here, though the forms already given, or Euler's equations, are those employed in general in hydrodynamics. Since the quantities a, y, 2, are functions of the initial coördinates of the point, we have

$$\frac{dp}{da} = \frac{dp}{dx}\frac{dx}{da} + \frac{dp}{dy}\frac{dy}{da} + \frac{dp}{dz}\frac{dz}{da}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

from these we have

$$\frac{dp}{dx} = \frac{1}{\Delta}\begin{vmatrix} \frac{dp}{da}, & \frac{dy}{da}, & \frac{dz}{da} \\ \frac{dp}{db}, & \frac{dy}{db}, & \frac{dz}{db} \\ \frac{dp}{dc}, & \frac{dy}{dc}, & \frac{dz}{dc} \end{vmatrix} \quad \&c.$$

but we have also

$$\frac{1}{\rho}\frac{dp}{dx}=X-\frac{d^2x}{dt^2}\&c.$$

Substituting the values of $\frac{dp}{dx}$ from these last equations in those giving the values of $\frac{dp}{da}$—and we have the Lagrangean equations of fluid motion, viz.

$$\left\{\frac{d^2x}{dt^2}-X\right\}\frac{dx}{da}+\left\{\frac{d^2y}{dt^2}-Y\right\}\frac{dy}{dx}+\left\{\frac{d^2z}{dt^2}-Z\right\}\frac{dz}{da}+\frac{1}{\rho}\frac{dp}{da}=o$$

with two similar ones containing b and c, respectively. The reader can see that from these equations we can readily pass to the forms given before. Where the forces X, Y, Z have a potential and there is also a velocity potential, these equations become

$$\frac{d^2x}{dt^2}\frac{dx}{da}+\frac{d^2y}{dt^2}+\frac{dy}{da}\frac{d^2z}{dt^2}\frac{dz}{da}=\frac{d(R-\pi)}{da}$$

$$\frac{d^2x}{dt^2}\frac{dx}{db}+\frac{d^2y}{dt^2}\frac{dy}{db}+\frac{d^2z}{dt^2}\frac{dz}{db}=\frac{d(R-\pi}{db}$$

$$\frac{d^2x}{dt}\frac{dx}{dc}+\frac{d^2y}{dt^2}\frac{dy}{dc}+\frac{d^2z}{dt^2}\frac{dz}{dc}=\frac{d(R_1-\pi)}{dc}$$

Differentiate the second of these equations with respect to c, the third with respect to b, and subtract the latter result from the former. It will be sufficient to examine only two terms of the result; we have then

$$\frac{d}{dc}\left\{\frac{d^2x}{dt^2}\frac{dx}{db}\right\}-\frac{d}{db}\left\{\frac{d^2x}{dt^2}\frac{dx}{dc}\right\}=\frac{dx}{db}\frac{d^2}{dt^2}\frac{dx}{dc}-\frac{dx}{dc}\frac{d^2}{dt^2}\frac{dx}{db}$$

$$=\frac{d}{dt}\left\{\frac{dx}{db}\frac{d}{dt}\frac{dx}{dc}-\frac{dx}{dc}\frac{d}{dt}\frac{dx}{db}\right\}=\frac{d}{dt}\left\{\frac{dx}{db}\frac{du}{dc}-\frac{dx}{dc}\frac{du}{db}\right\}\&c.,$$

the remaining terms will be obtained by advancing the letters. We have then a quantity which differentiated for t is $=0$. Performing similar operations on the remaing pairs of equations we arrive readily at the following equations, where C_1, C_2, C_3 denote quantities which are independent of the time.

$$\left\{\frac{dx}{da}\frac{du}{db}-\frac{dx}{db}\frac{du}{da}\right\}+\left\{\frac{dy}{da}\frac{dv}{db}\ \frac{dy}{db}\frac{dv}{da}\right\}+\left\{\frac{dz}{da}\frac{dw}{db}-\frac{dz}{db}\frac{dw}{da}\right\}=C_1$$

$$\left\{\frac{dx}{dc}\frac{du}{da}-\frac{dx}{da}\frac{du}{dc}\right\}+\left\{\frac{dy}{dc}\frac{dv}{da}-\frac{dy}{da}\frac{dv}{dc}\right\}+\left\{\frac{dz}{dc}\frac{dw}{da}-\frac{dz}{da}\frac{dw}{dc}\right\}=C_2,$$

$$\left\{\frac{dx}{db}\frac{du}{dc}-\frac{dx}{dc}\frac{du}{db}\right\}+\left\{\frac{dy}{db}\frac{du}{dc}-\frac{dy}{dc}\frac{du}{db}\right\}+\left\{\frac{dz}{db}\frac{dw}{dc}-\frac{dz}{dc}\frac{dw}{db}\right\}=C_3.$$

Let now u_0, v_0, w_0 represent the values of u, v, w for $t=o$, thus at this time we have

$$u=u_0,\ v=v_0,\ w=w_0$$
$$x=a,\ y=b,\ z=c;$$

Substituting these values in the above equations and they reduce immediately to

$$\frac{du_0}{db}-\frac{dv_0}{da}=C_1$$
$$\frac{dw_0}{da}-\frac{du_0}{dc}=C_2$$
$$\frac{dv_0}{dc}-\frac{dw_0}{db}=C_3$$

We have further, since u, v, w are functions of x, y, z,

$$\frac{du}{da}=\frac{du}{dx}\frac{dx}{da}+\frac{du}{dy}\frac{dy}{da}+\frac{du}{dz}\frac{dz}{da}\&c.$$

If now in our determinant Δ we denote the separate minors by A_1, B_1, Γ_1, and Γ_2, *i. e.*,

$$A_1=\frac{dy}{db}\frac{dz}{dc}-\frac{dy}{dc}\frac{dz}{da}\text{-}\&c.$$

substituting now the values of $\frac{du}{da}$ &c. in the above equations C_1, C_2, C_3 and noting these last abreviations we have, since for incompressible fluids $\Delta=1$,

$$A_1\left\{\frac{dv}{dz}-\frac{dw}{dy}\right\}+B_1\left\{\frac{dw}{dx}-\frac{du}{dz}\right\}+\Gamma_1\left\{\frac{du}{dy}-\frac{dv}{dz}\right\}=\frac{dv_0}{dc}-\frac{dw_0}{db}$$

$$A_2\left\{\frac{dv}{dz}-\frac{dw}{dy}\right\}+B_2\left\{\frac{dw}{dx}-\frac{du}{dz}\right\}+\Gamma_2\left\{\frac{du}{dy}-\frac{dv}{dz}\right\}=\frac{dw_0}{da}-\frac{du_0}{dc}$$

$$A_1\left\{\frac{dv}{dz}-\frac{dw}{dy}\right\}+B_1\left\{\frac{dw}{dx}-\frac{du}{dz}\right\}$$
$$+\Gamma_1\left\{\frac{du}{dy}-\frac{dv}{dz}\right\}=\frac{du_0}{db}-\frac{dv_0}{da}$$

Representing the quantities on the right hand side, as we appropriately may, by $2\xi_0$, $2\Gamma_0$, $2\delta_0$, and solving the equations for the quantities within the parenthesis, we have

$$\xi=\xi_0\frac{dx}{da}+\eta_0\frac{dx}{db}+\delta_0\frac{dx}{dc}$$
$$\eta=\xi_0\frac{dy}{da}+\eta_0\frac{dy}{db}+\delta_0\frac{dy}{db}$$
$$\delta=\xi_0\frac{dz}{da}+\eta_0\frac{db}{dz}+\delta_0\frac{dz}{dc}$$

If the quantities ξ_0, Γ_0, δ_0, which are the initial angular velocities of the particle of the fluid whose co-ordinates at $t=o$ a, b, c, are $=o$, we have that ξ, Γ, δ must also be $=o$, that is, we arrive again at the theorem that if there be no original motion of rotation in the fluid there will be none at any future time. It will be of interest to obtain the equations which were used by Helmholtz in his great memoir on vortex motion. These are simply obtained from our equations of motion. The first of these equations written out in full is

$$\frac{dw}{dx}=X-\frac{du}{dt}-u\frac{du}{dx}-v\frac{du}{dy}-w\frac{du}{dz}\ \&c.$$

Now supposing the fluid initially at rest to be set in motion by conservative forces and pressures from the exterior, the analytic conditions for this are

$$\frac{dX}{dy}-\frac{dY}{dx}=o\ \&c.$$

Therefore differentiating the first equation with respect to y and the second with respect to x, and subtracting we eliminate ω and the impressed forces, and have

$$o=\frac{d}{dx'}\frac{dv}{dt}-\frac{d}{dy}\frac{du}{dt}+\frac{du}{dx}\left\{\frac{dv}{dx}-\frac{du}{dy}\right\}$$
$$+\left\{u\frac{d}{dy}\frac{dv}{dx}-u\frac{d}{dx}\frac{du}{dx}\right\}+\ldots$$

from this we have obviously

$$\frac{D\delta}{Dt}=\frac{du}{dz}\xi+\frac{dv}{dz}\eta-\left\{\frac{du}{dx}+\frac{dv}{dy}\right\}\delta$$

and remembering that for incompressible fluids we have

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=o$$

this becomes

$$\frac{D\delta}{Dt}=\frac{du}{dz}\xi+\frac{dv}{dz}\eta+\frac{dw}{dz}\delta$$

And similarly

$$\frac{D\xi}{Dt}=\frac{du}{dx}\xi+\frac{dv}{dx}\eta+\frac{dw}{dx}\delta,$$
$$\frac{D\eta}{Dt}=\frac{du}{dy}\xi+\frac{dv}{dy}\eta+\frac{dw}{dy}\delta.$$

The principle of the persistence of initially irrotational motion obviously follows from these equations.

ON THE FRICTIONAL RESISTANCES OF PNEUMATIC FOUNDATIONS.

By A. SCHMOLL.

From "Zeitschrift des Vereines Deutscher Ingenieure," Abstracts published by the Institution of Civil Engineers.

In applying pneumatic foundations for bridge piers it is important to know the amount of fractional resistance encountered by caissons in various strata and at different depths, in order to determine the load necessary to overcome this resistance, or in case of light, insufficiently resisting soil, to determine the depth to which a pier must be sunk to carry the load.

Cast-iron cylinders or caissons with casements, can be detached from the chains by which they have been kept in position, as soon as the columns have well penetrated the ground, whereby the sinking proceeds quicker, and the quantity of loose earth, forced under the lower edge of the caisson into the excavating chamber, is considerably reduced. For the erection of four cylinders of the Kehl bridge over the Rhine, where no cofferdams were used, the quantity of sand and stone removed from the interior of the excavating chamber was about

21,000 cubic yards, while the space occupied by the subterranean portion of the cylinders was only 13,734 cubic yards, the difference arising from the influx of loose earth and sand into the excavations.

According to the experience of the author, not only the nature of the soil but also the shape of the column influences the frictional resistance, the latter being smaller for cast-iron cylinders and rectangular caissons than for caissons of an oblong section.

The details given relate to the application of cast-iron cylinders, or of wrought-iron riveted caissons with vertical sides and with casings reaching above the water level for strata which form the bed of the Seine, Rhine, and Danube. In determining the frictional resistance the following conditions must be observed:

1. The tube or caisson must be vertical.

2. It must be free on all sides, neither attached to the guide chains nor resting upon its lower edge, but only kept in equilibrium by its weight, by the friction on its circumference and by the internal air pressure.

To achieve these conditions the total weight of the cylinder must be less than the frictional resistance, plus the weight of the displaced water; or else sinking takes place without air being let off, and without the cutting edge being undermined. If these conditions be fulfilled, the air pressure shown by a gauge is recorded, the safety valve opened, and after the sinking motion of the caisson has begun, the air pressure is again observed and the valve rapidly closed. The beginning of the motion indicates that the effective air pressure, together with the friction at the circumference of the caisson, have become slightly smaller than the cylinder.

EXPERIMENTS ON CO-EFFICIENTS OF FRICTION.

As co-efficients of friction relating to materials and surfaces which occur in pneumatic foundations were unknown to the author, he determined the following by direct experiments made in March, 1876, in Vienna, with the aid of a dynamometer.

Description of Materials.	Co-efficients of Friction.			
	At the Beginning of Motion.	During the Motion.	Beginning of Motion.	During Motion.
	For Dry Materials.		For Wet Materials.	
Sheet iron without rivets on gravel mixed with sand.	0.4015	0.4583	0.3348	0.4409
Sheet iron with rivets on gravel and sand........	0.3965	0.4911	0.4677	0.5481
Cast iron (unplaned) " "	0.3677	0.4668	0.3646	0.4963
Granite (roughly worked) " "	0.4266	0.5368	0.4104	0.4800
Pine (sawn) " "	0.4088	0.5109	0.4106	0.4985
Sheet iron without rivets on sand................	0.5361	0.6313	0.3655	0 3247
" with " "	0.7269	0.8391	0.5156	0.4977
Cast iron (unplaned) " "	0.5636	0.6063	0.4744	0.3796
Granite (roughly worked) "	0.6473	0.7000	0.4728	0.5291
Pine (sawn) "	0.6633	0.7340	0.5787	0.4793

Each figure is the average result of at least ten experiments. Between two consecutive experiments there was an interval of from eight to ten minutes. All materials were rounded off at their face in sledge shape and drawn lengthwise and horizontally over the gravel or sand; the latter was well levelled and bedded as solid as it is likely to be in its natural position. The riveted sheet iron contained twenty-five rivets on a surface of $2.53 \times 1.67 = 4.22$ square feet, the rivet heads were half round and of $\frac{11}{16}$ inch diameter.

Contrary to the experience of General Morin with other materials, it follows from these experiments, that for the above-named rough materials, the resistance of friction from rest is smaller than the resistance during the motion.

As the author's experiments relate to materials with rough surfaces this remarkable fact can be explained by the supposition that during the motion the small cavities and depressions on the sheet iron, cast-iron, wood, and stone surfaces become filled with dry sand which adheres to the surface, so that the effect is nearly the same as though two surfaces of dry sand were in contact with each other.

But for the friction of the surfaces of the same materials (except granite) on wet sand the reverse occurs. The friction during the motion is smaller than the friction at the beginning of motion. Probably the wet sand forms a more solid bed for the wet bodies sliding over it, in consequence of which the small depressions of the latter are not filled with sand so easily as in the case of dry surfaces.

CALCULATION OF THE RESISTANCE OF FRICTION IN PNEUMATIC FOUNDATIONS.

First example: Experiments made on the 2d of June, 1863, with the upper part of column V of the railway viaduct over the Seine at Orival near Elbœuf. As soon as the cast-iron cylinder, standing in an extensive and rather uniform bed of gravel, and having ceased to move for thirty-two hours, completely fulfilled all conditions necessary for such an experiment, and was undermined about 6 inches, the workmen left the column and the safety-valve was opened. When the air pressure had sunk from 1.20 to 1 atmosphere above the normal pressure of the atmosphere, the cylinder began to move, and at once sank 13 inches. It would have sunk deeper if the experimenter had not arrested the escape of air by closing the safety-valve at the beginning of the motion. At the moment of sinking the water had risen in the lower ring to a height of 34 inches.

Some minutes after this first experiment a second and third was made with the same cylinder, when it sank 13 inches and 12.6 inches respectively, the total fall for the three consecutive experiments therefore being $38\frac{1}{2}$ inches.

The weight of the column, including iron, concrete, masonry, and woodwork, was 218 tons.

The loss of weight by immersion at the moment when friction was overcome and the column began to sink was

$$[(10.20 \times 10.1788) + (1.00 \times 10.3491) - (0.87 \times 9.7868)] \times 1000 = 105{,}659$$
kilogrammes (104 tons).

10.20 metres being the height of the immersed part of the column including the lowest ring, and 10.1788 square metres the superficial area of the cylinder (diameter = 3.6 metres, and circumference = 11.31).

Metre.

$\left.\begin{array}{l}1.00 = \text{height} \\ 10.3491 = \text{superficial area}\end{array}\right\}$ of the lowst ring.

9.87 = height of inner water level over the cutting edge.

9.7868 = superficial area of water in the lowest ring.

The resistance of friction was therefore:

$$222{,}231 - 105{,}659 = 116{,}572 \text{ kilogrammes (114 tons).}$$

The author further cites (Example 2) an experiment with the first caisson of the right abutment of the railway bridge over the Danube between Vienna and Stadlau (Austrian State Railway), 5th of November, 1868, the results being nearly the same as in No. 1.

It sometimes occurs that, when the cylinder is in clay soil, the water does not enter into the working chamber, although the compressed air may have been completely left out. This was the case in experiments made by the author during the sinking of the cylinder mentioned in Example No. 2, on the 31st of December, 1868, and on the 2d and 4th of January, 1869. If the sinking of the caisson, in consequence of a decrease of the pressure of the air in the interior has begun, and the water fails to enter into the working chamber, the loss of weight by immersion is determined in two ways, first, by multiplying the air pressure per unit remaining in the interior of the caisson, with the base of the caisson; and second, in assuming that the water column replaced by the sinking object, has only a height equal to the vertical distance of the surface of the clay from the external water level.

Of the two calculated results the larger one is considered to be the buoyancy or loss of weight of the immersed object,

and to be deducted from the total weight of the latter, the difference being the resistance of friction.

The third example cited is an experiment made with No. 1 column (right bank) of the bridge over the Danube near Stadlau, on the 4th of January, 1869. The total weight of the sinking column was 1,305 tons. The cylinder began to sink when the air pressure had fallen from 1 atmosphere to 0.85 atmosphere, and in consequence of a further escape of air it gradually sank in one minute 18½ inches. The caisson being in a layer of solid clay of 3.73 feet thickness before it began to move, the water failed to enter into the cylinder.

The weight of the displaced water column, taking its height 27.35 feet, was 594 tons, and the upper pressure of the compressed air inside the cylinder was 625 tons. Deducting the latter result from the total weight of the caisson the frictional resistance on the surface of the cylinder is 678 tons. The column was sunk into the soil to a depth of 18.76 feet.

If the water fails to enter into the working chamber after a complete escape of the compressed air, the upward pressure is represented by the weight of the displaced water, taking as its height the vertical distance of the external water level from the water-tight soil, and the amount of frictional resistance is obtained by deducting this pressure from the total weight of the object.

The examples from which the author obtained his results are recorded in a series of tables, which show clearly that, in homogeneous strata, the resistance per unit of frictional surface decreases with the increasing depth. From this it is to be concluded that the density and cohesion of these strata augment with increasing depth, and therefore the pressure upon the sides of the column becomes less than in the upper strata.

MATTERS AFFECTING THE USE OF STEEL.

By M. ERNEST MARCHE.

Read before the Iron and Steel Institute at Paris.

(1.) The manufacture of commercial steel produced by the Bessemer and Martin-Siemens processes, and of cognate articles, has been greatly developed of late, more especially during the last ten years, on account of the demand, on the part of railway companies, for steel rails, axles and tires, and, on the part of the navy, for sheet plates and profile thwarts, &c. This rapid increase in the demand for steel is the result of the reduction of the price of manufacture, and that reduction is attributable to improved processes, as well as to the augmented power of the apparatuses employed; but, even under these circumstances, so speedy a development would not have been obtained had not metallurgists succeeded in producing, at will and regularly, steel of a different but well-defined nature, and had not their customers attained, by means of methodical experiments, to a gradually complete knowledge of the physical properties of, and the mode of employing, each of the steel productions in question. Whilst the chemical analysis constituted the indispensable guide for the choice of the raw materials, as well as for the study of the various processes, the experiments in respect of the elasticity, malleability and power of resistance of the manufactured pieces could alone impart confidence to the parties making use of the articles, and enable them to extend the sphere of reasonable applications of commercial steels. The laboratory of the chemist, and the machinery employed in the experiments, were of equal utility to the metallurgist and the engineer.

(2.) The researches as to the physical properties of the steels were undertaken in three different points of view. Very numerous and beautiful experiments, of an eminently scientific character, were required from the adepts in natural philosophy, and from learned men, and they on their part operating with powerful apparatuses, and availing themselves of the most improved processes, whilst making thorough and minute observations of

all phenomena, as well as of the strains and the deformations, have been enabled to lay before the public very valuable documents, treating, as a whole, of the properties of steel. It is to that class of operations that we are indebted for the works of Mr. D. Kirkaldy, Mr. Knut Styffe, Colonel Bosat, Professor Bauschinger, &c. In some ironworks, too, their productions were subjected to a serious examination, limited, however, to the determination of certain characteristic properties (as compared with the chemical composition) of the process of manufacture, the degree of elaboration, &c. Those experiments led to the classification of those works, as exercising a great influence on the extension of industrial applications, for example, the Creusot, Neuberg, J. Cotterill, Terre Noire, and other works—a classification bearing, for example, on the question as to the quantity of carbon contained, or to the resisting power, and the elongations in regard to traction. Finally, railway board contractors and military and naval authorities have prescribed a series of different tests for the articles ordered by them, according to the form and destination of these articles, and the reports that have appeared supply elements for the careful study of the practical results in a commercial point of view. The Universal Exhibition of 1878 has already served to enrich the already valuable collection of the triple series of experiments already referred to, by enumerating those that have been made in regard of Swedish plates, by the Jernkontoret (Swedish Iron Association); those on the castings, irons and steels of the Reschitza Works (Hungary), undertaken by Professor Bauschinger, and the experiments, mentioned in the Exhibition Catalogue, of the Terre Noire, La Voulte and Besiges Foundries and Ironworks, as well as the results of the experiments conducted on some classes of steel, with variable ingredients of carbon, manganese, phosphorus, &c.

(3.) With such valuable materials at hand, and at the disposal of all who take an interest in the questions now before us, we are enabled to institute useful comparisons, reconcile several opinions, and dilate openly on a considerable number of facts; but, on the other hand, when we come to throw a complex glance at the *ensemble* of all those works, we are struck with the great difficulty of drawing several conclusions, or formulating mathematical precise laws, the existence of which, however, we cannot fail to perceive. With the exception of carbon, the other substances, such as manganese, phosphorus, silica, &c., do not afford us—despite all the experiments made—any exact notion as to their respective action on some of the properties of steel. Moreover, and with reference to the determination of the conditions under which the experiments should be conducted, there still prevails the greatest uncertainty, on account of the difficulty of establishing exactly the relation between the results of—for example—a traction experiment and the effects produced on the same metal by flexion, tension, or concession. If, when grouping together the thousands of experiments made in various countries, made by various experimentators and on steel produced in various works, it has been hitherto impossible to attain to any confirmation of the laws connecting the physical properties of steel, and admitted in the classification of the establishments or works producing that article (laws, however, which many enlightened persons consider as having been prematurely admitted), that impossibility may be chiefly attributed to the fact that those experiments do not allow of any *direct comparison*, and that previously to an analysis of those results. it would be necessary to make some corrections, in order to eliminate the complicated causes which affect them—the experiments not having been conducted under strictly analogous conditions. The results of the experiments of natural philosophers—say in regard to density—are referred, by way of correction, to a like atmospheric pressure and to a like temperature; but all those experiments should be referred to a similar form and size of sample, to a similar degree of elaboration of the metal, to a similar molecular condition, &c.

(4.) It must be admitted, however, that those corrections are very difficult to make, and that, in every point of view, it would be preferable to bring about a general understanding between the experimentators, the ironworkers and the engineers, as to the conditions of conducting everywhere experiments capable

of undergoing a direct comparison. The meeting of the Iron and Steel Institute having, as it has, an international character, through the presence of its members in Paris, appears to offer the most favorable opportunity for laying down the basis of a programme comprising questions relating to the resisting power of steel, and the conducting of experiments on general and analogous conditions, with the view of enabling us to institute comparisons in respect to all such experiments. Without any pretension to put forward the exact form of such a programme, we may be allowed to direct attention to a certain number of points of a nature (we think) to lead to useful discussion.

(5.) We shall not stop to point out the first difficulty one meets with, in endeavoring to compare the experiments made in different countries, viz., the varying systems of measurement, a difficulty which necessitates long calculations for the reduction of one set of figures into that of another nation. That difficulty is certainly not an obstacle, but it involves a loss of time, which might be avoided for the future; and we can only express here the hope that we shall all shortly be able, by adopting the metrical system, to express in the same figures the like facts and reports.

(6) When one has to proceed to the investigation of the physical properties of pieces or samples of steel handed to an experimentator, it would be necessary for him to know, in the first instance, in order to determine the nature of the experiments to which those pieces or samples are to be subjected, whether the object in view is to study the nature and general qualities of the steel *per se*, or to ascertain how it would act under certain given applications. In the former case it would be always necessary to test previously, by traction, the cylindical or rectangular bars of the section and length to be determined on. In the latter case the system of experiments would depend on the nature of the application itself, and, generally speaking, the flexion and the "shock" (concussion) at the flexion will form the most useful tests.

We shall commence by examining the conditions relative to the traction experiments.

(7) In subjecting to a traction strain a steel bar of given section Ω and of length l, the facts to be observed and the quantities to be determined—if a complete experiment is to be made—should the following:—(1) During the period of perfect elasticity, the observation of the momentary elongations under given loads will allow us to determine the coefficient of the module of elasticity, E. (2) When the elongations cease to be in proportion to the loads, then the limit of elasticity is attained, load per unity of section, beyond which permanent elongations are produced, L. (3) The charge or load is increased until the rupture is produced, under a charge per unity of section, R. (4) The two portions of the broken rod are brought together, and the total length is then measured, deducting therefrom the original length; the final elongation at the rupture is then ascertained, and it is generally expressed with regard to the original length by per cent., Λ. (5) Then measure also the section taken by the rod at the point where the rupture has occurred. The difference between the original section and the rupture section is called, by Mr. Kirkaldy, the contraction, expressed in so much per cent. of the original section, but we prefer explaining this phenomenon of the contraction of the rupture section by the relation which the contracted section bears to the original section—that is, striction Z. (6) By dividing the total charge which produced the rupture, not by the primitive section, but by the contracted section, we have the resistance per unity of section broken $\frac{R}{2}=F$. (7) If, in a great number of trials, one is content to determine the diverse quantities above mentioned, it is nevertheless necessary, in order to have an exact and complete knowledge of the nature of any steel, to observe, in addition thereto, the permanent successive elongations made by the bar under trial, under increasing charges from the one corresponding to the limit of elasticity to the charge of rupture or breaking load. The manner in which these elongations vary, with the excess of charge on the limit of elasticity, is ordinarily represented by the aid of a "Graphic" which gives the curve of the elongations and allows of the measure-

ment being made of the work of resistance to the rupture, of which we speak hereafter. (8) Finally, it is useful, in order that the test may be complete, to describe the exact form which the bar presented after the rupture, so as to observe how the elongation was divided, and what is the exact position of the rupture section. We have to make some observations on each of those quantities.

8. COEFFICIENT OR MODULE OF ELASTICITY, E.

The modulus of elasticity is the relation of the charge to the elongations, produced during the period of perfect elasticity. It is constant during that period.

The investigation of this coefficient of elasticity for castings, iron and steel, has given rise to numerous experiments, and the figures which have been arrived at are so unlike that they cannot be brought together and compared, in order to ascertain whether, in the special point of view which we are now taking up, viz., the properties of commercial steel, we could account for the influence of the composition and of the purity of those steels on the manner in which they act during the period of perfect elasticity.

The modulus of elasticity is generally admitted at:

For castings	10,000,000,000
" iron	20,000,000,000
" steel	20,000,000,000

According to Renleaux:

Iron, in the form of bars or wires,	20,020,000,000
" in thin plates	24,010,000,000
Cast-iron	30,005,000,000
Other kinds of steel	20,020,000,000

According to Kupffer:

Flat iron, English rolled	20,010,000,000
" " " forged	20,232,000,000
" " Swedish "	21,341,000,000
Soft-cast steel	21,330,000,000

We now give the value of the modulus of elasticity resulting from the trials made by Knut Styffe:

	Carbon.	
Bessemer forged steel of H0060	1.26%	21,177,000,000
" "	1.05	21,515,000,000
Wikmanshyttan steel	1.22	21,949,000,000
Krupp "	0.61	22,046,000,000
Puddled rolled steel of Surahammar	0.56	21,033,000,000
Lowmoor puddled iron	0.20	22,480,000,000
Dudley " "	0.09	19,972,000,000
Surahammar " "	0.14	21,853,000,000
" " "	0.20	21,418,000,000
Hollstahammer rolled iron	0.07	20,357,000,000
" "	0.07	21,660,000,000

These trials already show us that the influence of the carbon is completely null in the elastic elongations produced by the same changes or loads, the coefficient of the elasticity of steels containing more than 1 per cent. of carbon is not superior to that of the irons of Motala, Surahammar, &c., which only contain 0.07 to 0.2.

When it is proved, on the other hand, that the least variations of the amount of the carbon completely modify the data of the permanent deformation, one cannot but be struck with the fact that during the period of perfect elasticity, the momentary elongations are independent of the *nature* of the metal. They appear to be modified only by the purity of the metal, and by the degree of work to which it has been subjected.

The trials or tests of Mr. Bauschinger on a series of Fernitz steels are more conclusive in that respect, because they have been made on products of the same origin, only differing with regard to the tenor of carbon and tested under the same form.

On the traction test those steels, of which the tenor of carbon varied from 0.14 to 0.96, gave:

Tenor of Carbon.	Limit of Elasticity per MM.	Coefficient of Elasticity. E.
	K.	
0.14	29.50	22,550 millions.
0.19	33.10	21,700 "
0.46	34.50	22,550 "
0.51	34.05	22,100 "
0.54	34.90	21,550 "
0.55	33.	22,200 "
0.57	33.10	21,600 "
0.66	37.45	22,450 "
0.78	37.50	24,700 "
0.80	40.05	21,500 "
0.87	42.90	21,850 "
0.96	48.70	21,750 "
General Average		22,200 "

The experiments made at Woolwich, in 1870, by a Committee of Civil Engineers, resulted in for *steels* of various works in the limit of elasticity varying from 26 to 42ks. and the tenor of carbon of 0.30 to 0.90 per cent., an average coefficient of 20,627,000,000, the minimum being 20,160

and the maximum 21,060,000,000. Some irons tried under the same conditions gave 20,092,000,000.

From all those results, one may conclude that the coefficient of elasticity of steel, whatever may be its hardness, varies only from 20 to 22,000,000,000.

Looking at those tests in a practical point of view, we may come to the conclusion in regard to the results, that, although in some particular cases—for example, in investigating the action of manganese and phosphorus—it would not be unimportant to deduce some facts relative to the modulus of elasticity, nevertheless that part of the traction tests may be in general dispensed with, as being the most delicate, the longest in duration, and (on account of the great lengths of rod or bar required) the most expensive. Consequently, the experiments may be made on short lengths, and a larger number of tests obtained, with the same quantity of steel, within the like given time and at the like expense.

The experiments made by Professor Bauschinger on the steels of the Reschitza works (Hungary), the results of which, as well as the samples used, figure in the exhibition of the Government Railways, Austrian Company I. R. P. demonstrate how much attention and time would have to be devoted for determining certain points which after all throw no fresh light on the properties of the steels subjected to the experiments in question, inasmuch as with respect to all those steels, differing considerably as they do in regard to the tenor of carbon, the module of elasticity is comprised between 22,400,000,000 or 23,000,000,000.

We may at once state that Professor Bauschinger's experiments on the Ternitz as well as on the Reschitza steels demonstrate likewise that the module of elasticity is the same (whatever may be the hardness of the steel) at compression and flexion, and that it is represented by the same figure as the coefficient of elasticity at the traction test.

(9) *Limit of Elasticity.*—If, during the period of perfect elasticity, the elongations are in proportion to the lodes or charges, and disappear with the action of those lodes, the limit of elasticity is the equivalent ("value") of the last charge producing that effect; and under a heavier load than that limit of elasticity there is a permanent elongation.

But if there be a concordance of opinion with respect to the existence and definition of that limit of elasticity, it is otherwise with regard to the fixation of its value, for—however exact and correct the instruments employed—it is difficult to determine the precise moment at which the state of perfect elasticity passes to that of permanent deformation. More correctly speaking, the "precise moment" in question does not exist with reference to experiments on steel rods because, in point of fact, there is a period of break of elasticity, during which certain portions of the rod or bar undergo permanent elongations, whilst other portions resume their original length.

On account of the uncertainty attending the phenomenon, it is necessary to establish conventional rules for determining the limit of elasticity, and as those vary which are adopted by the various experimentalists it results that the limit of elasticity observed in respect of the same kind of bar of the same length—say in England and Sweden—will not be the same.

It is true that Wertheim and other experimenters agree in considering, as the limit of elasticity, the load or charge that produces a permanent elongation, equal to the 0.00005 of the original length, in other words—and with reference to a bar or rod of one meter—a permanent elongation of five hundredths of a millimeter; but, on the other hand, so feeble an elongation is considered by many experimenters as the possible effect of accessory causes, and they are of opinion that we cannot admit that the limit of elasticity is exceeded until such time as a more clearly defined permanent deformation is observable; and the extension under heavier loads testifies to the reality of the phenomenon.

Those considerations have led to the following special definition by Mr. Knut Styffe:

"If an iron or steel bar be gradually extended by successive loads, which at first are so small that they occasion no permanent elongation, but are gradually increased, and are always allowed to operate for as many minutes as each additional weight is per cent. of the whole load, then the author regards as the 'limit

of elasticity' that load by which, when it has been operating by successive small increments as above described, there is produced an increase in the permanent elongation which bears a ratio to the length of the bar equal to 0.01 (or approximates most nearly to 0.01) of the ratio which the increment of weight bears to the total load."

Without discussing that definition, we have only to remark that, as Mr. Knut Styffe, adhering to the opinion that the limit of elasticity cannot be considered to be attained until we can ascertain exactly, and measure the determined increments of the permanent elongation, he furnishes some estimates of the limits of elasticity considerably higher to those given in Wertheim's definition.

The following are a few examples taken from the tabular data of Mr. Knut Styffe:

	Limit of Elasticity.		
	According to the definition of Wertheim.		According to the definition of Mr. Knut Styffe.
		K.	K.
	under	16.7	20.8
Puddled iron of Motala..	"	15.8	21
" " ..	"	14.5	18.8
" " ..	"	15.3	18.8
" " ..	"	20	22.6
Puddled iron of Middlesborough-on-Tees	"	19.3	21.2
"	"	20.5	23.5
"	"	24.1	24.8
"	"	16.5	19.9
Puddled iron of Dudley.	"	16.1	20
" " .	about	28.9	51.3
Cast steel of Wilmanshyttan (carbon 1.22%).	"	28.9	51.1

The difference is from 3 to 4 kilos. in irons, and amounts to 22 kilos. in the very hard steel of Wikmanshyttan.

We only produce these figures in order to show the necessity which would exist of arriving at a mutual agreement as to the fixation of the limit of elasticity, and how imperative it is in all cases that in the publication of the series of trials the method of estimating that useful value should be pointed out.

The value of the limit of elasticity is, in point of fact, the primary manifestation of the degree of hardness or of malleability of a steel, and in many cases the basis of calculation of the dimensions of the pieces.

As, moreover, we have seen that tests on short bars are to be preferred, and that on short bars the definition of Wertheim cannot be applied, whilst, on the other hand, the definition of Mr. Knut Styffe appears to us to give too high a quotation of figures, one can see what interest would be attached to a plain agreement on the subject, easily formed and adapted to the tests of short bars.

MENTAL LOGARITHMS.—Some years ago, about 1863, Mr. Oliver Byrne, formerly Professor of Mathematics in the College of Civil Engineers at Putney, discovered an entirely new and ingenious method of arithmetical calculation of great practical importance to Engineers and others, and which was claimed to enable any one acquainted with the ordinary rules of common arithmetic to extract the roots of cubics, equations of the fifth degree, and higher equations; to determine angular magnitude and trigonometrical lines, to solve plane triangles without the use of tables, and, generally, to deal with almost innumerable problems which had previously been considered to require great mathematical skill, and an intimate acquaintance with the higher branches of the science. But owing to the discoverer having adopted a peculiar and unfamiliar system of notation in explaining the art, many have regarded the whole subject as unintelligible, if not useless. A complete remedy for this has now been found by Mr. Edward David Hearn, M.A., of Columbia College, New York, whose name is already known to mathematicians, as the author of an extension of Horner's method for the synthetic division of algebraic quantities with detached co-efficients, and of an elucidation of Suffield's method of arithmetical synthetic division. Mr. Hearn contributes to the October number of *Scientific Review*, an interesting paper on "Mental Logarithms," in which he demonstrates that all the developments of which Mr. Byrne's art is capable, are not only practicable without any departure from the ordinary Arabic notation, with which every schoolboy is familiar, but that the common notation really increases the speed at which the calculations on the new system can be performed.

AUSTRIAN GUN EXPERIMENTS.—The Budget Committee of the Austrian Delegation have rejected the items of 1,712,000 florins proposed for the adaptation of the Werndl rifles to stronger cartridges, 250,000 florins for experiments in the manufacture of steel and bronze guns, and 200,000 florins for providing twenty-five heavy guns for arming forts and for repairs in fortresses.

WATER-PRESSURE MACHINERY EMPLOYED FOR WATER SUPPLY.

By C. KROBER.

From "Der Praktische Maschinenconstructeur," Abstracts published by the Institution of Civil Engineers.

The water supply of Sigmaringen Castle, the residence of Prince Charles Antony of Hohenzollern, was formerly obtained from a source on the left bank of the Danube by means of pumps driven by a water-wheel. Owing to the imperfect construction of the machinery and the limited power, the water could not be raised to the upper storeys of the buildings. Improved machinery was therefore erected in October, 1876, the Roman tower, situated about the centre of the castle, being chosen as the most convenient place for the reservoir.

The highest water level of this reservoir was 183.5 feet above that of the source, and about 203 feet above the Danube. The quantity of water required to be lifted per second was 0.408 gallon, and the total quantity of water supplied by the source 10.867 gallons per second; thus there remained for the motor 10.867 $- 0.408 = 10.459$ gallons. The fall from the level of the source to the discharge of the proposed water motor was 13.1 feet. Taking the resistance of friction in the existing pressure pipe (of 4 inches diameter and 2,450 feet length) as equivalent to a loss of height of about 9.8 feet, the effective work to be done by the machinery was $0.408 \times 10 \times 193.3 = 788.6$ foot lbs.

The available water power $10.46 \times 10 \times 13.1 = 1370$ foot lbs. The machinery was therefore required to utilize $\frac{788.6}{1370} \times 100 = 57\frac{1}{2}$ per cent. of the available water power.

The application of a turbine with intermediate gearing would have occasioned a loss of effect amounting to at least 40 per cent.; a water-wheel would only have lost about 32 per cent. Adding to this a loss of effect of 15 per cent. by the pumps, the effect of the whole machinery was estimated at 55 per cent. These considerations induced the engineer to use direct-acting water-pressure machines, and although these had probably not yet been applied for so small a head of water (13 feet), no doubt was entertained as to their superiority over turbines or water-wheels. Experiments on the loss of effect arising from friction of the pistons and from other sources, were made by Messrs. Sulzer Brothers, and the result justified them in undertaking the construction of the machines under guarantee to lift 0.408 gallon of water per second into the high level reservoir with the available water power.

To enable repairs being made without a complete interruption of the water supply, the machinery was constructed in duplicate. The machines were arranged horizontally, the pumps were single-acting plunger pumps, the piston rods of the motor serving as plungers of the pumps. By attaching the pump cylinders immediately to the covers of the central or pressure cylinder, only one stuffing box was required on each side of the motor to separate the two water spaces.

The starting of the machinery completely justified the expectations of the engineer. The measurements and experiments made by him showed, that for the normal work it was necessary to throttle the headwater, and that the required quantity of 0.408 gallon could be pumped into the reservoir by 9.8 gallons of water at a pressure corresponding to a height of 10.66 feet. Taking again the resistance of friction in the tubes as equivalent to a loss of height of 9·8 feet, the machines utilize

$$\frac{0.408 \times 193.3}{9.8 \times 10.66} \times 100 = 75$$

per cent. of the consumed water power.

The indicator diagrams taken from the pumps as well as from the pressure cylinder, showed a perfect uniformity of pressure. The pressure pistons have 1 foot $3\frac{1}{16}$ inches, and the pump pistons $3\frac{1}{4}$ inches diameter. The stroke is 3.3 feet.

THE STRENGTH OF MATERIALS.

By WILLIAM KENT, M.E., Pittsburgh, Pa.

Written for VAN NOSTRAND'S MAGAZINE.

II.

COMPRESSIBLE STRESS.

A compressive stress, or push, applied to a piece of material is a force which tends to shorten it. In testing the compressive resistance of metals or other materials, testing machines similar to those used in tests of tensile resistance are used, or the tensile machine may be adapted to compressive tests by means of a couple of yokes or like mechanical device.

What is meant by the term "compressive strength" has not yet been settled by the authorities, and there exists more confusion in regard to this term than in regard to any other used by writers on strength of materials. The reason of this may be easily explained. The effect of a compressive stress upon a material varies with the nature of the material, and with the shape and size of the specimen tested. While the effect of a tensile stress is always to produce rupture or separation of particles in the direction of the line of strain, the effect of a compressive stress on a piece of material may be either to cause it to fly into splinters, to separate into two or more wedge-shaped pieces and fly apart, to bulge, buckle or bend, or to flatten out and utterly resist rupture or separation of particles. A piece of speculum metal under compressive stress will exhibit no change of appearance until rupture takes place, and then it will fly to pieces almost as suddenly as if blown apart by gunpowder. A piece of cast iron or of stone will generally split into wedge-shaped fragments. A piece of wrought iron will buckle or bend. A piece of wood or of zinc may bulge, but its action will depend upon its shape and size. A piece of lead will flatten out and resist compression till the last degree; that is, the more it is compressed the greater becomes its resistance.

Air and other gaseous bodies are compressible to any extent, as long as they retain the gaseous condition. Water, not confined in a vessel, is compressed by its own weight to the thickness of a mere film, while when confined in a vessel it is almost incompressible. It is probable, although it has not been determined experimentally, that solid bodies when confined are at least as incompressible as water. When they are not confined, the effect of a compressive stress is not only to shorten them, but also to increase their lateral dimensions or bulge them. Lateral strains are therefore induced by compressive stresses.

The weight per square inch of original section required to produce any given amount or percentage of shortening of any material is not a constant quantity, but varies with both the length and the sectional area, with the shape of this sectional area, and with the relation of the area to the length. The "compressive strength" of a material, if this term be supposed to mean the weight in pounds per square inch necessary to cause rupture, may vary with every size and shape of specimen experimented upon. Still more difficult would it be to state what is the "compressive strength" of a material which does not rupture at all, but flattens out. Suppose we are testing a cylinder of a soft metal like lead, two inches in length and one inch in diameter, a certain weight will shorten it one per cent., another weight ten per cent., another fifty per cent., but no weight that we can place upon it will rupture it, for it will flatten out to a thin sheet. What then is its compressive strength? Again, a similar cylinder of soft wrought iron would probably compress a few per cent., bulging evenly all around, it would then commence to bend, but at first the bend would be imperceptible to the eye and too small to be measured. Soon this bend would be great enough to be noticed, and finally the piece might be bent nearly double, or otherwise distorted. What is the "compressive strength" of this piece of iron? Is it the weight per square inch

which compresses the piece one per cent., or five per cent., that which causes the first bending (impossible to be discovered), or that which causes a "perceptible" bend?

So confusing is this whole matter of compressive strength that there is scarcely a single published figure on the compressive strength of wrought iron that can be relied upon. Wood's resistance of materials has the following:

"Comparatively few experiments have been made to determine how much wrought iron will sustain at the point of crushing. Hodgkinson gives 65,000, Rondulet 70,800, Weisbach 72,000, Rankine 30,000 to 40,000. It is generally assumed that wrought iron will resist about two-thirds as much crushing as to tension, but the experiments fail to give a very definite ratio."

Mr. Whipple, in his treatise on bridge building, states that a bar of good wrought iron will sustain a tensile strain of about 60,000 pounds per square inch, and a compressive strain, in pieces of a length not exceeding twice the least diameter, of about 90,000 pounds.

In a "Pocket Companion" of tables appertaining to the use of wrought iron, published by Carnegie Bros. & Co., of Pittsburgh, the following values are given in a table of crushing strength of materials, said to be deduced from the experiments of Major Wade, Hodgkinson, and Capt. Meigs:

American	wrought	iron,	127,720	lbs.
"	"	" mean	83,500	"
English	"	"	65,200	"
			40,000	"

On the page next after this table is the following:

"Experiments upon wrought iron give a mean crushing stress [strength] of 74,250 lbs. per square inch."

When the best authorities differ so widely in their views of the compressive strength of wrought iron, can it be wondered that engineers have so long hesitated to use the material in compression, and that when they do use it they dare not use it with economy? If the United States Board appointed to test Iron and Steel, which seems to be about to close its labors for want of an appropriation to continue them, were to do nothing else than investigate this matter of the compressive strength of wrought iron, and establish rules governing its use to resist compressive forces in structures, they would confer a benefit on the country and the world which would be worth ten times the amount of the appropriation necessary for the whole work.

Stoney states that the strength of short pillars of any given material, all having the same diameter, does not vary much, provided the length of the piece is not less than one, and does not exceed four or five diameters, and that the weight which will just crush a short prism whose base equals one square inch, and whose height is not less than 1 to 1½ and does not exceed 4 or 5 diameters, is called the *crushing strength* of the material. It would be well if experimenters would all agree upon some such definition of the term "crushing strength," and insist that all experiments which are made for the purpose of testing the relative values of different materials in compression be made on specimens of exactly the same shape and size. An arbitrary size and shape should be assumed and agreed upon for this purpose. The size mentioned by Stoney is definite as regards area of section; *viz*: one square inch, but is indefinite as regards length; *viz:* from 1 to 5 diameters. In some metals a specimen of 5 diameters long would bend, and give a much lower apparent strength than a specimen having a length of 1 diameter. The words "will just crush" are also indefinite for ductile materials, in which the resistance increases indefinitely if the piece tested does not bend. In such cases the weight which causes a certain percentage of compression, as five, ten, or fifty per cent. should be assumed as the crushing strength.

For future experiments on crushing strength, three things are desirable; first, an arbitrary standard shape and size of test specimen for comparison of all materials; secondly, a standard limit of compression for ductile materials, which shall be considered equivalent to fracture in brittle materials; thirdly, an accurate knowledge of the relation of the crushing strength of a specimen of standard shape and size to the crushing strength of specimens of all other shapes and sizes. The latter can only be

secured by a very extensive and accurate series of experiments upon all kinds of materials, and on specimens of a great number of different shapes and sizes. Hodgkinson has been the chief experimenter in this direction, but his researches have not been nearly so extensive as to give us all the desirable information on this point. A standard size for compression tests, and a standard limit of compression, assumed equivalent to fracture, have never yet been agreed upon and have probably never even been proposed.

The writer proposes, as a standard shape and size for a compressive test specimen* for all materials, a cylinder one inch in length, and one half square inch in sectional area, or 0.798 inch diameter; and for the limit of compression equivalent to fracture, ten per cent. of the original length. The term "compressive strength," or "compressive strength of standard specimen," would then mean *the weight per square inch required to fracture by compressive stress a cylinder one inch long and* 0.798 *inch diameter, or to reduce its length to* 0.9 *inch if fracture does not take place before that reduction in length is reached.* If such a standard, or any standard size whatever, had been used by the earlier authorities on the strength of materials, we never would have had such discrepancies in their statements in regard to the compressive strength of wrought iron as those given above

The reasons why this particular size is recommended are, that the sectional area, one half square inch, is as large as can be taken in the ordinary testing machines of 100,000 pounds capacity, to include all the ordinary metals of construction, cast and wrought iron and the softer steels; and that the length, one inch, is convenient for calculation of percentage of compression. If the length were made two inches many materials would bend in testing, and give incorrect results. Even in cast iron, Hodgkinson found as the mean of several experiments on various grades, tested in specimens $\frac{3}{4}$ inch in height, a compressive strength per square inch of 94,730 pounds, while the mean of the same number of specimens of the same irons tested in pieces $1\frac{1}{2}$ inches in height, was only 88,800 pounds. The best size and shape of standard specimen should, however, be settled upon only after consultation and agreement among several authorities. The United States Board appointed to test Iron, Steel, etc., or the American Society of Civil Engineers might easily fix upon such a standard.

After fixing upon a standard test piece, by which all materials might be compared, tests should be made of all sizes and shapes other than the standard, to determine what relation existed between their apparent strength per square inch and that of the standard. When these results were obtained and formulated, a test might be made of any material of any shape and size, and by applying the formulæ of reduction, the "compressive strength of standard specimen" be predicted, and from the latter could be calculated the strength of any shape and size which it might be proposed to use in a structure.

Some of the results already obtained in the direction of determining the relation of length and diameter to apparent compressive strength will now be given; taken chiefly from Wood's Resistance of Materials.

As above stated, in Hodgkinson's experiments the increase of the length of test specimens decreased the apparent compressive strength per square inch from 94,730 to 88,800 pounds.

Fairbank and Tate, in testing small cubes and cylinders of glass, found a compressive strength for the cubes of 18,401 and for the cylinders of 30,153 pounds per square inch.

Hodgkinson, in experiments on long square pillars, found that the compressive strength varied as the 3.59 power of the side of the square, as a mean result; the extremes being the 2.69 and the 4.17 powers. From his experiments, the following table of the absolute strength of columns was obtained, in which

P = crushing weight in gross tons.

d = the side of the column in inches, or external diameter.

d_1 = the internal diameter of the hollow, in inches.

l = the length in feet.

Kind of column.	Both ends rounded the length of the column exceeding 15 times its diameter.	Both ends flat, the length of the column exceeding 30 times its diameter.
Solid cylindrical columns of cast iron..........	$P=14.9\frac{d^{3.76}}{l^{1.7}}$	$P=44.16\frac{d^{3.55}}{l^{1.7}}$
Hollow cylindrical columns of cast iron....	$P=13\frac{d^{3.76}-d_1^{3.76}}{l^{1.7}}$	$P=44.34\frac{d^{3.55}-d_1^{3.55}}{l^{1.7}}$
Solid cylindrical columns of wrought iron......	$P=42\frac{d^{3.76}}{l^2}$	$P=133.75\frac{d^{3.55}}{l^2}$
Solid square pillar of Dantzic oak..........	....	$P=10.95\frac{d^4}{l^2}$

The above formulas apply only in cases in which the length is so great that the column breaks by bending and not by simple crushing. If the column be shorter than that given in the table, and more than four or five times its diameter, the strength is found by the following formula:

$$W=\frac{PCK}{P+\frac{3}{4}CK}$$

in which

P=the value given in the preceding table.

K=the transverse section of the column in square inches.

C=the modulus for crushing in gross tons per square inch.

W=the strength of the column in gross tons.

The "modulus of crushing" is defined by Prof. Wood as "the pressure which is necessary to crush a piece of any material whose section is unity and whose length does not exceed from one to five times its diameter"—a very indefinite quantity, as already shown in the case of wrought iron!

Prof. Wood states that it is found by experiment that the resistance of short pieces (blocks) to crushing varies nearly as the transverse section of the piece. Gen. Gillmore, however, in experiments on stone, found that the strength per square inch of section of cubes of different sizes varied nearly as the cube root of the side of the cube.

In some experiments made by Mr. J. Tangye on the compressive strength of wrought iron, a bar of soft Lowmoor iron, 8 or 9 inches long, was planed on opposite sides to a thickness of $\frac{3}{4}$ inch, and subjected to pressure on one side under a steel die $\frac{1}{2}$ inch square. The following are the results of the tests, and they prove clearly that a unit of iron has a much greater power of resistance when it forms part of a large mass than when it is isolated in the manner customary in experiments on compression:

Load per square inch.

20 tons, no impression.

24 " slightest indentation, sensible to finger nail.

28 " indentation visible, edge followed by finger nail.

40 " indented about $\frac{1}{64}$th of an inch.

These experiments certainly throw a doubt upon the statement that the resistance of short pieces varies as the transverse section.

Gordon's Rules for Flexible Columns. (From Clark).—The first and second formulas given below were deduced by Lewis D. Gordon, from the results of Hodgkinson's experiments. As here given, they show the total breaking weight of a cast iron column. The succeeding formulas for strength of columns of wrought iron and steel were constructed on the basis of Gordon's formula by the authorities named.

For solid or hollow round cast iron

$$\text{columns } W=\frac{36a}{1+\frac{r^2}{400}}$$

For solid or hollow rectangular cast iron

$$\text{columns } W=\frac{36a}{1+\frac{r^2}{500}}$$

For solid rectangular wrought iron

columns $W=\frac{16a}{1+\frac{r^2}{3000}}$ (Stoney).

For columns of angle, tee, channel, or cruciform iron $W=\frac{19a}{1+\frac{r^2}{900}}$ (Unwin).

For solid round column of mild steel

$$W=\frac{30a}{1+\frac{r^2}{1400}} \quad \text{(Baker).}$$

For solid round column of strong steel

$$W=\frac{51a}{1+\frac{r^2}{900}} \quad \text{(Baker).}$$

For solid rectangular column of mild steel $W=\frac{30a}{1+\frac{r^2}{2480}}$ (Baker).

For solid rectangular column of strong steel $W=\frac{51a}{1+\frac{r^2}{1600}}$ (Baker).

In these formulas W = the breaking weight in tons of 2240 lbs., a = sectional area of the material in square inches, r = the ratio of the length to the diameter, the diameter being the least dimension of the section or that in which it is most flexible.

The pocket book of Carnegie Bros. & Co. (mentioned above) gives a formula for the strength of wrought iron columns, based upon Gordon's formula as follows:

$$W=\frac{FA}{1+\frac{1}{4500}\left(\frac{l}{h}\right)^2}:$$

in which W = breaking load in pounds, F = 36000 pounds, A = sectional area of column in square inches, l = its length in inches, and h = its diameter in inches.

A similar pocket book, by the Phœnix Iron Co., of Philadelphia, gives the formula in this shape:

$$W=\frac{FA}{1+\frac{1}{3000}\left(\frac{l}{h}\right)^2}$$

in which F = 50000 pounds.

The following example will show the discrepancies in results obtained by these two formulas and the one of Stoney:

Given a wrought iron column one inch square in section and ten inches long. Required its breaking weight?

A = 1 square inch. $\frac{l}{h}=r=10.$ $r^2=10.$

$W=\frac{16\times 1}{1+\frac{100}{3000}}=15.489$ tons $=34{,}695$ lbs. (Stoney).

$W=\frac{36000\times 1}{1+\frac{100}{4500}}=\ldots\ldots$ 35,218 lbs. (Carnegie Bros. & Co).

$W=\frac{50000\times 1}{1+\frac{100}{3000}}=\ldots\ldots$ 48,389 lbs. (Phœnix Iron Co).

showing a difference of nearly 40 per cent. between the lowest and highest results. The columns of a building which were designed with the use of one of these formulas might cost 40 per cent. more than if designed with the use of another.

Enough has now been given to show that our knowledge of the subject of compressive strength is very indefinite and unreliable. The greatest necessity exists for a comprehensive series of experiments which shall serve as a standard for reference and comparison. Such experiments would be too costly to be undertaken by any individual, and as the matter is of national importance, it is well worthy the attention of the government.

In making experiments upon compressive strength, even greater care is required than in experiments on tensile strength. In tensile tests, the tendency of a ductile specimen is always to pull into the line of strain, and this to some extent (but not entirely) corrects the error caused by wrongly placing the piece in the testing machine. In compressed tests the tendency is just the reverse; the effect of a push is always to cause the piece to tend to bend out of the line of strain, and this can only be prevented by having the line of strain pass exactly through the axis of the specimen. The test specimen should, therefore, be placed in the machine with the utmost accuracy, care should be taken that the bearing of the piece on the compression blocks is a true one, and that in pulling or pushing together the compression blocks they shall have no tendency

to move sidewise or in any other direction than that of the line of strain.

TRANSVERSE STRESS.

Tests by transverse stress are in general much more easily made than tests by either tensile or compressive stress. An elaborate testing machine is not necessary. The bar or beam to be tested is placed on two supports, which are a measured distance apart and perfectly level, and weights applied to the middle till the piece breaks. Steel rollers are frequently used for supports, rolling on a horizontal plane and kept a constant distance apart during the test. The use of rollers obviates the error due to friction of the bar upon the supports when the latter are fixed.

In testing bars which require considerable weight to break them, some mechanical appliance has to be used to assist in placing the weights on the bar without shock. A testing machine, designed for transverse tests alone, built by Messrs. E. & T. Fairbanks & Co. for the Mechanical Laboratory of the Stevens Institute of Technology, consists of an ordinary platform scale, with a heavy timber foundation, carrying a heavy cast iron beam with two cast iron supports which may be placed any distance apart up to five feet. The bar to be tested rests on rollers, and the pressure is applied by means of a screw. This machine was largely used in the experiments on strength of alloys, made by the United States Board appointed to test iron, steel, etc., and a complete description of it will appear in the report on these tests when published. It has been used up to 7,000 pounds pressure, and it will test bars up to five feet in length. Measurements of deflection are made to the $\frac{1}{10000}$ of an inch by means of the micrometer screw apparatus, with electrical contact, which was described under the head of tensile tests. By this machine and measuring apparatus very important scientific results have been obtained in reference to the effect of time upon tests, in elevating the elastic limit and in causing increase of deflection and decrease of set, which are described at length by Prof. Thurston in the Transactions of the American Society of Civil Engineers, 1875 and 1876.

The new 40,000 pound tensile testing machines, built by Riehle Bros. of Philadelphia, have attachments by which transverse tests may be made on bars 12 inches in length between supports, and the new 100,000 pound machine of John L. Gill, Jr., of Pittsburgh, has a transverse attachment that will take a bar of any length from 10 to 40 inches. The manufacturers of wrought iron beams have had tests made and published in their "pocket books of information" on the transverse strength of their beams, the results of which tests may in general be relied on for practical purposes. There exists no such discrepancy in published figures of transverse strength of beams as that which has been shown to exist in figures of the compressive strength of columns.

In recording results of transverse tests, it is important sometimes to note the time taken in the tests. A remarkable instance of the increase of apparent strength of wrought iron, by keeping it under strain for three weeks, has already been noted in discussing the influence of time upon tensile tests. Bars of tin and of ductile alloys will show a much less transverse strength if considerable time is taken in testing than if the test is made rapidly, while the reverse is true in tests of wrought iron and soft steel.

The effect of time in a transverse test, varies according to the method of testing. In testing by dead loads the load remains constant, and if the bar is strained beyond its elastic limit the deflection may increase with time. This increase of deflection may continue for several minutes or for several days and then entirely cease; or it may continue with increasing rapidity until the bar breaks or bends. In Prof. Thurston's tests by transverse stress some curious phenomena have been observed. The rate of increase of deflection in some instances grew smaller and smaller for several hours, then for a while it remained constant, and then increased till the bar broke. It would appear as if there was a certain definite deflection for each material, after passing which a continuance of the load which caused it will increase the deflection indefinitely, at an increasing rate, until bending or rupture takes place, but that before that deflection is reached the rate of increase of deflection

will be a decreasing one, tending to a cessation of that increase.

In testing by a pressure screw and platform scale, or similar testing machine, the action of a dead load is imitated only when the machine is so operated as to keep the scale beam constantly balanced, the deflection meanwhile increasing, as the pressure screw requires to be advanced to keep the beam balanced. If the pressure screw is not advanced, and the deflection is therefore held constant, the bar will exhibit a decrease of resistance to that deflection, which decrease, in every ductile and non-elastic material, will continue with a decreasing rate until the deflection has become a permanent set, and the scale beam indicates no resistance at all; in elastic materials the decrease of resistance will soon cease, and the bar, acting like a perfect spring, will keep the scale beam balanced at a figure somewhat below that indicating the load which primarily caused the deflection. In a perfectly elastic material, as hard steel or glass, no decrease of resistance (or only a very slight one) takes place. The study of these phenomena opens an interesting and almost unexplored field of physical research.

A standard size and shape of test specimen would be desirable for transverse tests, but it is not so necessary as in compressive or tensile tests, since the relation existing between the dimensions of bars and both their ultimate strength and deflection under loads has been determined, both mathematically and by experiment. The strength of bars of rectangular section is found to vary directly as the breadth of the specimen tested, as the square of its depth, and inversely as its length. The deflection under any load varies approximately as the cube of the length, and inversely as the breadth and as the cube of the depth. Represented algebraically, if S = the strength and D the deflection, l the length, b the breadth, and d the depth.

$$S \text{ varies as } \frac{bd^2}{l} \text{ and } D \text{ varies as } \frac{l^3}{bd^3}.$$

For the purpose of reducing the strength of pieces of various sizes to a common standard the term *modulus of rupture* (represented by R) is used. Its value is obtained by experiment on a bar of rectangular section supported at the ends and loaded in the middle and substituting numerical values in the following formula

$$R = \frac{3}{2}\frac{Pl}{bd^2}, \text{ in which}$$

P = the breaking load in pounds, l = the length in inches, b the breadth, and d the depth.

The *modulus of rupture* is sometimes defined as the strain at the instant of rupture upon a unit of the section which is most remote from the neutral axis on the side which first ruptures. This definition, however, is based upon a theory which is yet in dispute among authorities, and it is better to define it as a numerical value found by application of the formula above given.

Knowing the value of R for any material the weight necessary to break a beam of that material, loaded in any of the ways specified below may be found from the following formulas:

For a beam fixed at one end and a load P at the other

$$P = \frac{1}{6}\frac{Rbd^2}{l} \qquad (1)$$

The same beam with a load W uniformly distributed over the length

$$\frac{1}{2}W = \frac{1}{6}\frac{Rbd^2}{l} \qquad (2)$$

Beam supported at its ends and loaded in the middle

$$P = \frac{2}{3}\frac{Rbd^2}{l} \qquad (3)$$

Beam supported at its ends and loaded uniformly

$$P = \frac{4}{3}\frac{Rbd^2}{l} \qquad (4)$$

Beam fixed at both ends and loaded in the middle

$$P = \frac{4}{3}\frac{Rbd^2}{l} \qquad (5)$$

Beam fixed at both ends and loaded uniformly

$$P = \frac{2Rbd^2}{l} \qquad (6)$$

These formulas are all deduced mathematically (see Wood's Resistance of Materials) and are confirmed by experiment, except the last two, in which it seems that

the mathematical investigation has not included all the conditions, for Mr. Barlow found by experiment that equation (5) should be $P=\frac{Rbd^2}{l}$.

The value of R for any material depends upon the tensile and the compressive strength, and if these (per square inch) are not equal it is always greater than the one and less than the other. In testing a beam by transverse stress (supported at the ends) the upper side is compressed and the lower side is extended, the surface located at some position between the compressed and extended sides, which receives no strain, being called the neutral axis, or neutral surface.

The strength of beams of other than rectangular section, the deflection of beams of various shapes and sizes, the relation existing between transverse strength and tensile and compressive strength, and many other interesting branches of the subject of transverse strength are treated of at great length by various authorities, but the limits of this paper will not allow of more than a mere mention of them here. A recent work entitled "Transverse Strain," by R. G. Hatfield, is an invaluable book for architects and others interested in the subject of which it treats, who are not professional engineers. Various theories of the relation between transverse strength and tensile and compressive strength are found in works on resistance of materials. A theory on this subject is given by the writer in a note published in VAN NOSTRAND'S MAGAZINE, October, 1877.

SHEARING STRESS.

Shearing stress is a force tending to draw one part of a solid substance over another part of it; the applied and resisting forces acting in parallel planes which are very near to each other. It acts like a pair of shears. Materials under a variety of circumstances are subjected to this stress—as rivets in riveted plates, pins and bolts in spliced joints, beams subjected to transverse stress, bars which are twisted, and in short all pieces which are subjected to any kind of distorsive stress in which all parts are not equally strained. Shearing may take place in detail, as when plates or bars of iron are cut with a pair of shears, when only a small portion is operated upon at a time; or it may be done so as to bring into action the whole section at a time, as in the process of punching holes into metal, where the whole convex surface of the hole is supposed to resist uniformly.*

The total resistance to ultimate shearing, when all parts of the resisting surface are brought into action at once, is found to vary directly as the section. The experiments on large sections, however, have not been sufficiently numerous to make this certain for all cases. From a table of shearing strength of various metals, given in Good's "Resistance of Materials," on the authority of various experimenters, it appears that the shearing strength of wrought iron is about the same as its tenacity, of cast steel it is a little less than its tenacity, of cast iron it is double its tenacity and about $\frac{2}{5}$ its crushing resistance, and of copper it is about $\frac{2}{3}$ its tenacity. Clark considers that the shearing strength of wrought iron may be taken at about $\frac{4}{5}$ of its tenacity. Clark also gives the following: "Rankine states that the shearing strength of cast iron is 12.37 tons per square inch; Stoney found by experiment, that it varied from 8 to 9 tons per square inch. Both may be correct, as cast iron is very variable in tensile strength. It is probable that its shearing resistance is by reason of its comparative incompressibility equal to its direct tensile resistance."

Engineer-in-Chief, William H. Shock, U. S. N., found that the shearing resistance of ordinary round bar iron of commerce averaged 17.81 tons, or 39894 pounds per square inch, on bolts of from $\frac{1}{2}$ inch to 1 inch diameter.

M. C. Little found the resistance to shearing by parallel cutters of bars 3 inches by $\frac{1}{2}$ inch and 1 inch thick, and to punching 1 and 2 inch holes through bars $\frac{1}{2}$, 1 and $1\frac{1}{2}$ inches thick, varied from 19 to 22.35 tons per square inch of area cut.

In view of the conflicting statements given above, it may well be supposed that our knowledge on the subject of shearing resistance is almost as indefinite as that of compressive resistance. Some light is thrown upon the reason of the

* Wood's "Resistance of Materials."

discrepancies by the following fact, taken from Clark:

"Oak treenails firmly held, of from 1 to 1¾ inch diameter, were found by Mr. Parsons to have a shearing strength of about two tons per square inch of section. For the development of so much resistance, Rankine deduces that the planks connected by the treenails should have a thickness of at least three times their diameter. Treenails of 1¾ inches diameter in a three inch plank, bore only 1.43 tons per square inch, and in a six inch plank 1.73 tons."

Here it is plainly shown that the strength of the treenail depended not only on its diameter, but also on the bearing it had in the plank, and it seems almost certain, therefore, that the varying firmness of the bearings used by different experimenters has been the cause of the discrepancies in their results. The following illustration may show how different results might be obtained on the same piece of iron:

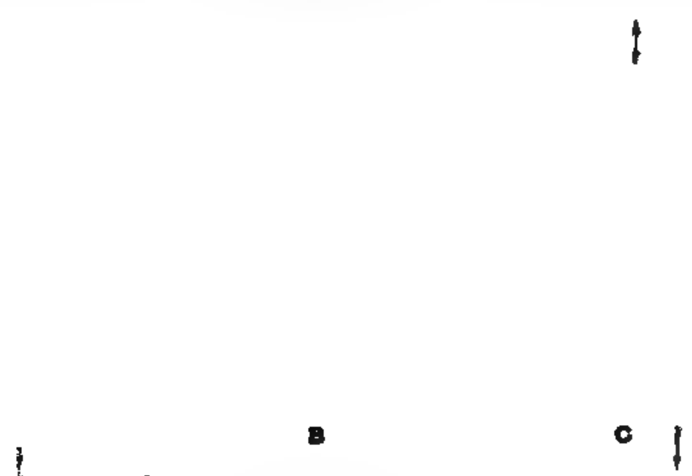


Fig. 6.

A represents a case of true double shearing. The iron bolt is being sheared off directly in the lines of the sliding surfaces aa' and bb', the sliding plates being of hardened steel with true corners. Test B might appear exactly like test A to the ordinary observer if the bending, which is exaggerated in the cut, were very slight. The only difference here is, that the sliding plates are of soft iron, and the bolt fits itself to a bearing along the lines cc', and the result is a combination of tensile, shearing and bending stresses. It would be impossible to say from such a test as this, what the actual shearing strength of the bolt would be. C represents a case of single shearing in which plates of soft iron are used with a similar effect to that shown in B, but still worse. The dotted line shows that the line of strain of the testing machine is at an angle to the line of sliding, which further complicates the result.

A committee of the American Railway Master Mechanics' Association, made the following experiments on riveted plates:

Six pieces 1¾ inches wide and $\frac{5}{16}$ inch thick, cut from the same sheet, were punched and riveted together with the best ⅝ inch rivets, one rivet to each pair, with the following results:

	lbs.
No. 1 broke in center line of hole under	17,828
No. 2 " " " "	17,828
No. 3 " " " "	17,143
The average breaking strain being	17,599

Six pieces, duplicates of those last mentioned, were *drilled* and riveted together, one ⅝ inch rivet to each pair.

No. 1 *sheared the rivet* under	17,143 lbs.
No. 2 " "	16,457 "
No. 3 " "	15,428 "
The average *shearing* strain being	16,342

Prof. Wood remarks concerning these tests, that "it is evident that drilled holes cause the rivets to be sheared more easily than punched ones." If these sets of tests are considered as tests of the

strength of ⅝ inch rivets, the latter set show the rivets to have an average strength of 16,342 pounds, or 53,231 pounds per square inch, while the former shows their average to be *more than* 17,599 pounds, or 57,326 pounds per square inch. Different methods of testing, therefore, give different results. Which is the correct method? And how many tests of which the results are published were made by the correct method?

The writer is not aware that any standard method of making tests of shearing strength, or standard size of test specimens, has ever been proposed. For testing the transverse shearing strength of bolts and rivets, the use of the double shearing plates, shown at A Fig. 6, is probably the best method; the plates being made of hardened steel, and the holes drilled in them just large enough to allow the bolt to enter with a sliding fit. The best thickness of the plates or the relation of thickness to the diameter of the holes would have to be determined by experiment before the proper standard could be fixed, as the tests of treenails by M. Parsons, above mentioned, show that the thickness of the bearing has an influence upon the results. As it is not entirely certain that the resistance of various sections of the same material to shearing stress is exactly proportional to the area of section, experiments to determine the relation of shearing resistance to area of section and determine the best size and shape for a standard test specimen are needed, in order that the results obtained by different experimenters may be compared.

It has been stated above that in the process of punching metal the "whole convex surface of the hole is supposed to resist uniformly." This is probably true only in punching thin plates. In punching bars, blocks or nuts whose thickness exceds the diameter of the punch, it is found that the punch may be entered into the upper side of the metal a considerable distance before the lower side of the plate shows any signs of being strained. This was plainly shown in some experiments on punching made by Mr. David Townsend, published in the *Journal of the Franklin Institute*, for March, 1878. In these experiments rectangular blocks of iron 1¾ inches thick were punched only partly through, and then, after withdrawing the punch and planing away one-half of the metal to a plane passing through the axis of the hole, it was found that the effect of the partial punching was to crowd the portion of metal displaced by the punch into those portions beneath and around the hole, but that when the punch had not entered more than half the thickness of the block the fibers of the metal at the bottom of the block appeared to be uninfluenced in any way by the punching. It seems evident from these experiments that the resistance to punching is not simply resistance to shearing, but is compounded with compressive resistance, and that the convex surface of the hole therefore does not resist uniformly and simultaneously, but to some extent in detail, or one portion after the other.

It would appear probable, also, that in shearing large bolts, or large sections of any kind, that the whole section may not resist simultaneously. If this is the case large sections would show a less apparent shearing strength per square inch than small sections. The relation of resistance to section may also vary with the nature of the material.

It is plain that the subject of shearing strength is a complex one, and further experiments are required to render our knowledge upon it at all definite and reliable.

TORSIONAL STRENGTH.

A torsional stress applied to a piece of material is a force that tends to turn or twist it. In all cases in which a force is applied at one point on a shaft to turn it, and there is a resisting force at another point, the shaft is subjected to torsional strain. The wheel and axle is an example. To produce torsion without bending, a *couple*, whose axis coincides with the axis of the piece, must be applied to it. If only a single force is applied, the result is a combined bending and twisting. A hand-wheel, operated by two hands placed exactly opposite on the rim, and each hand exerting the same effort to turn the wheel, is an example of the application of a couple. A hand-wheel operated by only one hand on the rim is an example of the application of a combined bending and twisting force.

A torsional stress tends to break a

piece by combined shearing and tensile stresses. In twisting a rope, for instance, it is plainly seen that rupture will take place by tension of the fibers. In twisting certain metals, such as soft steel, the piece appears to break chiefly by shearing, the fracture being a comparatively plane surface, perpendicular to the axis of the piece.

The exact relation between torsional strength and tensile and shearing strength has not yet been determined, but approximate theoretical relation may be derived mathematically, and still closer approximations to the true relation may be obtained by experiment. The relation varies with the ductility of the material. In a material which is perfectly elastic until rupture the resistance of each fiber varies directly as its distance from the axis of the piece. In a material which is very ductile, the final resistance of each fiber is nearly the same, whatever its distance from the axis of the piece. Of two pieces of metal of the same size and the same tensile and shearing strength, the one which is the more ductile will offer the greater resistance to torsion.

Resistance to torsion is expressed in terms of *moment*, or force multiplied by distance, as foot pounds. The amount of force P necessary to twist the head off the bolt shown in the cut, will depend upon the distance from the axis of the bolt at which the force P (or the couple PP′) is applied to the wrench handle. If it is applied at two feet from the axis, it will take only half as much force as if it is applied at one foot from the axis, but the product of pounds into feet, or "foot pounds," will be the same.

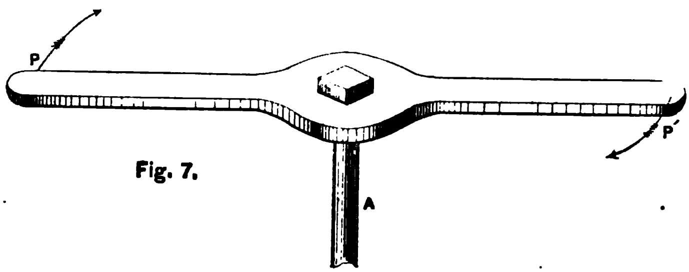


Fig. 7.

The resistance to torsion of a cylinder of any material, varies as the cube of its diameter. A demonstration of this fact may be found in Wood's "Resistance of Materials," page 208. The following formulas are given:

$$Pa=\tfrac{1}{2}\pi Jr^3, \text{ for brittle materials} \quad (1)$$

$$Pa=\tfrac{2}{3}\pi Jr^3, \text{ for ductile materials} \quad (2)$$

in which P is the force applied at the distance a, the product Pa being the moment, usually expressed in foot pounds, π equals 3.1416, r is the radius of the cylinder, and J the *modulus of torsion*, a quantity which is found by experiment, and varies with each material. Formula (1) is based on the supposition that the resistance of each fiber varies directly as its distance from the center, and formula (2) on the supposition that the resistance of each fiber is the same. Both formulas are deduced mathematically, by processes which may be found in Wood's "Resistance."

In experiments on cylindrical specimens, to obtain merely the torsional strength of a material, the quantity Pa is found, and from the result of the experiment the quantity $\tfrac{1}{2}\pi J$ (or $\tfrac{2}{3}\pi J$) is calculated, which is the strength of a piece of one inch radius or two inches diameter. Having this quantity, or modulus, the strength of any cylindrical piece of the same material is found by multiplying the modulus by the cube of the radius.

Prof. R. H. Thurston has recently invented a testing machine which determines the strength, ductility, resilience and homogeneity of materials by torsional strain. A complete description of this machine, with an account of results obtained by it may be found in a paper by the inventor, published in the *Journal of the Franklin Institute* in 1874.* The principal features of the machine which recommend its adoption for general use are, its extreme simplicity

* *Van Nostrand's Magazine*, Vol. XI.

and cheapness, its convenience of operation and its autographic registry. By the latter a strain diagram is made, which shows the resistance of the material at every instant of the test. The value of the graphic method of recording tests has already been shown, in treating of tests by tensile stress, and the automatic graphic record of this machine is especially valuable, as it is more accurate than any plotted diagram.

The autographic record of a torsional test of any material is almost precisely similar to a strain diagram made by plotting the results of a tensile test, as shown in the plate given with the discussion of tensile stress, page 48; but the vertical distances of any point of the curve from the base line represent foot pounds of moment, and the horizontal distances from a vertical line at the origin represent angles of torsion. As in the plotted diagrams the vertical distances indicate strength, the horizontal distances ductility, the inclination of the initial portion of the curve from the vertical is a measure of the coefficient of elasticity, the point at which the curve begins to bend toward the horizontal marks the limit of elasticity, the area included between the curve and the base line measures the work done upon the specimen in breaking it, or the resilience, and the regularity of the curve indicates the homogeneity of the material. The strength expressed in foot pounds of moment, has a direct relation to the tensile strength, varying somewhat with different kind of material, and when this relation is known, the distance of the curve from the base line is a measure of tensile strength with a sufficient approximation to correctness for nearly all practical purposes. The ductility, expressed in the angle of torsion, has a direct relation to the extension of the exterior fibres of the specimen, depending upon the size of the latter, hence the angle of torsion is a measure of extension. It is thus seen that a test by torsional stress of any material can be made to give a record of all the qualities which render it available in construction. This is done by Prof. Thurston's machine with the utmost convenience and accuracy. The writer has made several hundred tests with this machine and can therefore speak from experience. It is now in use in several manufacturing establishments and institutions of learning. A torsion machine, somewhat similar to that of Prof. Thurston, but lacking the feature of automatic registry, has been invented by W. E. Woodbridge, M. D. It was used by the inventor in an extensive series of experiments on steel wire, the results of which, with a plate and description of the machine, are given in a report by him to the government.* The writer is not aware that this machine has been used by other experimenters, but it appears to be well designed, and capable of doing accurate work.

RESISTANCE TO CONTINUED AND TO REPEATED STRESSES, TO SUDDEN SHOCKS AND TO VIBRATIONS.

The preceding discussions have chiefly had relation to the strength of materials as determined by tests of limited duration, and in which the stress is usually applied gradually or without suddenness, and repeated but a few times, if at all.

The conditions to which materials of construction are subjected when in actual use are quite different in some respects to those in which the test specimen is subjected. They have to endure uniform strains through many years, or many centuries, as in buildings; uniform strains with variable strains and sudden shocks at intervals, as in bridges; rapidly recurring strains of different kinds repeated millions of times, as in reciprocating parts of machinery; incessant tremors and vibrations, as in railroad cars; and sometimes every variety and combination of all of these.

As it is manifestly impossible to subject a material to a test which shall fulfill all the conditions of actual use, it is desirable to know what relation exists between the resistance of each variety of material used in construction to steady strain applied for a limited period, to its resistance to continued or repeated stresses, to sudden shocks, and to tremors or vibrations. Thus questions like these may arise: If a rod of iron one inch square in section will resist a steady pull of 50,000 pounds for one hour or one day without breaking, what steady pull will it resist without breaking

* "Report on the Mechanical Properties of Steel, chiefly with reference to Gun Construction on the Woodbridge system." By W. E. Woodbridge, M.D. Appendix F. to the report of the Chief of Ordnance. Washington, 1875.

for a hundred years? What load will it resist for a hundred years if one-half the load is gradually placed on and taken off the rod a certain number of times each day? What load will it resist if applied suddenly? What load will it resist if the suddenly applied load is repeated a number of times, say a million or a billion times? What load will it resist if the load falls a certain number of feet before striking? What load will it resist if this load falls a certain distance a certain number of times? These and a hundred similar questions might be asked concerning this one rod of iron, and the same might be asked of rods of all kinds of material and of all sizes.

The field of inquiry here presented is almost illimitable, but it is an eminently practical one. Materials of construction are used every day which are subjected to just such various conditions as those above mentioned, but the only knowledge we have concerning the resistance of these materials is that derived from tests of small specimens for a limited time, and that derived from observations of materials in use under similar circumstances. On some of these points our knowledge is already definite enough for all practical purposes. Thus it is safe to assume that a stone pillar, resting on a stone foundation, that supports a certain load for one day will support it for a thousand years; if the elements do not destroy the stone; as the pyramids of Egypt and the columns of Greece appear as strong to-day as they were when they were erected. They are living witnesses of the strength of the materials that compose them. The endurance of iron under similar circumstances may be considered still a disputed question. If 50,000 pounds will break a bar in a test taking a limited time, 45,000 pounds may break it if applied long enough, but it may not. It is reasonably certain that there is some weight less than 50,000 pounds which it will sustain indefinitely, but it is also certain that a very much less weight than this will break it if it is repeatedly applied and removed.

Roebling, in his report on the Niagara railroad bridge* states that he found that the iron in the old Monongahela bridge, after thirty years' service was in such good condition that he used it in the new bridge. He also found that the iron in another bridge, over the Allegheny river, was in good condition after forty-one years of service. On the other hand, iron rods used in stiffening the frames of some Ohio river steamboats have been found to break after less than ten years service, and on inspecting the broken rods they were found to be brittle and worthless throughout, although they were of excellent quality when put into the frames.

Resistance to Continued Steady Stress.—The question of the endurance of iron and other materials under steady stress is, as has been stated above, still a disputed one, and the records of experiments bearing upon the subject are somewhat conflicting. The results of the experiments of Vicat, which showed that an iron wire loaded to three fourths of its ultimate strength broke after remaining loaded for thirty-three months, seems to be at variance with the results obtained by later experimenters; which show that iron offers an *increased* resistance to steady stress with time. A full discussion of this subject would extend this paper to an unreasonable length, but the conclusions to which the writer has come, after reviewing a vast amount of the work of recent authorities, may be briefly expressed thus:

Each material has a certain limit of strength (not the initial elastic limit, but usually beyond it) *within which limit it will endure a steady stress indefinitely; but any stress which causes it to pass this limit will, if continned long enough, cause it to rupture.* The exact limit for each material has yet to be determined by experiment, but it is probable that with glass, hard steel, cast iron, and all brittle materials, the limit is but little short of the point of rupture, and that these materials will endure indefinitely a strain which is almost sufficient to cause rupture. With materials like lead, tin, or other soft and ductile materials, it appears that the limit is reached at a stress which is only a small fraction of what they will bear in a test which lasts but a few minutes. Evidence of this has already been given in the remarks upon the influence of time on tensile tests. In these metals, "of an inelastic, viscous character, which do not show an eleva-

* *Journal Franklin Institute*, 1860, Vol. LXX, p. 361.

tion of the elastic limit under strain, and which offer an increased resistance when the rapidity of distortion is increased," there seems to be a *flow of particles* which almost any load will cause, and which almost any load will continue until rupture takes place. They are therefore unsafe metals to use in construction.

With metals like wrought iron and soft steel, Vicat's experiments seem to show that they also exhibit viscosity and flow, and that therefore long continued strain might cause their ultimate rupture; but later experiments, by Prof. Thurston, Commander Beardslee, U.S.N. and others, indicate that, *within certain limits*, no appreciable flow takes place (or if it does take place it soon ceases), but on the contrary that an increase rather than a diminution of strength occurs. This latter phenomena has been named the "elevation of the elastic limit." Still later experiments by Prof. Thurston, however, indicate that both phenomena can be shown by the same material; the elevation of the elastic limit is exhibited in the beginning of the test, or perhaps throughout the test nearly to the end; and later on, flow is exhibited, and rupture finally can take place under a steady and not increasing, or even under a decreasing load. Much light has been thrown upon the subject of endurance of continued strain by experiments made within the last five years, but there yet remains a wide field of research.

Resistance to Repeated Stress.—It is only within the past few years that any really scientific investigation has been made of the resistance of materials to repeated steady stresses. The results of these investigations plainly show that if a piece once resists without breaking any given stress, it is by no means certain that it will resist an indefinite number of repetitions of that stress.

As the result of a long series of experiments for the Prussian government, A. Wöhler, in 1858, first pointed out that a load much less than that necessary to cause rupture by a single application would cause rupture if repeated a sufficient number of times; and that it was not sufficient, in experiments on materials of construction, to learn only the rupturing strength for a single application of load, but that it was necessary, for a safe foundation for calculation, to experiment upon the resistance to stress frequently repeated. The results of his researches are embodied in the following general statement, which is known as Wöhler's law*:

"*Rupture may be caused, not only by a steady load which exceeds the carrying strength, but also by repeated application of stresses, none of which are equal to this carrying strength. The differences of these stresses are measures of the disturbance of continuity, in so far as by their increase the minimum stress which is still necessary for rupture diminishes.*"

Unwin states, in reference to Wohler's researches, that they show that the safety of a structure, subjected to a varying amount of straining action, depends upon the *range of variation* of stress to which the structure is subjected, and on the number of repetitions of the change of load. It has hitherto been assumed that the safety depends only on the maximum intensity of the stress, but this must now be considered to be erroneous. Every machine, subjected to a constant variation of load must be designed to resist a practically infinite number of changes of load. In order that it may do so, the greatest intensity of stress must be less than that for a steady load, and less in some proportion which depends upon the amount of variation the stress undergoes in its successive changes†.

Weyrauch remarks upon Wöhler's law, that in the general form already given it is without doubt correct, and it may even be considered as a *long-known* result of experience, since we continually make unconscious use of it. "If one endeavors to break a beam walled in at the end with the hand, and a single pull proves insufficient, he naturally ceases, and pulls again and again, and when this fails per chance accomplishes the fracture by bending to and fro. The force of the arm is not greater in the second case than in the first, but we do not even need so great a force. We have long known therefore that by alternate stress in opposite directions, where the differences of stress are greatest, the force necessary for rupture is less than for repeated stress in a single direction, and still less than for a single application of such a

* Weyrauch, Strength and Determination of the Dimensions of Structures. New York, 1877.

† Unwin, Elements of Machine Design, London, 1876.

stress. . . . . There remains still much room for the further development of Wohler's law. In his experiments, the stress was repeated very rapidly; the strains however require a certain time in order to reach their full intensity; we disregard now impact proper. What influence has the rapidity of the repetition, what influence the rapidity of the increase of stress, and what the duration of the individual stresses? The questions are not as yet satisfactorily answered."

Wohler found that a bar which is alternately subjected to compression and tension will endure a much smaller number of repetitions of strain than the same bar to an equal amount of tension or compression alone. Certain bars of wrought iron and steel were equally safe to resist varying bending and tensile straining actions repeated for an indefinite time when the maximum and minimum stresses had the following values:

FOR WROUGHT IRON.

In tension only from + 18,713 to + 31 pounds per square inch.

In tension and compression alternately, from + 8317 to − 8317 pounds per square inch.

FOR CAST STEEL.

In tension only from + 34,307 to + 113,436 pounds per square inch.

In tension and compression alternately from + 12475 to − 12475 pounds per square inch. + represents tension, and − compression.

Wohler's experiments have been completely confirmed by those of Spangenberg*. The latter states that numerous experiments confirm Wohler's second deduction, viz: "*Differences* of strains at the extremes of vibrations are a sufficient cause of rupture," and that as the strain increases the *differences* which are sufficient to cause rupture become less. Experiments showed that variations of stress between the following limits may take place with equal security:

Iron	between +160 Ctr.† and −160 Ctr. " +800 " " − 0 " " +440 " " +240 "	per sq. in.

* Spangenberg. The "Fatigue of Metals." Translation: Van Nostrand, New York, 1876.

† A centner=110.2 pounds Avoir. and 1 German square inch = 1.0608 English square inch

Axle Cast Steel	between +280 Ctr. and −280 Ctr. " +480 " " − 0 " " +800 " " +350 "	per sq. in.

Spring steel not hardened	between +500 Ctr. and 0 Ctr. " +700 " " 250 " " +800 " " 400 " " +900 " " 600 "	per sq. in.

and for shearing resistance,

Axle Cast Steel	between 220 Ctr. and −220 Ctr. " 380 " " 0 "	per sq. in.

The following is one of the tables given by Wohler showing the effect of repeated bendings in one direction:

HOMOGENEOUS IRON.

Maximum strain in ctr. per sq. in.	Number of bendings before rupture.
550	169,750
500	420,000
450	481,975
400	1,320,000
360	4,035,400
320	sound after 3,420,000
300	sound after 48,200,000

Unwin remarks: "Unfortunately, Wöhlers experiments, although extensive, do not furnish decisive rules for practical guidance. They afford an explanation of the apparently high factors of safety which in certain cases experience has shown to be necessary, but they are not complete enough to indicate precisely the factor of safety to be chosen in different cases. Nor, indeed, could rules be obtained without the most careful comparison of the results of researches of the kind begun by Wöhler with the actual stresses found to be safe in practice, in a great variety of cases."

For a discussion of the researches of Wöhler and Spangenberg, the reader is referred to Dubois's translation of Weyrauch on "Strength and Determination of Dimensions of Structures." The same work gives formulas derived from the results of Wohler's experiments, by Gerber, Shäffer and Launhardt, for the dimensioning of structures subjected to repeated stresses and to alternation of tension and compression, but the formulas are not yet generally adopted in practice, and the accuracy of the numerical constants which enter into them is doubtful.

The present state of our knowledge upon the subject of resistance to repeated stresses is singularly defective. The

Germans have been the only experimenters in this field (a limited research by Sir William Fairbairn, in England, perhaps alone excepted) and their results, as presented to us in English translations, are so beclouded with discussions of theories that they are not appreciated by the ordinary practical reader, and they have not yet found their way into English or American text-books.

There is scarcely any subject of more importance to the engineering profession. Wohler's investigations are of immense value as far as they go, but they must be supplemented by still more extended investigations before they can be available for general practice. The field is such a large one that it would take a century for private experimenters to explore it. The research can be efficiently made only under the direction or patronage of government.

Resistance to Shock.—Materials are frequently called upon to resist sudden and violent shocks. As a shock is caused by the sudden arrest of a *moving* force or load, it cannot be measured like a steady strain in pounds; but its amount can be expressed in units of work or energy, as foot-pounds, or the product of mass into velocity or of force into space.

The subject of resistance to shock was treated of by the writer in a note published in the *Metallurgical Review* of October, 1877.* The following extracts from that note may be quoted here:

"The resistance to shock is not merely a resistance to an external force, but to force moving through space or to a mass moving with a velocity. It is a resistance to energy, which is measured by the product of the force into the space through which it moves, or by the product of one-half the moving mass which causes the shock into the square of its velocity.

"Also, the resistance to shock is not merely a resisting force, but a resisting force moving through space; a *work* measured by the product of the mean resisting force into the space through which it acts. The space through which the resisting force acts, in tensile strain produced by shock, is the extension.

"If rupture does not take place, equilibrium must exist between the shock-producing energy and the shock-resisting energy, or between the work of the shock and the work of the resistance. Expressed in symbols,

$$FS = \tfrac{1}{2}MV^2 = RS',$$

in which F is the force causing the shock and S the space through which the force acts, M the mass of the moving body and V its velocity, R the mean resistance, or resisting force, and S′ the space through which the resistance acts."

* Vol. I. p. 190.

The product of the mean resistance of a material to steady strain into its ultimate ductility (or amount of extension or other distortion before rupture) furnishes us an approximate measure of its shock-resisting capacity. This product is termed the "resilience,"* and it is expressed in foot-pounds or other similar unit. In the graphic method of recording results, heretofore described, the resilience is represented by the area of the diagram. This measure, however, is only approximate, as the time occupied in the test may have an influence upon the amount of the resilience. We know that soft metals, such as tin and zinc, show greater resistance to rapid than to slow strain, while their ductility under either rapid or slow strain is the same. The resilience is greater therefore under rapid than under slow strain. If the time is made as short as the endurance of a shock, a small fraction of a second, the resilience may be still greater. With wrought iron and steel probably the reverse is the case, as we know that they offer greater resistance to slow than to rapid distortion—the ductility remaining, as far as we know, nearly constant. The resilience under strain rapid enough to constitute a shock might be much less than the resilience under steady strain. This is apparently indicated by the results of experiments on iron and steel armor plate, the plate cracking or "starring" under the impact of a shot fired with a high velocity, to a much greater degree than would probably happen under the impact of the same number of foot-tons produced by a greater load at a smaller velocity.

Experiments on the resistance of materials to shock are not numerous. In practice, car axles are tested by placing

* Some writers have used this term to designate elastic range, or "spring."

them on supports at the ends, and dropping a heavy weight on them in the middle. The deflection caused by the blow or by successive blows or the number of blows they will stand without breaking is taken as a measure of their shock-resisting capacity. In the steel works, specimens are frequently tested by blows from a steam hammer, applied in various ways. No experiments, it is believed, have yet been made to determine what precise relation resistance to shock bears to resistance to steady strain and to ductility. As already stated, the product of the two latter affords an approximate measure of the former, but the precise relation probably depends upon the velocity of the shock producing force. The impact, $\frac{1}{2}MV^2$, may be a constant quantity (M and V^2 being variable, but their product constant) but the resistance to impact, RS', may possibly not be constant, R varying as some function of V.

Resistance to Repeated Shocks.—The single violent shocks treated of above usually occur only in what are called accidents, but in all constructions repeated shocks, tremors or vibrations much less than that sufficient to cause rupture are of constant occurrence; and a knowledge of the resistance of materials to these repeated shocks is a much more important matter than a knowledge of their resistance to a single heavy shock.

Wohler's and Spangenberg's experiments teach us something in regard to the effect of repeated steady strains, but there have been scarcely any experiments on the effect of repeated shocks, and nearly all our information on this subject is the result of common observation and experience. We know that in breaking a piece of cast iron if one blow of a sledge does not accomplish the result, several will. It is well known that pieces of machinery may be in use for years subjected to light shocks repeated millions of times, and will at some time break under a lighter shock than they have repeatedly experienced. Prof. Wood states: If repeated, shocks upon metals are quite certain to produce fracture sooner or later. All metals in use have their life. They can sustain only a certain amount of service."

It is generally believed that repeated shocks will change the mechanical condition of a material, and render it weak; as in the case of iron, by changing the structure from fibrous to crystalline. The evidence upon this subject is, however, conflicting, and direct experiments will have to be made before it can be satisfactorily settled.

Fairbairn says: "We know that in some cases wrought iron subjected to continuous vibration assumes a crystalline structure, and that then the cohesive powers are much deteriorated; but we are ignorant of the causes of this change." In another place he says; "I am inclined to think that we attribute too much influence to percussion and vibration, and neglect more obvious causes which are frequently in operation to produce the change." The fact is, in my opinion, we cannot change a body composed of a fibrous texture to that of a crystalline character by a mechanical process, except only in those cases where percussion is carried to the extent of producing considerable change of temperature.

The late John A. Roebling, in his report on the Niagara Suspension Bridge, in 1860, gives as his opinion that "a molecular change, or so-called *granulation* or *crystallization*, in consequence of vibration or tension, or both combined, has in no instance been satisfactorily proved by demonstration or experiment." This seems to be in direct conflict with the testimony of most authorities, and with a great accumulation of facts learned by common observation.

Prof. Wood, in a review of this subject, remarks: "These several facts, though apparently somewhat conflicting, show quite conclusively, that some metals will crystallize under certain conditions; that under certain conditions they may be strained millions of times without being damaged, or at least without being broken; that under certain conditions strains and shocks combined may produce crystallization; that shocks when severe will weaken metals and, if they are sufficiently numerous, will produce rupture. Much evidently remains to be learned upon this subject."

As already stated, the product of tensile strength and ductility is an approximate measure of the capacity of a material to resist a single heavy shock. We know little or nothing however con-

cerning the relation of strength and ductility to resistance to repeated shocks. It is generally believed that the material which is best able to resist a single heavy shock is also best able to resist a succession of lighter shocks; and this belief is acted on in practice. Thus in certain kinds of machinery pieces which are subjected to innumerable vibrations or light shocks are made of the very finest and most ductile iron or of the softest steel.

Car axles, which are subjected to continuous "hammering" while in service, are made of steel which must be so soft as to resist without breaking one or more blows of a heavy weight falling a number of feet. The percentage of carbon in steel for axles is therefore kept under a certain figure in order that the steel may be soft enough to stand the test without breaking.

It is tolerably certain that the axle which will resist the test of heavy blows without breaking will also not break at the beginning of its service by an accident which causes it a very heavy shock, such as a derailment, but will rather bend; but it is not at all certain that this axle would have a longer "life" in ordinary service than one that is less ductile, and that would be broken under the test by blows. Neither is it certain that the very soft steel used in vibrating pieces of machinery would have as long a life as harder steel used in place of it.

The most valuable contribution to our knowledge upon the subject of the resistance of steel to repeated shocks, is that given by Mr. William Metcalf, in a letter to the *Metallurgical Review* for December, 1877. It is all the more valuable, since it is directly in opposition to the common belief mentioned above, viz: that soft steels are the best adapted to resist repeated shocks and vibrations. The importance of the subject will justify the reprinting here of a brief extract.

"This action of resistance to vibration we first observed at the Crescent Steel Works about three years ago, and it was a complete surprise to us, as up to that time we had always used the mildest steel to resist such strains.

"The piston rods of steam hammers used on steel always break. In hammers where the life of a wrought iron rod was about three months, a mild steel rod was found to last about six months. To improve upon this still milder rods were tried, and four to five months' use obtained, to the surprise of everybody. An accident caused the hurried use of a rod much higher than any ever tried before, probably containing .60 carbon. Immediate provision was made for its replacement by a mild rod, its destruction being expected in a few weeks. The high rod ran over two years, or about four times as long as the average of milder rods.

"The next case was that of steel for small pitmans, where the test required was that a machine should run $4\frac{1}{2}$ hours, at a rate of 1200 revolutions per minute, unloaded, before the pitman broke. These pitmans were unforged in the middle, and consisted of a piece of straight round bar with a head welded on each end, the middle of the piece being left as it came from the rolls. This explanation is necessary in order that it may be understood that no accidents of forging affected the results.

"The first trial was with .53 carbon steel: mean time of six trials, 2 hours $9\frac{1}{8}$ minutes. Second trial, .65 carbon steel: mean time of six trials, 2 hours $57\frac{1}{2}$ minutes. Third trial, .85 carbon steel: mean time of three trials, 9 hours 45 minutes, or more than double the requirements. This was satisfactory and the trials were stopped.

"These trial pitmans were all of uniform quality except as to carbon. This led to the trial of a set of twelve pitmans taken from ingots which were carefully analyzed by Prof. J. W. Langley, who published a paper on the results in the *Am. Chemist* of Nov., 1876.

"These pitmans were of a finer quality of steel than the above.

The .30 C. ran 1 h. 21 m. heated and bent before break'g
" .49 " 1 h. 28 m. " " " "
" .53 " 4 h. 57 m. broke without heating.
" .65 " 3 h. 50 m. broke at weld where imperfect.
" .80 " 5 h. 40 m.
" .84 " 18 h.
.87 carbon broke in weld near the end.
.96 " ran 4 h. 55 m. and the machine broke down."

Should Mr. Metcalf's results be found on further experiment to hold good for all kinds of steel it may well be doubted whether the practice of using soft steel for car axles is the best practice, and also whether engineers who have used the softest steels in bridges have used the materials which offers the greatest security to human life. The increasing

use of Bessemer and Siemens-Martin steels in structures and the substitution of these materials for wrought iron renders the subject one of vast importance. A thorough series of experiments on the resistance of the various steels to repeated shocks and on the relation which this resistance bears to tensile strength, ductility and other mechanical properties, and to chemical composition would be of far greater value to the world than the researches of Wöhler and Spängenberg on resistance to repeated steady loads.

It may be well to explain why a knowledge of the relation between resistance to repeated shocks and other mechanical properties is important, as well as a knowledge of the absolute value of this resistance for various materials: Direct experiments on resistance to repeated shock will be somewhat difficult, and they may take a long time to make; while experiments on tensile, compressive or torsional strength, or on ductility may be made at most in a few hours. If then it can once be settled what relation exists between these kinds of strength and ductility and resistance to repeated shock for all materials, it may then be possible to tell with precision from the "strain diagram" furnished by a tensile or torsional test of any material what is the capacity of that material to resist repeated shocks, without subjecting it to direct experiment.

It would be eminently proper that the series of experiments mentioned above should be made either by government or under its patronage; but in view of the increasing use of Bessemer steel in construction, and the immediate necessity existing that we should know something of its ability to resist continued shocks before putting it into structures, and also of the proper kind of steel to be used under different conditions, it seems probable that it would be to the interest of the Bessemer steel companies to make such experiments for their own benefit.*

Conclusion.—All the various forms of stress to which materials can be subjected have been treated of at length, but the subject is by no means exhausted. Much might be said of the effect of different conditions in increasing or decreasing strength. Of these are: influence of temperature of the metal when cast; of mass of casting; of temperature of piece when tested; of amount of work done on piece; of reheating, rerolling and welding; of annealing; of remelting; of compression while in the fluid condition; of cold-rolling; of removing the outside surface; of punching or drilling holes in plates, etc. The influences of these conditions have been discussed by several writers on strength of materials, and it is not necessary to extend this paper further by treating of them here. As a most important practical fact it may be well to state that the process of compressing metal while in the liquid condition has been found to cause a great increase in strength, and it is likely that it will be extensively adopted in the manufacture of steel. The process of cold-rolling has been found to increase the strength of bar iron in some cases as much as 100 per cent.

A table of the strength of various materials might be appended, but such tables already exist in abundance, and the writer would rather discourage, than otherwise, reliance on published figures of strength. In the first portion of this paper several examples were given to show the unreliability of such figures. In all important structures the material to be used should first be tested; there should be no guesswork in regard to the strength of a bridge rod or any piece on which the safety of life may depend.

If the result of these articles shall be to show to manufacturers and users of materials how recklessly and incorrectly tests of these materials have been and are being made, if they shall tend in any degree to bring about some reform in the common method of testing, and if they shall help make more general the belief that no material should be used in an important structure until specimens of it are first subjected to test, their object will be accomplished.

* The writer has recently designed an apparatus for the use of Bessemer steel works and others, for the purpose of testing the relative resistance of different grades and tempers of steel or other metals to long continued and repeated small shocks, by which a number of pieces can be tested at once, and the test of each piece made in a few minutes or in several years as desired. He hopes soon to have such an apparatus built and to publish results obtained from it.

A PROPOSAL is receiving favor for the construction of a railway between Alyth and Braemar in Scotland.

THE GAS ENGINES AT THE PARIS EXHIBITION.

By M. ARMENGAUD. Jr.

Abstract of the Author's Address at the Conference du Trocadéro.

Translated from "Revue Industrielle" for Van Nostrand's Magazine.

The attention of most of those in attendance at this Conference has doubtless been drawn at the Exposition to certain machines, which in their general appearance and mode of working resemble steam engines. But upon approaching them it is readily seen that the similarity is only apparent; the movements of these motors is accompanied by a series of slight detonations, and a near inspection reveals the fact that attached to each is a perforated chamber within which burns a flame which seems to give life to the machine. These are the gas motors. The motive power is derived from illuminating gas and not from steam.

It is proposed to present to the conference some explanations of the structure and mode of working of the different engines of this class exhibited at the Champ de Mars, and to consider the possible future achievements of the inventors and constructors of these engines if they continue the progress so clearly made manifest by this exposition.

Such a programme would hardly seem to possess sufficient interest to hold the attention of an audience not composed exclusively of engineers. But it is proposed to relieve the subject of its technical character as far as possible, and present the more general form of discussion. The task is rendered easy by the very happy circumstance that in the working of a gas engine we find a summary of the most beautiful applications of scientific principles.

A gas motor possesses the essential organs of the steam engine; the cylinder which receives the gaseous fluid; the piston which by aid of rod and crank transmits the pressure to the shaft—the fly-wheel which regulates the motion and the pulleys and belts by which the power is conveyed to the machines to be driven.

The gaseous fluid is a mixture of gas and air in such proportions as is most susceptible of explosion when brought in contact with an ignited body. The mixture is exploded by the little flame before mentioned, and the products of the combustion, suddenly dilated by the heat, urge the piston and develop the motive power. The gas on burning combines with the oxygen of the atmosphere; that is to say, the carbon is converted into carbonic dioxide and the hydrogen into water. This change is completed only under the condition that the volume of air is sufficient to afford the proper amount of oxygen; the gas requires about seven times its volume of air. The combustion may be completed in the open air or in a closed space. An example of the first kind is afforded by the burners in metallurgical works. The gas and air arrive separately and by small quantities at a time, so that their union is effected without sensible noise. If, however, the air and gas are mixed in advance and ignited in a closed space, then the combustion takes place throughout the whole mass at once; a strong detonation and a probable rupture of the envelope immediately follow. Of such a kind are the explosions which result from the careless leaving a gas pipe open in a closed room. The gas diffusing throughout the room becomes capable of exploding as soon as the volume of gas is equal to one-twentieth the volume of the air.

The flame is not absolutely necessary to effect the explosion, as an electric spark will serve the same purpose, but it is necessary to establish a point of intense heat somewhere in the mixture.

The effects of gas explosion are explained, as are those of powder explosions, by the enormous increase of volume of the combustion products at the instant of burning.

In the case of the mixture most suitable for explosion, seven parts of air to one of gas, the explosion develops 10180 units of heat for each kilogram of gas used, or 6000 units for each cubic meter. This heat is instantly communicated to the gaseous products raising their temperature, about 2700 degrees.

Now as gas dilates to the extent of $\frac{1}{273}$ of its volume for each degree of rise of

temperature, it follows that a heat of 2700 degrees would expand the mixture to nearly ten times its normal volume.

When a mixture of twelve parts of air to one of gas is exploded in the cylinder of a gas engine, the heat developed by the explosion is about 1400 degrees, and the gases develop a tension of six atmospheres. The effect on the piston is the same as would be produced by the introduction of a gas or vapor under a pressure of six atmospheres, and exactly as in the case of such vapor do these gaseous products of explosion by their expansion urge the piston of the engine. In this way is the work of gas engines developed.

Gas engines have many points of resemblance to steam and hot air engines. In one as in the others it is the expansion of a gaseous fluid which originates the work of the engine. Whatever the nature of the fluid, its elastic force is employed to move the piston. This fluid then is only an intermediate agent employed to convey heat, and giving up a portion during the process of transportation. The heat which disappears is converted into mechanical work and is the source of power in the motor. This correlation of heat and motion is one of the most beautiful conceptions of modern science.

The essential differences in the various kinds of motors above mentioned reside in the different modes of communicating heat to the intermediate gaseous body.

In steam engines heat is first employed in converting water into steam and then in augmenting the heat of the steam. The vaporization is effected outside of the cylinder, that is to say, in the boiler and some time before the steam acts upon the piston. In hot air engines the heat which dilates the air is equally apart from the cylinder.

It is quite otherwise with gas motors, for in these the heat is developed within the cylinder and in the midst of the gaseous mass and only at the moment that power is wanted. As there is no storing up of the heat nor any transportation of it, these causes of loss are avoided.

The advantages of such a mode of production and utilization of heat are apparent. While in the steam engine, the time of firing up, of evaporating the water, and of raising the steam to a proper temperature is necessarily consumed, in the gas engine it is only required to admit the gaseous mixture by a single turn of the stop-cock. Furthermore, *the gas engine consumes fuel only while it is doing work.* In this quality it possesses a great advantage over other heat engines in those branches of industry which require only an intermittent application of power.

Although the conception of the employment of a detonating mixture as a source of power is a simple one, the practical realization is not without complexities. The heat developed by the explosion of the mixed gases in the old Lenoir gas engine was communicated to the products of the explosion and the excess of air too suddenly; the expansion was too violent and in no way like the motive force in steam or hot air engines. Its ill effects were only mitigated by admitting air in large excess before the explosion. Such sudden shocks were not in keeping with the regular motion required of a motor.

Furthermore, the heat tended to escape as rapidly as it was generated by conduction through the sides of the cylinder. A testing of the cylinder resulted, which was only partly avoided by a current of water on the outside.

The heat developed in the cylinder of the gas engine tends to disappear in two ways: one by conduction through the sides of the cylinder and subsequent radiation; and the other by the expansion of the gaseous products left after combustion. The first of these is useless; the other useful. Utility demands that the first should be counteracted and the second developed. One step towards the accomplishment of this object is attained by allowing a very rapid expansion of the gases in the cylinder. Gas engines ought, therefore, to work with high velocity.

Loss of heat is also occasioned by the escape, from the cylinder, of the mixture at a high temperature. It is clearly necessary furthermore to develop all the heat of which the original combustible gas is susceptible. This is attained by its complete combustion, which depends upon the method of mixing with air and igniting.

Such are the requirements for good service in gas engines. We propose to

explain to what extent they are fulfilled in the gas motors of the Exposition, first glancing at the history of previous experiments in this direction.

As with all other great inventions many nations claim the honor of producing the first gas engine. A very complete and impartial history of the subject has been written by M. H. Tresca, who presides over this Conference. From this history are largely drawn the notes here presented.

The application of gun powder as a motor preceded the use of both steam and gas. Before thinking of the employment of steam, Papin constructed an engine to be driven by cannon powder, following suggestions made by Huyghens in 1678, and Hautefeuille in 1680. In the machine described by Papin in 1688, and which contains a piston and valve, he did not seek to utilize in a direct manner the expansion of the gas, but only the force which is the direct result of this expansion. It was upon the same principle that he devised a little later a steam engine.

John Barber, in 1791, was the first who proposed to produce a motive force by burning hydrogen or other inflammable gas. But his engine was without a piston and was only urged by the force of a jet leading from the vessel in which the explosion was produced.

Then came the inventions of Thomas Mead and Robert Street in 1794, who employed—one the gas resulting from combustion of some substance in a fire, and the other the volatile vapor produced by dropping petroleum or terebenthine or similar substances into the cylinder.

To Philip Lebon, who invented gas illumination, is to be ascribed the honor of devising the principles of construction and working of the gas engine

It is most remarkable that in the machine described by Lebon in his patent of 1799, there are two pumps one for the air supply and the other for the inflammable gas. These imply a certain state of compression before the explosion, an idea which is one of the most salient points of the improved motors of the Exhibition. Lebon also devised the use of the electric spark for igniting the mixture, a method employed in the Lenoir engine.

Experiments by Rivaz in 1807, by Samuel Brown in 1823, and by Talbot in 1840, only need be mentioned here. A patent in this latter year by MM. Demichelis and Monnier describes for the first time the use of gas. It was generated in an apparatus which formed part of the engine. In some patents coal gas was replaced by vapor from petroleum and by other volatile liquids. Other inventors proposed to use air which had been carburetted by passing through naphtha, benzine or similar liquids.

Omitting the names of many experimenters of this period, Hugon (1858). and Lenoir (1859), may be mentioned as being entitled to the honor of first applying gas motors to industrial purposes. These two forms were known and enjoyed celebrity in 1867. Several Hugon engines figured in the French section in the Exposition of that year.

It is unnecessary to describe here the details of these machines. It is sufficient to say that they utilized directly the expansion of the gases. The piston at each stroke admitted on one side the mixture which was ignited at the middle of the stroke, and expelled on the opposite side the products of the combustion. The essential differences of the two systems was in the distribution and mode of igniting the gases. Hugon employed a gas flame for this latter purpose, and Lenoir an electric spark from a Rumkhorff coil.

The Hugon engine is supplied with an apparatus for carefully adjusting the quantities of air and gas supplied to the cylinder; but it does not appear that any advantage in the way of economy of consumption of gas has been gained over that afforded by the Lenoir engine.

Next to these forms came the engine of Otto and Langen of Cologne. This is a vertical engine, the former were horizontal. The principle of action of the Otto engine is different inasmuch as it utilizes the effect of expansion in an indirect manner only.

The piston rod urges the shaft only in an intermittent manner; it is geared to a pinion on the shaft which engages the shaft by means of a friction pulley only on the down stroke. On the up stroke the piston is driven violently by the expansive force of the gases till the pressure of the mixed products equals the atmospheric pressure, and is carried beyond this point by acquired momentum. It

stops only when the work of the atmospheric resistance has absorbed the accumulated work in the piston. There results a rarefaction under the piston, so the down stroke is urged by atmospheric pressure aided by the weight of the piston. This descent is the effective stroke of the engine, for it is only then that the piston rod is connected to the driving shaft.

This method of employing the effects of the explosion only indirectly has yielded good results economically; affording one horse power for a consumption of one cubic meter of gas per hour, in place of 2.7^{m} as in the Lenoir and Hugon engines. But the noise of the engine as in the former cases is quite unendurable, and has led to its rejection.

Claude Segré, an Italian engineer, and M. Schmitz, the engineer of the Paris gas company, have made a thorough study of these systems of motors, and with the result of the following classification into two general divisions.

1st. That system in which the expansive force of the gases acts directly upon the piston and, through this, upon the other moving parts, as in the Lenoir and Hugon engines, and

2. That in which the force of the explosion urges the piston (which for the moment is free) until a partial vacuum is created below it, when the atmospheric pressure is brought to act, and at the return stroke producing the effective work. This includes the Otto and Langen engine.

Sometimes both plans of action are exhibited in the same engine. An example of this kind is afforded by the Gilles motor, a specimen of which was exhibited in the English section of the Exposition. This belongs to an improved form of which there are three kinds, represented respectively by; 1st, the motor of Mr. Otto, of Cologne; 2d, the engine of Louis Simon of Nottingham; and 3d, the engine of M. Bisschof.

In the engines of Messrs. Otto and Simon, the mixture of gas and air is compressed before explosion, so that the initial pressure at the moment of ignition is twelve atmospheres; double that obtained in the Hugon and Lenoir engines where the mixtures before explosion were at a tension of atmospheric pressure only.

Furthermore, in these new forms the ignition of the gases is gradual.

The two ideas of previous compression of the mixed gases and a gradual combustion, instead of a violent explosion, distinguish the improved engines from the old forms.

We will describe these systems more particularly:

The Otto engine resembles externally a single acting steam engine. It has a single horizontal cylinder, open at one end and closed at the other, with a head furnished on the inner side with a conical cavity. The piston is connected by a crank with the shaft of the fly-wheel. Behind the cylinder is the supply chamber, which is furnished with a direct connection with the main shaft. The piston at the in-stroke does not reach the end of the cylinder, but leaves a space equal to about two fifths of the capacity of the cylinder. This is the compression chamber.

The cylinder serves the double purpose of compression pump and working cylinder, which is perhaps not the least of the characteristics of the new system.

The complete cycle of motions in the Otto engine is accomplished only by two complete revolutions of the working shaft, or four strokes of the piston. It comprehends the four following phases: viz.

1st. The piston makes an up-stroke drawing in the explosive mixture of gas and air;

2d. The inlet cock closes and the piston returns, compressing the gaseous mixture;

3d. At the moment the down-stroke is completed and while the tension of the gases is somewhat above two atmospheres, the mixture is inflamed and the consequent expansion causes the piston to make an up-stroke;

4th. The piston returns driving out the expanded and cooled gases.

Thus of four strokes of the piston, only one (the third) conveys motive force to the shaft.

The second consumes power; the other two have no appreciable effect.

Such a method of working calls for a heavy fly-wheel, the accumulated work of which accomplishes the compression of the gases.

A special regulator to the engine inter-

cepts the supply of gas and delays the ignition whenever the velocity becomes too great. Furthermore, the engine works without noise—a great advantage over the Otto and Langen motor.

The effective working power of the engine is of course the difference between that afforded by the expansion of the gases and that absorbed by the compression. The indicator diagrams show a regular curve of pressures very different from the line of abrupt changes exhibited by the Lenoir engine.

The regular decrease of pressure in the Otto engine is due to the method of burning the mixed gases. The combustion is retarded so that the heat developed is absorbed by the gases, at a rate that is in better accord with the motion of the piston.

M. Otto has accomplished this by his method of mixing and admitting his gases. He employs two different mixtures; one of fifteen parts air to one of illuminating gas, which he calls his "feebly explosive mixture;" the other of seven parts air to one of gas is called his "strongly explosive mixture."

During the working of the engine, and at the moment when the gases are about to be ignited, the contents of the cylinder are: products of the preceding explosion, atmospheric air, hydrogen and hydro-carbon gases. These are not uniformly diffused, but owing to the position and action of the valves, the most combustible portion is at the bottom of the cylinder at the point of ignition, and the combustibility probably decreases quite regularly from the bottom of the cylinder to the piston.

The result of this condition is a *prolonged* explosion, and the force of expansion is less of the nature of a shock than in the previous engines.

In order to insure combustion with proper rapidity, a jet of "strongly explosive mixture" is made to traverse the mass at the critical moment.

Such is the principle of action of M. Otto's engine. It satisfies the theoretic conditions indicated above. That it employs heat to good advantage is indicated by the small loss shown by the outside cooling of the cylinder. Experiments prove a heating of 35 litres of water per hour for each horse-power, the water being raised from 10° to 85° C.

This is 2,520 units of heat; as 6,000 units are generated by the combustion, the loss is 42 per cent. The loss in the Lenoir engine, according to M. Tresca, was 85 per cent.

The principle of compression was employed in the hot air engines. Ericsson and Franchot both applied it in their motors. In these engines it has a double use: at first, to diminish the temperature to which the air must be raised, and so diminish the heating which is pernicious to these engines, and then also a reduction of the size of the parts of the engine for a given power is accomplished.

But if the previous compression is so advantageous in the working of the engine, it may be asked is it equally advantageous to its practical performance? In other words, is the advantage gained an adequate return for the work expended in compressing the gas?

Calculation shows that the same quantity of gas not compressed would yield a greater amount total of motive force, but on the other hand the loss by compression is compensated by a better utilization of the heat generated.

By means of the measurement of the lost heat mentioned above, it is found that the Otto engine is nearly three times as effective as either the Lenoir or Hugon engine.

It is not surprising therefore that in place of 2.7 metres of gas consumed for each horse-power per hour, the Otto engine yields the same power by the consumption of one cubic metre or probably less in large engines.

The principle of action of the Simon engine is essentially the same as the Otto, but there are some notable differences in the details. The compression is performed in a separate cylinder; upon the admission to the working cylinder the mixture is ignited by a gas flame kept constantly burning. The cylinders are vertical and the two piston rods connect with the same horizontal shaft.

The admission of the explosive mixture and the escape of the products of combustion are managed by valves worked by cams on the working shaft. The mixed gases are admitted in a series of small charges and inflamed successively, thereby insuring a gradual expansion. This method affords also a good

economy of heat, a very small quantity of water sufficing to keep the cylinder cool.

The regular variations of tension in the working cylinder were sufficiently shown by the indicator diagrams taken during the Exhibition. The curve exhibits some slight sudden variations upon the opening of the inlet valve, then it rises to full pressure and remains for a time constant; then falls nearly to atmospheric pressure at the opening of the escape valve. It is claimed that the consumption of gas is less than one cubic metre per horse-power per hour.

One of the peculiarities of M. Simon's plan, is the use of steam in connection with his gaseous mixture.

It was an idea of the earlier inventors —M. Hugon among the first, to reduce the excessive heat in the cylinder by a jet of water. But it proved difficult to regulate. M. Simon introduces a jet of steam from a boiler heated by the escaping gases. The water supplied to the boiler is first used to cool the working cylinder. Thus the heat which tends to escape is utilized. Furthermore, the steam in the cylinder tends to absorb with useful effect any excess of heat resulting from the explosion, and also acts to some extent as a lubricant to the piston.

M. Simon claims to have obtained results superior, in point of economy of working, to those obtained by the Otto engine.

The Bisschof engine belongs to the class that utilize the effects of an explosion to drive the piston. The cylinder is vertical and the piston rod connects with the shaft in such manner as to utilize in the fullest degree the effect of expansion. No water is employed for cooling the cylinder; the result is secured by constructing the cylinder with projecting ribs or flanges so as to expose an abundance of radiating surface.

Engines of small size only have thus far been constructed on Bisschof's system. Most of those made have been designed to run sewing machines with a capacity of $\frac{1}{16}$ to $\frac{1}{2}$ a horse power. The working of these costs, in Paris, two cents an hour for the former, and only five cents for the latter size.

It was the intention of the inventor to solve the question of furnishing a light motor for domestic use. Its advantages are:

1st. It employs no water.

2d. It possesses great stability without specially prepared foundations.

3d. It utilizes as completely as possible the force of the explosion, by the long stroke of the piston.

4th. All shock is avoided by cushioning the air on the down stroke.

The cylinder is heated by a gas burner a few minutes before starting, otherwise the moisture, resulting from the explosion, might deposit within the cylinder and in the absence of oil rust it. By starting with the cylinder heated this is avoided.

The Ravel motor could not be shown at the Exposition under conditions that would permit its working.

In this engine, which is called by the inventor *moteur a centre de gravitè variable*, the explosion of the mixture lifts a heavy piston. The cylinder is furnished with trunnions which turn in bearings and which being prolonged form the axis of the machine. An explosion chamber, either attached to the end of the cylinder or independent of it, receives the explosive mixture where it is ignited by a gas flame; the heavy piston is thrown upward; the reaction forces the cylinder to oscillate like a pendulum, and as at each fall of the piston a new explosion gives it a new impulse, the oscillations are continued. The time of fall of the piston is made to agree with the vibration of the cylinder, by varying the resistances to the rise of the piston. No experiments have been tried with this engine which would justify the expression of a definite judgment in regard to its efficiency, though the principle of its action does not forbid the expectation of a good degree of economy.

Having thus reviewed the gas engines of the late Exposition, we arrive readily at the conclusion that the progress in this direction since 1867 has been very great. The improvement, is evidently not alone owing to the ingenious mechanical devices, but also to a closer study of the utilization of heat. The science of thermodynamics teaches us that heat and motion are effects of the same cause; they are equivalent. All loss of heat is loss of work or energy, and, in case of the motors just consider-

ed, the heat produced is a measure of the gas employed and the cost of the motive power.

The expense of working, therefore, depends upon the proper utilization of the heat. This expense has of late been much reduced, the consumption of $2\frac{1}{2}$ cubic meters in the Lenoir engine yielding the same useful effect as is obtained by one cubic meter in the Otto engine.

Some improvement had previously been made it is true in Otto and Langen engines, but it was counterbalanced by such an intolerable noise that their use has been generally abandoned. They are in vogue only in Germany, and are employed there to the astonishment of those people who attribute to our neighbors on the Rhine a more delicate musical ear than ours.

The motors of Otto and Simon exhibit superior qualities in the way of economical working. If at present the Bisschof engine does not possess the same advantage, it still merits a favorable appreciation for having furnished, in so complete a manner, a solution of the problem of supplying a motor of one-man power.

What is more reasonable than to expect of our gas companies a supply of heat and force as well as light? Many light industries which require machinery might then be pursued at the home of the artisan.

But to realize this extension in the application of gas engines, it is necessary that the fuel should be cheaper. If heating quality alone were to be considered, gas could now be afforded at lower cost. A ton of coal yields about 300 cubic meters of illuminating gas, but about double that quantity of combustible gas of good heating quality could be obtained from the same amount of coal.

The question of supplying a cheaper gas for heating purposes, is worth considering by our gas companies, and also whether the same service pipes may not serve the double purpose, furnishing one kind of gas by day and another by night.

In thus eulogizing gas engines, it is not to be concluded that they are recommended to replace steam engines. There is a use for all. Gas motors will compete with small steam engines or with water engines; and in places where neither gas nor water power is easily obtained, hot air engines may properly be preferred.

In concluding this glance at recent progress, the wish may be expressed that this advance towards the amelioration of human toil shall not be checked, but that the end may be soon reached in which all of man's intelligence shall be demanded, and almost nothing of his muscular strength. Although we are yet far from this desideratum this consoling thought remains to us, when from this point of view we compare our epoch with antiquity. In place of the sad inscription "Sale of Slaves" in the market place, we read on modern sign-boards "Motive-power to sell."

REMARKS BY THE PRESIDENT, M. TRESCA.

I ask permission to call attention a little more particularly to two points in the discussion which has been so well presented by M. Armangaud.

If I estimate correctly the old engines of Hugon or Lenoir consuming 2,500 litres per horse-power per hour, would cost, at the present price of gas in Paris, 75 centimes per hour, as one horse-power is 75 kilogrammeters per second, it follows that power at the rate of one kilogrammeter per second costs one centime per hour. If we reckon the work of a man turning a crank at five kilogrammeters, it results from the above estimate that the cost of this labor performed by the gas engines is five centimes per hour.

The later improvements however yield such results that one man-power per hour is afforded for two and a half centimes. But the machines of the Exposition of 1867 yielded as good results as this. There has been no great improvement in economy of consumption in ten years. The problem presented now is to furnish an economical motor whose dimensions are only such as is required for driving light machines, and of which the consumption of fuel shall be in the same proportion as the larger engines.

The gas engine would seem to be dependant upon gas distribution. But a good substitute is afforded by a mixture of volatile vapors with atmospheric air, and the results obtained are nearly or quite equivalent to those from the con sumption of gas.

I add these remarks because the gas engine thus modified is a motor in which

heat is transformed into work in the best possible manner, and I have far more confidence in the future success of these modifications of the gas motor than I have in the hot air engines, which require so much more space and do not conform readily to the theoretic conditions of the problem.

DISCHARGE OF SEWAGE INTO THE SEA.

By HENRY ROBINSON, C. E.

From "Journal of the Society of Arts."

A very general impression prevails that if a town is situated close to the sea it is necessarily in a more advantageous position than inland towns, respecting the disposal of its sewage, as it has only to avail itself of its proximity to the sea to get rid of its sewage by discharging into it. That this is an erroneous impression the experience of most of our watering places proves, and it is therefore desirable to offer a caution to those who are contemplating adopting a similar course. In the Local Government Board Blue-book of 1876, one of the conclusions arrived at is as follows:—"That towns, situate on the sea-coast, or on tidal estuaries, may be allowed to turn sewage into the sea or estuary, below the line of low water, provided no nuisance is caused; and that such mode of getting rid of sewage may be allowed and justified on the score of economy." This has been often quoted as encouraging the adoption of this method of sewage disposal, and it is to be regretted that the report gives no data whatever (such as are abundantly available) by which the qualifying expression, "provided no nuisance is caused," would be shown to apply to a great number, if not the majority of cases. It might have been stated that, to avoid a nuisance, the sewage must be discharged into the sea at a point not only below low water, but where there is a well-ascertained current which would carry it permanently seaward. A point of discharge complying with these conditions is but seldom found to exist close to the town, but has to be reached by long and costly outfall sewers, or rather tunnels. At the outfalls there should be a continuous movement seaward during the twenty-four hours, instead of an oscillating action to and fro, resulting in a return of the sewage and its deposition along the shore, not only at the outfall and in its immediate neighbourhood, but also at distant places to which the tide carries. The writer has had occasion to inspect many watering places where the foreshore is being distinctly polluted in this way. At first the mischief is not great, and only traces of the sewage are visible; but in time it becomes serious, and the knowledge of the existence of sewage pollution on the foreshore causes the place to be avoided by those who hitherto have resorted to it. The grievance is not a merely sentimental one, as the exhalations along the foreshore from sewage accretions at low tide involve not only offensive smells, but also a danger to health.

The difficulties attending the discharge of sewage into the sea would be diminished were it not that it has a higher temperature and a lower specific gravity than sea or river water, which causes it to rise to the surface; and if it is not carried seaward at once, part of the suspended solid impurities are deposited on the coast wherever there is still water and no tidal current, whilst the rest of the suspended, together with the dissolved, impurities float on the surface, and are carried backwards and forwards by every tide, decomposing and liberating gases (sulphuretted hydrogen being one of the most offensive) injurious to health and polluting the air.

In some cases, by means of long outfall sewers, the sewage is carried clear away from the place producing it, as at Brighton. These practically become elongated cesspools, in which noxious gases are generated, and are liable to be forced back into the town drains, and thence into the houses. In these long

outfalls, also, the solids deposit and involve both expense and difficulty to remove. Even if the places producing the sewage really get rid of it in this way, they are frequently simply transferring it to others, a set of the tide carrying it so as to cause mischief and nuisance elsewhere. No better illustration of this can be given than the experience of Margate. The authorities there proposed, after much competition amongst rival engineers, to adopt a scheme by which the sewage was to be discharged into the sea in a bay about a mile and a half eastward of the town, where it turned out that there was practically no current seaward, so that, had the scheme been carried out, the coast there would have been permanently polluted, as the sewage would have risen and dropped with the tide, evolving all kinds of dangerous and offensive gases, which would have effectually driven visitors away, and have depreciated to a serious extent the value of the neighboring property. Ramsgate is in a similar difficulty, and many other places could be cited where it is a matter of serious concern how to deal with the sewage. The authorities are compelled to drain their towns, and the very effort they make to comply with the sanitary requirements of the day appears to involve them in almost greater difficulties. There is only one way safely of dealing with sewage at seaside places where the tidal currents are not clearly favorable, and that is to deodorize the sewage before it is discharged into the sea.

The authorities of Glasgow have had the question of how to get rid of their sewage under consideration for a long while. A Royal Commission investigated this case, and although the result of this was to advise the adoption of a scheme to carry the sewage twenty-seven miles in a tunnel to the sea, at enormous cost, and although this advice was similar to that previously given, the authorities took the matter into their own hands, and appointed a committee of their body, which has recently presented an exceedingly able and interesting report, giving the results of their investigations. The conclusion they arrive at is not to adopt the recommendations to discharge their sewage into the sea, but to discharge it into the River Clyde after it has been purified by chemical treatment.

Where there is a risk of nuisance, either to the place to be drained or to its neighbors (which is equally important), by discharging sewage into the sea, a clarification and deodorization of the sewage can be easily and cheaply effected. No attempt to arrest the solids in catchment tanks can possibly be satisfactory, inasmuch as they only remove a very small portion or the solids, and become huge cesspools, which have to be cleared out at intervals, with a certainty of causing great nuisance. Filtration is also not admissible, as the filters soon get inoperative, and become in addition as great a nuisance as catchment tanks. By deodorizing the sewage the first difficulty is overcome, as the sewage is no longer offensive.

There has hitherto been much prejudice against chemical treatment, which is, however, disappearing, as it has been abundantly proved that sewage can thereby be deprived of its offensive properties by simple and inexpensive means. The disposal of the semi-fluid sludge has been a difficulty which the writer has had to give much attention to, and he has employed several methods of converting it into a portable form. The plan which he has found the best is to remove a great part of the moisture from the sludge by means of a simple filter press. A model of this press (which is an automatic modification of an old construction of press) has been placed in the Exhibition of Sanitary Appliances. By an appliance of this kind, the sludge has the bulk of its water pressed out, and the consequent reduction both in mass and consistency enables the sludge to be better removed and utilized, or dealt with in any other way.

An amusing result has followed an exhibition of gas apparatus at Wolverhampton. Some of the exhibitors decline to accept the medals awarded them for the very adequate reason that the judges were utterly incompetent. In one case a medal was awarded to a stove which could not even be entered in the class, and which, as a matter of fact, was not even in the exhibition at the time the award was made. One of the jurors did not know what was meant by a Bunsen burner! And yet the public buy in blind faith in medals.

THE SANITATION OF WORKSHOPS.

From "Iron."

At the late Sanitary Congress at Stafford, an address on Workshop Sanitation was read by Dr. J. T. Arlidge, which, though its object was to treat alone of the sanitary conditions and regulations at work where pottery is made, contained so many observations and suggestions of interest to all classes of workshop owners and employers that we think its reproduction here will be profitable. The following is the text:—

Among the hygienic conditions and surroundings of various manufactures, experience and observation have convinced me that we must make a distinction between those inseparable from, and those only incidental to, the manufacturing processes themselves, and that for the greater number of occupations and operating more energetically as factors in inducing disease, the latter play the more important part. This statement, in other words, amounts to this—that with regard to the multiform occupations pursued in this country, the nature of the occupations is undeservedly blamed, in the case of the majority of them, for the ill consequences to the health of those engaged in them. Now I propose in this paper to deal with a manufacture in which the essential conditions and surroundings are really fraught with danger to health; nevertheless, I shall be able to show that the non-essential or accidental surroundings are accountable for very much. To guard myself against misunderstanding, however, I need remark that the line between essential and non-essential conditions of manufacture cannot, in all particulars, be distinctly drawn; and that, in more or fewer of them, some will overlap others.

For example, take the case of dust, which is the most prolific agent in inducing disease amongst potters, we may fairly regard it as inseparable from the processes of the trade, because the material cannot be handled in accordance with the necessities of trade without giving off fine dust; yet, on the other hand, the amount of dust thrown off may be lessened by care, attention and cleanliness, and, further, may be dispersed and driven away by efficient ventilation. Still, notwithstanding this inability to accurately separate the essential from the non-essential conditions of manufacture, I regard it as important and practically most valuable to make the attempt; and firstly, let me briefly sketch the conditions or surroundings of the pottery manufacture, which are non-essential, accidental, or outside the calling, but yet are answerable for more or less sickness, suffering and death amongst those engaged in it. Of this whole class of morbific agencies I may predicate that they are preventable. I do not assert that practically they can all be removed and annihilated, but that we possess known remedies against them capable of being more or less efficiently applied.

The first in order I shall name is defective construction of workshops. In respect of this matter the manufacturers in the Potteries suffer, as do also those in other places, whatsoever be the occupation carried on, by the sins of their forefathers. They inherit old buildings erected ere sanitation had made good its existence. Their demolition and reconstruction is an affair representing capital expended and the tendency is to "make them do" until better days dawn, or progressive dilapidation enforces destruction. Now, a fairly diligent inspection of the buildings used for the making of china and earthenware—or, to use the comprehensive term of the district, of the banks—will reveal too many shops or workplaces wretchedly built, at times in bad repair, with wet dirty floors, damp walls, low ceilings, scarcely high enough for a tall man to stand under, full of draughts, yet not ventilated, and with every trade appliance or fitting covered with a coating of dust of variable thickness. If these be the features of but a few, there are more to be found which, though less unfit as places of labor, are yet far from being what they ought to be.

Defective elevation of rooms, and in its train draughts and bad ventilation are far too generally found. Another common structural defect is the absence of

a plaster ceiling in workshops having others over them, and, as a consequence, the precipitation of dust from the upper floor to that beneath it. Another prevailing structural defect is to be found in the unpaved yards, and in the absence of waterspouts and drains to conduct rainwater away into the sewers. The consequence is wretchedly sloppy and muddy courts and approaches, and concurrently wet feet and drabbled dresses, and the accessories—colds, coughs and rheumatism. Again, efficient ventilation is a thing far too widely neglected in pottery workshops, particularly when considered in connection with the prevalence of dust, pervading especially those shops in which the clay is thrown, moulded, or turned, or in which scouring and dipping of the ware are carried on. The position, the size and the construction of the windows as to the mode of opening and closing them, are such as to make ventilation difficult, or even almost impossible. Add to these impediments to ventilation the frequent mode of warming by what are called pot-stoves, which, standing about the middle of the rooms, heat and unduly dry the air around them by their highly-heated or even red-hot iron surface, whilst causing at the same time very little atmospheric movement upwards through the small iron chimney flue, and yet ever and anon dispersing into the apartment an inconvenient supply of smoke. It is no new thing to condemn these pot-stoves as insanitary; but, for my part, I am almost equally disposed to condemn their modern innovators, the hot-water and steam-pipes, which have found their way into several recently erected factories. For this system of warming, when adopted, as too frequently happens, without corresponding arrangements for ventilating, is but a system of enervating the work-people submitted to it, and provocative of evils as great, perhaps, as that of the cold it is used to guard against. Indeed, I maintain that it is better for a man at active employment to do without artificial warmth from a heating apparatus of the sort in question. The mischief to contend against is the separation of the heating from the ventilating process. When the former is attained apart from the latter, or the latter is let shift for itself, the air of the apartment soon becomes insufferable, doors and windows are opened, and draughts and colds follow, and inflict their injurious consequences upon people peculiarly sensitive to them by reason of the heated, almost motionless, air previously surrounding them. So far as concerns the pottery hands, it must be admitted they are difficult to deal with in this matter of ventilation. They have a morbid dread of cold air, and are unlucki y in large numbers the victims of chest affections. As a consequence, they are prone to stuff up ventilators, to cover over any sort of apertures by which the outer air is to be admitted into the room, and to deliver themselves up to a warm, stagnant, effete air laden with particles of dust. That man would be a public benefactor who could invent a simple plan for ventilating and warming the workrooms of potters, and persuade them at the same time to give it a fair trial. Overcrowding of shops is a further insanitary condition not essential to any process in the manufacture of pottery. According to my observation, it is likewise a widely spread one; but I trust that the better defined sanitary requirements of the new Factory and Workshop Act will gradually diminish this evil.

In the manufacture of pottery the use of machinery has heretofore found little application. It is now slowly making its way; but for its adoption to any large extent it will be necessary to replace old factories by new ones, the arrangement and relative situation of the shops of most old factories rendering the adaptation of machinery impracticable or else too costly. The use of machinery would exert a beneficial influence upon the health aspects of the manufacture. The introduction of presses for preparing the clay, in place of the old process of evaporation and plunging, by men exposed constantly to an atmosphere of steam, and to the influx of cold air at the same time from the half-enclosed shed in which the work was carried on, is a movement in favor of the health of potters. So, again, is the adoption of the pug-mill, to replace in great measure, or wholly, the process of wedging the clay. I can point to other examples, in the use of steam to turn the jiggers, the turners' lathes, and the thrower's wheel; and I could well wish to see the like mechanical

agency more widely patronized; for my experience as infirmary physician has shwon me, in numerous instances, the injurious results to health of girls and young women employed in lathe and wheel turning.

Not actually pertaining to machinery, yet allied to it, are the arrangements now pretty generally used in substitution of the old "hot closets," in and out of which boys were kept continually running, with great detriment to health, from the excessive heat and ever-recurrent exposure to chills. These improvements in machinery and in mechanical appliances must be seen to be understood. I will now pass to another set of conditions unfavorable to health, for which neither the manufacturing processes nor the structural defects of the factories are to blame. I allude to the habits, the food and the clothing of the workpeople. But to sufficiently discuss these matters would carry me far beyond the limits of a paper like the present. Let me say, in brief, that there is very far too much indulgence in intoxicating drinks, irregularity in living, indifference and carelessness in the selection and preparation of food, neglect of cleanliness, pride and folly in dress. I do not say these defects are peculiar to potters. I fear they prevail generally among factory hands as a class; but I fancy they are, as a whole, more productive of disease among potters than among other artisans, because by their association with concurrent causes of sickness found in the occupation itself, their power to do mischief is enhanced. For instance, the most fertile source of disease is the dust of potteries, and the morbid resultant affections of the lungs, wherefore the indulgence in alcoholic drinks, which load the vessels and lungs with carbonaceous matter, and otherwise prejudice the respiratory act, prepares the way and facilitates the action of the active morbific agent, the inhaled dust.

I will now turn to the conditions essential to the occupation of potters, or if not positively essential, at least practically inseparable at present from it. The first to be noted is dust. This is evolved from the materials used in the manufacture of pottery; the china and blue clays, the stone and the powdered flint. It is a mineral dust, and extremely fine, consisting mainly of silicious particles. It enters the air-passages, and finds its way into the bronchial tubes, great and small, where, as a direct irritant, it sets up a slow, inflammatory action, ending in altered pulmonary tissue—so altered by condensation and other changes as to be useless for the purpose of respiration. This dust is apparent in the air, and is seen covering all objects in the workshops as well as the clothing of the potters themselves. Some processes are more dusty than others, but none of them in which the clay is worked up into the forms desired are free from it. In the finishing departments of painting, gilding and burnishing, the clay is transformed into china and earthenware, and no dust consequently come from it. We have, therefore, an obvious and direct cause of disease existent in the dust given off from the yet unbaked clay, and its evolution to some extent appears inevitable. As a matter of course, it is used in a moist or plastic condition; but its surface rapidly dries, and readily breaks down into a fine powder on touching, as shown by its soiling the hands. In some processes, particularly in that of turning, the lathe separates fine shavings from the surface of the vessels, still in the "green" state, and with them more or less of the clay in fine particles and in powder. The disposition of the surface to dry and form a pulverulent film is increased by the heat of the work-rooms. Therefore, for preventive measures, we must seek such as will lessen the heat of shops, and the formation on the surface of the unbaked ware of a dry, powdery film, and withal such ventilation as shall disperse the dust generated, and expel it from the work-rooms. Airy, well ventilated, and comparatively cool shops constitute the primary hygienic requirements in the manufacture of pottery. Subsidiary to these are plans for laying the dust by sprinkling the floor, the observance of all means of cleanliness, both of the shops and the persons employed; but it is painful to add that these minor measures to remove a patent cause of disease are greatly neglected. I would throw it out suggestively how far it might be possible to remove the dust from the atmosphere by employing from time to time forcible jets of steam. These we may imagine would induce the precipitation of much dust and lessen the dry heat of the air.

China-scouring has been especially singled out as most noxious to those engaged in it, by reason of the dust generated and the pecularly irritating quality of that silicious dust. It is an occupation of women, and consists in scouring the flinty dust from the surface of the china cups and other articles. It is very provocative of lung disease, although not in the terrible ratio some have reported. The mode in which the work is carried on seems certainly to place it within the scope of remedial measures. These must be in plans for withdrawing the dust from the workers as they sit at the bench with the ware before them, and it seems to me very feasible to effect this by having a perforated bench allowing the dust to fall through, or rather to be forcibly drawn through into an inclosed trough or cylinder below, in which a constant process of exhaustion of the air is effected by the rapid revolution of a fan at its open end, driven by machinery. The perforated bench and subjacent trough have been adopted, but the withdrawing force relied upon has been the suction power of a heated chimney with which the trough has been connected. But this plan is better in theory than in practice. The chimney has not a constant and equal draught, and if the shop become heated when the chimney is cool, the current will be reversed, and no dust will be extracted through the perforations.

Another expedient preventive of the ill consequences of dust inhalation is the use of respirators worn over the mouth and nose. There can be doubt of their efficiency in sifting the dust from the air before it enters the respiratory passages, but the difficulty is to get the workpeople to use them. There is on their part a recklessness and an indifference to consequences; the respirators increase the effort to breathe, and they feel hot and uncomfortable. But what is more fatal to their adoption is, they are innovations on the usages of trade, and, above all, they subject their wearers to the jibes and jeers of their fellow workers, and no class of people are more sensitive to jests, or have so little moral courage in regard to the introduction of novelties as factory workmen and workwomen. In very many factories the evils of dust might be reduced by the re-arrangement of shops and of the several branches of labor. As said before, some branches are more dusty than others, and it is not uncommon to carry on two or more branches in the same workplace, so that all therein employed are exposed to the whole of the disadvantages of processes which otherwise would be restricted to a few.

But, besides dust, there are other conditions of labor inherent in it. For instance, the position taken during work is unfavorable to the normal action of the chest walls. This happens to throwers and pressers. In the handle-making branch there is rapidly repeated pressure upon the lower part of the chest; and in all the finishing processes the position in sitting is more or less constrained. On the other hand, in the case of lathe treaders, there is undue fatigue in standing and in the exertion, by a sort of perpetual jumping on one foot, of treading the footboard or treadle.

There is yet one more department of pottery labor destructive of health which I must mention in order to give my paper any semblance of completeness. It is the department of dipping. In order to put the impervious glaze on the ware after firing in the biscuit-oven, it is dipped into a mixture of lead and borax, combined with other materials, which have previously been all fused together into a vitreous substance, then ground down into fine powder, and mixed with water. The lead of the glaze is, of course, a poisonous ingredient, and, as a consequence, those engaged in dipping the ware are the subjects of severe colic and of paralysis in various and often very aggravated forms. Could lead be eliminated from the glaze one great source of disease of the potter's craft would be abolished. I am told that the proportion of lead used is not so great as formerly, and that an increased quantity of borax may be substituted for the poisonous metal. Whether this be so or not is a question for chemists, and it would be a happy day could these scientific gentlemen contrive a new form of glaze of non-poisonous quality. Probably some lead is absorbed by the skin of the dippers, whose hands and arms are, practically speaking, immersed in the glaze during the period they are at work. But my impression is that the lead finds its way into their sys-

tem chiefly in the form of dust inhaled by the mouth and nose. The glaze most rapidly dries on the absorbent surface of the ware, whilst the clothes of the worker become bespattered with it, and every surrounding object betokens the formation and presence of dust. Here, again, the like remedial measures claim attention as in the case of dust from clay. There is need of airy shops and good ventilation, and, above all, of cleanliness of person and of surroundings. Most manufacturers supply their dippers with means of washing, with towels, soap and nail-brushes, and the Factory Act forbids the partaking of meals in dipping houses. In fact, preventive measures are of the first importance in this department. If experience indicates that some individuals are much more liable to be affected by the lead poison than others, it proves even more forcibly that by care and cleanliness on the part of the men its ill effects may be largely obviated. The use of medicines calculated to eliminate the poison is well known to the workmen, and another excellent means of preserving their health is at their command in the way of hot water, vapor, and Turkish baths. Respirators would serve them a good turn, but, as with other pottery artisans, their prejudices stand in the way of their use.

I have now made a rough survey of the principal un-hygienic conditions of the pottery manufacture. The general lesson to be derived from it is that those conditions are largely remediable and preventable; that one group has most to do with structural and mechanical expedients to remedy it, and that the other is largely bound up with the willingness of the workpeople themselves to lessen and remove them.

APPARATUS FOR ILLUSTRATING THE ACTION OF LOADS UPON TRUSSES.

By R. FLETCHER, Thayer Professor of Civil Engineering, Dartmouth College.

Written for VAN NOSTRAND'S MAGAZINE.

In a course of engineering instruction, doubtless some educators have felt the need of a simple apparatus for illustrating the principles of transmission of stress in trusses. A rigid model of any kind has only a very narrow range of utility. A flexible model, representing only a single form of truss, has greater value but is still insufficient, owing to the number of typical forms in use, each involving one or more distinctive features.

Whatever opinions may be held as to the necessity of using models for purposes of instruction, few would deny the advantages of an apparatus combining the following features:

1. Joints so made as to secure perfect flexibility, excepting unavoidable sliding friction,—thus according with the usual hypothesis upon which computations are based.

2. Concentration of weight at the joints, in accordance with another convenient assumption,—the weights representing either "fixed load" alone or both "fixed" and "moving load" combined.

3. Tension members so made and connected as to be incapable of transmitting any compression, and compression members incapable of transmitting tension.

4. Members interchangeable and capable of being combined so as to form different trusses of several of the various types constructed upon the principle of pin-connections.

5. An arrangement by which some form of dynamometer may be inserted in place of any member, and the previously computed stress on such part verified.

Models of a single form of truss, made flexible and having a dynamometer which may be inserted for verification of strain, are already in use in one or more engineering schools. But, so far as the writer is aware, there is nothing of very general adaptation according fully with the above conditions. To meet all of these requirements the writer has recently

devised an apparatus, which appears to have sufficient novelty and utility to justify the following brief description.

The first condition is realized by a form of joint so clearly shown in Fig. 1 as to need little additional explanation.

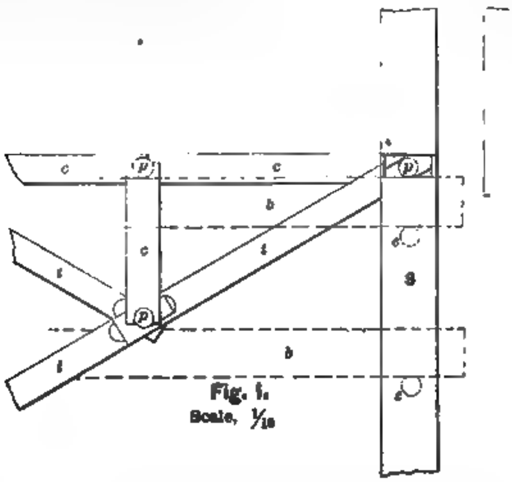


Fig. 1.
Scale, 1/12

The pins, *p*, are made of brass tubing filled with lead. They are six inches long, one inch in diameter and weigh 1.18 lbs. each. Their weight aids very considerably in securing conformity with the second condition. It is almost needless to remind the reader that a model is strained by its own weight only in very small proportion to the strain due to gravity upon the actual structure which the model represents. Assuming for an approximation that the strength of material varies directly as the cross-section, then, if the model were constructed to a scale of $\frac{1}{12}$ the relative weights of model and structure are nearly as $1:(12)^3=1:1728$. But the sections of corresponding parts are as $1:(12)^2=1:144$; hence, if W is the weight of the model, the weight to be put upon it so as to strain it to the same extent that the actual structure is strained by its own weight is

$$W\times1728\div144-W=12W-W.$$

Similarly for any other scale of construction, using the cube and square of the ratio. Accordingly it might be desirable to make the weight of the pins sufficient to bring about this proper proportion of stress. In the case before us, however, such exact relation was hardly practicable with the scale adopted and materials used, without employing too much weight. Pendant weights of lead, of five and three pounds each were made to be hung from the pins or any other part of the frame so as to increase the strains as much as might be required.

The method of meeting the third requirement is shown in Fig. 2. Pieces of

Fig. 2. Scale, 1/4

the form marked 1 simply rest upon or against the pins by virtue of the compressive stress which they transmit.

They cannot oppose any resistance to an effort to further separate the pins. Pieces of the form marked 2 can transmit, evidently, only tensile stress which would tend to increase the distance between the pins; they cannot oppose the slightest resistance to any compressive effort which would crowd the pins towards each other along the lines of the pieces. Compression members should be colored so as to be clearly distinguished

from tension members when in place. By this means much is added to the effectiveness of any combination for illustration.

The fourth condition is fulfilled by carefully proportioning the pieces so that some or all of those used for one combination may constitute parts of one or more other combinations. Some of these other combinations may require a few special pieces not needed elsewhere. The same piece may do service in two or more combinations by having one or more openings at proper intervals for the pins of that combination, as shown in Fig. 2. It is hardly necessary to remark that the planning of the pieces as to dimensions, adjustment of distances from center to center of pin openings, etc., for so many different forms, requires great care, and the workmanship must be of the best, in order to secure accurate fitting of all the parts. The mechanic who makes the pieces should work from carefully prepared drawings of the parts, on a large scale.

It is immediately obvious that such combinations can illustrate nothing as to practical details of construction, but simply the principles of transmission of stress, which is the sole object in view.

Fig. 1 represents the elevation and end view of a part of a Fink truss combination and one support; *p*, the pins; *c*, compression members, which are supposed to be colored; *t* tension members; S one end support consisting of two uprights resting upon a common foot-frame; in this case the uprights are seven inches apart, transversely. The supporting pins at the ends may have to be longer than the others, and there is no need that they should be so heavy. The different parts of the frame must be in duplicate, as seen in the end view, else no symmetrical arrangement in reference to a median plane could be made, and couples would result, destroying equilibrium.

The figure also shows the arrangement for assembling the parts. Strips of thin board, *b*, rest upon buttons, *e*, which are placed against the inner faces of the posts. Each button turns eccentrically upon a wire axis which is prolonged and bent upwards, as shown in the end view, so as to prevent the strip from slipping off. The buttons are so disposed that, when they are turned upwards the strips will have their upper edges exactly on such a level as to just support the pins of the particular form to be assembled. A pair of strips is needed for each chord of parallel-chord trusses, and no more than the four would be necessary for any of the more important forms. The pins being properly placed on the strips, the parts may be rapidly put in position. To aid in placing the end supports at the proper distance apart for a given combination, suitable marks may be made upon the strips. The best disposition of parts for any one combination having been found, it is well to have this indicated by a clear diagram to be kept with the apparatus and serve as a guide for all future assembling of the same form; otherwise there may be frequent confusion and vexation from not remembering the best arrangement of the parts. In some cases a certain order is especially necessary in order to avoid interference of pieces with each other. After the pieces are all in place, the buttons, *e*, are turned half way around, when the strips are lowered a distance equal to double the eccentricity of the buttons, and they may then be removed, leaving the truss supported only by means of the end pins. The illustration represents a "deck" or under-grade truss. For over-grade trusses the posts are continued upwards above the supporting pins and buttons, *e*, are suitably disposed for all the combinations of this kind which are desired. The upper parts of the posts may be made detachable as shown in the end view, and thus put out of the way when not required.

To secure greater stiffness transversely it is well to have a $\frac{3}{4}$ inch hole exactly in the middle of each piece, as shown in Fig. 2. Snugly fitting wooden or metal pins may be inserted after the assembling and passed through each two opposite pieces, thus forming connected pairs throughout the combination. This feature is not indicated in Fig. 1, in order to avoid complexity.

The fifth requirement calls for a dynamometer of special construction. It must be sensitive enough to indicate small differences of strain and yet not be too much extended by the greatest strain it may have to transmit when in place in the truss. It would be desirable, al-

though by no means necessary, to have one capable of indicating both tension and compression. In this particular case the utmost limit of extension, in order not to have too much yielding of the truss under strain was fixed at $\frac{3}{4}$ of an inch, but the actual extension is generally less than half an inch. The form adopted is that of the ordinary "spring balance" with a dial face, the spring being especially made to meet the above conditions. It indicates tension only. The cylinder in which the spring works and the rod which draws the spring, each has a cross-head at the end. Upon each of the latter a pair of eye-bars is secured, the eye-openings fitting the pins of the combination. To adjust for the different lengths of various positions in the same or different combinations, the eye-bars have square holes at carefully ascertained intervals, which fit square ends of the cross-heads. In this case the distance from center to center of pin-openings may be varied from eight to eighteen and a half inches. It may be objected that the yielding of the dynamometer, and consequent distortion of the truss, will make the strains different from those computed for the perfect form. But proper allowance may be made for this—the exact strains being quickly computed by the graphical method,—or the effect may be neglected and the approximate indication taken as sufficient.

The combinations possible with the apparatus thus described are the following:

I and II. The ordinary queen-post truss erect and inverted. With either of these the distortion of a partially braced truss may be shown, and the different effects of bracing by struts or ties. This form is 24 inches long and 8 inches high. These and the following dimensions are from center to center of extreme pins.

III. A Fink truss of four bays, erect. Length 56 inches, depth, 16. Some of the advantages of the Fink system may be strikingly illustrated by this frame.

IV. The same form inverted. This may be inclined at any desirable angle to illustrate its use for roofing purposes, by having a properly arranged bearing on a vertical wall for one end, and rods which retain the bearing pin at the other, or lower end, and hook into staples in the same wall. By this may be shown, also, the effect of changes of load in altering the character of the stress in certain parts.

V. An eight-panel, parallel-chord truss of the Pratt or Whipple type, length 64 inches, depth 14 inches. By this all the principles of counterbracing may be fully illustrated, using few or many counters and distributing or concentrating load to exhibit the action in all cases usually considered, the model showing immediate distortion when insufficiently braced. The dynamometer affords means of illustrating the principles concerning maximum and minimum stress or web members and chords.

VI. The same form inverted. A comparison of any of the inverted with the corresponding erect forms will exhibit clearly the greater stability of those having the lower position of the center of gravity.

VII. A triangular or Warren truss of four panels; length $64\frac{1}{2}$ inches, depth 14 inches. This will illustrate the peculiarities of the system. One piece of each type (1 & 2, Fig. 2) must be inserted where a member is liable to take both kinds of stress, in order to have constant equilibrium. The differences between this and the quadrangular or panel system become so obvious as to need no further comment from the instructor.

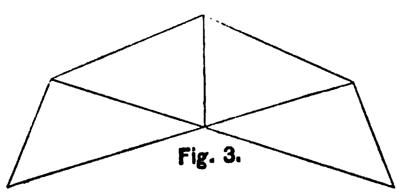


Fig. 3.

VIII. A roof truss of the form shown by Fig. 3; span 30 inches, height 17. This will illustrate the effect of wind pressure on one side in changing the nature of strains due to weight alone in certain pieces on the opposite side.

By having a few extra pieces, the following combinations may be added to the above, if deemed desirable: Howe truss, erect and inverted; triangular truss with vertical suspension members; common king-post truss, erect and inverted. Others might be included also, but there is no advantage in having a great number of forms, since a few is amply sufficient for illustration of leading principles.

Much more might be given in detail,

and other applications described, but the above, which is offered more by way of suggestion than a complete elucidation of that which is capable of further extension and improvement, is sufficient for the interested reader.

THE CONSTRUCTION OF CONVEYOR SPIRALS.

By S. W. ROBINSON, Professor of Mechanical Engineering, Ohio State University.

Written for VAN NOSTRAND'S MAGAZINE.

CONVEYOR SPIRALS may be made in various ways: of cast iron, wrought iron, or wood, separately or combined, and with one or more wings. The object of the present paper is to give some simple rules for making the wings of sheet metal, such as shall be adapted for attachment to a wooden core or spindle, and when thus attached, to stand, at each element, exactly radial. Such wings will be truly helicoidal, and of the form that would be generated by a line kept constantly perpendicular to the axis of the helicoid, and moved along it with a constant velocity and with a constant rate of rotation.

Spiral wings of such form will be warped surfaces, and evidently cannot be formed from flat sheet metal without swaging. For instance, suppose a helicoid be made of a flat strip without any core, and with the axis in the middle of the strip. By hammering the edges from the middle line out, and not the middle line, the edges will become puckered and forced into some twisted shape like an ordinary auger bit. By careful management, a true helicoid is obtained. It is evident that the amount of stretch required increases on each side with the distance from the axis. It is easily seen from this, that if an annular sector, or bent strip, had been taken in place of the straight one, it might have been made helicoidal by stretching the edges and not the middle in a similar manner, resulting in such a helicoidal wing as would fit upon a core.

The rules we seek are, therefore, such as give the proper curvature of the annular sector and the proper stretch for the edges of it for a given case.

RADIUS OF THE ANNULAR STRIP.

Let the annular strip be represented by the figure ABFH, the circular edges being struck from the center O. AB is the radial width of the spiral wing, and AE the radius of the core of conveyor, or spindle upon which the wings are mounted.

Let the annular segment be so shaped that a narrow portion of it lying along the concentric circle line CI will neither need swaging nor bending edgewise.

Let S = the length of any truly helicoidal line which makes one complete turn; r, its radius; and P, its pitch.

Then

$$S^2=P^2+4\pi^2r^2,$$

as can be seen by supposing the spiral developed by unrolling the circumscribing cylinder upon a plane.

Any other spiral line of the same pitch will also give,

$$S_1^2=P^2+4\pi^2r_1^2.$$

Subtracting one from the other, member by member, and

$$s^2-s_1^2=4\pi^2(r^2-r_1^2)$$

or

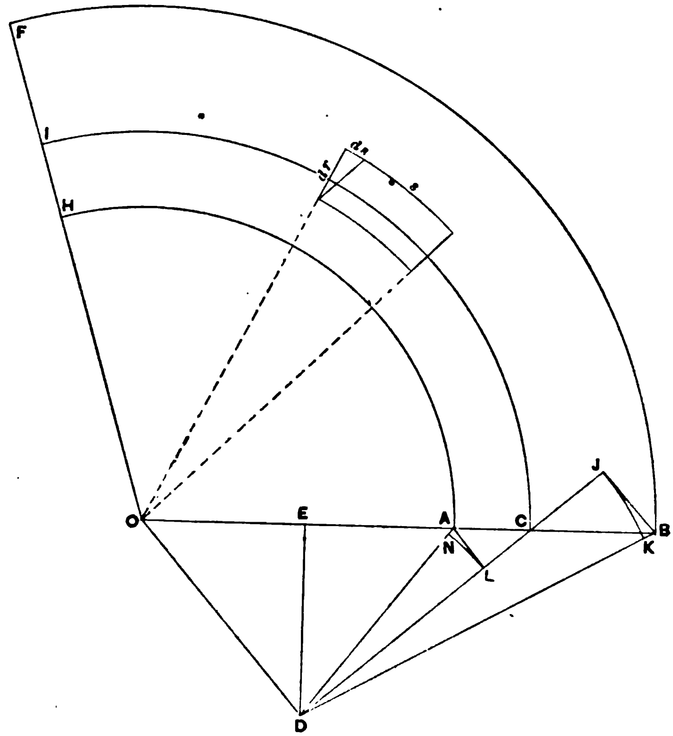
$$(s-s_1)(s+s_1)=4\pi^2(r-r_1)(r+r_1).$$

If the spirals are taken with only a small difference dr, in the radii, we have $r-r_1=dr$ and $r+r_1=r_12r$ and at the same time $s-s_1=ds$ and $s+s_1=2s$ $\therefore$

$$sds=4\pi^2rdr.$$

The same result would have been obtained by differentiating the first equation above.

Let these two spirals be supposed to lie very near to the line CI. The space between them may be regarded as that above mentioned, which is not hammered nor bent, but simply twisted. It is a sort of neutral element to the spiral wing. A small portion of it is shown in the figure, from which we see, observing that s and ds vary together, we have,



$$ds : dr :: s : CO.$$

Combining this with the last equation, we get

$$CO = r + \frac{P^2}{4\pi^2 r}$$

But $r = CE$, according to what precedes, and hence

$$CO - CE = EO = \frac{P^2}{4\pi^2 . CE}$$

from which EO may be computed, as soon as the point C is assumed. If it is taken at the middle of AB, the inside edge, AH, will require much more stretching than BF, as will appear below.

We may obtain the point O graphically by drawing a perpendicular ED, making

$$ED = \frac{P}{2\pi},$$

and then drawing CD and DO, right angled at D. This is proved from the fact that this construction gives

$$CE : ED :: ED : EO.$$

$$\text{or } EO = \frac{\overline{ED}^2}{CE} = \frac{P^2}{4\pi^2 . CE}$$

as required. Hence O is determined.

THE LENGTH CI FOR A COMPLETE TURN.

The first equation gives the length for a complete turn

$$s = \sqrt{P^2 + 4\pi^2 r^2} = 2\pi\sqrt{\frac{P^2}{4\pi^2} + r^2}$$

The first term under the last radical is $\overline{ED}^2$, and the last term is $\overline{CE}^2$. Then the radical equal CD, and $2\pi \times CD$ = a circle described with CD as a radius.

Hence the length, CI, if the annular sector sufficient for a complete turn, is the circumference described with the radius CD.

THE AMOUNT OF SWAGING.

The edges must be lengthened to change the flat to a warped surface. To simply change the length of the edges and not the width of piece, a hammer with a wedge shaped pene may be used with the line of the pene crosswise of piece. But even this will increase the width slightly, and for this reason the piece may be cut out a little narrow. A ball pene will require still greater allowence for expanding width.

To form a particular spiral, a definite

amount of saving is necessary, which amount must be known in order to shape the wing without a pattern.

To determine the amount of stretch to be given to the edges by the hammering, we see from the first equation above, that the final length of AH for a complete turn will be,

$$S=\sqrt{P^2+4\pi^2\overline{AE}^2}=2\pi\sqrt{\frac{P^2}{4\pi^2}+\overline{AE}^2}$$
$$=2\pi.AD.$$

And the length before stretching will be,

$$AH=CI.\frac{AO}{CO}=\frac{AO}{CO}.2\pi.CD$$

and the stretch will be,

$$S-AH=2\pi\left(AD-CD\frac{AO}{CO}\right)$$

By drawing AL perpendicular to CD, or parallel to DO, we have from similar triangles,

$$DL : CD :: AO : CO.$$

$$\therefore DL=CD\frac{AO}{CO}$$

the same as the second term in the parenthesis for the value of stretch. Hence,

$$AD-CD\frac{AO}{CO}=AD-DL,$$

or,

$$=AD-DN$$

if a circular arc LN be struck from a center D.

Hence, the stretch required for the side AH, of a piece AF just sufficient for a complete turn is,

$$S-AH=2\pi(AD-DN)=2\pi.A.N.$$

or equal the circumference of a circle struck with the radius AN.

For pieces AF of a length to make a fractional part of a turn, of course, a corresponding fractional part of the circumference to AN will be necessary for the stretch.

In a similar way it may be shown that the stretch for the outside edge BF will be,

$$2\pi.BK$$

where BJD = a right angle, and DK = DJ.

As regards the choice of the point C, nothing appears above to indicate that it is limited in position. In choosing it near A will throw most of the hammering upon the outer edge, and *vice versa*. A little inquiry will naturally lead to selecting that point which will result in giving the same stretch per inch to the two edges AH, and BF. This will make the total stretch of AH to BF, as AO is to BO, because AH : BF : : AO : BO. Now if CD be drawn so as to bisect the angle ADB, the little triangles, or spandrels, ALN and BJK will be similar, and

AD : BD : : AL : BJ : : AN : BK : : AC : BC : : AO : BO : : AH : BF, since DO is parallel to AL and BJ.

Hence for the same stretch per inch along AH and BF let BC bisect the angle ADB.

The percentage of the stretch will be obtained by dividing AH by the circumference to radius AN, or since AH = circumference to AD, the percentage will be $\frac{AN}{AD}=\frac{BK}{BD}$ for the case of bisected angle ADB.

RULE.

Hence the following rule may be stated for laying out and forming from sheet metal, the wings for conveyors. Referring to the figure:

Make AE = radius of core,

" AB = radial width of wing or blade,

" DE = pitch divided by 2π, and perpendicular to AO,

" DC = bisect the angle ADB,

" DO = perpendicular to DC.

" O = center of arcs for edges AH and BF of piece,

" CI = neutral line, $=2\pi.CD$, for complete turn,

" $\frac{AN}{AD}$ = stretch per unit for each edge, $=\frac{BK}{BD}$.

SANITARY SCIENCE.

A SKETCH BY ROBERT RAWLINSON, C. B.

From "The Builder."

Sanitary science includes works and arrangements necessary to comfort and health—the individual must be cared for. History, for the main part, however, records the deeds and misdeeds of emperors and kings, but the condition of the people is most fully recorded in descriptions of epidemics, pestilence and famine, aggravated by negligence of sanitary science, as also by war.

The strength of a nation has hitherto been estimated by the numerical force of armies and their power to destroy; but sanitary science, when fully understood and reduced to practice, will reverse the order as above enumerated, and the history of the future will take most note of the condition of the people, and the strength of nations will be estimated by a standard of health and intelligence rather than by numbers of armed men compelled to live unproductive lives at the cost of civilians.

Sanitary works must be useful generally; and, to be so to the fullest, they must be simple in construction and in their arrangements. Supervision must also be ample, economical, and unceasing.

The prime works for aggregated populations will be main-sewering, house-draining, scavenging, and a safe and proper disposal of excretal and of scavenged refuse, with a full and pure water-supply, and with public lighting. Villages and isolated houses must be sewered, drained, and have a supply of pure water.

Arterial drainage, the conserving of rivers, and land-drainage come within the scope of sanitary science.

Road and street surface formations are most important works to facilitate general and local traffic, and must be provided.

Having stated these general requirements, it may be instructive to describe briefly some of the great sanitary works of former times; as also some of the existing sanitary works in Great Britain, on the Continent, and in other parts of the world.

Main Sewering and Draining.—Main sewers and drains were constructed in some of the cities of antiquity, and some public buildings were drained, but we have no evidences, written or in buried ruins, that can warrant conclusions that any city of antiquity was completely sewered, including house-draining. There is no evidence that land-draining, in detail, was ever practised by any nation of antiquity, as is now considered necessary by British agriculturists.

Water-supply.—There is abundant evidence, both in history and in ruins, that works of water-supply were executed. Springs of water were utilized, wells were dug, natural lakes were made available, and streams and rivers were led by artificial canals contouring the intervening country. Reservoirs, bunds, and tanks of vast area and capacity were constructed, from which water was drawn for domestic purposes, but chiefly for land irrigation.

There are ruins of aqueducts of great age, especially in India and in Italy—the principal ruins in Italy being those near Rome. There is no reason to conclude that the laws of hydraulics were not understood in early times, as water placed under pressure would rise to its head. The metals lead, silver and gold were used for pipes and valves, in baths, and in the formation of fountains, as also for jets; the use of iron upon a great scale for main conduits and service-pipes is undoubtedly modern. The Romans, however, used lead for main-pipe conduits, necessarily limited in diameter and capacity to suit the strength of the metal.

Pipes of earthenware, moulded on a block by hand, or raised on a potter's wheel, were also made and used at an early period for water-conduits to supply towns, baths, and fountains; in some cases earthenware pipes were bedded in hydraulic lime concrete, so as to enable them to resist pressure, and to be used for street mains, as also to be used as inverted syphons to cross valleys.

Rome, and some cities of Rome, were supplied with water in vast volume, distributed to public and private baths, as

also to public and private fountains; but there is no written record, or other evidence, proving that a domestic supply of water by separate and special service was ever given to the masses of the people.* Main roads were formed by the Romans in Italy, as also in the great Roman provinces, streets were also paved in the great Roman cities, but we have no evidence that at any period highways, roads, and streets were general and were formed and paved as perfectly as those which at present exist throughout the several countries of Great Britain, France, parts of some other Continental nations, and in some North American towns.

Improvements in sanitary science are, for the most part, modern, and up to this date (1878) have had exceptional application. In British India roads, railways, and canals have been formed. Calcutta, and some other Indian cities, have been partially sewered, and works of water-supply have been executed for Calcutta, for Bombay, for Nagpore, and for some other Indian cities. In the Australian colonies works for water-supply have been executed, and main sewerage works are at this time under consideration. There has been considerable movement on the Continent of Europe in main sewering and in water-supply; as also, in Great Britain; but in few towns have sanitary works been fully completed; probably not in any one are sanitary works, sanitary appliances, and supervision as complete as these might be and ought to be.

Complete sanitary works, appliances, and supervision should provide as under:—

For Towns.—The sewering and removal of subsoil-water so as to dry the basements of houses; main sewering of the town and draining of the houses, with a daily disposal of excretal and scavenged refuse, so that there shall not be any removable cause of nuisance. There must also be the formation of good roads in the city and in the suburbs, with the paving of all street and yard surfaces, in such manner, and with such material, as shall best prevent accumulation of dirt, diminish noise, and admit of cleansing by washing. Public lighting must be ample and general, as proper lighting is useful for police purposes.

The main sewers of a town should be of such dimensions, and be laid at such gradients, as will transmit sewage evenly and regularly at rates not in any case less than one mile per hour. The sewers should also be fully ventilated, and have at command ample flushing power. In all main and tributary sewers side-junctions for house-drains should be provided in the first construction, and all houses should be drained, but the drains should not traverse house basements. Water-closets and sinks should be against external walls, and should have means for lighting and ventilation from the open air. Cesspools and cesspits should be abolished. If a drain must traverse a basement from front to back, as in London and in other street houses, such drain must be absolutely air and water-tight, and be fully ventilated externally, both front and back. Cast-iron pipes will be safest to use for such drains beneath basements. Back draining is, however, the true mode of sewering and draining streets of houses.

Water-Supply.—A town supply of water should provide for each house and each tenement, having a separate service-pipe and tap within the walls, the service to be high-pressure and constant; a volume equal to fifteen gallons per head of a population, with sound service-pipes and taps, efficiently supervised and maintained in repair, will be equal to the requirements of ordinary populations for all purposes, public and private.

Great mistakes have been made in the design and construction of sanitary works which it may be useful to indicate. As previously stated, the science is new, and some men who attempt to practice it have not learned the alphabet, not having had time, opportunity, or inclination. About 1850 three of the principal English engineers were required by the corporation of the city of London to advise and report upon the best forms and cross-sectional dimensions adapted for main sewering. In their report flat bottoms to sewers are recommended, as being the easiest to cleanse by hand labor, and, according to the rules laid down by these eminent men, no main sewer was to be constructed which was not of suf-

* The volume of water brought into London per day is about 110 millions of gallons. It is recorded that the twenty aqueducts which supplied Rome poured in 300 millions of gallons per day, or nearly the dry-weather flow of the river Thames over Teddington weir. Ancient authors are not, however, always to be relied upon.

ficient dimensions to allow men to enter. House-drains were not to be less in diameter than 12 inches. This report was accepted by English and by European engineers generally, hence the tunnel and canal-like sewers of Paris and of Brussels, executed for small sections of these cities at vast cost in proportion to the areas sewered. Many of the first formed sewers in London were large and flat-bottomed; scores of miles of such sewers remain to this day. The late John Roe and John Phillips first worked out the problem of egg-shaped sewers, with side-entrances and flushing chambers.

Since the year 1850, when I first introduced small sewers, laid at right lines from point to point, with side-entrances, manholes or lamp-holes at each change of line or gradient, as also at junctions of sewers, many entire towns have been so sewered by thousands of miles of small sewers and drains, laid on the principles above indicated. The first town so sewered was Alnwick, in Northumberland, under the supervision of Hugh M'Kie, C. E., the present most able city engineer for Carlisle.

Defective sanitary works may be illustrated by Paris, which may be taken as an example of a city laid out above ground on a most magnificent scale. The main streets are in right lines, open and well paved; the houses are regular in their architecture, lofty, and being built of white stone, are clean-looking and imposing. Paris may therefore be termed the finest-looking city in Europe. Street scavenging and watering are also admirable; but there are many sanitary sores in Paris; as, notwithstanding its big and costly sewers, it is a city of cesspools which are emptied at intervals, carts, or iron tanks on wheels having been specially contrived for this purpose. The covers of these huge cesspools are to be seen at the surface in the newest and most architecturally imposing streets.

The supply of water to Paris is abundant in volume, and is used profusely for public purposes; as in grand cascades and fountains, road-watering and street channel washing; but in quality the water is very hard, being to some people absolutely painful to wash in; the distribution within the house and tenement is not general. The true use of water is to supply, at high pressure, every house and tenement to the entire abolition of hand-carriage; and the true use of sewers is to remove all waste water and all human excreta at once, whilst unfermenting, to some outlet or outlets to be pumped and applied to land. At present there are the labor and cost of all the separate pumping from cesspools in detail, after putridity has set it, and the removal by horse-power of tanks and refuse which in the aggregate involves far more cost and power than dealing with the sewage if delivered at outlets by a full system of main sewers and house-drains, as water flowing down a conduit finds the necessary power to do so in gravity.

London is an example of a vast city, sewered almost in its entirety, the houses being drained also into the sewers; cesspools and common privies having been abolished, though not all filled up, water-closets being the rule. Taking the entire population at some three and a half millions of inhabitants, there is one water-closet to about every five persons; or, there are about 700,000 water-closets in London, soil-pans which are used as water-closets being included.

The water supply of London is also most abundant, and is far softer and consequently purer than the water supplied to Paris. It is true that in London there are no vast cascades or grand fountains, neither are the streets so well watered and washed, nor the street channels so fully flushed and washed, as in Paris; but the city at large is vastly better sewered, has water at high pressure, which is taken within and to the top of the best class houses, and is delivered within the tenement, or near the door of the poorest inhabitant. There are no large recognized cesspools in use, cesspool refuse carts, nor water-carriers. There are, however, defects both in sewering, in house-draining, and in water-supply in London, the result of defective information and divided jurisdictions. Parish vestrydom is not an apt learner: the local governing system is old, the form of election is old, and the ideas and prejudices are old. London has not been improved, laid out, and reconstructed under imperial authority, but has grown up under Dogberry blundering and speculative building, house-draining being the builders' work, the parish authorities having, and very

improperly, only extracted a fee for each drain entering a sewer, without taking the responsibility of seeing that each house-drain did properly join and enter the sewer as paid for, the results being no junctions in some cases, bad junctions in many cases, and no house-drain ventilation in most cases. The street-sewers are ventilated, but not house-drains as a rule. There are many miles of old flat-bottomed, broken-sided, shallow, and defective sewers in some of the richest London parishes—sewers which accumulate refuse and require to be hand-cleansed; but there are also very many miles of well-formed sound sewers, which, by the ordinary flow or by flushing, transmit sewage through the new intercepting and low level sewers to be poured wholesale in the crude state into the Thames at Barking and Crossness, the solids so scoured along, and in suspension, amounting to many thousands of tons per annum. Taking the ascertained results of some towns where the solids of sewage are intercepted, the volume and weight of silt and sludge annually washed into the Thames must amount to 200,000 tons or more.

Modern sanitary works have in some respects disappointed expectation, as reductions in local death rates do not appear to have followed closely upon the expenditure of money and the execution of works: but it is probably overlooked that sanitary works of the most perfect design and execution, with the best regulations fully carried out, do not in themselves solve all the problems of human health. There are the wretched houses, hovels, and tenements of the poor in every town and village of Great Britain, so frequently described by the editor of the *Builder*, in which excessive overcrowding takes place, and out of which inevitably proceed pauperism, crime, and disease. Sanitary science does very little to improve these dens of foul air and misery, unless it entirely supersedes them by new and better dwellings, and the old cottages are improved off the face of the earth, and this is both a costly and a slow progress; then there are spirit and beer drinking, tobacco smoking and chewing, with defective feeding and want of proper clothing amongst the poor, and want of sense to clothe properly amongst the rich, to make up the sum of human neglect and human misery. When these things have been studied, learned, and considered, wonder at continued excess of disease will diminish.

As the strength of a nation is in the health of the people, it must be the duty of governments to see that means of health are secured to every child born into the world. There is no value apart from human life, and there is no form of property so valuable as human life, and as the poor cannot provide their own dwelling-places, and as experience from the first dawn of history proves that defective tenements produce disease in excess, it must be a prime duty of a governments to legislate, order, and regulate, that health shall be possible within the cottage.

The inhabitants of a city cannot, however, all be treated as are criminals in jails; consequently, the health attainable in a model prison is at present exceptional; but sanitary works, properly supervised, can prevent raging epidemics by removing the soil in which the disease, seed, or germ, fructifies. Typhus fever, cholera, and other zymotic diseases, may decimate a filthy, drunken, demoralized population, and yet never enter the county model prison. The black-death, plague, and sweating-sickness of Mediæval Europe appear no more in England, but haunt the foul cities of Asia, and yellow fever now rages in the foul southern cities of America, teaching a lesson terrible to study, but instructive to learn.

THE report of the committee of the British Association, on the best means of developing light from coal gas, is largely occupied by a tabulated series of results obtained with different kinds of burners, such as the rat-tail, union or fish tail, batwing and argand, and also the influence of globes of different sizes, shapes and materials. The report is strongly in favor of cannel rather than common gas, on account of its comparatively small influence on the atmosphere of apartments and the smaller proportion of sulphur it contains. The report also advocates the burning of gas at a comparatively low pressure, and the use of district governors to equalize the pressure in different levels of towns, and of regulators in houses and street lamps, to give the exact pressure calculated to give the best photometric results.

PROCESSES FOR DETERMINING THE ORGANIC PURITY OF POTABLE WATER.

By Dr. TIDY.*

From "Iron."

The processes for estimating the organic matter in water naturally divide themselves under two great heads. (1) Where the organic matter is estimated from a water residue; (2) where it is estimated in the original water, *i. e.*, from the water itself and not from the residue after evaporation. It is here to be remarked that if it be possible, by any process or processes, to estimate the organic matter in the original water, such processes must have manifest advantages over those where the quantity of organic matter is deduced from a residue.

The Ignition Process.—The first in respect of date, last in respect of value. This process to be of value presupposes three things. First, that no organic matter is lost and none gained during the evaporation of the water. In this point the process fails, because there may be both a loss of the actual organic matter present in the water during evaporation; such loss being either physical, particles being carried off mechanically with the steam, or chemical, from decomposition; and again, resulting from the introduction of extraneous organic matter in the shape of impurities floating in the laboratory atmosphere. Secondly, That all the organic matter is burnt off by the ignition of the residue; and thirdly, that nothing but organic matter is lost by ignition. With regard to both these points the ignition process also fails. The process is not quite abandoned, else there would be no need to refer to it; some analyses having been, not long ago, put in evidence where the loss on incineration was entirely relied on as the indication of organic purity or impurity.

Combustion Process of Drs. Frankland and Armstrong.—The objections to this process are twofold. First, general, and secondly, special. The general objection is its impracticability, arising from the manipulative skill required, and the length of time consumed in conducting the process. These two latter objections may seem to have some weight at first sight, but very little on further consideration. It is admitted that a certain amount of manipulative skill is required, but the possession, by practice, of this manipulative skill constitutes a man an analytical chemist; and the sooner so delicate and difficult a work as water analysis is taken out of the hands of those who imagine themselves professional chemists after a few lessons in a laboratory, the better for chemistry and sanitary science. As regards the time required, rapidity ought not to enter into the calculation if accuracy or delicacy be thereby imperilled. This objection of impracticability is therefore worthless. The question of evaporation is then reviewed, including the ingenious devices of Professor Bischof and Dr. Mills. Until we have definite evidence that no organic matter is oxidized, and also that no organic matter is volatilized or destroyed by the heat required in evaporation, it follows that evaporation must always constitute a possible source of error in any process where the determination is made on the residue. Indeed, we cannot resist the conclusion being forced upon us that after all our trouble we may be simply estimating the harmless organic matter, that which was poisonous and disease-producing having been carefully got rid of by our previous work. The next difficulty, the addition of the solution of sulphurous acid, is an important one. From a careful series of observations the author concludes that if the nitrogen present as nitrates does not exceed 1 part in 100,000, or 3.15 gramme of nitric acid per gallon, the reduction of the nitrates is complete on boiling with 10 or 20 cc. of the sulphurous acid solution. If, however, the quantity present amounts to 1.5 per 100,000, some difficulty was experienced in effecting complete reduction, even when 60 cc. of sulphurous acid solution were used. The next step is the combustion of the residue, the deter-

* Chemical Society, Dec. 5, 1878.

mination of organic carbon, and is quite accurate; but the same value does not attach to the determinations of the organic nitrogen, and therefore it is doubtful how far we are justified in drawing conclusions as to the source of the organic matter, *i. e.*, whether it is vegetable or animal, from the proportion which exists between the organic carbon and nitrogen. However, the process has so much improved on acquaintance that the author believes it to be of great value.

The processes where the organic matter is estimated from the water before evaporation are two: The Ammonia Process and the Oxygen Process. *The Ammonia Process* consists of the comparative determination of the nitrogenous organic matter by the quantity of ammonia (albuminoid ammonia) yielded by the destruction of the organic matter; this destruction is effected by boiling the water in the presence of potassic permanganate and a large excess of caustic potash. Waters are divided by the authors of this process into three classes: Waters of extraordinary organic purity, yielding 0.000 to 0.5 part per million of albuminoid ammonia; safe waters, yielding 0.5 to 0.10; and dirty waters, yielding more than 0.10. The objections made to the process by Dr. Frankland are, that the conversion of the nitrogen of the organic matter into ammonia is seldom or never complete, and that the proportion of nitrogen converted into ammonia in a series of nitrogenized bodies varies widely. There is a certain force in these objections. If the organic matter of all water was alike, it would be of no importance whether the whole or a definite part of the nitrogen was converted; but inasmuch as in all probability the organic matter of one water is not the organic matter of other waters, the circumstance that bodies yield their nitrogen as albumenoid ammonia in vastly different proportions constitutes an objection of some importance. Nevertheless, this objection can be overrated; if it can be shown that the ammonia process indicates clearly that the yield of albuminoid ammonia keeps pace with the purity or impurity of waters, and that it is sufficiently delicate to indicate the finer grades of purity, the fact that piperin yields all its nitrogen, whilst thein yields one-fourth, is of little significance in water analysis, whatever interest such facts possess for the scientific chemist. It must be remembered that Mr. Wanklyn has repeatedly and distinctly condemned the course taken by certain chemists in regarding the process as a method for the quantitative determination of nitrogen, and asserts that his process only answers the question: Is this water wholesome or is it not? whilst it leaves untouched the question: How much organic nitrogen does this water contain? The author proceeds to notice some practical difficulties in the details of the process. It is practically impossible to prepare the alkaline solution of potassic permanganate absolutely free from ammonia, and it is always necessary to estimate the quantity of ammonia in the permanganate solution and deduct this from the total amount obtained in the actual experiment. It is a matter of extreme difficulty to effect the complete (*i. e.*, as complete as the alkaline permanganate solution is capable of effecting) decomposition of the organic matter by boiling the water with the permanganate solution. Thus, you distil a water with permanganate until ammonia ceases to be evolved, and then leave the apparatus carefully protected from contamination for a few hours; on redistilling, a second yield of ammonia, often equal or even larger than the first, is obtained; and so, again and again, fresh quantities of albuminoid ammonia may be obtained until every drop of water in the retort has been distilled over. This is a serious difficulty, and has, in one case at least, led one analyst to report a water to be of "extraordinary organic purity," whilst a second classes the same water as a "dirty water" and entirely unfit to drink. Sometimes, too, ammonia seems to disappear. Thus, the permanganate solution is known to yield a certain quantity of ammonia on distillation. It is added to a water, and the two distilled, when the distillate from the water, plus the permanganate, contains less ammonia than the distillate from the permanganate alone. As regards Neslerizing, a serious error may creep in from the fact that eyes are very far from being equally sensitive in observing and in classifying tints. Thus, out of a large number of average men observing, 60 per cent. failed to arrange a series of Nesler test

solutions in the proper order of their tints. The presence of free ammonia, within certain limits, the author regards as of little importance, and entirely disagrees with Mr. Wanklyn when he regards the presence of more than 0.08 part of ammonia per million as evidence that ammonia proceeds from the fermentation of urea. As a general conclusion from a wide experience, the author thinks that the ammonia process gives fairly concordant results when uniformly conducted, *i. e.*, given solutions made by the same person, the same hands to manipulate, the same eyes to judge of the tint-depths. But this is almost a fatal objection to its general employness; for, if the author's statements be true, in the ammonia process every man must be a law unto himself, whilst one man's law is no one else's law. Hence comparisons are rendered impossible. As a rule, the albuminoid-ammonia process enables you to say whether a water be of excellent quality or of an exceptionally bad quality; but in those more delicate and difficult cases where a water is not what may be termed excellent, but, nevertheless, is not "dirty," in the opinion of the author the ammonia process absolutely and entirely fails.

The Oxygen Process.—This process, when properly carried out, is much relied on by the author. He deprecates most strongly the ordinary method of using it. The proper plan of using the permanganate is the following: Into two 20-oz. flasks, cleaned by rinsing with sulphuric acid, and then thorough washing under the tap, place 500 septems (1 septem = 7 grains = $\frac{1}{1000}$th gallon) of the water, add to each 20 septems of dilute (1 in 3) sulphuric acid and 20 septems of the permanganate solution (2 grains in 1000 septems). Note the exact time at which the permanganate solution was added; at the same time two similar quantities of distilled water are to be treated in precisely the same manner. At the end of one hour and of three hours the oxygen used up by the water is to be determined. To the flasks, after standing the appointed time, add a sufficiency of potassic iodide (1 in 10), and then a standard solution of sodic hyposulphite (5.4 grains in 1000 septems) until the whole of the free iodine is removed, judging of the exact spot by the addition, towards the end of the experiment, of a few drops of starch solution. By deducting the quantity of oxygen equivalent to the hypo-solution used from that in the quantity of permanganate originally added, we obtain the quantity of oxygen used by the water. The blank experiments with distilled water give the value of the hypo-solution. It is obvious the samples of water must have a pink tint at the end of the one hour or the three hours, otherwise fresh experiments must be made with larger doses of permanganate. The author then proceeds to consider the interference of various substances with the process. He concludes that the only important errors which can arise would be due to the presence of ferrous salts, sulphuretted hydrogen and nitrites. The presence of the first two substances would be sure to be discovered in the analysis, and by taste or smell; the nitrites act immediately on the permanganate solution, and any decolorization taking place during the first five minutes must be due to nitrites and allowed for. Besides, even if a careless manipulator was to miss the iron, the sulphuretted hydrogen and the nitrites, and estimate the whole as oxidizable organic matter, he would simply condemn a good water, but could never, by using the oxygen process, pass a bad water as harmless. It is admitted that permanganate fails to oxidize some substances, such as urea; but, nevertheless, the quantity of oxygen used affords evidence of the relative quantity of matter in the water which is likely to be injurious, and this is what we want in water analysis, as it enables us to speak with confidence as to the use or rejection of a water for drinking purposes. The quantity of oxygen used during the first hour, as compared with that used in the first three hours, gives valuable information as to the relative quantities of putrescent, easily oxidizable matter, and of non-putrescent and less easily oxidizable matters. The author also recommends, as a valuable accessory, the tint of the water as seen viewed through a 2-feet tube 2 inches in diameter, daylight reflected from a white card being used. This tube is of special value in determining whether a water is peaty or not. In some cases the tint gives a clue to the quantity of organic matter present. The author has collected and plotted out in curves the results obtained by Dr. Frankland using the combustion process, and

these obtained by Dr. Letheby and himself using the oxygen process, and those obtained with the ammonia process, with the waters of the eight London companies, since 1870. He finds that the curves of the oxygen and combustion processes are strikingly concordant, whilst that of the ammonia process agrees with neither. The author has also divided these results into classes, making, as far as possible, classes (with the three methods of estimating the organic matter) which should be comparable with each other. Thus, Class I. contains waters of great organic purity, which require less than 0.05 parts of oxygen per 100,000 to oxidize their organic matter, or which give less than 0.1 part per 100,000 of organic carbon and nitrogen, or which yield less than 0.05 parts per 1,000,000 of albuminoid ammonia. Compared in this way, out of 1686 experiments, the oxygen and combustion processes tell the same tale in 1418 cases, and of the rest a large proportion have differences depending on the third decimal place. The results by the ammonia process correspond, save in a very general way, neither with those of the oxygen process nor with those obtained by the combustion process. The author has analyzed, moreover, 200 miscellaneous waters, using the three processes in each case. Of these, 94 were placed in the first class by the oxygen, and 92 by the combustion process, whilst only 42 were placed in the first class by the ammonia process. It must be understood that the author deprecates most strongly the judging of a water by one constituent without reference to its complete analysis and natural history, and has only instituted these classes for the sake of comparing results. The relative value of the three processes may be briefly summed up as follows. The Ammonia Process furnishes results which are marked by singular inconstancy, and are not delicate enough to allow the recognition and classification of the finer grades of purity or impurity. The errors incidental to the process form an array of difficulties which become infinitely serious, seeing that the range (from 0.05 to 0.1 parts per million) between pure and dirty waters is comparatively small. The Combustion Process has all the evils of evaporation to encounter, but the organic carbon estimation is trustworthy; the organic nitrogen determination, however, scarcely yields absolutely trustworthy evidence on which to found an opinion as to the probable source of the organic matter.

The Oxygen Process avoids the errors incidental to evaporation; its results are constant and extremely delicate, it draws a sharp line between putrescent and probably pernicious, and the non-putrescent and probably harmless, organic matter. By it a bad water would never be passed as good. As far as the three processes are concerned, the oxygen and combustion processes give closely concordant results, whilst those yielded by the ammonia process are often at direct variance with both.

AMERICAN IRONMONGERY.

From "The Engineer."

Our correspondents in the Sheffield and Birmingham districts have often referred to the competition experienced by the manufacturers in their districts with the cheaper productions of American manufacturers and factors. For the evidence of that competition we need not now go to our producing districts. We find it in every ironmonger's shop in town and country, and particularly in furnishing ironmongery stores. If we ask for locks, gas standards, roller-blind fittings, small brackets, hooks and hat pegs, domestic apparatus and tools, substantial toys, and very many other things, we are shown American productions. The reason for this is not sufficiently obvious in all cases, though in many cheapness is the explanation. In some things, as for instance small brackets and pegs, the patterns are new in their lightness and suitability for their intended

purposes. But in the case of many things, the reason for the preference which English ironmongers plainly show for American goods is not superficial. It is not altogether the lowness of price at which he can buy the goods, for the low price at which he sells them prevents any heavy profit. The real reasons are, however, not far to see. To begin with, hardly any English small castings are anything like as fine in surface, light in pattern, and cleanly turned out as are these American things. Small English castings often show the joint in the mould in which they are cast, fins are often not absent; and they are either turned out uncoated, or are daubed with a common black or dipped into a commoner. Most often screw holes are too large or too small. The countersunk part is large and flat, while the actual hole is small. Hence a small reamer is required to prepare the holes for the screws, and few carpenters would feel themselves prepared to go and fix a set of door and window furniture without a counter-sinking rose bit, and even then he must fix with screws with bright heads, which soon rust. All this the Americans have changed. Their castings are light, though strong in design, they are clean, and are touched up on an emery or grindstone, and are nicely coated with a clear brown varnish of great toughness and strength. The holes are almost invariably properly prepared to receive the screws for fixing. The screws themselves are colored to match the ironwork and, at the same time, prevent rusting. The holes, too, are arranged so that where the greatest strain comes there are the most screws. Now these are reasons which affect the purchaser only, but there are other reasons which affect the ironmonger, and which explain why he is so ready to show his customer the American articles. All the small articles to which we have referred are sent out by the English manufacturer done up in separate papers, or in paper packets tied up with string. Thus when the ironmonger wants even one article, or only wishes to show one to a customer, he has to undo a string, unfold paper or papers, do these papers up again, tie them, and re-arrange the label on the package. Instead of this old bungling way of keeping store and serving customers, the Americans supply their articles in paper boxes, sufficiently strong to last out the sale of the articles, one or two at a time. These boxes are easily and neatly stacked, the labels are fixed once for all, and to open one and show its contents, or take some out and reclose the box, is the work of a moment. This question is one of much greater importance than it at first sight appears. Folding up and re-tieing parcels is irksome and exceedingly uninteresting work, and is such as is not done very quickly at any time. Piles of packages, which have been taken down to show customers, collect on the counters to be done up "presently," because they cannot be done up while the customer is being served. These have to be done up, and some one must do it. Here is an important saving. One London ironmonger, whose sales in furnishing ironmongery in a moderate size shop consist of about one-half American articles, recently assured us that he had, from the saving of labor in this way, been able to dispense with about one-fourth the assistance he would otherwise have required for the increased business done in small articles. Again, not only do tradesmen and assistants appreciate the saving in time in serving customers, but the work is so very much cleaner for them, and the work of clearing up after closing is reduced almost to *nil*. The same ironmonger also assured us that ready as is the sale of these American articles, he is unable to get them as fast as he could wish. The receipt of the goods is irregular, and is not likely to be regular as those from English makers. He would therefore prefer buying of the English maker, if he would produce at the same price equally well designed and finished articles, packed as conveniently as by the American competitor. In a recent note we recorded the relative consumption of paper by the different nations of the world, and observed that the consumption per head of the American population was far in excess of that of any other country. Part of this excess is probably explained by the large use of paper in the manufacture of boxes for packing. These, of course, it may be said, only take the place of our paper, but this is not the case. A considerable number of articles are here wrapped up in very thin paper, and en-

closed in one strong wrap. The same article would in America, and by American factors, he sent out in boxes containing about a dozen, or dozen pairs of the same articles, and the boxes weigh more than the paper wrap. How much preferable the box system is may be acknowledged by every one, and before Americans satisfy English demand by regular supply, it will be well for our manufacturers to look about them, and produce clean, light, well-finished articles and apparatus, packed and delivered in a manner tempting not only to the consumer, but to the retail dealer. Are Englishmen to be beaten on their own ground in this way because men and masters cannot or will not depart from old errors of reasonless custom? There seems something of disgrace in the fact that the producers of another land should have, even so far as they have, taken the place of home producers? The retail ironmongers tell us, when speaking of the fine, clean, finished castings, that "the English cannot do it." Cannot do it! Surely to be told this ought to make some of the more energetic and ingenious of our manufacturers prove by works that it is false. To a certain extent it is false, for we know of founders who produce fine machinery castings of the most excellent cleanness and finish. This is, however, far from general, and though it is not true that English manufacturers cannot produce clean, fine, well finished, and cheap work, it is true that they cannot, or at least have not, produced it in accordance with that most important condition, price.

REPORTS OF ENGINEERING SOCIETIES.

Engineers' Club of Philadelphia.—At the recent annual meeting the following gentlemen were elected officers of the Engineers' Club of Philadelphia, to serve during the year beginning January 11th, 1879:

President—T. C. Clarke; Vice-President—J. B. Knight; Corresponding Secretary and Treasurer—Charles E. Billin; Recording Secretary—Herman Hoops; Directors—William G. Neilson, D. McN. Stauffer, Rudolph Hering, Coleman Sellers, Jr., Percival Roberts, Jr.

At the last meeting of the Northern Architectural Association Mr. W. H. Dunn read a paper on "Concrete." He described the many forms in which concrete could be used in building constructions, showing its adaptiveness for foundations and fire-resisting floors and roofing, and the possibility of a much more general application of the material. In accounting for the shyness which the public had entertained for concrete work, he thought that the early promoters had much to do with the false impression, their great argument being "cheapness and expedition," and this seemed rather characteristic of bad work. From his experience concrete was neither so very cheap nor yet so expeditious for house building, its virtue consisting more of "utility and usefulness," and, instead of being so expeditious, was in reality a slow setting material, and when treated in its true and legitimate way was most reliable. Under the section of "Flooring," Mr. Dunn referred to the several works in which he had introduced concrete for large spans, mentioning that he had recently executed a floor with a clear span of 21ft. by 18ft. 6in. without any intervening supports. In describing the application of different kinds of concrete, he stated, that for malting working floors he had used a compound of riddled marl, slaked lime, and Portland cement, which, whilst forming a durable floor, also retained and gave moisture; thus assisting the growth of the malt. The paper was illustrated with working drawings of works in concrete he had executed.

At the meeting of the Scientific and Mechanical Society, held recently, Mr. I. Bowes read a paper on the utilization of blast furnace slag and other waste products. After briefly alluding to the great saving now effected at many collieries by washing the small coal and making it into coke instead of burning it in the pit heaps as formerly, and to the useful products made from what was once the refuse of gas manufacturers, he spoke of what was now being done in the manufacture of useful commodities from blast furnace slag. About 30 cwt. of slag is made for every ton of pig iron, and in the Cleveland district alone from three to four millions of tons of this slag are made annually. Some millions of tons have been deposited on the banks of the River Tees and in forming an immense breakwater, which stretches out into the sea some miles at the mouth of the river. A few years ago Mr. Charles Wood, of Middlesbrough, Mr Woodward, and others, commenced making bricks, paving sets, concrete, and other articles from it, and two companies are now at work making these articles. About three millions of bricks are made annually, and sent principally to London by water carriage; and streets and crossings have been paved with these sets in several towns of the North of England. At some of the furnaces on the west coast the same articles have been produced from slag, and buildings, river walls, water courses, &c., constructed from the articles made. Glass works are now in operation at blast furnaces in Northamptonshire, where the slag is run direct from the iron furnace into the glass furnace, mixed with other materials, and then used for making bottles and other articles of glass. Mr. A. Jacob, borough engineer of Salford, is now using the sets to pave alongside the tram rails which are being put down in Eccles Old road.

IRON AND STEEL NOTES.

Protection of Iron from Oxydation.—We have on many occasions drawn the attention of our readers to various devices suggested for preventing the oxydation which invariably occurs when iron is exposed to the action of the weather. The two latest of these, namely, those of Professor Barff and Mr. Bower, depend upon the production of a superficial coating of magnetic oxide, and, however excellent in their way, can scarcely be termed ornamental. When any other color than bluish-grey is required they are wholly inapplicable, and the material has to be painted in the ordinary way. We have lately had submitted to us some specimens of ironwork, suited to the most exacting requirements of ornament, and at the same time capable of withstanding, apparently for many years, the action of the weather, prepared by a process invented by Mr. J. B. A. Dodé, of Paris, which seems to promise exceedingly well for those many cases wherein ornamental effects are desired as well as simple protection from corrosion. For example, the railings surrounding the British Museum, and the lamp-posts of new designs used in the City, are periodically painted and gilt at short intervals, the expense, in comparison with the effect and durability, being enormous. It has often struck us, while watching the elaborate and tedious process of gilding ironwork in the open air, that some less wasteful method ought to be adopted. Such a method is certainly that we are about to describe, and without saying it is the best, we are bound to say it is a good one, and has satisfactorily stood every test we have applied. As a new process it has yet to prove how time and weather affect it; but when this is done there is little doubt that "platinized iron" will become as familiar a term as "oxidised iron."

The principle of Mr. Dodé's method is to coat the surface to be protected with a thin film of borate of lead having a little oxide of copper dissolved in it, and having also suspended in it bright scales of precipitated platinum. A red heat is employed to fuse the composition, which is either applied with a brush or employed as a bath, in which small articles may be dipped. Its effect is, to cover the iron with a thin glassy coating of a bright grey tint, not far removed from that of polished iron itself, and unaffected by sewer gases, dilute acids and alkalis, and the heat of a kitchen fire. Modifications of the composition give the means of imparting different colors to the coating, and these are as easy of application as the platinum grey just mentioned. The effects are really very good, and show how ornamental an iron grating of neat pattern, or an iron frieze might be in front of a gallery in a large building, or in any equally elevated position. There is, too, an opportunity for the City or Cathedral authorities to venture upon an experiment which deserves trying. The railings round St. Paul's Cathedral are to be redecorated, simultaneously with the improvement of the churchyard. Instead of adopting the present expensive method of gilding, why not coat them with Mr. Dodé's composition, at (we are informed) about $\frac{1}{10}$ of the cost?

Cost is in all cases a most important feature of preservative operations. We are told that the cost of platinizing is about equal to that of applying three coats of paint, and about one-tenth of that of electro-plating with nickel. These statements have reference to Paris prices, which as a usual rule are not lower than English. A detailed account of the treatment of eight stoves is as follows:—

	fr.
1 litre preparation (retail)	3.75
1st furnace operation	3.20
Reagents for platinizing	4.00
2nd furnace operation	3.20
Manipulation, wear and tear, &c.	1.85
	16.00

Or 2 fr. per stove. The trade is quite willing to pay four times this rate, as to electro-nickel a stove costs from 32 fr. to 40 fr. For some special manufactures the new process appears to promise particularly well, and we understand that the Val d'Osne Company have expressed a highly favorable opinion of it. Such manufacturers, who can treat their castings before they cool, will find a still further economy in what already seems to be a very cheap and efficient process.

RAILWAY NOTES.

A Portuguese railway of considerable importance, known as the Beira Alta Railway, and forming a junction with the Northern Railway at Pampilhosa, is about to be built. It will cross the Busacs range, pass through the valley of the Mondega to Guarda, and terminate at Villar Formosa, on the Spanish frontier. The total length of the road will be only 125 miles, but the line will be important because it will open up a fertile and well populated district, at present without any railway facilities, and also form the missing link of the direct or international route between Lisbon and Paris by the north of Spain. The present railway communication between the two capitals is 1449 miles in length, while by the Beira Alta line it will be only 1171 miles, or 273 miles less. The contract for its construction and working has just been given to the Société Financiere of Paris, which binds itself to open the road in four years. The Portuguese Government grants a cash subsidy of £8225 per mile, the total estimated cost being £11,719 per mile. For this and other aid, in the way of lands and buildings, and exemptions from duties and imposts, it will exact a "transit duty of five per cent. on fares for freight and passengers."

Some interesting statistics have recently been published on the railways of the world, by Prof. Neumann-Spallart, of Vienna. During the last three decennial periods the length of the railroads in Europe has risen from 9000 kilometers to 154,200. In that total the share of Germany is 30,000 kilos.; Great Britain and Ireland, 27,500; Austro-Hungary, 24,800; France, 23,400; Russia, 18,000, &c. The result is that in this quarter of the world the number of kilometers of lines is 150 to each superficies of 1000 square kilos., and 4 4-5 kilos.

per 10,000 inhabitants. Those averages are, however, exceeded in Great Britain, Belgium, France, Switzerland, Holland, &c. In America, the United States commenced in 1830 with 42 kilos.; at present they possess 128,000, or 133 per 1000 square kilos, and 28 per 10,000 inhabitants. The professor, however, states that this result has been attained by a loss to the shareholders of 4,000,000,000 marks (1f. 25c.) from 1872 to 1877. The other States in that part of the world only possess 17,000 kilos of railway, of which 7000 belong to Canada. In Asia, China remains closed to that system of communication, while British India, including Ceylon, has 11,000 kilos., or 46 per 1000 square kilos., and half a kilo. per 10,000 people. Africa has 2800 kilos., of which 1800 belong to Egypt. Australia has 4000 kilos., principally situated in the part of the continent which contains the colony of Victoria, then Tasmania, and finally New Zealand. In Oceania, Otaheite has a little railroad. The capital invested in all the railroads of the globe exceeds £3,500,000,000. Those lines dispose of 62,000 locomotives, 112,000 passenger carriages, and 1,500,000 goods trucks. They carry yearly 1,500,000,000 persons and 1,600,000,000 tons of merchandize annually.

The Federal railway department has recently published an interesting and elaborate statement of the position of the Swiss railways in the years 1874 and 1876. In the former year the total length of railways in the territory of the confederation was 1020 miles; in 1876, 1472 miles, being an increase of 452—nearly 50 per cent. The principal lines are the Aargau Wohlen-Bremgarten, with 215 miles; the Jura-Berne-Lucerne and the Boedeh, which, together with the Berne State lines, count 318 miles; the Swiss North-Eastern, 319 miles; the Swiss Western and its subsidiary lines, 351 miles; and the Swiss Union, with the lines Toggenbourg and Wald-Ruti, 195 miles. The remaining mileage is made up of ten smaller lines from the Rohrshach-Heiden and the Uethberg, with their five miles each, to the Gothard and the Swiss National, with their forty and forty-seven miles respectively. In this list are included only lines that were actually working at the time in question—the 31st December, 1876; but the increase since that date is not considerable. The cost of constructing these lines is put down at £33,606,300, and the capital employed in working them at £28,728,000. The rolling stock consisted of 549 locomotives and 1662 carriages, capable of accommodating 73,243 passengers; and 8352 wagons, with a carrying capacity of 84,605 tons. The distance run by these 549 locomotives in 1876 was 8,857,550 miles—14,310 miles each. The material served to transport 23,815,207 passengers, 82,036 tons of luggage, 694,694 head of cattle, horses and dogs, and 5,669,364 tons of merchandize. The receipts from passenger traffic amounted in 1874 to £831,200; in 1875 to £928,948; and in 1876 to £976,800. The total receipts from all sources, in which 1874 reached £2,086,000, had increased in 1876 to £2,427,240; but this increase was far from being commensurate with the increase in mileage; for, whereas in the former year the total receipts per kilometer were £1375, they had sunk in 1876 to £1141 per kilometer. It is a notable fact that in late years there has been a steady diminution in the number of passengers traveling first-class, and an increase in the numbers of those traveling second and third-class. In 1876, out of every 100 passengesrs 6.99 took first-class, 33.63 second-class, and 59.38 third-class tickets. It is stated that the market value of eighteen companies, with an aggregate paid-up capital of 43 millions sterling, does not exceed 23 millions—a depreciation of 20 millions—an enormous loss for so small a country as Switzerland.

Improvement in Railroad Tracks.—Anaxamander Herring, of Cohoes, has invented a new and useful mode of securing an elastic railroad track or bearing, and destroying the effect of the percussive force of railroad cars upon the rails and supports, and upon the cars.

The nature of the invention consists in the employment of sand between the parts of the pillow-blocks which support railroad-rails on iron bridges, or elevated railways.

By this invention an elastic track on iron bridges can be secured without the aid of metal and rubber springs or combustible materials, such as wood; at the same time the effect of the percussive action of cars upon the metal railroad-bridge heretofore experienced is destroyed; the shaking and loosening of the parts of the bridge structure, as well as the jarring of the rail-cars is also avoided, this result being due to the fact that the sand serves to absorb the blows due to the weight and motion of the cars as they roll over the track, while it serves to effectually deaden the sound experienced in elevated railways.

Mr. Herring's claim is as follows:

1. Sand as a cushion for railroad-rails, car-bodies, and other analogous uses, applied between the upper head of a drum or chamber, and a piston.

2. Sand as a cushion for rails and other objects, applied between the heads of a drum or chamber, and the upper and lower ends of a piston.

3. The drum or chamber, packed with sand and provided with a piston, the stem or rod of which is fastened to a foundation, and having a railroad-rail shoe or chair formed on its upper head.

4. The drum or chamber packed with sand and provided with a piston, and having sand-supply holes.

ENGINEERING STRUCTURES.

The Avonmouth Dock.—At the meeting of the Institution of Civil Engineers, held on Tuesday, November 12, the first paper read was on "The Avonmouth Dock," by Mr. J. B. Mackenzie, M. Inst. C.E.

Bristol at an early period of history was one of the chief shipping ports in the kingdom. Down to the era of ocean steamers, it was accounted only second in importance to the port of London, but subsequently declined to a comparatively subordinate rank. The paper de-

scribed various schemes by different engineers for the improvement of the port from the time of Smeaton to the year 1860, of which only one by Jessop had been carried out. In 1864, Mr. Brunlees, Vice President Inst. C.E., recommended a scheme for a dock at the mouth of the Avon, which had been previously suggested. It was undertaken by the Bristol Port and Channel Dock Company in 1868, and was completed in 1877. The dock was on the Gloucestershire side of the Avon; from the anchorage of King Road in the Bristol Channel to the entrance lock, the distance was only 1,000 yards. The entrance channel from the Avon to the lock was about 350 yards in length by an average width of 70 yards, with a depth at high water of equinoctial spring tides of 44 feet, and of 40 feet at ordinary spring tides. The dock was 1400 feet in length, and 500 feet in width, giving a water area of about 16 acres, and a length of quay wall of 3200 feet. The south end was not protected by a wall, but was finished off with a slope of 2½ to 1. The range of an ordinary spring tide was 39 feet, while that of an ordinary neap tide was 19 feet. A special feature of the tides was the quantity of mud which the water held in suspension. The complete silting up of the old entrance of the Avon a few years ago, and the opening of the present Swash Way, was a striking example of mud settlement and accumulation. A temporary embankment, to exclude the tide during the construction of the works, was made by tipping silt excavated from the dock over the ground. A wooden truss was used to exclude the tide while the outer clay dam was being removed. It proved satisfactory; and the leakage from the tide was easily kept under by a small force pump. The mouth of the lock had a wing wall on each side, extending about 159 feet beyond the roundheads, and diverging from a line parallel with, and 100 feet distant from, the center line of the lock, at an angle of 11 deg. 30 min. Rubble masonry faced with rough ashlar was employed. The walls were 49 feet in height from the top of the footings to the coping, 23 feet 6 inches wide at the base, and 7 feet wide at the top. The face was battered to a radius of 150 feet, and the back had two steps 18 inches and two 12 inches wide. The footings were also of rubble masonry, and rested on sand; the inverts were of brick. The clear length between the inner and the outer gates was 454 feet. This length was divided by a pair of gates into two locks, the inner one being about 50 feet longer than the outer one. The foundations of the lock were laid upon a bed of fine gray sand underlying clay at an almost uniform level, and at a depth of about 6 feet under low water of equinoctial spring tides. The frequent occurrence of springs in this sand was a source of some trouble and difficulty. The apron in front of the lock was a mass of lime concrete, mixed with blocks of stone of 2 tons to 3 tons weight, and surrounded by walls of Portland cement concrete. The lock gates consisted of oak heel and mitre posts, except the outer pair of gates, which were of greenheart, with ribs, intermediate posts, and walings of pitch pine and Memel. The gates were 2 feet 8 inches thick at the heel and mitre posts, and about 2 feet 11 inches thick at the center of the leaf, exclusive of the walings. The back of the gates, when shut, formed a continuous arc of a circle from one hollow quoin to another, the radius of which was 50 feet. The ribs and intermediate posts of the upper gates were differently arranged to those of the middle and outer gates. The height of the dock wall was 40 feet, and the depth of the foundations below the dock floor varied from 2 feet 6 inches to 19 feet. The footings were of lime concrete 22 feet 6 inches in width, and were carried up 2 feet above the dock floor. From this level to the top, the wall was built of rubble masonry, faced with dressed stone. Two failures of parts of the dock wall, caused by the wall slipping forward and sinking, were then described, and the remedial measures pursued, also the modifications introduced in the subsequent work. The earthwork chiefly consisted of clay. Upwards of 1,750,000 cubic yards of material were shifted from the dock basin, lock, entrance channel, and foundations. Of this quantity, about 150,000 cubic yards were dredged from the entrance, and discharged from hopper barges, at a shallow part of the Bristol Channel, about three miles from the works. The average cost of the excavations, including a portion of the pumping expenses, was about 1s. 6d. per cubic yard. The average price for rubble masonry was about 20s. per cubic yard. The Portland cement concrete consisted of one part of Portland cement, three parts of sand and gravel, and five of stone broken to a small size, and the whole mixed with large blocks of stone. The average price of this concrete was about 16s. per cubic yard. The lime concrete used for the foundations was mixed in the proportions of six to one, viz., one part of lime, two parts of sand, two of ashes, two of broken stone, and cost about 10s. per cubic yard.

Darien Canal.—An interesting lecture was recently given by Rear-Admiral Dan'l Ammen, U.S. Navy, before the American Geographical Society in New York, "On the proposed Inter-Oceanic Ship Canal across the American Isthmus." The Admiral stated that after an examination of the reports of Lieut. Wyse of both seasons, he was confirmed in the opinion expressed in a paper previously presented to the society "that no possible route exists comparable with what had been presented in the surveys made by order of our Government." The able reports of Commander E. P. Lull and Civil Engineer A. G. Menocal, U.S. Navy, on the Nicaragua route, are, he said, "sufficiently full for examination and criticism by the civil engineer or the expert. There has been given throughout a careful consideration to that vital question in the construction of an inter-oceanic ship canal in that region—an ample and studied provision to prevent any considerable quantity of surface drainage entering the canal, and the feasibility of accomplishing this object on the located route, as compared with other routes, is, in my belief, a most important point in its favor." In conclusion, he said: "To the courage, devotion, and ability of cultured officers as leaders, and to their assistants, we are

indebted for much substantial information. It is impossible for any one having no personal knowledge of the Isthmus to appreciate the difficulty of making surveys in that region." A summary of distances and estimates of cost, as given in the report of Civil Engineer Menocal, is as follows:—Western division, from Port Brito to the lake: Distance, 16.33 miles, estimated cost £4,337,555. Middle division, Lake Nicaragua: Distance, 66.50 miles, estimated cost £143,131. Eastern division, from lake to Greytown: Distance, 108.48 miles, estimated cost £5,005,200. Construction of Greytown harbor, £457,126. Construction of Brito harbor, £567,746; total, £10,515,543. A true economy, however, will be to consider the cost of the canal, including the interest on dormant capital, as double the estimated cost, in round numbers at £20,000,000.

ORDNANCE AND NAVAL.

ARTILLERY TESTS.—Artillerists, whose judgment could not be ignored having expressed the opinion that with the new 4-ton 6-inch Armstrong gun-hitting power almost equal to that of the 9-inch gun of the service could be obtained, a series of experiments has lately been made for the purpose of testing the accuracy of the view.

The first round was fired with a shell of Whitworth steel, weighing 80 lbs., with a charge of 33 lbs. This gun, when fired for velocity only, had 70 lbs. and 64 lbs. projectiles, the calibre being that of the 64-pounder gun of the service. With these projectiles velocities of 2,000 feet per second and 2,070 feet per second were attained respectively, and that with a very light strain on the gun—namely, 15 tons per square inch. In adding 10 lbs. to the weight of the shells, a less velocity with the same charge would necessarily be attained. The velocity with 33 lbs. and an 80 lbs. shell was 1,792 feet at the point it had reached when its speed was taken. This was rather less than the muzzle velocity, which may be put at 1,800 feet. Its target was an unbacked plate 10 inches thick. The steel shell passed completely through the plate and buried itself 8 feet in the sand behind it. If we suppose that the force which remained in the projectile and carried it 8 feet into the sand was sufficient to have overcome the resistance due to one more inch of iron—and this is most probable—we have a gun of only 4 tons throwing a shell of 80 lbs. weight with a force sufficient to pierce by far the larger number of ironclads now afloat. The hole made in this plate was 6.04 inches in diameter, and the shell was hardly at all altered in shape. Nothing could be more satisfactory as to the quality of the metal; and another fact of great interest presented itself. The gas check was found to remain firmly attached to the shell, though it had passed through 10 inches of solid iron.

For the second round another shell of Whitworth steel was taken, and the charge was 36 lbs., instead of 33 lb. as in the first round. The additional 3 lb. of powder raised the velocity of the projectile to 1,887.5 feet per second at the point where the observation was made. The initial velocity would, therefore, have been nearly 1,900 feet. This ought to have given a penetration of about 11 inches into the plate. But, unfortunately, the quality of the steel was not equal to that of the first shell. Instead of retaining its form, the projectile set up considerably, thus wasting, in the alteration of its own form, the power that ought to have been spent upon the plate. The result was a penetration of only 9.6 inch. No certain argument can be drawn from one misfortune; but, so far as it went, the opinion of those who object to the expensive steel projectiles was confirmed. A long series of rounds must be fired before any definite conclusion can be drawn. It is possible, however, that steel shells may be found superior to iron when the target is within their power to pierce, but inferior in effect when the iron plate resists complete penetration.

The third round was fired with exactly the same charge as the first—33 lbs., and the weight of the projectile was also the same (80 lbs.); but it was of chilled iron instead of Whitworth steel. The velocity was rather higher—1,819 feet against 1,792 feet, a difference hardly appreciable. The effect was equal as far as could be judged. The plate was easily penetrated, the diameter of the hole being 6.06 inch. The head of the shell was found in the sand entire, save that the extreme point was broken off. The body of the shell was broken into numerous pieces, which all passed through the plate. The advocates of chilled iron shells assert that, though they always break up in passing through strong iron plates, the penetration is not affected thereby to any appreciable extent; for, as the hardness of the metal prevents the shell from setting up—that is, bulging—the pieces keep together and act as a whole while passing through the plate. We should be sorry to speak dogmatically on such a subject, but we cannot believe that no force is lost in breaking up the shell. Something must be wasted, but that something may be very small, especially as it is known that the tension existing in a chilled shell is sometimes sufficient in itself to break up the mass even without the shock of striking an iron target.

However this may be, the fourth round gave results which must be deemed satisfactory by all who hold to the chilled shells of the service. This time a charge of 36 lbs. was used, as in the second round, and the weight of the shell, as usual, was about 80 lbs. The observed velocity was 1,919 feet per second, and the target was beyond the power of the projectile to penetrate, being an unbacked plate 12 inches thick. The behavior of this shell was very interesting. As usual, it broke up; the head, without altering its form, rebounded 30 or 40 feet from the plate, which it could not entirely pierce. The body of the shell broke up into five or six pieces. But when the effect on the target was examined it was found that the projectile had penetrated no less than 11.8 inches, against the 9.6 inch of the Whitworth steel projectile in the second round, and that the back of the plate was

cracked in front of the hole. Little more force would have been required to complete the penetration of the target, and the effect produced was almost exactly the same as that obtained by firing the 9-inch service gun of 12 tons with a charge of 65 lbs. of powder. In order to show clearly the progress made by Sir William Armstrong in the construction of guns, let us place these two pieces in juxtaposition :—Service gun—12 tons weight, 9-inch calibre, 65 lbs. charge, 250 lbs. projectile; new gun—3 tons 18 cwt., 6-inch calibre, 36 lbs. charge, 80 lbs. projectile. If we carry back our thoughts a few years and remember that the heaviest gun in the service, the 68-pounder of 5 tons 12 cwt. could make no impression on the 4.5 inch plates of the early ironclads, and that in 1866 the Austrian ships carried ordnance which were totally ineffective against the Italian vessels, the sides of which were pitted with the marks of shot fired at close range, we shall see how extraordinary has been the development of the art of destruction. The most interesting comparison that could be made with ordnance that were actually supported by artillerists in America, and even by some in our own country as late as 1867, would be that of the 15-inch American smoothbore with the new 6-inch Armstrong. The American gun weighed 19½ tons, nearly five times as much as the 6-inch. Its charge was 60 lbs., or, as an extreme case, 100 lbs. With 60 lbs. the shot (for no round shells could be used against plates) was utterly foiled by the 8-inch target, and with 100 lbs. it succeeded in penetrating it. The little 6-inch of less than 4 tons would pierce that target with ease, as the 9-inch service gun did.

RECENT ADDITIONS TO THE BRITISH NAVY.—One of the most powerful armor-plated ships afloat was added to the list of vessels composing Her Majesty's navy by the purchase from the Thames Ironworks Company of the steam-ram, Memdouhiye, built for the Turkish Government, but detained for the last twelve months as contraband in the Victoria Docks, Blackwall. The vessel was handed over by Sir Peter Rolt, chairman of the Ironworks Company, and Captain Comyn, who has had charge, and was to have taken the vessel out to Turkey, delivered over his command to the representative of the Board of Admiralty. As a ship of the Royal Navy, she is to be named the Superb. Her length is 340 feet, and her beam 60 feet. She carries a raised fighting battery amidships, and the battery, which is about 100 feet by 60 feet, is pierced for twelve 18-tons guns. The guns are not on board, but in other respects she is quite ready for sea. The battery is covered with 12-inch wrought-iron plates on the usual teak backing, carried down some depth below the water-line, and raised only a few feet above what appears to be the main deck, but is really only a spar deck of unarmored timber. The true main deck is below, and is made of iron, the saloons and quarters for officers and men being upon it, but it is assumed that in time of action all hands will be in the armored enclosure, leaving the unarmored portion to its fate. The hatchways of the main deck close by water-tight iron doors, and there are upwards of sixty water-tight compartments in the lower decks and the double iron skin. It is proposed to mount four 12-ton guns on the main deck, which is about 6 feet clear of the water, the battery guns having a freeboard of 10 feet. The engines are by Maudslay and Field, and are on the direct-action principle, and of 7,000 horse-power. The ship is fitted with Paul's steam steering gear, worked either from the pilot tower on the upper deck or between the battery, with hand-gear and reserve appliances in case of accident. The steering apparatus is very powerful, and the ship is said to have behaved splendidly in her trial trip. Her saloons are spacious and handsomely furnished. She resembles the Alexandra, the flagship of the fleet in the Mediterranean, except that the Alexandra has a double battery with two 25-ton guns in the upper portion. The Superb has a burden of 5,349 tons, and will require a ship's company of 800 officers and men. Seen afloat, only the upper swell of her prow is visible, but the ram extends some distance under water. Another valuable ship will shortly be added to the effective strength of the navy, the Triumph having undergone a comprehensive overhaul and refit at Portsmouth. She has been furnished with an entirely new set of boilers from the Keyham yard, and her machinery has been thoroughly repaired and renovated by the contractors, Messrs. Maudslay, Sons and Field. The old cylinders have been removed and new cylinders and cylinder covers have been fitted, the bearings have been adjusted and refitted with white metal, the condenser tubes have been taken out and examined, and the whole of the copper piping has been tested and renewed where found to be necessary. The superheater, which is gradually being superseded in our men-of-war, has been removed, and a fresh-water donkey engine has been added to her complement of engines by Messrs. Brotherhood and Hardingham. The hull of the ship has also undergone important changes in order to bring her up to the requirements of modern warfare. She has been, for the first time, fitted with the Whitehead torpedo, two special ports having been cut in each bow and the usual racers and overhead gear provided for the carriages and the transport of the projectiles from below. The torpedo engine was manufactured at the Portsmouth yard. A Gatling gun has been placed on the foretop in addition to the one which she carried throughout her late commission in the Mediterranean in the maintop; and, as a further protection against the attacks of boats and small craft, she has been armed with four 20 pounder torpedo guns, which are mounted on the spar deck amidships. Shell gratings have been fitted in the wake of the boilers, in order to prevent fragments intruding into the stokeholes and disabling the machinery. The block compressors on the main deck have been removed, and new cable controllers have been fitted on the upper deck, the coal bunkers have received additional ventilation, and the steam steering gear and the steam capstan have had all their defects made good. As the shell for the 12-ton guns has been lengthened to the ex-

tent of 2 inches, this alteration has necessitated important readjustments being made in the shell-room for the storing of the new projectiles. The other changes which have been effected mainly consist of the cabin rearrangements, which were necessary to convert the Triumph into an admiral's ship. The cost of repairs and improvements to the hull alone amounts to something like £20,000. The ship made a six hours' continuous full-power trial of her main engines, which are of the return connecting-rod type, with surface-condensers and work with steam at 30 lbs. pressure. The new boilers, considering the state in which they were found on the preliminary trial, behaved pretty well, a little priming only being exhibited at the beginning. Notwithstanding, however, the fact that the blast was kept wide open during the six hours, considerable difficulty was experienced in maintaining a full head of steam, and during the latter part of the trial it was deemed expedient to raise the links and work the steam more expansively. The new bearings also gave a little trouble. The average pressure of steam in the boilers was 27 lbs. to the square inch; the mean vacuum recorded was 26.4 inch in the forward, and 27.9 inch in the after condenser; the mean pressure of steam in the cylinder was 15.27 lbs.; and the mean revolutions per minute 63.46. The maximum power developed was 4,287 horses, but at one time it fell as low as 2,859. The mean indicated power developed was 3,556.61, and the approximate speed realized 12 knots. The trial, which was under the superintendence of Mr. Warriner and the officers of the Portsmouth Steam Reserve, was considered scarcely satisfactory.

BOOK NOTICES.

The Transmission of Power by Compressed Air. By Robert Zahner, M. E. New York: Price 50 cts.

This important subject is treated with sufficient fullness to satisfy the wants of the practical engineer within the limits of this small volume. The essay which is reprinted from Vol. XIX of this Magazine forms No. 40 of the Science Series.

The use of compressed air as a motive power is rapidly extending, but is yielding results that are far short of what is to be expected, when the theoretical conditions are well understood.

This *petite* volume is well designed to prepare the way to a better knowledge of so much thermodynamics as is involved in the practical problem.

The Commercial Products of the Sea. By P. L. Simmonds. London: Griffith & Farran. Price $8.00.

This octavo of 490 pages is full of interesting information respecting the marine contributions to food, industry and art; these three divisions of the subjects being treated separately as "parts" of the work.

The Food Products are, of course, the results of the many so-called fisheries and are numerous and wide spread.

The Industrial Products include Sponges, Shells, Fish Oils, Sea Weeds and Salt.

The contributions to Art are represented by Pearls, Corals, Tortoise-Shell and Amber.

The illustrations are good and the statistics seem carefully prepared.

The Geology of New Hampshire. By C. H. Hitchcock. Concord, N. H.: E. A. Jenks. Price, $10.00.

Vol. III of this work alone is of recent publication. It contains a valuable contribution to general Lithology.

The entire work, consisting of three large volumes and an unwieldy Atlas, will be regarded with more interest than State Reports usually are. It is an invaluable contribution to the sciences, if we may use the plural, of Geology, Lithology, and Physical Geography.

The maps are beautifully clear and many of the illustrations in the text represent the best style of pictorial printing.

Instructions for Testing Telegraph Lines. Vol. I. By Louis Schwendler. London: Trübner & Co. Price $4.00.

This work is designed particularly for practical telegraphers but may be read profitably by all students of Electrical Science.

The requirements for applications in any use of Electricity involve an acquaintance with the principles demonstrated and fully illustrated in this book, as may be seen by this abstract of the contents. Wheatstone's Bridge, its sensibility; best practical management; Differential Galvanometer; explanation of the method; sensibility of the method; measurement of its resistances; best practical arrangement. Line testing; Regular testing; fault testing.

The Appendices treat of Ohm's Law; Kirchoff's Corollaries; Cable Testing, etc.

The typography of the book is excellent.

Leisure-Time Studies. By Andrew Wilson, Ph. D. New York: R. Worthington.

This is a series of pleasant essays, chiefly upon biological subjects. All are profitable reading, and nearly all will prove exceedingly interesting to the general reader, if he have in the slightest degree a liking for natural history.

To the young inquirer after scientific facts, nothing could be more profitable than such essays as: A Study of Lower Life; The Sea Serpents of Science; Some Animal Architects; What I Saw in an Ant's Nest; and a Summer's Day.

A few good illustrations embellish the work.

A Manual for Engineers and Steam Users. By John W. Hill, M. E. Providence: William A. Harris.

A good deal of valuable information is condensed into this compact little volume. That the information is of the right kind for all interested in employing steam, and that the rules, formulæ and tables are accurate, is sufficiently attested by the statement that it is prepared by Mr. Hill, the well-known expert, of Cincinnati.

The book is the property of Mr. William A. Harris, and is designed chiefly as a guide to those who use the Harris-Corliss engine.

MISCELLANEOUS.

Telephonic Determination of the Magnetic Meridian.—M. H. de Parville substitutes a bar of soft iron at least 1 meter (39.37 inches) in length for the short magnet of an ordinary telephone. The apparatus still transmits sounds, but with an intensity which varies with the direction of the bar, the sound being most intense in the receiver when the transmitter is parallel to the dipping needle. The sound is more or less completely extinguished when the transmitter is perpendicular to the magnetic meridian. If such a telephone is provided with a resonator it can be used not only to find the direction of the magnetic needle, but also for the approximate determination of the variations in magnetic intensity. This method seems applicable, on shipboard, for the correction of the compass, especially when the indications of the needle may be deceptive on account of the neighborhood of magnetic rocks or of islands rich in iron ores. The inventor also suggests the use of a bar of soft iron several meters in length, having at one end a magnetic coil with a self-registering apparatus. The pitching of the vessel would excite induction currents, and the diagram on the register would reveal the direction of the vessel so as to check the indications of the compass.—*Comptes Rendus.*

Elephants for African Travel.—The *Academy* notes that a successful experiment has lately been tried in the equatorial provinces of Egypt, which may not improbably ere long revolutionize the mode of transit in Eastern Africa, and solve a problem which has hitherto puzzled travelers. About a year ago, at Colonel Gordon's request, a few trained elephants were sent to Khartum, where they arrived in due course, having marched along the banks of the Nile. A report has been received in Cairo from Colonel Gordon stating that he had despatched them to the military station of Lardo, about 11 deg. south of Khartum, and six miles north of Gondokoro, and that they had accomplished this distance in 84 days. A not unimportant advantage to be derived from the employment of elephants in this manner was demonstrated by the fact that the negroes along the line of march were frightened by them, and made no attempt to attack the party. The elephants have gradually learned to live on leaves and grass, as the wild elephants do, and keep in first-rate condition without the different kinds of food to which they had previously been accustomed. Colonel Gordon consequently advises travelers going into the interior of Africa from Zanzibar to use elephants, and thus to avoid the necessity for a host of porters, who are a never-ending source of delay and annoyance. It may be remembered that the question of employing elephants in African exploration was discussed after the reading of Mr. H. B. Cotterill's paper on the Nyassa, for the Society's African Section on the 28th of May last.

The employment of alkaline manganates for imparting to light woods in furniture and floors an attractive, uniform, and durable walnut brown, is highly recommended by M. Viedt. The action depends upon the decomposition of salt in the pores of the wood, with the separation in them of very finely-divided brown hydrate peroxide of manganese, and an addition of magnesium sulphate to the solution is found to hasten the reaction. In practice, the following method is said to be successful. Equal parts of manganate of soda and crystallized Epsom salts are dissolved in twenty to thirty times the amount of water, at about 144 deg., and the planed wood is then brushed with the solution; the less the water employed, the darker the stain, and the hotter the solution, the deeper it will penetrate. When thoroughly dry, and after the operation has been repeated, if necessary, the furniture is smoothed with oil, and finely polished, the appearance being then really beautiful. Before smoothing, however, a careful washing with hot water will have the effect of preventing the efflorescence of the sulphate of soda formed. In the treatment of floors, the solution may be employed boiling hot, and, if the shade produced is not dark enough, a second application of a less concentrated solution is made; after it is quite dry, it is varnished with a perfectly colorless oil-varnish. On account of the depth to which the coloring solution penetrates a fresh application is not, says the *Lumber Gazette*, soon required.

One of the most suggestive illustrations that can be adduced as showing the advances made within the last forty years in marine engine economy is derivable from an examination of data of recorded averages of Atlantic steamships; and more especially of those of the Cunard paddle-wheel steamer Britannia, in 1840, and the White Star screw steamer Britannic in 1877. Of the first vessel the average duration of passage was fourteen days and eight hours, and the consumption of fuel 544 tons, the daily consumption thus being 38 tons. Assuming the average cargo at 225 tons, this gives 48.35 cwt. of coal per ton of cargo; and the average speed in knots per hour being 8.3, the consumption per knot was 3.8 cwt. The indicated horse-power was 740, and consumption per horse-power, 4.7 cwt. The Britannia displaced but 2,050 tons, and this must be taken into account in comparing her with the Britannic, whose displacement is more than four times as great, or 8,500 tons. That vessel, in 1877, showed an average passage of seven days ten hours and fifty-three minutes, an average daily consumption of fuel of 100 tons, or total consumption of 745 tons. Her cargo is 3,350 tons; consumption of fuel per ton of cargo, 4.45 cwt.; average speed, 15.6 knots; consumption per knot, 5.3 cwt.; indicated horse-power, 4,920; consumption per horse-power, 1.9 cwt. In other words, we are now enabled to transport fifteen times as much freight across the ocean in one-half the time at an expenditure of less than one and a half times as much coal as in 1840.

—*The Engineer.*

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ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION.

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II.

§ 2.

THE POTENTIAL.

It will perhaps be as well to make a few remarks here concerning the theory of the potential. It is not the purpose in these pages to go into that subject with any degree of fullness, but as there are a few leading principles which frequently recur a brief statement and derivation of them may be of assistance to some readers.

We have already observed one fact concerning the velocity potential, viz., that if it is constant over a closed surface containing a certain definite region that it will be constant throughout this region, and in particular, if it be $=o$ over the surface it will be $=o$ throughout the contained region. When $udx+vdy+wdz$ is an exact differential we have seen that by making it equal to $d\varphi$ we can replace the quantity

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}$$

by

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}.$$

Now let σ denote any closed surface, then if ν be the outer normal to this surface we have $\frac{d\varphi}{d\nu}\,d\sigma$ for the rate of flow outwards through the element $d\sigma$ in unit of time, then the total flow outwards in time dt is equal to

$$\int\int\frac{d\varphi}{d\nu}d\sigma.dt$$

where the integration extends over the whole surface. If the space enclosed by σ be full both at the beginning and end of this time we, of course, have

$$\int\int\frac{d\varphi}{d\nu}d\sigma=o.$$

This is the equation of continuity for the whole region. Applied to the element of volume $d\tau$ or $dx\,dy\,dz$ this gives us

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}=o \text{ or } \Delta^2\,\varphi=o$$

If we denote by r the distance between any two points x, y, z, and a, b, c, *i. e.*

$$r^2=(x-a)^2+(y-b)^2+(z-c)^2$$

and by m a constant we see that the equation $\Delta^2\varphi=o$ is satisfied by

$$\varphi=\frac{m}{r}$$

or $\frac{m}{r}$ is a particular solution of the partial differential equation.

The general solution is a homogeneous function of the quantities x, y, z of the degree i, where i is any positive integer. It is also well known that to every solution of the degree i then corresponds one of the degree $-(i+1)$ expressed by $\frac{\varphi^i}{r^{2i+1}}$.

The expression φ_i is a Solid Spherical Harmonic of the degree i. The expression obtained by dividing φ_i by r^i which will be a function only of two quantities, viz., the angles θ and ψ, is a Spherical Surface Harmonic of the same degree.

The quantity φ is now the Potential of the mass m upon the point (x, y, z). At infinity the Potential with its derivatives vanishes, but is finite and continuous throughout the space except at the points in which the masses are found, *i. e.*, for $x=a$, $y=b$, $z=c$. Let now in the space under consideration, which is supposed to be continuously filled with masses, m denote a mass placed at a given point A and let r denote the length of a line drawn from this point to any other B—then we know that the attraction of the mass m upon the point B is given by—$\frac{d\varphi}{dr}=\frac{m}{r^2}$.

Now suppose the line AB drawn to an infinite distance, and further that the space which contains the masses m is bounded by a closed surface. The line AB will cut the surface an even number of times. Suppose now a sphere of radius unity to be described with center A, and then let the line AB describe a conical surface cutting the element $d\omega$ from the surface of the sphere, and ds_1, ds_2, &c., from the given closed surface. Let ε denote the angle between AB and the outer normal to the surface; then where the line issues from the surface cos. ε will be positive and where it enters cos. ε will be negative. We have now

$$ds=\frac{r^2 d\omega}{\cos.\ \varepsilon}$$

The normal force on the element ds is given by R cos. $\varepsilon\, ds$; but R cos. $\varepsilon\, ds$ is equal to

$$\left(\frac{dX}{dx}+\frac{dY}{dy}+\frac{dZ}{dz}\right)dx\,dy\,dz$$

and therefore for the whole surface

$$\int\int \text{R}\cos.\ \varepsilon\, ds=\int\int\int\left(\frac{dX}{dx}+\frac{dY}{dy}+\frac{dZ}{dz}\right)dx\,dy\,dz$$

Now since $\text{R}=\frac{m}{r^2}$ we have

$$\text{R}\cos.\ \varepsilon\, ds=\pm\, m d\omega.$$

As the point A is within the surface, the line AB first issues from the surface, giving a positive value of $md\omega$ after that alternate positive and negative values of $md\omega$ which destroy each other so that we have simply

$$\Sigma\, \text{R}\cos.\ \varepsilon\, d\omega=md\omega$$

and

$$\int\int \text{R}\cos.\ \varepsilon\, d\omega=m\int\int d\omega=4\pi m$$

Now

$$4\pi m=4\pi\int\int\int\rho\, dx\,dy\,dz$$

$$\therefore\ \frac{dX}{dx}+\frac{dY}{dy}+\frac{dZ}{dz}=4\pi\rho$$

or

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}=-4\pi\rho \text{ for } X=-\frac{d\varphi}{dx}\ \&c.$$

This equation holds throughout the entire space which is filled with the masses m.

If we represent an element of the space under consideration by $d\tau$ and the density by ρ we may write

$$\varphi=\int\frac{\rho d\tau}{r}$$

from which

$$\frac{d\Phi}{dz}=\int\rho\frac{d\frac{1}{r}}{dz}d\tau=-\int\rho\frac{d\frac{1}{r}}{dc}d\tau$$

$$=-\int\frac{d\frac{\rho}{r}}{dc}d\tau+\int\frac{d\rho}{dc}\frac{d\tau}{r}$$

and finally

$$\frac{d\varphi}{dz}=\int ds\frac{\rho}{r}\cos.\left(\frac{n}{z}\right)+\int\frac{d\rho}{dc}\frac{d\tau}{d}$$

The second of these integrals is evidently a quantity of the same kind as φ; the first is the potential of a mass which is spread out upon the surface $\int ds$ and giving the surface density

$$\rho\cos.\left(\frac{n}{z}\right).$$

Suppose now that φ' be the potential with reference to the point (x, y, z) of a mass that is spread out upon a surface giving the surface density μ; we have

$$\varphi' = \int \frac{\mu d\sigma}{\sigma}.$$

Assume the rectangular axes so that z is normal to the surface; assume on d a point infinitely near the surface, and suppose a circular cylinder with radius P′ and having the axis of z for its axis of figure to cut the surface; suppose P′ indefinitely small but infinitely large with respect to the ordinate z. Let φ_1' denote the portion of φ belonging to that part of surface included in the cylinder; the remaining portion $\varphi' - \varphi_1'$ will not become infinite or discontinuous by z becoming either$=o$ or passing through zero.

$$\varphi_1' = 2\pi\mu \int_o^{P'} \frac{\rho' d\rho'}{\sqrt{\rho'^2 + z^2}}$$

$$\varphi_1' = 2\pi\mu[\sqrt{P'^2 + z^2} - \sqrt{z^2}]$$

By neglecting infinitesimals we have

$$\varphi_1' = o$$

or φ' remains finite and continuous if the point under consideration passes through the surface, *i.e.*, if $\sqrt{z^2} = \pm z$. Further.

$$\frac{d\varphi_1'}{dz} = 3\pi\mu \left\{ \frac{z}{\sqrt{P'^2 + z^2}} - \frac{z}{\sqrt{z^2}} \right\}$$

or since with respect to z P′ is infinitely great,

$$\frac{d\varphi_1'}{dz} = -2\pi\mu \frac{z}{\sqrt{z^2}}$$

or if z be positive

$$\frac{d\varphi_1'}{dz} = -2\pi\mu$$

if z be negative

$$\frac{d\varphi_1'}{dz} = +2\pi\mu.$$

But $\varphi' - \varphi_1'$ is continuous and also $\dfrac{d(\varphi' - \varphi_1')}{dz}$; consequently when z changes from positive to negative passing through zero $\dfrac{d\varphi'}{dz}$ changes suddenly by the amount $-4\pi\mu$. Now as we have taken z in the direction of the normal this fact can be expressed as follows: Call the inner normal ν_1 and the outer normai ν_2 then we have

$$\frac{d\varphi'}{d\nu_1} + \frac{d\varphi'}{d\nu_2} = -4\pi\mu$$

which may be called the *characteristic equation* of φ' at the surface.*

Suppose now that we have a surface over which a mass is so distributed as to give rise to the surface density that we have denoted by μ giving the n the potential φ'. We will use, for the present, the symbol φ to express general function, which satisfies the equation,

$$\frac{d^2\varphi}{dx^2} + \frac{d^2\varphi}{dy^2} + \frac{d^2\varphi}{dz^2} = o$$

And we will take now U to represent the potential in the space under consideration, V for what we have denoted by φ', the potential of a surface over which a mass is distributed, as mentioned above. Now we will introduce a new quantity W, which we proceed to define: At every point of the surface that we have spoken of conceive normals (positive) to be drawn, lay off on these infinitely small lengths, and through these points conceive another surface to pass, the elements of which correspond to the elements of the first surface and consider that on each element of this new surface a mass is distributed equal to that on the corresponding element of the first surface but of opposite sign. Represent by k the negative product of the density of the mass on the element $d\sigma$ of the first surface, by $d\sigma$; then if (a, b, c) denote the element $d\sigma$ and W is the potential of this element at the point (x, y, z); we have

$$W = \int k \frac{d\frac{1}{r}}{d\nu} d\sigma$$

$$\text{Since, } \frac{d\frac{1}{r}}{dr} = \frac{d\frac{1}{r}}{dr}\frac{dr}{d\nu}$$

we have

$$W = -\int \frac{k d\sigma}{r^2} \cos. (r\nu)$$

We can also express this in another way. Conceive a sphere of unit radius described about (x, y, z) as center, also conceive a cone on $d\sigma$ as base and having (x, y, z) as vertex, to cut the sphere, the area of the included portion being $d\Sigma$, then

* Maxwell, Elec. and Mag., p. 82, Vol. I.

$$\frac{d\frac{1}{r}}{d\nu}d\sigma = \pm d\Sigma$$

the upper or lower sign to be taken according as cos. $(r\nu)$ is positive or negative. The cosine can only change its sign by $(r\nu)$ passing through $\frac{\pi}{2}$ evidently this is the case only when the point can lie on a tangent to the surface; then supposing that cos. $(r\nu)$ does not change its sign, we have

$$W = \pm \int k d\Sigma$$

when the upper or lower sign is to be taken according as cos. $(r\nu)$ is positive or negative. If the above condition is not fulfilled, the surface may be divided into parts so that each part can satisfy the imposed conditions; then will W begin as the sum of the corresponding expressions for each part. In order to examine whether or not discontinuity occurs in the value of W, by the point to which it refers, approaching indefinitely near the surface —coinciding with it or passing through it—we will choose the axis so that z is normal to the surface, and assume a point on z indefinitely near the surface now by exactly the same process as that before employed, we see that for a negative z we have

$$W_1 = -z\pi k$$

and for a positive z

$$W_1 = +z\pi k$$

W_1 corresponding to the small portion cut out of the surface by a circular cylinder of indefinitely small radius P, which is nevertheless infinitely large as regards z. Now since W_1 is independent of z, W cannot become infinitely great by z becoming infinitely small; and since W_1 suddenly changes by $4\pi k$, W does so likewise. From our equation for W, we have for W_1 since it refers to an indefinitely small portion of the surface for which k is constant,

$$W_1 = -k\int\frac{d\frac{1}{r}}{dz}d\sigma = 2\pi k\int_0^P \frac{z\rho d\rho}{(\rho^2+z^2)^{\frac{3}{2}}}$$

since

$$r = \sqrt{\rho^2+z^2} \text{ and } \frac{d\frac{1}{r}}{dz} = \frac{d\frac{1}{r}}{dd}\,\frac{dr}{dz} = -\frac{1}{r^2}\,\frac{dr}{dz},$$

or

$$W_1 = z\pi k\left\{\frac{z}{\sqrt{z^2}} - \frac{z}{\sqrt{P^2+z^2}}\right\}$$

from this follows

$$\frac{dW_1}{dz} = -z\pi k\frac{P^2}{(P^2+z^2)^{\frac{3}{2}}}$$

or, since z^2 is negligable with respect to P^2,

$$\frac{dW_1}{dz} = -2\pi k\frac{1}{P}$$

The second member being independent of z, $\frac{dW'}{dz}$ for $z=o$ is finite and continuous. Thus we see that the potential W for this *double layer* is finite always, but changes suddenly by the amount $4\pi k$ on the point to which it refers passing through the surface in the direction of ν; the quantity $\frac{dW}{d\nu}$ is, however, finite and continuous. With a perfectly arbitrary co-ordinate system the quantities $\frac{dW}{dx}$, $\frac{dW}{dy}$, $\frac{dW}{dz}$, will in general suffer discontinuity, since k is in general not constant over all the surface; if k be constant the differential co-efficients will suffer no discontinuity at the surface.

Suppose that we have two functions U and V of x, y, z, which with their derivations are single valued and continuous in the space under consideration, which is bounded by a closed surface. We have the identical equations

$$\frac{dU}{dx}\frac{dV}{dx} + U\frac{d^2V}{dx^2} = \frac{d}{dx}\left(U\frac{dV}{dx}\right),$$

$$\frac{dU}{dy}\frac{dV}{dy} + U\frac{d^2V}{dy^2} = \frac{d}{dy}\left(U\frac{dV}{dy}\right),$$

$$\frac{dU}{dz}\frac{dV}{dz} + U\frac{d^2V}{dz^2} = \frac{d}{dz}\left(U\frac{dV}{dz}\right).$$

Add and multiply by the element of the space $d\tau$, we have by changing in the second member a volume into a surface integral,

$$\text{(A)}\int\left(\frac{dU}{dx}\frac{dV}{dx} + \frac{dU}{dy}\frac{dV}{dy} + \frac{dU}{dz}\frac{dV}{dz}\right)d\tau$$
$$= -\int d\tau\, U\,\Delta V - \int d\sigma U\frac{dV}{d\nu}.$$

This equation expresses what is known as Green's Theorem. By an interchange of U and V, which from the nature of the functions can be effected, the first member of this equation will not change. We will have then

$$(B)\int d\sigma\left(U\frac{dV}{d\nu}+V\frac{dU}{d\nu}\right)=\int d\tau(V\Delta U-U\Delta V)$$

If U and V are velocity potentials this gives,

$$\int d\sigma\left(U\frac{dV}{d\nu}-V\frac{Ud}{d\nu}\right)=o$$

In the preceding equation (a) suppose U=V, then it gives,

$$\int d\tau\left\{\left(\frac{dV}{dx}\right)^2+\left(\frac{dV}{dy}\right)^2+\left(\frac{dV}{dz}\right)^2\right\}=-\int d\sigma V\frac{dV}{d\nu}$$

or

$$2T=-\int V\left\{\frac{dV}{dx}\cos.(\nu x)+\frac{dV}{dy}\cos.(\nu y)+\frac{dV}{dz}\cos.(\nu z)\right\}d\sigma$$

An expression for the energy in the form of a surface integral.

"The irrotational motion of incompressible fluid in a simply connected closed space Σ is completely determined by the normal velocities over the surface Σ. If Σ be a material envelope, it is evident that an arbitrary normal velocity may be impressed upon its surface, which normal velocity must be shared by the fluid immediately in contact, provided that the whole volume inclosed remain unaltered. If the fluid be previously at rest, it can acquire no molecular rotation under the operation of the fluid pressures, which shows that it must be possible to determine a function φ, such that $\Delta^2\varphi=o$ throughout the space inclosed by Σ while over the surface $\frac{d\varphi}{d\nu}$ has a prescribed value limited only by the condition

$$\int\int\frac{d\varphi}{d\nu}d\sigma=o$$

"By Green's theorem if $\Delta^2\varphi=o$,

$$\int\int\int\left\{\left(\frac{d\varphi}{dx}\right)^2+\left(\frac{d\varphi}{dy}\right)^2+\left(\frac{d\varphi}{dz}\right)^2\right\}d\tau=\int\int\varphi\frac{d\varphi}{d\nu}d\sigma$$

the integration on the right hand side extending over the surface Σ that on the left hand side over the volume. Now if φ and $\varphi+\Delta\varphi$ be two functions, satisfying Laplace's equation, and giving prescribed values of $\frac{d\varphi}{d\nu}$, then the difference, $\Delta\varphi$ is a function also satisfying Laplace's equation, and making $\frac{\Delta d\varphi}{d\nu}$ vanish over the surface of Σ. Under these circumstances the surface integral in the preceding equation vanishes, and we infer that at every point of Σ, $\frac{d\Delta\varphi}{dx}$, $\frac{d\Delta\varphi}{dy}$, $\frac{d\Delta\varphi}{dz}$ must be equal to zero. In other words, $\Delta\varphi$ must be constant and the two motions identical. As a particular case, there can be no motion of the irrotational kind within the volume Σ, independently of a motion of the surface."

The line described by a point in the fluid, moving always in the direction of the resultant velocity, is, as has been mentioned for a simple case, called a stream line. If φ denote the velocity potential, we have obviously for the differential equations of a stream line,

$$\frac{dx}{\frac{d\varphi}{dx}}=\frac{dy}{\frac{d\varphi}{dy}}=\frac{dz}{\frac{d\varphi}{dz}}$$

These lines evidently cut at right angles, the surface φ=const.

If the normal velocity at every point of this surface is equal to zero, we must have

$$\frac{d\varphi}{d\nu}=o$$

and this equation will represent a surface which cuts at right angles the surface φ=const., and is consequently made up of stream lines; such a surface is appropriately termed a surface of flow. The following properties of such surfaces, though not directly bearing upon the matter in hand, will possibly be of interest:

Suppose that we have for the equation of a family of surfaces

$$f(x, y, z)=q,$$

q being a variable parameter by giving constant values to which we obtain the equation of each member of the family. Make

$$\left(\frac{dq}{dx}\right)^2+\left(\frac{dq}{dy}\right)^2+\left(\frac{dq}{dz}\right)^2=\frac{1}{\nu^2}$$

Then we have for the direction cosines of the normal in the direction in which q increases

$$\cos.(\nu x)=\nu\frac{dq}{dx}, \cos(\nu y)=\nu\frac{dq}{dy},$$
$$\cos.(\nu z)=\nu\frac{dq}{dz}$$

if N be the component of the flow normal to the surface, u, v, w being the components parallel to x, y and z respectively, we have

$$\mathrm{N}=\nu\left(u\frac{dq}{dx}+v\frac{dq}{dy}+w\frac{dq}{dz}\right).$$

Hence, if N be zero, there will be no flow through the surface, which may then be called a surface of flow; we have then for the equation of such a surface,

$$u\frac{dq}{dx}+v\ \frac{dq}{dy}+w\ \frac{dq}{dz}=o.$$

If there be another family of surfaces whose parameter is q', and these are surfaces of flow, then

$$u\ \frac{dq'}{dx}+v\ \frac{dq'}{dy}+w\frac{dq'}{dz}=o$$

If we have still a third family of surfaces whose parameter is q'' that are surfaces of flow, then again,

$$u\ \frac{dq''}{dx}+v\ \frac{dq''}{dy}+v\ \frac{dq''}{dz}=o$$

eliminating u, v and w between these, and we obtain

$$\begin{vmatrix} \frac{dq}{dx}, & \frac{dq}{dy}, & \frac{dq}{dz} \\ \frac{dq'}{dx}, & \frac{dq'}{dy}, & \frac{dq'}{dz} \\ \frac{dq''}{dx}, & \frac{dq''}{dy}, & \frac{dq''}{dz} \end{vmatrix}=o$$

which is only satisfied by making $q''=$ some function of q and q'. Suppose that we have only the two first of these equations viz.:

$$u\frac{dq}{dx}+v\frac{dq}{dy}+w\frac{dq}{dz}=o$$
$$u\frac{dq'}{dx}+v\ \frac{dq'}{dy}+w\ \frac{dq'}{dz}=o$$

By eliminating u, v, w in turn from these equations we arrive at the following where Φ is an undetermined function of q and q',

$$u=\Phi\left(\frac{dq}{dy}\frac{dq'}{dz}-\frac{dq}{dz}\frac{dq}{dy}\right)$$
$$v=\Phi\left(\frac{dq}{dz}\frac{dq'}{dx}-\frac{dq}{dx}\frac{dq'}{dz}\right)$$
$$w=\Phi\left(\frac{dq}{dx}\frac{dq'}{dy}-\frac{dq}{dy}\frac{dq'}{dx}\right)$$

When one of the functions represented by q and q' is known, it is possible so to determine the other, that Φ shall be = unity. The flow in the direction of the normals to these surfaces being $=o$, this flow can only take place along the surface, and the intersection of the two surfaces will be a line of flow or stream line. A tube of flow or a stream filament, is a tube whose bounding surfaces are made up of lines of flow. If the two parameters q and q' have a series of values given to them, they will form a double system of surfaces dividing space up into a number of tubes, each of which will be a tube of flow. We will go further into the consideration of stream lines in another place.

§ 3.

PLANE WAVES.

Having thus briefly stated some of the more simple of the general properties of the equations of fluid motion we will now proceed to examine some of the problems which present themselves most naturally to the student. The first case that we shall take up is that of the motion of water in *plane waves* when the excursions of each particle are very small.

When a body of water originally in a state of rest is endowed with a wave motion each particle of the mass has a motion of oscillation or, describes a closed curve in such a manner as to cause the particle of water, after a certain definite lapse of time, to resume its original position on the surface of the wave. By wave, is to be of course, understood simply the *forms* which the water assumes under the action of the disturbing force. Plane waves are those in which the motion of every particle is parallel to a certain fixed plane, and they may be generated by bringing a solid body, *e.g.*, a cylinder, in contact with the surface of the water contained in a rectangular canal of uniform depth, and in such a manner that the line of contact shall be at right angles to the length of the canal.

Plane waves will be generated the instant the contact takes place; these will travel along the entire length of the canal; impinge on the ends and return; we will in our problem, however, limit ourselves at first to the case of a canal of indefinite length and then need not take account of the phenomena at the ends. Our mass of water being then supposed, contained in a canal as described, we will now proceed to the mathematical examination of the waves generated by such a disturbing force as has been mentioned.

Assume the axis of z vertical and positive downwards, the axis of x parallel to the length of the canal, and that of y at right angles to its sides; the origin being on the surface of the water at rest. Let now x_0, y_0, z_0 denote the initial values of the co-ordinates of a particle and let u, v, w denote the displacements which the particle undergoes in the directions of x, y, and z respectively. For plane waves advancing in the direction of x, we have of course $v=o$, and the equation of continuity assumes the form

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dz^2}=o$$

the values of u and w being

$$u=\frac{d\varphi}{dx},\quad w=\frac{d\varphi}{dz}.$$

We have now to express φ as a function of x, z, and t, and, from the nature of the motion, periodic with respect to the last.

The simplest way of expressing this periodicity, will be by introducing in φ a factor which shall be a trigonometric function of the time. We may assume φ in the form

$$\varphi=\varepsilon^{\pm\sigma z}(x)f\,{}^{\sin.}_{\cos.}\left\{F(t)\right.$$

when σ is a constant, and the forms of f and F are to be determined, the latter is easily obtained by the following considerations. We know that φ must not change if we increase t by the time of oscillation, denoting this by τ, and φ becomes

$$=\varepsilon^{\pm\sigma z}f(x)\,{}^{\sin.}_{\cos.}\left\{F(t+\tau)\right.$$

Now in order that φ may be a periodic function of t, we must have

$$F(t+\tau)=F(t)+2\pi$$

which gives by expansion

$$\tau F'(t)+\frac{\tau^2}{1.2}F''(t)+\ .\ .\ .\ =2\pi$$

from which since τ is constant,

$$F''=F'''=,\ \&c.\ .\ .=o$$

$$\therefore \qquad F(t)=\int_o^t F'(t)dt\frac{2\pi}{\tau}t$$

and φ now assumes the form

$$\varphi=\varepsilon^{\pm\sigma z}f(x)\,{}^{\sin.}_{\cos.}\frac{2\pi}{\tau}t.$$

Now for the determination of f; substituting this value of φ in the differential equation of continuity $\Delta^2\varphi=o$ and it becomes

$$\frac{d^2f}{dx^2}+\sigma^2f=o$$

Integrating this and we have for f the equation

$$f=A\sin.\sigma x+B\cos.\sigma x$$

when A and B are the constants of integration. This gives us now

$$\varphi=\varepsilon^{\pm\sigma z}\left(A\sin.\sigma x+B\cos.\sigma x\right){}^{\sin.}_{\cos.}\frac{2\pi}{\tau}t.$$

This obviously may be written in this form

$$\varphi=\left\{\begin{array}{l}
\varepsilon^{\sigma z}\left(a_1\sin.\sigma x\sin.\frac{2\pi}{\tau}t+b_2\cos.\sigma x\cos.\frac{2\pi}{\tau}t\right)\\
+\varepsilon^{\sigma z}\left(b_1\cos.\sigma x\sin.\frac{2\pi}{\tau}t+b_2\sin.\sigma x\cos.\frac{2\pi}{\tau}t\right)\\
+\varepsilon^{-\sigma z}\left(\alpha_1\sin.\sigma x\sin.\frac{2\pi}{\tau}t+\beta_2\cos.\sigma x\cos.\frac{2\pi}{\tau}t\right)\\
+\varepsilon^{-\sigma z}\left(\beta_1\cos.\sigma x\sin.\frac{2\pi}{\tau}t+\alpha_2\sin.\sigma x\cos.\frac{2\pi}{\tau}t\right)
\end{array}\right.$$

By supposing the constants a, b, ... positive we can, by making the proper ones vanish and establishing certain relations among the remaining ones, obtain an expression for φ which shall contain as a factor the sine of the sum or the cosine of the difference of the quantities $\frac{2\pi}{\tau}t$ and σx. It is of course desirable to introduce the quantity x into the trigonometric factor as the form of φ is alike unaltered if we increase t by the time of

oscillation or x by the wave length. Then making

$$a_1=b_2=\alpha_1=\beta_2=o$$

and for a simple advancing wave making,

$$b_1=a_2=a_1,\ \beta_1=\alpha_2=\alpha_2$$

we have

$$\varphi=\left(a_1\varepsilon^{\sigma z}+a_2\varepsilon^{-\sigma z}\right)\sin.\left(\frac{2\pi}{\tau}t+\sigma x\right).$$

We have here before spoken of φ as the velocity function—there is a manifest appropriateness in this case in calling it the *wave function*—a name that we shall adopt for the present.

The quantity

$$\left(a_1\varepsilon^{\sigma z}+a_2\varepsilon^{-\sigma z}\right)$$

is called the *amplitude* of the wave, and is evidently the maximum value of φ; τ is the *periodic time*, or *period*, after the lapse of which the values of φ recur; and σx determines the *phase* of the wave at the moment from which t is measured. It is evident that if we have any number of wave functions φ', φ'' . . . which satisfy the differential equations

$$\Delta^2\varphi'=o,\quad \Delta^2\varphi''=o, \text{ \&c.}$$

that this sum must also satisfy the equation

$$\Delta^2\Sigma\varphi=o$$

or any number of wave functions may be compounded into one resultant by simple addition.

Before proceeding to the general problem we will examine the simple case of only one wave function. From the value given above for φ we have

$$u=\sigma\left(a_1\varepsilon^{\sigma z}+a_2\varepsilon^{-\sigma z}\right)\cos.\left(\frac{2\pi}{\tau}t+\sigma x\right)$$

$$w=\sigma\left(a_1\varepsilon^{\sigma z}-a_2\varepsilon^{-\sigma z}\right)\sin.\left(\frac{2\pi}{\tau}t+\sigma x\right)$$

From these we see at once that the displacements u and w satisfy the equation of an ellipse whose semi axes are

$$\sigma\left(a_1\varepsilon^{\sigma z}+a_2\varepsilon^{-\sigma z}\right)$$

$$\sigma\left(a_1\varepsilon^{\sigma z}-a_2\varepsilon^{-\sigma z}\right).$$

These values of φ, u and w can however, be further simplified by finding what relation exists between the quantities a_1 and a_2. To find this relation, we proceed to examine the forces which act on any particle of the fluid; these are well known to be of the form

$$-\frac{d^2u}{dt^2},\ o,\ -\frac{d^2w}{dt^2}+g,$$

and the elementary equations of motion thus become

$$\frac{1}{\rho}\frac{dp}{dx}=-\frac{d^2u}{dx^2}$$

$$\frac{1}{\rho}\frac{dp}{dy}=-\frac{d^2w}{dt^2}+g$$

from these we have

$$p=\rho\left\{\int-\frac{d^2u}{dt^2}dx+\int-\frac{d^2w}{dt^2}dz+\int gdz\right\}+\text{const.};$$

but

$$-\frac{d^2u}{dt^2}=-\frac{d}{dx}\frac{d^2\varphi}{dt^2}=\left(\frac{2\pi}{\tau}\right)^2\frac{d\varphi}{dx},$$

$$-\frac{d^2w}{dt^2}=-\frac{d}{dz}\frac{d^2\sigma}{dt^2}=\left(\frac{2\pi}{\tau}\right)^2\frac{d\varphi}{dz},$$

therefore,

$$p=\rho\left\{\left(\frac{2\pi}{\tau}\right)^2\varphi+gz\right\}+\text{const.}$$

The constant is evidently p_o the initial pressure, the first part of the second number of this equation being the increase of pressure at the time t above what it was at the initial instant.

As we suppose ourselves limited to the case of very small motions, we may regard u and w as quantities of the first order; then it is obvious from the form of the expressions obtained for these quantities that σ is of the same order—the other factors being in general of finite magnitude. Then any terms which contain σu, or σw being quantities of the second order, may, with reference to those of the first order of magnitude, be discarded.

Mr. E. Easton, C.E., President of Section G British Association, has issued his address in pamphlet form, accompanied by several of the papers on the same subject—"River Conservation"—which were read at the meeting. The question is an important one, in view of the many disastrous floods which have occurred in recent winters, and information given in this handy form is always attractive.

THE MISSISSIPPI JETTIES.

ABSTRACT OF OFFICIAL REPORT.

[The interest felt by the public, and especially by engineers, in the works undertaken by Mr. Jas. B. Eads, at the South Pass of the Mississippi, prompt us to insert, nearly in full, the report of the Board of Engineer Officers appointed by the President of the United States, under Act of Congress of June 19, 1878, to examine and make a "full report" on the same. The Board was composed of Gen. Barnard, Macomb, Tower, Wright, and Col. Merrill.—EDITOR.]

ARMY BUILDING,
NEW YORK, January, 22, 1879.

HON. GEO. W. MCCRARY,
Secretary of War, etc.:

SIR: The board constituted by Special Orders No. 228, Headquarters of the Army, Adjutant-General's Office, Washington, D. C., October 22, 1878 (of which a copy is annexed, marked 1), assembled at New York on the 11th of December, 1878, and after examination and discussion of the section of the act of Congress by which its proceedings are to be governed, and of other documents referring to their prescribed duties, adjourned to meet at the mouth of the South Pâss of the Mississippi on the 30th of December.

The section of the act (section 4, act approved June 19, 1878) requires that—

"The board shall visit the works in process of construction by said James B. Eads, at the South Pass of the Mississippi River, and make an examination of the same, and make a full report of the progress made in the construction of the works, the probable cost of their completion, and the results produced, or that may properly [probably?] be produced by them, their probable permanency, and of the advisability of any modification of the terms of the act under which said Eads is constructing said works, so far as regards dimensions of channel through the jetties, and of the terms of payment for the same; which said report shall be submitted to the Secretary of War, to be presented at the next session of Congress."

The subjects upon which the board is required to make a full report are these:

1. Progress.
2. Probable cost of completion.
3. The results produced.
4. The results that may probably be produced.
5. Their probable permanency.
6. The advisability of any modification of the act, &c.

To discuss these subjects in the order named:

1. PROGRESS MADE IN THE CONSTRUCTION OF THE WORKS.

This requires reference to the original design, which was to construct, starting at marginal points on its banks above where the normal channel depth (30 feet) of the pass itself begins to diminish, dikes or parallel piers (as they are commonly designated) extending thence to the deep water of the Gulf, thus confining the outflowing water in a channel of the same width (nearly) as that of the pass itself until it reaches the deep water of the Gulf and thereby prolonging *through* the "bar" and to the Gulf the normal depth (nearly) which the pass maintains between its natural banks. These parallel piers or dikes, technically called "jetties," are essentially, then, an artificial prolongation of the natural "banks" to deep water in the Gulf. From the land's end (East Point) of eastern shore to 35 feet depth in the Gulf (which was about 300 feet beyond the crest of the bar) was a distance of 11,941 feet, which figures define the length of the east jetty as originally designed and marked out by piles.

The natural bank on the west side of the South Pass extended seaward 4,000 feet farther than the natural eastern bank: the initial point of the west jetty was, therefore, taken about that distance below the origin of the eastern one; hence the required length of the west jetty was 8,000 feet nearly.

The average width between banks of the pass itself is about 700 feet; which width, by the act of March 4, 1875, is fixed as the minimum between the authorized jetties.

The origin of the west jetty was established some distance (about 600 feet) from the west bank; hence it became

necessary to connect this point with the natural west bank by a dam of this length, built at right angles to the jetty, to which the name of the Kipp dam was given.

The original design therefore consisted of "east jetty," 11,941 feet in length (2½ miles, nearly), a "west jetty" 8,050 feet in length, and the "Kipp dam." The plan of "construction" was, in its main features, essentially that developed by long experience in Holland for dikes, dams, and jetties, on like yielding substrata, viz., a broad foundation stratum of willows or other suitable brush, formed into mattresses" (technically so called), on top of which was built a superstructure of tapering section, of alternate strata of mattresses and stone or gravel.

If we except 330 feet in length of extreme end of the originally designed east jetty, and 280 feet in length of the west jetty (reducing the total lengths of these jetties to 11,600 and 7,770 feet respectively), the jetties, throughout the lengths just mentioned, and the dam have been actually *built up* to the level of average high-water or somewhat above that level, and, owing to subsidence, supplementary elevations of willows and stone have from time to time been added. At present the seaward ends for about 1,500 feet are overflowed at high-tide, and even at low-water through a portion of less extent.

Pages 8 and 9 of the annual report upon the improvement of the South Pass, give for east and west jetty the actual height (July 1, 1878) above "average flood-tide" throughout their whole length. It will be seen that the outer ends and a few points higher up are now at various depths below high-tide. The extreme end of the east jetty is reported by Captain Brown to be 11 feet below; at the date of our visit the lower ends of both jetties for 1,000 feet or more were more or less submerged at flood-tide. Higher up the two jetties were found to be from .3 to 1 foot or 1½ feet above high-tide.

Page 10 gives the same particulars for the "Kipp" dam, which is throughout from ½ to 1-foot above flood-tide.

After the mention just made of "supplementary elevations" added from time to time to the jetties, to compensate for subsidence, it is proper to say a few words on this point.

Owing to the well-known character of the formation at the month of the Mississippi, much subsidence was expected. We have no accurate record of the total settlement of the various parts from the commencement. But on page 19 of the annual report, already cited, Captain Brown has given them for the year ending July 1, 1878. The amount of depression—superficial destruction by storms being eliminated, due to actual settlement *and* compression of willows is, beginning at East Point, one-half foot, gradually increasing to 3⅛ feet at the extreme end of the east jetty.

It is impossible to eliminate the compression of willows to show how much of the above is due to pure *settlement;* but, from the corresponding depression of the heads of the piles along the center portions of the jetties, it would seem probable that it is mainly due to "settlement." On page 2 of the fifth report, Major Comstock gives the settlement of certain piles along the last 1,000 feet of the east jetty, counting from the *old end*, and therefore covering the outer six or seven hundred feet of this jetty as it is at present. Between July 18 and October 21, 1876, three months, the outermost pile had settled 2.55 feet, the innermost 0.67, the gradation being progressive.

On the west jetty the observations began about 270 feet within the present end and extended *back* along 250 running feet of the jetty, the observed settlements graduating from 1.60 to 0.80 feet. Owing, however, to the great subsidence of mattresses and piles at the outer ends, as first fixed, the terminal points were (as before stated) withdrawn 330 and 280 feet respectively, by which withdrawal the jetties now terminate inside the crest of the old bar—the eastern on the very edge, the western at 200 feet inside of it.

Inasmuch as a full statement of the "progress" of the construction can scarcely be made without reference to the repeated partial reconstructions on account of subsidence, we have been led to develop that subject so fully. For details of actual construction, we refer to the third report of Major Comstock, and to map No. 2 of the same report, which exhibits various sections of the jetties as built, and makes it unnecessary that we

should swell this report by further description.

Up to November 1, 1878, there had been consumed in the construction of the two jetties and of the Kipp dam, as stated by the engineers employed by Mr. Eads, 310,830 cubic yards of mattresses and willows and 54,565 cubic yards of stone, mostly small stone.

By decision of the Attorney-General, January 17, 1877 (Ex. Doc. 28, Part 1, H. R., Forty-fourth Congress, second session), the channel through the shoal at the "head of the pass" is made a part of the "South Pass," through which a "navigable depth" is exacted by the act of March 3, 1875. The work at this locality had, however, been undertaken by Mr. Eads simultaneously with that on the jetties. A deflecting dike or "catch-water" was designed and commenced, running from the eastern margin of the entrance to the pass, a distance of about 3,000 feet, in a course starting northerly, curving to the westward; but a channel into the west entrance developing itself, which vessels began to use, the plan was abandoned, and the channel east of the island was closed by a dam. Dikes (called "T-head dams" on charts) were run out from the island and from the west bank, and they now define the present channel or entrance. The island or eastern T-head, about 1,600 feet long (originally 800 feet longer) running north-west by north from the head of the island, consists at present only of a row of piles and a single layer of mattresses on the bottom.

The west T-head, 800 feet distant from the eastern one, starts from a point about 1,200 feet above the origin of the latter and 400 feet distant from the west shore, with which it is connected by a dam. Except the last 350 feet, this T-head is built up above flood tide with five or six tiers of matresses loaded with stone on a double foundation layer of two matresses side by side. The extension of 350 feet has at present its foundation layer only. The dam above mentioned has been built up above high-water level and loaded with stone.

The permanent dam, 550 feet long, extending from the lower part of the island to the east shore and stopping the old east channel, has been built up above high-water and well loaded with stone.

The foregoing described constitute the system of "works" properly belonging to the head of the pass. Auxiliary thereto, mattress "sills," so called, have been laid on the bottom across the two great passes. The one across Southwest Pass runs from the west shore (from which a spur-dam about 400 feet long is first thrown out) to a point near the upper end of the west T-head. The entire length, including spur-dam, is about 3,200 feet. The one across the North-east Pass runs from the end of the old east dike to the opposite shore, length about 3,000 feet. They consist of a single mattress layer 70 feet wide and about 30 inches thick, weighted with stone.

In these various works at the head of the pass have been consumed 141,100 cubic yards of mattresses and willows, and 10,755 cubic yards of stone, as stated by Mr. Eads' engineer.

The foregoing is deemed a sufficiently full report of the "progress made in the construction of the works."

In what precedes we have made no mention of the wing-dams which have been constructed at various times. Their maintenance not being contemplated as permanent works, it is understood they have already subserved the purpose intended, of accelerating the process of channel formation.

2. PROBABLE COST OF COMPLETION.

Inasmuch as the act of Congress by which the contract was made with James B. Eads for improving the South Pass of the Mississippi River, expressly stipulates that the contractor should have perfect freedom as to the means to be used in obtaining the depths and widths of channel named in the act, estimates of the cost of completing the works could only be made after the board had been officially informed by the contractor of the methods which he purposed using. An official letter requesting this information was sent to the contractor (copy attached and marked A) as soon as the board arrived at Port Eads. His reply (copy attached and marked B), with the accompanying drawings, fully set forth his present plans and methods of completing the works. These plans are necessarily subject to modification, should experience in carrying them into effect

indicate a necessity therefor. Whether such a necessity will arise cannot be foreseen, and therefore estimates must be based on the plans as they now stand.

The following is a brief statement of the work proposed:

1. The top of the east jetty is to be raised to the height which the contractor deems desirable, which varies from 1½ feet above average flood tide at East Point, to 7 feet 9 inches above the same plane at the sea end. The upper part of this jetty, from the point 500 feet below its origin (to which point the jetty may be considered as finished) to a point 9,200 feet below, is to be raised to the level of 1½ feet above average flood tide, and finished by a rounded paving of riprap stone. The next 1,000 feet in length is to be capped by a low wall of rubble masonry. The remaining portion of this jetty to its sea end, a distance of 1,550 feet, is to be capped with large blocks of concrete built in place, on which at a later date a continuous parapet of concrete is to be built. The river and sea slopes of this jetty and its sea end are to be protected by mattresses covered with stone, additionally strengthened at the sea ends and for some distance back by crib-works of palmetto logs filled with stone.

2. The west jetty is to be treated in a similar manner, the changes in the method of finish being made at points opposite those at which the changes are made on the east jetty. The protections designed for the sea end and for the slopes of the west jetty are not so extensive as those for the east jetty, the latter being apparently considered as more exposed to injury.

3. The training-walls at the head of the South Pass (called on the maps "T-head dams") are to be improved. The eastern training-wall, on which but little work had been done, is to be raised above the surface of the water. The portion at the head of the western training-wall, now consisting of piles and one layer of mattresses, is to be completed.

4. The obstruction now in the Southwest Pass is to be increased by the superposition of other mattresses until the cross-section of this pass is made about 12,000 square feet less than it was after the original sill had been laid.

5. The dam closing Grand Bayou is to be maintained by such additional work as may from time to time become necessary.

Mr. Eads estimates the cost of doing the work thus summarily indicated at $349,641.

The board have carefully gone over the details of this estimate, and believe that it is substantially correct. They differ from Mr. Eads in some minor items of cost, but these differences are amply covered by the $58,273 allowed for contingencies. The board is, therefore, of the opinion that the work indicated by Mr. Eads can probably be done for his estimate, provided no extraordinary contingencies intervene.

This at once brings up the question whether the completion of the indicated works is a substantial completion of the original project, and may be so considered in questions of compensation.

When may the works at the South Pass be considered as completed?

A careful study of the act of Congress, under whose authority the original contract was made with Mr. Eads, shows that there is no mention of any specific work to be done by the contractor. The act authorizes him to construct "such walls, jetties, dikes, levees, and other structures, * * * as he may in the prosecution of said work deem necessary."

It also expressly states, that "said Eads shall be untrammeled in the exercise of his judgment and skill in the location, design and construction of said jetties and auxiliary works." The only limitation on the contractor is the provision that the jetties "shall not be less than 700 feet apart."

The whole contract is based on results. Certain specified sums were to be given to the contractor for certain depths of channel obtained by him. It was only required that the jetties should be "permanent and sufficient" "to create and permanently maintain" these various depths, the test of the permanent completion of the work being the creation and maintenance for 20 years of a channel 30 feet deep and 350 feet wide. This is the standard of completion established by law, and there is no power, short of the power that made the law, that can change the standard.

3. THE RESULTS PRODUCED.

The average width of the South Pass

between banks, is about 700 feet; the depth, so long as that width is maintained, is about 30 feet. At East Point, where the eastern bank terminates, and where the width was already increased to 850 feet, the bank confinement ceased; thence to the crest of the bar, 2½ miles distant, the depth gradually diminished to about 9 feet (average flood tide). The last half mile before reaching the outer crest having, nearly uniformly, only this small depth.

The results produced by the works may be stated as follows (omitting the tabular statement of the report for want of room):

The maximum bar-depth that has been obtained prevailed December 14, 1877, when it was 23.7 feet. At the date of the latest survey, December 28, 1878, it was 23 feet. This slightly diminished bar-depth by no means indicates actual retrogression in the progress of "results." On the contrary, there has been constant progressive general improvement in the jettied channel, at no time more evident than at the present.

At the date last named, a depth of 24 feet, with a channel-width of 300 feet, extended down to within 2,000 feet of the jetty ends; and the same depth with a channel-width of 200 feet, almost to the very ends. Thence to the same depth outside was a distance of but 60 feet with a navigable channel of 23 feet intervening.

The 25-foot channel has nearly the extent of, and not much less width than, the 24-foot channel. From its terminus inside to the same depth outside of the bar, there is but an interval of 160 feet.

The 26-foot channel extends (with a break of only 150 feet) down to within 1,000 feet of the jetty ends. Above the single interruption mentioned, which is 3,000 feet from the ends, the 26-foot channel has in its narrowest parts 100 and 150 feet width; in the widest, 350 and 700 feet, the latter at the site of the so-called "deep hole."

The depth of 27 feet is found at various points in the channel down to very near the jetty ends.

If we compare the above with the chart of a year's earlier date (December 5, 1877) we find a general improvement of navigable channel through the lower 6,000 feet. There is 24 feet where there then was 22 feet; that is to say, a general increment of the channel depth by 2 feet, accompanied by rectification and widening of areas of lesser navigable depths; there being for the 22-foot depth 230 feet width at the bar and a general width within of over 400 feet.

(Even greater increase of depth amounting to 6, 7, and even 10 feet are stated to have taken place in the upper portions of the jetties channel.)

At the *head of the passes* the result of the works has been the procuring of a channel-depth of 22 feet where there was, over the shoal, but 14 or 15 feet. If we compare the present condition with the chart of December, 1877, we find that while the actually navigable depth is not much changed, the distance between the 24-foot curves at T-heads and above the same respectively has increased from 350 to 800 feet; this augmentation of the *bar*-width taking place both inwardly and outwardly, but in much the greater proportion outwardly (*i.e.*, on the Southwest Pass margin). Between the training walls the channel may have somewhat improved, not by increase of maximum depths, but by the diffusion of the current more equally over the intervening space.

4. RESULTS THAT MAY PROBABLY BE PRODUCED.

It is a difficult matter to respond satisfactorily to this requirement of the law inasmuch as the efficient causes cannot be precisely defined or measured. Reference to opinions of engineers who have recommended the resort to jetties and developed their views as to depths which should be obtained by the means proposed, furnishes one basis of judgment. Reference to the *actual results* combined with the progress and present condition of the works furnishes another. The trial of the jetty system at the South Pass, or at least a further study of the subject, before undertaking the construction of a ship-canal, was first recommended in the minority report of the Board of Engineers of 1873, with the expression of opinion that 25 feet at low-water might be attained; the practicability of terminating the jetties inside the bar crest (instead of encountering the great expense and doubtful practicability of

prolongation to deep water) being assumed or supposed probable.

The Board of Engineers constituted by act of June 23, 1874, "to determine the best method of obtaining and maintaining a depth of water sufficient for the purposes of commerce, either by a canal from said river to the waters of the Gulf, or by deepening one or more of the natural outlets of said river," proposed, by the extension of jetties, 900 feet apart, to the depth of 30 feet outside, to obtain provisionally a channel-depth which would, as was estimated, gradually shoal by bar advance in about ten years to 25 feet depth, when the jetties must be extended 1,000 feet seaward to reach 30 feet depth again. The present jetties are about 950 feet apart, and terminate, the eastern almost an the outer edge of the bar, where there was originally but 15 or 16 feet of water; the western, about 200 feet within the outer edge, where there was but 7 or 8 feet of water. The conditions, therefore, by which the engineers of the board of 1874 expected to get, provisionally, 30 feet depth, do not, in the existing arrangements, fully obtain.

Mr. Eads, in his letter to this board, herewith appended, states:

"I believe the natural pass is sufficient to create and maintain a channel through the jetties that shall have a central depth of thirty feet when the jetties are fully consolidated and all leakage through them is prevented; but I do not believe such volume will produce a channel of greater magnitude."

Though the ground for this belief is not here stated by Mr. Eads, it is understood that he relies on the fact that the pass, in its natural condition, maintains a depth of 30 feet in its channel. He believes that the jetties will carry out that depth, undiminished, to the sea. This, too, was admitted or assumed by the board of 1874, *provided always*, the jetties, 900 feet apart or less, be extended to or beyond that natural depth in the Gulf. This proviso is not fulfilled, as we have seen, by the existing jetties.

The foregoing refers to *a priori* opinions. Turning now to the results actually and progressively obtained, coupled with the stage of construction and present condition of the jetties, the facts of the case have been stated under the proper head.

The jetties, since their commencement, have produced an increase of bar (or minimum navigable) depth from 9 to 23 feet; and if the last twelve months have shown no actual increment of that particular element, yet there has been (as already fully set forth) a most decided improvement throughout the whole jettied channel length. There is ground to look for further improvement, coupled with increase of bar-depth, which, to 25 feet, requires the cutting through of a bar of only 160 feet width. But the jetties have not yet (as seen in our statements on "progress, &c.") acquired their full action. The outer ends, though more than once raised, are still submerged. According to Captain Brown, Eighth Report, p. 31, "at least 20 per cent. of the water passing the land's end at East Point, escapes over the jetties and through the meshes of the mattresses at average flood-tide." Mr. Corthell, resident engineer under Mr. Eads, estimates the escape at 25 per cent. By far, the greater portion of this escape takes place along the lower 1,000 or 1,500 feet of length. Moreover, the temporary effect of the operations at the jetties at Grand Bayou, and at the head of the passes, has been to diminish the discharge of the pass by 10 or 12 per cent. The raising and consolidating the jetties at their outer ends will in great measure prevent the loss attributed to imperfect confinement; the volume originally entering the pass at the head may probably be restored.

The foregoing considerations, and the facts already stated under the head of "the results produced," induce us to think that if the jetties were well consolidated and raised sufficiently high to prevent leakage and overflow, a considerable increase of navigable depth would result. We cannot state that in our opinion it is a "probable result" that the depth of 30 feet will be attained, as assumed by Mr. Eads. What the limit will be, cannot be positively announced. That it may attain 25 or 26 feet is all that we can venture to expect, as a depth which shall permanently maintain itself; and, as past experience shows annual fluctuations amounting to about 2 feet, a permanent channel 25 or 26 feet will require an occasional channel of 27 or 28 feet.

With regard to the head of the passes, the considerable widening of the bar or shoal between 24 feet inside and 24 feet outside during the last twelve months, is an unfavorable result. The work designed by Mr. Eads, and embraced in his estimate for "completion," and now commenced, consisting mainly in the raising by additional mattresses the sill of the Southwest Pass, with the view of restoring the lost inflow to the South Pass, and deepening the entrance, may accomplish the result expected. We have not full confidence, however, that that measure alone will do so.

In connection with the "probable results" of jetty construction upon which we are directed to report, there is one to which pre-eminent importance has been attributed and which should not be here overlooked—that of bar advance. The Board of Engineers of 1874, in recommending the jetty construction at the South Pass, assumed that the normal rate (supposed to be 100 feet per annum) would be maintained after the pass was jettied, and hence that, to maintain a depth of at least 25 feet, the jetties must be prolonged every ten years. One of the main arguments used against the resort to the jetty system has always been that a greatly increased rate of bar advance will ensue. On this point we refer to the Seventh and Eighth Reports of Captain Brown, and to his Annual Report of June 30, 1878. By the Seventh Report, page 30, it is shown that over a fan-shaped area of 1¼ square miles immediately seaward of the ends of the jetties, there had been between June 20, 1876, and June 22, 1877, a mean "fill" of four-tenths of a foot. Parts of this area had scoured (*i. e.*, had become deeper); in 4 of the 21 sections there had been, on the contrary, a large "fill"; the average total result having been, as just stated, a slight shoaling of 0.4 foot. Over this same area (page 15, Annual Report) there had been in the subsequent twelve months from June, 1877, to July, 1878, a scour (*i. e.*, increase of depth) averaging 1.8 feet.

By the table, p. 33, of the Eighth Report, it is shown that in that portion of the 30-foot curve, lying seaward, between the prolongations of the jetties, an advance of 132 feet had taken place so early as October, 1875, soon after the jetties were commenced and before any impression had been made on the bar. During the subsequent period of two years there had been fluctuations, the advance reaching (July 28, 1877) the magnitude of 242 feet, followed by a retrogression to 108 feet, December 15, 1877. This last was followed by another retrogression (table, page 18, Annual Report) to 60 feet in the early summer of 1878, succeeded by an advance, July 15, to 140 feet.

The foregoing, referring to the portions of the 30-foot curve included between the prolongations of the jetties, affords no proof of *progressive* advance. If, in this connection, we take into account the position of the 30-foot curve outside the jetties (and this is evidently a better test), there is shown, instead of advance, an absolute retrogression. Or, again, if we have reference to deeper curves, Captain Brown's surveys (Annual Report, table, p. 15) show that from June, 1877, to July, 1878, the 40, 50, 60, 80, 90, and 100 foot curves had drawn in towards the ends of the jetties the respective distances of 117, 228, 190, 65, 71, and 183 feet; the 70-foot curve alone showing advance into the Gulf (46 feet). The actual results, therefore, so far as we know them, do not justify the predictions of accelerated bar advance. On the contrary, they show a disappearance of bar material from the front of the jetties.

5. PROBABLE PERMANENCY.

Wave (or storm) *action* of the sea and *decay* or *destruction by the teredo* of the willow mattresses are the principal destructive elements to be mentioned. An additional element of deterioration, viz., settlement not peculiar to the location, but supposed to be so prominent as to involve the question of permanence, must also be noticed.

The jetties, except the extreme ends and contiguous portions, for about 1,500 feet inward, are so well sheltered by shoals that wave action, except on those portions, has little effect. On the sea-ends the effect has been considerable, but mainly superficial, destroying more than once the upper course or courses of mattresses and washing off and scattering the stones (mostly small) which have been repeatedly applied to the top surface. Wave action is by no means as

violent here as in similar exposures on the Atlantic coast. We see no reason to doubt that the thick concrete capping Mr. Eads is now commencing to apply (work having already begun on the upper portions), flanked by enrockments of heavy stones on palmetto-log grillages overlying the original marginal mattresses, will resist sea-action.

Wood of all kinds considerably submerged is sufficiently secure against decay.

Experience here shows that for about 1,700 feet inwards from the jetty ends the teredo destroys rapidly all exposed wood (including in this term the willows of the mattresses) lying more than four or five feet below the surface of the water. Evidence enough of its attacks upon piles and willows exists. But the teredo does not attack wood where the free access of sea-water is impeded. Those portions of a stick buried in mud or sand, or packed around with mud or sand, are secure. We have no reason to believe that the teredo has penetrated or can penetrate far into the interior of the mattress courses; we have pretty good reason to believe that the foundation mattresses are, and will, remain secure; and probably also the bulk of the interior of the masses of willow-work.

In what we have said under the head of "progress" we have given sufficiently full details concerning settlement. It is still very great at the outer ends, though very much less in all those portions more than two or three thousand feet from those ends. That additional superficial applications of stone or concrete will be necessary to the structures we must expect.

In the ordinary sense of the word permanency, *i.e.*, capability of endurance of destructive forces, the works may be said to possess the attribute, to a reasonable degree, for work of the kind thus situated. As regards the outer ends it is yet early to predict to what extent or how long renewals of height to compensate the still progressing settlement must be resorted to.

6. ADVISABILITY OF ANY MODIFICATION OF THE ACT AS REGARDS TERMS OF PAYMENT AND DIMENSIONS OF CHANNEL.

Under this head the board states substantially that, in view of the fact of Mr. Eads having already received $1,686,000—that Congress has already authorized (by Act of June 19, 1878) advancing $1,000,000 ($686,000 of the total already received being on this account) over and above what, under the terms of the contract has been earned—the board does not admit his plan for further immediate payment; but recommends an additional advance, for carrying on the work, of $250,000.

The board considers it "premature to recommend at this time any changes in channel dimensions, as required by the contract," and "in view of its recommendation that Mr. Eads be provided with sufficient funds to complete his work according to his own programme, and of his expressed ability to obtain the depth and width of channel prescribed" declines to recommend any essential modification. But in compliance with one of Mr. Eads' suggestions it advises as follows:

"As every additional foot in depth of channel is of substantial benefit to commerce, we would suggest the advisability of a change in the terms of payment in the original act, so as to allow of payments for each additional foot gained instead of for every two feet."

SCHWEDLER'S TRUSS.—This truss is so designed as to avoid the necessity for the use of counterbraces. It is symmetrical with respect to the middle of the span. The heights of posts on either side are to be calculated according to the following formula, which is exact and is believed to be new

$$H_x = \frac{Cx(n-x)}{n+\theta x}$$

where n=number of panels

" H_x=height of xth post

" θ=max. ratio of live to dead load

and C is a constant determined for an assumed depth at midspan. If $\theta(n-2)>4$ the value of H_x attains its maximum H before reaching mid span. In this case H is to be used for all posts from this to midspan inclusive. The values of H_x may be laid off above the line of the supports below, part above, and part below or entirely above for roof truss.

WM. M. THORNTON.

A PRACTICAL THEORY OF VOUSSOIR ARCHES. PART II.

BY WM. CAIN, C.E., Carolina Military Institute, Charlotte, N. C.

Contributed to VAN NOSTRAND'S MAGAZINE.

III.

GROINED ARCHES.

80. Let ABCD, Fig. 17, be the plan of a groined arch, $\overline{AC}$ and $\overline{BD}$ representing the groins; the elevation is shown, at BMC, of the front face AD. There are abutments at A, B, C and D, one of h is shown at A in plan.

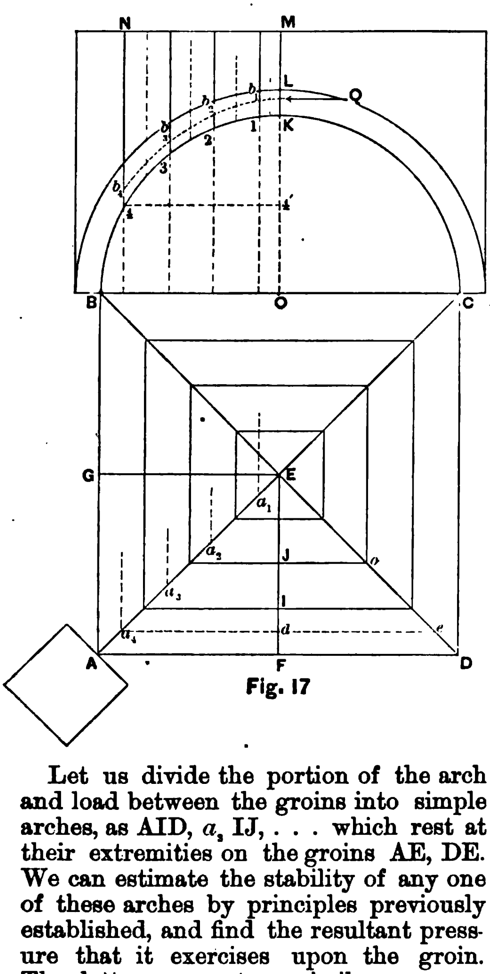


Fig. 17

Let us divide the portion of the arch and load between the groins into simple arches, as AID, a_2 IJ, . . . which rest at their extremities on the groins AE, DE. We can estimate the stability of any one of these arches by principles previously established, and find the resultant pressure that it exercises upon the groin. The latter supports a similar pressure from each side; the resultant of these two pressures, which is generally oblique, can then be decomposed into horizontal and vertical components, which are the forces to be used, in their proper positions, in ascertaining the stability of the simple arch constituting the groin, and also of the abutment against which it leans.

81. An example will render this clearer. The dimensions are given in meters, though any unit may be taken. Let $\overline{AD}$ $=7.54$; the arc AFD in plan, a semicircle whose radius is thus, $\overline{OB}=3.77$; depth of keystone $\overline{KL}=0.47$; the height of surcharge above it, LM$=1.26$. Divide the semi groin AE into a number of equal parts, four in the figure, and suppose, each simple arch, as AID, to terminate at the middle, a_4, of its corresponding division. Project up a_1, a_2, a_3, a_4 to b_1, b_2, b_3, b_4, in elevation. Then on this supposition the weight AIF sustained at a_4 is represented in elevation by MNb_4K, supposing the joints vertical. Similarly for arch a_3 JI, etc. Pass a curve of pressures now through the top of middle third limit at the crown KL and the lower middle third limit about the joint of rupture, taken approximately at b_4 in this case, from which the resultant at $(a_4\ b_4)$ can be found. It is sufficiently correct, and is on the side of safety for the other arcs, as a_3 IJ, to retain the same value and position of Q at the crown. We thus find from the diagram for arc AID the resultants in amount, position and direction at the points $(a_3,\ b_3)$, $(a_2,\ b_2)$, $(a_1,\ b_1)$ of the groins, due to all the arcs in the space AEF.

In the following table of volumes and centers of gravity, $v=$volume of trapezoid lying just to right of joint to which it refers$=$width$\times$mean height $\times\overline{IF}$.

In this case IF$=$JI$=0.94$. l is the distance of the center of gravity of the trapezoid from the crown, and m the corresponding moment. V is the volume from the crown to the joint to which it refers found by cumulating the numbers in column v. Similarly M is formed from m, and the quotients $\frac{M}{V}=C$,

give the distances of the centers of gravity of these volumes V from the crown.

Joint.	v	l	m	V	M	C
1	.78	.24	.1872	.78	.1872	.24
2	1.68	.96	1.6128	2 46	1.8000	.73
3	2.04	1.91	3.8964	4.50	5.6964	1.27.
4	2.74	2.87	7.8638	7.24	13.5602	1.87
	7.24		13.5602			

Laying off $1^m.87$ from the crown to the left on Q prolonged and drawing from this point a straight line to b_4, we have the direction of the resultant at b_4. Its amount is found by laying off $P_4=7.24$ on the vertical, through the point 1.87 to left of the crown, downwards, and then drawing a horizontal line to the resultant, which may now, as well as Q, be scaled off. We thus find Q=5.45.

Next, combining the forces at a_1, a_2, a_3 and a_4 due to the arcs on either side of the groin AE, we have for the vertical components of the resultants at a_1, a_2, a_3, a_4, 1.56, 4.92, 9., 14.48, respectively, being double the numbers given in column V above.

The horizontal component at each point is $Q\sqrt{2}=7.7$.

82. It is evident that the greater the number of divisions IF, JI, &c., the more accurate the result. It is well to test the above volumes analytically.

Call the radius, $OB=r$, $\overline{KM}=c$, and the variable distance $Ed=a_4d=x$. Since any arc as a_4de of the intrados has a radius r, its *rise*, when the half chord is x, $=(r-\sqrt{r^2-x^2})$.

For simplicity we shall introduce one approximation, viz: that the areas of the circular spandrels is nearly that of parabolic spandrels of same encompassing rectangle. This is on the side of safety.

We have, calling $44'=x$,

$$\text{area KMN } 4=\frac{x(r-\sqrt{r^2-x^2})}{3}+cx.$$

On multiplying this by $\triangle x=$ elementary width of a slice, we get the approximate volume of one half of a simple arch, $\parallel \overline{AD}$, and at a distance x from E. The limit of the sum of such slices, between the limits $x=r$ and $x=o$, gives the total volume of the part AEF

$$\therefore \text{ volume AGFE}=\tfrac{1}{4} \text{ groined arch}$$

$$=\lim\sum_{0}^{r}\left(\tfrac{2}{3}xr-\tfrac{2}{3}\left\{r^2-x^2\right\}^{\frac{1}{2}}x+2cx\right)\triangle x$$

$$=\left(\frac{r}{9}+c\right)r^2$$

In this case $r=3.77$ and $c=.47+1.26=1.73$

$\therefore$ vol. AGFE=30.53.

From the table we find this volume to be 2(.78+2.46+4.50+7.24)=29.92, which differs but slightly from the above, and is somewhat less, as it should be.

83. It will conduce to clearness to lay off on an elevation of the groin and abutment, Fig. 18, the forces just found, directly over a_1, a_2, . . . at the same

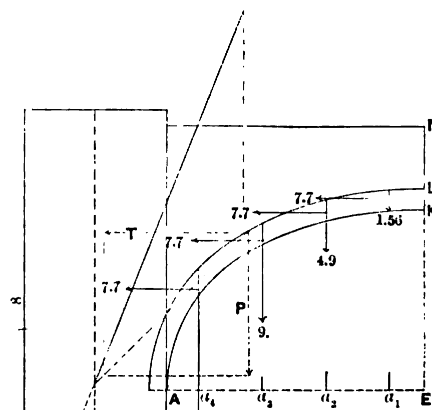


Fig. 18

heights as b_1, b_2, . . . are above the springing, viz., 4.03, 3.76, 3.15 and 2.12. The distance t of the resultant T of the horizontal forces above the springing is thus,

$$t=\frac{7.7(4.03+3.76+3.15+2.12)}{T=30.8}=3.26$$

from which T can be located. Similarly take moments of the vertical forces about A, to find the distance, p that their resultant, P, acts to the right of A.

$$\therefore p=\frac{14.46\times.68+9\times2.01+4.9\times3.35+1.56\times4.69}{P=29.92}$$

$$=1.73$$

The resultant of T and P passes outside of the arch ring above the springing. On combining it in turn with the weight of the abutment $8\times3\times2$, the final resultant cuts the base at s, $\frac{1}{6}$ the depth from the outer edge. The abutment may be increased in size to cause s to pass as near the center as may be desired.

84. The arch ring of the groin in the actual example has a depth of 0.^{m}94, being double that of the ring as drawn; which may thus be supposed to represent its middle half.

To test its stability, combine the resultant of the forces 7.7 and 1.56, being the pressure on the joint midway between a_1 and a_2, with the resultant of the next two concurrent forces, 7.7 and 4.9, to find the resultant on the joint midway between a_2 and a_3; next, combine this last resultant with that of the next two concurrent forces and so on. The final resultant on the springing joint should coincide with the resultant of P and T just found.

The line of pressures is thus found to keep very near the center line down to a_4, below which it passes out of the arch ring, on the extrados side.

The heavy backing will exert horizontal forces to modify this line of pressures, probably keeping it in the arch ring near the springing; for otherwise the intrados joints about the springing must open; but this cannot happen unless the extrados joints open about a_3. If the backing prevents the latter, the former cannot occur; but if no joints open, the line of pressures must lie in the middle third; so that the arch ring is stable.

85. It will be observed, that no pressure at the crown is needed to ensure stability. In fact, if any were supposed it would only cause the final resultant on the springing joint to depart still more from the arch ring.

For other dimensions than those given in this example, a horizontal thrust at the crown of the groin may be needed. For example, when the line of pressures just found falls below the middle third at any joint.

In this case, if we desire that the curve of pressures pass through a particular point in the lower middle third limit, as the one vertically over a_4; let the horizontal thrust, H at the crown KL act at the upper middle third limit. Call the lever arms of H, T and P about the lower point vertically over a_4, h, t and p; we have,

$$\mathrm{H}h+\mathrm{T}t-\mathrm{P}p=o$$

from which H can be found and the curve of pressures located as before.

The true curve keeps within the middle third, and, as before explained, conforms nearly to the maximum and minimum of the thrust in the limits.

86. It is more usual to place the abutments as in Fig. 19. The space between them is usually covered with simple arches as ABCD. The horizontal thrusts

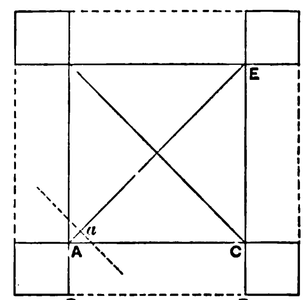


Fig. 19

of the two leaning against one abutment, acting at the crown joints are combined into one, $Q'\sqrt{2}$ acting directly over the center of the abutment, and in the direction of a diagonal, as EA. The weights of the two semi-arches acting at their centers of gravity are combined into one, 2P′, acting at a on the diagonal AE. On drawing now a section of the abutment along AE produced and laying off the forces $Q'\sqrt{2}$, 2P′, T, P, H if any, and the weight of abutment, in their proper positions and combining these forces into one resultant, we ascertain if the center of pressure at the base of the abutment lies within proper limits: the middle third, or whatever limit is chosen from practical considerations. It will be found that the addition of the encompassing arches conduces to stability, the effect of the downward force 2P′, more than counteracting the effect of the force $Q'\sqrt{2}$.

87. The groined arch investigated in art. 81, *et seq.*, is considered by Scheffler; but the analytical solution proposed by him is too rough an approximation to be commended; and besides it errs, in part on the safe side, and in part otherwise, so that it is not known whether the final result is on the safe side or not, especially as the line of pressures is made to touch the contour curves.

For definitions of groined and cloistered arches, domes, &c., the reader is referred to Mahan's Civil Engineering; which book like-

wise contains descriptions and drawings of several noted bridges.

CLOISTERED ARCHES.

88. In the cloistered arch, shown in plan, in Fig. 20, AB, BD, DC and CA, are straight lines, whilst EF is a simple arch

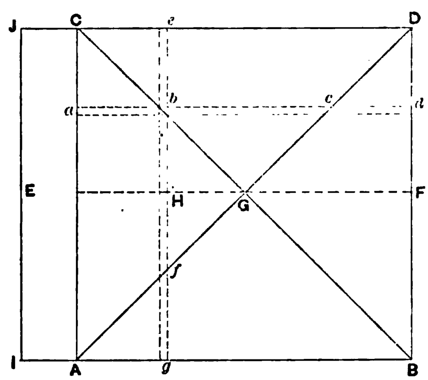


Fig. 20

of span EF, and rise equal to the height of the crown at G above the springing. AD and BC are the groins, forming the reentrant angles on which the smaller arcs, as *ab*, *be*, *cd*, *fg*, etc., meet with an inclined tangent. Thus *ab* is precisely similar in form to the part EH of the full center arch EF. The elements *bc* or *fb* are thus horizontal. Now the thrust at the crown G of the simple arch EF of small width, is horizontal, and is computed as for a simple arch. The arcs *ab* and *cd* sustain at *b* and *c* horizontal thrusts communicated through the horizontal element *bc*. When the centers are struck, the tendency to fall causes pressure on the voussoirs in four directions, $\perp$ and $\parallel$ AB, so that the elements, as *bc* and *bf* of the cylinders sustain a uniform horizontal compression in the directions *bc* and *bf*, and the voussoirs composing these elements sustain likewise an inclined thrust (except at the groins, where it is horizontal), in a direction perpendicular to the elements, whose amount is easily determined by the methods affecting simple arches.

89. If the above hypothesis be granted, it becomes an easy question to investigate the stability of a cloistered arch and its abutments, one of which is shown at AIJC.

Thus divide EG into any number of equal parts, and find by usual methods the weights and the positions of their centers of gravity, from the springing AC to any joint, in place of from G to the joint, as hitherto.

Part of the table made out then directly applies to each partial arc, as *ab*. On the elevation of the semi-arch EG and of each partial arch, as *ab*, pass curves of pressures, lying within the middle third, if possible; assuming the horizontal thrust at the top of the arch at the upper middle third limit as a first trial for the arc EF. With the tables made out as above, the resultant at the abutment must be combined, in turn, with the weights from the abutment to the joint considered, to find the centers of pressure on those joints.

We thus find the various horizontal thrusts, acting at the groins CG and AG in a direction $\perp$AC. On multiplying each of these thrusts by its vertical distance above the springing, and dividing the result by the sum of the thrusts, we find the vertical distance above the springing at which the resultant of the horizontal thrust T, of the part AGC acts. Similarly, find the horizontal distance to the resultant of the vertical forces, P acting on the part AGC; this resultant representing the weight of AGC. On combining these resultants, T and P acting in the vertical plane EG, with the weight of the abutment, as shown in Fig. 18, we ascertain whether the center of pressure on the base of the abutment falls within proper limits. Since the arc EF causes the greatest thrust, unless the abutment is made to act as one piece, as supposed above, its width should vary, being greatest at E and diminishing to nought, theoretically, at C; the intermediate widths being found in the usual manner from the thrusts of the partial arches resting there. When the backing of the arches can resist a horizontal thrust, the curve of pressures for the smaller arches may be assumed (for safety in designing the abutment) to pass through the centers of the joints at the summit and abutment, or even lower at the summit and higher at the abutment; especially if the stones are not cut to fit snugly.

The writer has not met hitherto with a proper solution of either the groined or cloistered arch, and therefore commends the above to the attention of engineers.

DOMES.

90. The soffit of the dome will be supposed to be generated by revolving a curve about the vertical line representing the rise of the arch called the axis, so that every horizontal section of the soffit is a circle. The extrados may be generated by revolving a similar curve or any other figure about the axis. If we pass two meridian planes, making a small angle, ψ with each other, through the axis, we cut from the dome and backing, if any, a solid FC, Fig. 21, being a part

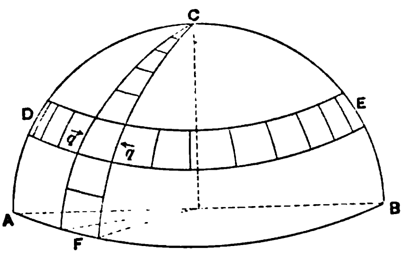


Fig. 21

of a wedge-shaped figure whose soffit is a *lune*. This solid, for want of a better name, we shall call a *lune solid*.

Now pass conical joints, perpendicular to the soffit, at certain distances apart: the part of the dome proper, as DE, lying between any two conical joints, will be called a *crown*.

91. We shall introduce the discussion by a quotation from Dr. Scheffler.*

"The authors who have treated the question of domes (NAVIER: *Résumé des leçons sur l'application de la mécanique*, part 1, No. 349; RONDELET: *Art. de bâtir*, liv. ix, sect. vi, chap. ii, etc,) have commonly divided the dome into lune solids (as defined above), and apply afterwards to these solids the same principles as to simple arches with vertical loads and horizontal thrusts. Now this view is entirely erroneous. It does not make known the influence of the forces which acts upon an arch of this character, and it implies this condition, impossible to realize; that the materials sustain at the summit a finite thrust upon an edge infinitely small.

"It is necessary, on the contrary, to divide the dome ABC, Fig. 21, into crowns, (as defined above), DE resting on the inclined bases of cones of revolution.

"It is proper first to examine the conditions of equilibrium of such a crown; which can moreover form the superior part of a dome open above.

"There are developed in these crowns horizontal pressures q, q, whose directions are normal to the joints of the crown, and more intense in the upper than in the lower crowns.

"When we afterwards consider the lune solid CF, limited by meridian planes, it is necessary to combine the two forces q, q into a single horizontal force Q, acting outwards. It is necessary in all cases that the horizontal thrust at the upper joint may be null.

"This is evident for an open dome; for the dome closed at top, which is only a particular case of the open dome, it results from the fact, that the surface of the joint at the summit reduces to a line, which cannot support a finite pressure."

92. Let Fig. 22 represent a lune solid of the dome considered, and let P_1, P_2,

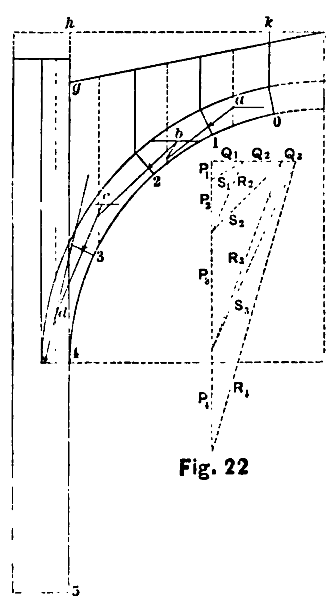


Fig. 22

P_3, P_4, laid off in order on the vertical line P_1 P_4, represent the weights of voussoirs 1, 2, 3, 4, respectively, with their loads. Let us assume, for the present, that the forces q, q, of the preceding figure act at the centers of the voussoirs; so that the forces Q_1, Q_2 . . , act through the centers of their corre-

* "Theorie der Gewolbe;" also a French translation, "Theorie Des Voutes, &c."

sponding voussoirs, 1, 2, . . . , and horizontally to the left in Fig. 22.

Now the horizontal thrust at joint o is null. The weight P_1 of the first voussoir and load, acts through a and does not meet joint 1, so that there is no stability unless the "crown" including this voussoir, in tending to fall, exerts a horizontal pressure. The resultant Q_1 of the pressures exerted on both sides of voussoir 1 should be so great that when combined with P_1, the resultant shall cut the joint to which it refers, and make with the normal to this joint an angle not less than the angle of friction. These two conditions hold for every joint. If no joints open the resultants will lie in the middle third. Now if Q_1 be made so large that a line drawn through $a \parallel S_1$ the resultant of P_1 and Q_1, satisfies the above conditions, the point where it cuts joint 1 may be regarded as a *possible* center of pressure.

If the above conditions are not satisfied for an assumed value of Q_1, Q_1 must be increased.

Now extend the line through a, just drawn to intersection with the vertical through the center of gravity of the second voussoir and load, whose weight is P_2; from this point draw a parallel to R_2, the resultant of S_1 and P_2, and extend it upwards to intersection b, with the horizontal through the center of the second voussoir along which Q_2 acts. On drawing through b a line to some point on joint 2; a parallel to it, in the force diagram, gives S_2, and cuts off Q_2, as shown in the figure. As before, if S_2 does not make an angle with the normal to joint 2 less than the angle of friction, Q_2 must be increased and the line through b made parallel to S_2 thus found. Similarly we proceed for other joints, until finally we get to a joint, as 3, below which no more forces of the type Q are needed to prevent the resultants on succeeding joints from falling *below* certain limits. The part of the lune solid below joint 3, called the "*joint of rupture*," thus acts as any simple arch; therefore we determine the resultant on joint 4 by combining S_3 and P_4—*i. e.*, by drawing through d a parallel to R_4 of the force diagram, the resultant on joint 4, to intersection with that joint.

Similarly, if we combine the resultant S_3 on joint 3 with the weight of the entire abutment, we find the centre of pressure on joint 5, which should lie within the middle third.

It will be observed that the resultants on joints 1, 2, 3, . . . , are represented in magnitude and direction by the lines S_1, S_2, . . . , of the force diagram, and in position by the arrow heads on the drawing of the arch.

The scale of the arch ring should be as large as can conveniently be drawn, since the lines determining the directions of the resultants are very short, and cannot be well shown on the small diagram. On that account, the preceding directions have been made full to conduce to clearness.

93. Scheffler now says, in effect, that if the voussoirs were absolutely incompressible, Q_1, Q_2, . . . should each in turn be the least that will cause stability and should therefore pass through the upper edges of joints 0, 1, 2, . . . ; and the resultants S_1, S_2, . . . should pass through the lower edges of joints 1, 2, etc., if the conditions affecting sliding are fulfilled. (On this point, see art. 106.)

But we know that actual voussoirs are compressible, so that if, as is usual, the actual resultant on the springing joint passes to the left of the center, the outer edge is most compressed, and to allow this the haunches must spread and the top of the arch descend, so that about joint 3 the line of pressures passes below and at the top, above the center line. This is all the more evident if the springing joint opens at the inner edge. In the previous figure, the forces S_1, S_2, . . . , were drawn through the lower middle third limits. Now, if a dome acts like a stone bridge in a lowering of the crown, it would seem that the line of pressures there should lie above the center, so that its most probable position is at the upper middle third limit at the crown, and at the lower middle third limit at the joint of rupture, if the line of pressures can just be inscribed in the inner third of all the joints from the crown to the foundation of the abutment. When the line of pressures can be drawn within narrower limits, it is probably so confined actually.

An illustration of this view is given in article 105 following, which see.

It is plain that the forces q, q do not necessarily act at the centers of the vous-

soirs, as assumed. Their positions are indeterminate. Their least values for the same crown, consistent with no joint opening, corresponds to positions on the upper middle third limit, distant $\frac{1}{3}$ width voussoir from upper joint; their greatest values correspond to the lowest limiting positions.

As it is the object of the investigation to ascertain if the proper thickness of the arch ring and abutment have been chosen, it is well to err on the safe side in our hypotheses. In testing the stability of the abutment, it seems best to consider Q_1, Q_2, . . . , as acting at the centers of the voussoirs; the resultants S_1, S_2, . . . , at the centers of the joints. The latter hypothesis implies a uniform compression on the joints down to the joint of rupture; whereas, in fact, the line of pressures, as explained above, must pass below the center at this joint, giving generally a less total horizontal thrust.

As a modification of the above hypothesis, we may assume that the resultants S_1, S_2, . . . , are tangent to the center line, from the crown to the joint of rupture. It will be found that this involves raising Q_1, Q_2, . . , slightly above the centers of the voussoirs. The construction is much simplified by this assumption which will be illustrated more fully in Art. 100.

94. From the definitions of arts. 90 and 91, and a plan of a voussoir bounded by the two meridian planes whose included angle in arc is ψ, and which is solicited by the two horizontal forces q, q, (acting perpendicular to its vertical faces), whose resultant is Q, we have,

$$\tfrac{1}{2}Q = q \sin \tfrac{1}{2}\psi.$$

If a=half span, and ε=horizontal width of voussoir at the springing, then $a\psi=\varepsilon$. When the angle ψ is small, *i. e.*, when ε is made small enough, we have from the above equation,

$$q = \frac{a}{\varepsilon} Q;$$

from which q_1, q_2, . . can be computed, as soon as Q_1, Q_2, are found by the construction above.

95. *Numerical Example.*—Let us take the half span (Fig. 22) equal to 9.42, the depth of arch ring 0.94; and let the inclined line fg limit the load, the point f being 12.24 above the center e of the soffit, and g, 1.98 lower than f. The radius fk=1.86. Now divide the horizontal hk into six parts, each 1.26 wide, in place of three as before; drop verticals through the points of division, and from their intersection with the extrados draw the joints 0 to 6. We shall suppose approximately that the figures so formed are trapezoids, whose area equals the mean height multiplied by the width.

But now each trapezoidal solid, included between the two meridian planes, has a different thickness. Since the plan of the solid cut from the dome by the two meridian planes is a triangle, if we call its thickness at the middle of the springing joint 1, we find the thickness at the other mean verticals by multiplying 1 by the ratio of the horizontal distance of the mean vertical from ef to the radius of the center of the springing joint about ef. By regarding each trapezoidal solid as having the thickness at its mean vertical, we introduce an error which diminishes indefinitely with ψ, and can thus be made as small as we please.

In the following table, column (1) refers to the joint, column (2) gives the height of the trapezoidal solid, column (3) its width, column (4) its thickness, and column (5) their product representing the forces P_1, P_2, ...

(1)	(2)	(3)	(4)		(5)
1	2.65	1.26	.25	P_1	0.83
2	2.83	1.26	.38	P_2	1.35
3	3.23	1.26	.50	P_3	2.03
4	3.92	1.26	.63	P_4	3.11
5	5.03	1.26	.76	P_5	4.82
6	7.05	1.26	.89	P_6	7.90
7	10.99	0.94	1.00	P_7	10.33

The volumes of the voussoirs and loads can be exactly determined by the principle of Guldinus (see Weisbach's "Mechanics," Coxe, Vol. 1, p. 126), that *the contents of a solid of rotation is equal to the product of the generating surface and the space described by its center of gravity while generating the body.*

For greater accuracy the voussoirs and loads may be considered separately, and their common center of gravity and volume found by combining them afterwards.

The above principle we shall use in some subsequent constructions.

The construction is now proceeded with exactly as described for Fig. 22, which is in fact a drawing to these dimensions.

The induced forces Q_1, Q_2, . . . , were conceived to pass through the centers of the voussoirs. The resultants S_1 S_2, . . . , on the joints were made first to pass through the lower middle third limits, and afterwards through the center of those joints. In both cases the joint marked 3 in Fig. 22 was the joint of rupture: the resultant on the springing joint, in the first instance, coinciding with the resultant as drawn in Fig. 22; in the last case passing nearly through the extrados. The total horizontal thrust in the first case=7.75; in the last, 8.13. If we take the width of abutment at 3, its height above the springing 10.99, its depth below it 7.01; its mean thickness is $\frac{10.92}{9.9}=1.1$, and its total volume, including a part of the arch is $3\times18\times1.1=59.4$; which combined with the resultant S_6 on the joint marked 3 in Fig. 22 cuts the base, for the first case noted above, only 0.05 outside of the middle third, in the last case 0.18 outside.

The force $Q_2=2.3$ is the largest of the forces Q_1, Q_2, . . . , whence by art. 94, $q=\frac{a}{e}Q_2=9.9\times2.3=22.77$ cubic units of stone. The force q acts on an area of 1.22 square units. There is evidently no danger of crushing from the horizontal thrust around the second crown from the top, as stone will bear on a square unit a pillar of a square unit section and several thousand units high. Similarly for the resultants on all the joints. We conclude that with the backing used, or by increasing the depth of arch ring at the springing about one-third, the arch will be stable. A very slight increase in the width of abutment will prevent any joint from opening.

In the preceding example any unit of length may be taken, as foot, meter, etc.

96. We took the thickness of voussoir at springing as unity. Since ψ should be a very small angle for greater accuracy in the assumptions about center of gravity and the values of q, etc., it may be thought that the thickness is too great. But since multiplying the weights of voussoirs and abutments by the same quantity does not change their ratios, the thickness taken is really immaterial except in finding q, since the centers of gravity are assumed to lie in the same verticals on the vertical projection of a medial meridian section, no matter what the thickness may be.

In truth, the assumption about the position of the center of gravity of a voussoir and load is more nearly realized when the thickness is appreciable.

97. When the soffit and exterior surface of the dome are both surfaces of spheres having the same center A (Fig. 23), the volumes of the voussoirs are

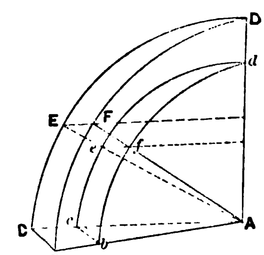


Fig. 23

easily obtained. Thus let $r=AD=$radius of outer sphere; one half its volume is $\frac{2}{3}\pi r^3$. The wedge ABCD is $\frac{1}{n}$ of this volume, if BC is $\frac{1}{n}$ of the circumference $2\pi r$ $\therefore$ volume of wedge $ABCD=\frac{2\pi r^3}{3n}$.

Now divide the altitude AD into m equal parts (2 in the Fig.) and pass horizontal planes through the points of division; the surface of the lune BCD is divided into m equal parts, by geometry, and the pyramids formed on these parts, as bases, with the center of the sphere as the common vertex, are therefore equal.

The volume of such a pyramid, as A—BCEF, is consequently, $\frac{2\pi r^3}{3mn}$.

Similarly the volume of a pyramid of the sphere, whose radius, $r'=\overline{Ad}$ is that of the soffit, and which is bounded by the same planes as any one of the preceding pyramids is, $\frac{2\pi r'^3}{3mn}$, since these pyramids, as $Abcef$, are diminished images of the preceding ones; every corresponding line being diminished in the ratio of r' to r. It follows that if

horizontals are drawn through the ends $f, \ldots,$ of the edges of the second set of pyramids, they will divide the distance $\overline{Ad}$ into m equal parts. The same holds for similar pyramids of any sphere; so that if the center line of the arch ring on the sectional elevation, as de in Fig. 24,

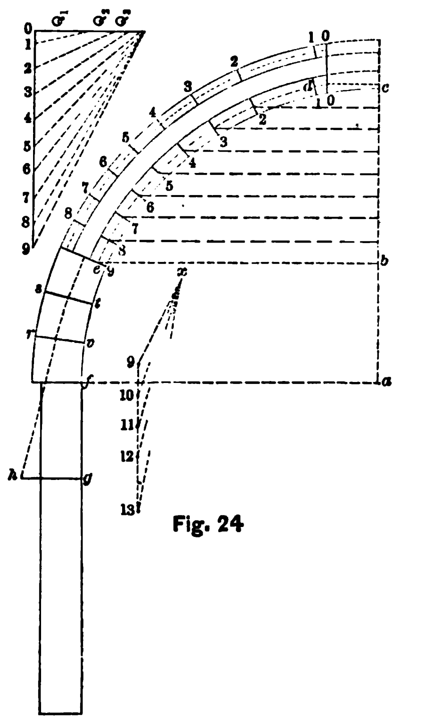


Fig. 24

is divided into equal parts and horizontals be drawn through the points of division to de and the joints be drawn through the latter points and the center of the sphere, the voussoirs so formed are all equal in volume.

Recurring to Fig. 23, we see that the volume of the voussoir Be = difference in volume of the two pyramids $=\frac{2\pi}{3mn}(r^3-r'^3)$.

98. In the open dome however, as Fig. 24, it is not always convenient to divide the rise ac into such equal parts that certain of the points of division will lie on the horizontals through d and e. We proceed then as follows: By geometry the area of the zone formed by revolving an arc as rs (Fig. 24) about the rise ac is equal to the altitude h of rs multiplied by $2\pi r$, r being the radius $\overline{as}$ of the sphere. Pass now two meridian planes through ac, whose included angle is $\frac{1}{n}$ of a circumference. The part of the zone included between them has an area $\frac{2\pi rh}{n}$; so that the pyramid formed on this base with a vertex at a, has a volume $\frac{2\pi r^2 h}{3n}$.

Similarly the pyramid having the part of the zone represented by tv as a base, has a volume, $\frac{2\pi r'^2 h'}{3n}$ where r' = radius at and h' = altitude of arc tv. Therefore the volume of the voussoir $rstv$ included between the meridian planes and the conical joints rv and st is,

$$V=\frac{2\pi}{3n}(r^2h-r'^2h')$$

where r and h are the radius and altitude of the exterior arc, r' and h' of the interior. As before shown, if the altitudes of the type h are made equal in successive arcs, the values h' will all be equal. The divisions into equal altitudes can best be made along the center line in consequence of what follows.

99. Let us refer again to the inner dome de (Fig. 24).

If we pass horizontal planes midway between the horizontals drawn, also pass conical joints through their intersection with the center line de, we divide the previous voussoirs exactly in half, so that *the centers of gravity* of the first voussoirs lie on these supposed intermediate conical joints. They also lie nearly on the center line de, and the error of so regarding them can be made as small as we choose by sufficiently diminishing ψ, the angle included between the two meridian planes and the height of voussoir.

The centers of gravity of the voussoirs will therefore be assumed to be on the center line of the elevation of the medial meridian section de, at the intersections of the horizontals drawn midway between the first horizontals drawn that divide bc into equal parts.

100. Fig. 24 represents a meridian section of the Church of St. Peter, at Rome. The dimensions given by Scheffler, as I understand them, are as follows:

The radius of the soffit is 72 feet, and of the outer surface, $\overline{as}$ = 83.8. At 31

feet above the springing, the structure is composed of two domes, the outer having a thickness of 2.6 feet, the inner being 4.1 thick at d and 5.1 at e, so that the center line de is described from a center slightly below a on the vertical $\overline{ac}$ produced. The dome has an opening at top 12.4 radius, and the lantern supported at the top is equivalent in weight to a block of stone 2.1×56.6, of which the outer shell supports one-third and the inner two-thirds. The first voussoir at the top, in both shells, is made 2.1, horizontal width; the altitude of the center line, $\overline{bc}$, for the part de, is then divided into 8 equal parts and the joints drawn as in the figure, a similar construction applying to the outer shell. The part below the two shells is similarly divided into 3 equal parts. Applying the formula just deduced in Article 98, measuring the altitudes on a drawing to a scale of 4 feet to the inch, we find the volumes of the voussoirs like $rstv$, $\frac{2\pi}{3n}$ 28696, the voussoirs of the outer shell, $\frac{2\pi}{3n}$ 3957, except the top one, whose volume is $\frac{2\pi}{3n}$ 359. cubic feet. The volume of the top voussoir of the inner shell is $\frac{2\pi}{3n}$ 501 cubic feet. The voussoirs 2 to 9 of the inner shell, were each, in turn, assumed to have an outer surface concentric with the soffit, of radii equal to the mean radii of the outer surface for the voussoir considered, *i.e.*, equal to 72 + mean thickness in feet of voussoir. We thus find the volumes of voussoirs 2 to 9 equal to the constant $\frac{2\pi}{3n}$ multiplied in turn by 4695, 4805, 4915, 5000, 5124, 5206, 5267 and 5400.

The volume of the lantern, by the law of Guldinus (art. 95) $= 2.1 \times 56.6 \times \frac{2\pi \times 13.45}{n} = \frac{2\pi}{3n}$ 4794, one-third of which is added to the volume of voussoir 1 of outer shell and two thirds to that of voussoir 1 of the inner shell. The part fgh, 10.3 wide and 23.7 high, has a volume, $10.3 \times 23.7 \times \frac{2\pi \times 77.15}{n} = \frac{2\pi}{3n}$ 56500. This part has not the full width of the bottom voussoirs as drawn in the figure.

We now lay off on vertical lines, the weights just found, omitting the common constant $\frac{2\pi}{3n}$.

The loads affecting the outer shell are laid off to its left; those pertaining to the inner shell just below its center about (not shown in figure).

To be on the safe side, we shall assume that the resultants on the joints from the summit to the joint of rupture are tangent to the center line of the ring. Thus for the outer shell, draw through the points 1, 2, inches, of the force lines, parallel to tangents to the center line at joints 1, 2, . . . (or $\perp$ to radii). These lines cut off successive distances on the horizontal through o, equal to the radial forces Q_1, Q_2, . . . exerted by the successive crowns 1, 2, . . .

We find that below joint 6 there is no longer a radial force needed; so that below that joint the curve of pressures is continued to joint 9 as in a simple arch.

Similar results were found for the inner shell. The centers of pressure on joints 9 of the outer and inner shells are at the outer middle third limit for the outer shell, and slightly above the center line for the inner shell. This necessitates spreading about joints 6, so that the line of pressures there is below the center line, so that the actual horizontal thrust is less than estimated, as stated above.

Now combining the resultants at joint 9 into one, laying off $\overline{x9}$ equal and parallel to it, its position being found by moments, we continue the line of pressures as per dotted line to joint hg. The successive volumes of voussoirs are laid off on the force line 9 . . . 13. This second force diagram is drawn to a smaller scale than the preceding.

101. At joint hg this line of pressures passes outside the joint so that the dome cannot be regarded as sufficiently stable in itself. If rotation occurs the line of pressures would approach the extrados at the summit, the intrados at the joints of rupture and the extrados at joint hg.

By encircling the dome just above the springing by a band of iron of sufficient cross section, stability may be assured. The band may be applied above the springing if desired. It evidently is much less effective in preventing de

formation of the dome if applied below the springing as was done in this case.

The total horizontal thrust of the lune solid (being $\frac{2\pi}{3n}$ × the horizontal component of $\overline{x9}$) is $Q=39600\frac{2\pi}{3n}=13200\psi$ cubic feet of stone $=924.\psi$ tons, if we put the weight of a cubic foot of stone at 157 lbs. $=.07$ ton.

If this is to be entirely destroyed by the iron band, so that the resultants below the springing will all be vertical, we have the strain on the band by art. 94, when ψ is small

$$q=\frac{Q}{\psi}=924 \text{ tons.}$$

Now iron, exposed to a dead strain alone, may safely be subjected to a strain of 7½ tons per square inch; so that the bar may have a cross section of 123 square inches.

If the iron stretches $\frac{1}{12000}$ of its length for every ton per square inch, the ring whose diameter is 168 feet will elongate 0.33 feet, so that the diameter is increased 0.04 feet. There will consequently be a slight deformation of the arch, in consequence the top of the abutment moves slightly outwards, and the pressure on its base is not generally vertical; *i. e.*, the iron band has not totally destroyed the horizontal thrust. The action of the band is like that of a radial force acting inwards and equal, or nearly equal, to the total horizontal thrust.

If the hoop encircles the dome just below the springing of the two shells, it will prevent deformation of the arch also; for there can be no spreading outwards at this point, any tendency that way being met by the resistance of the hoop, which thus supplies sufficient horizontal force to force the line of pressures below it to keep within the joint areas a certain distance, dependent upon the spreading of the arch at the hoop. If this spreading is inappreciable, then the hoop exerts force enough to restrain the line of pressures to the centers of the joints nearly below it.

There is therefore no necessity in the voussoir dome for additional hoops below the first, unless the first is unable to destroy the outward thrust. The problem is then really indeterminate of ascertaining the precise amounts of the strains sustained by the several hoops. The one nearest the joint of rupture of course will sustain by far the greatest part; the hoops, at joints where no spreading would occur, if they were not applied, not sustaining any. The resultants on the abutment joints cannot approach the outer limits without the top of abutment moving outwards; as this is prevented by the top hoop principally, it is evident that the one hoop should be placed not far below the joint of rupture. It would seem best to make these hoops of steel as it does not stretch as much as iron. Wire cables with a means of tightening would be especially convenient.

102. We see that the thrust of the type Q is greatest on voussoirs 1 of both shells. Thus for outer shell, Q_1

$$=16900\frac{2\pi}{3n}=5633\ \psi$$

Thus for inner shell, $Q_1=6000\ \psi$, which multiplied by .07 gives the thrusts in tons. By art. 94, we have $q=\frac{Q_1}{\psi}$ or 394 tons and 420 tons respectively. Now voussoir 1 of outer shell has an area of 6 square feet; the lower voussoir 1 an area of 10.45 square feet, so that the pressures per square foot are 66 and 40 tons respectively, which good stone can stand easily.

The thrusts Q_1 of both shells is probably less than assumed, for the force diagrams indicate a small value for Q_2—in fact for the lower shell Q_2 nearly vanishes—but the compression around the ring of the first crown would necessarily bring the second more in action thereby increasing its thrust. The tendency then is to equalize more nearly the values of the thrusts Q_1, Q_2, . . . than as given by our construction.

103. It is evident that the greatest economy is subserved, by the employment of one or more thin domes to a short distance below the joint of rupture, as is the practice generally in large domes. One shell would suffice if the weight of lantern (if any) could be carried by it; otherwise two or more should be used.

104. The abutment below joint *hg* is counterforted so as to present a greater width than shown in the figure. The introduction of the hoop of course prevents any movement in it, so that the

stability of the whole structure is assured.

105. We shall give now the spherical dome closed at top to illustrate the view taken in art. 93 of the position of the actual line of pressures, besides other points not noticed before. This dome, Fig. 25, has a thickness of one fifteenth the span.

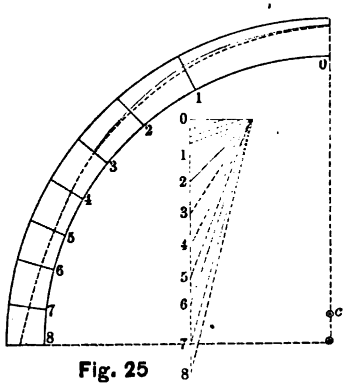


Fig. 25

Divide the altitude of the center line of the ring into eight equal parts, draw horizontals &c. as before. The lune solid is thus divided into eight equal parts which lay off on the force line 0 . . . 8.

Now a dome of this kind fails by rotating about the outer edges of joints at the crown and abutment, and the inner edges about joint 4; each lune solid separating from the others and acting as simple arches.

For a dome then of small stability the line of pressures passes nearly through the top, the intrados edge at joint 4 and the outer edge at joint 8.

For a dome of greater stability its position depends upon the amount of spreading at the haunches, and the consequent rocking at joint 8. If a line of pressures can just be inscribed in certain limits, equally distant from the center, then it is probable, from a consideration of the way in which an arch settles, that the actual line of pressures touches these limiting curves towards the extrados side at the top and abutment, and next the intrados side at the joint of rupture.

Now if such a curve can be inscribed in the middle third no joints will open, and we may conclude that the dome has sufficient stability.

Draw an arc of a circle through the upper limit at the summit and the lower limit at joint 4 with a center c and assume that this arc coincides with the line of pressures a certain distance from the top, the resultants being tangent to it.

Then at some joint as 2 continue the pressure line down to the springing with the horizontal thrust found at 2. If the line so found does not keep within the middle third, let it be commenced at another joint, until one is found that will satisfy the conditions. On dividing voussoir 2 into four others, it was found that a line of pressures, continued as for a simple arch, from where the arc above cuts the upper joint of voussoir $1\frac{3}{4}$, keeps almost entirely in the middle third, as shown by the dotted line: cutting joints 4 and 5 at the lower limits and joint 8 at the outer limit.

This is therefore a *probable* curve of pressures as determined from considerations of how an arch settles.

106. The construction given in art. 93, gives the lines of pressures, that beginning at the summit, first makes Q_1 a minimum, and then, for the same line of pressures in order, Q_2, Q_3, . . . *But this does not make, necessarily, the total thrust*, $Q_1 + Q_2 +$ . . . , *in the lower part of the arch a minimum.* In fact, this total thrust determined in this way, restricting the line to the inner third is found to be $\frac{1}{4}$ to $\frac{1}{3}$ greater than the thrust, determined as follows, that corresponds to the *minimum of the total horizontal thrust*, $Q_1 + Q_2 +$. . . , within the limits taken; as first given, in effect, by Prof. Eddy, in his "New Constructions in Graphical Statics." Take the upper middle third limit as the line of pressures down to a joint ($1\frac{1}{8}$ in this case), where the horizontal thrust may become constant, as for a simple arch, the pressure line below this point remaining within the inner third and just touching the inner limit at some point. The line so drawn coincides nearly with the first below joint 3. As mentioned in art. 93, the thrusts Q_1, Q_2, . . . will, by this construction, be really slightly outside of the inner third limits. The point where the simple arch begins is higher than in the previous cases. If the center line of the arch ring is assumed for the line of pressures, a certain distance from the summit the joint of rupture is lowered,

i.e., a less part of the lune solid acts as a simple arch, and the horizontal thrust is increased.

If the curve of pressures is taken to coincide with an arc starting at the upper limit at the crown as before, and lying below the previous arc, the thrusts Q_1, Q_2, . . . near the crown are lessened; but the total horizontal thrust will be greater than as found in art. 105, as is evident.

Now, of all such arcs it is impossible to say which one, if any, is the true line of pressures, since this is a function of the deformation of the arch.

It may be remarked further that the direction of the thrusts near the summit are most likely more inclined than drawn above, since, by the construction above, a very small crown of voussoirs next the summit exerts a comparatively large thrust; so that the upper crown is compressed sufficiently to bring the next crown more in action, and so on down. If we divide voussoir 1 into four equal ones, we find that the circumferential thrusts *per square unit* on each crown going from the top are proportional to 2.5, 2.3, 1.9, and 1.8 respectively, so that the unit strains decrease going from the summit as stated. It would certainly give a large stability to apply the method of art. 100 and require that the line of pressures so drawn should keep within the inner third below the joint of rupture. The arch cannot fall unless the line of pressures nearly touches the contour curves; hence, of the infinite number of positions it can take, it would seem that the first just drawn should offer sufficient stability, though it may not be exactly the true one.

107. If there is a weight at the summit, since its effect is to move the point o of the force line upwards and thus increase the thrusts in the top crowns, the line of pressures with a constant horizontal thrust must commence nearer the summit than before. The reverse happens when any weight is taken from the top of the dome, as for an open dome. In the former case, a small weight at top will cause nearly the whole of the lune solid to act as a simple arch.

CONICAL DOME.

108. Let Fig. 26 represent a meridian section of a conical dome. Divide the altitude $\overline{cd}$ into eight equal parts, and pass horizontal planes through the points of division giving the joints 1, 2, . . . Midway between these joints draw the horizontal radii of the center line shown by the dotted lines. If we call the horizontal thickness of the ring $ab=t$, the vertical distance between any two joints h, and the mean radius of the center line between these joints by r, we have for the volume of the voussoir included between the two joints and two meridian planes, making an angle ψ with each other,

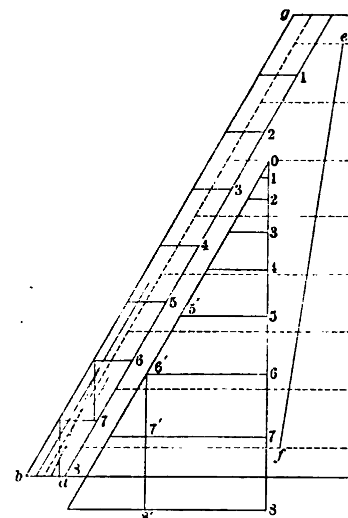


Fig. 26

$$V=htr\psi,$$

according to the law of Guldinus.

The weights of the successive voussoirs vary therefore as r. Therefore lay off on the vertical force line, 0 . . . 8 successive distances, proportional to the dotted radii, beginning with the first voussoir. The line $\overline{ef}$ cuts off $\frac{1}{4}$ of these radii counting from $\overline{cd}$.

Next draw the line $\overline{06'} \parallel \overline{bg}$ and pass horizontals through the points 1, 2, . . . of the force line. If the resultants on the joints are assumed to coincide with, or be parallel to the center line, the hypothenuses of the triangles just formed represent the strains on the joints. Thus $\overline{06'}$ is proportional to the strain on joint 6, and $\overline{66'}-\overline{55'}$=horizontal radial force Q exerted by the sixth ring.

Now if the arch is assumed practically undeformable, the center line is the line of pressures, and the force diagram is sufficiently correct.

For compressible materials however, if the top and bottom are kept from moving horizontally, the middle of the dome from the compression of its rings tends to move inwards, which requires that the line of pressures there lies outside of the center line, and, as a consequence, inside the center line at top and bottom. Its exact position is indeterminate and is dependent upon the method used in building (whether with or without centers or supports) also on the fitting of the stones. If the abutments spread, as probably happens, the wedge-shaped solids, included between the meridian planes, tend to separate next the base; and the inner edges of joints next the abutment must bear the most, *i.e.* the line of pressures there is inside of the center line.

We see, therefore, that the assumption that the center line is the line of pressures, is on the side of safety, so that the force diagram above may be used in determining the dimensions of abutments or hoops to withstand the horizontal thrust.

109. The least horizontal thrust, consistent with no joints opening, may be found as follows (also see Eddy's New Constructions, &c.):

Assume the centers of gravity of the voussoirs to lie on the center line midway between the joints, which hypothesis can be made as near the truth as we choose by sufficiently diminishing ψ. Now combine the thrust at joint 6 , 06 (as found above), supposed to act at the exterior middle third limit, with the weight of voussoirs below it; if the resultant, $\overline{08'}$, strikes the base at the inner limit, the horizontal thrust, $66' = 88'$, is a minimum in order that no joints open. The dome, however, may be perfectly stable when joints open, so that the smallest thrust consistent with stability is less than the above. In the above figure it was found, on a second trial, that at joint $6\frac{1}{4}$ the horizontal thrust first became constant, so that the part of the dome below it exerted no circumferential thrust on the hypothesis. The point of contact with the outer limit is probably nearer the middle of the dome section, as the yielding is greatest there.

The two constructions given thus indicate limits between which the true thrust is found.

110. The preceding construction is the concluding one for this series.

In what has preceded we have been careful to state clearly the hypotheses which are introduced, and to criticise them in the light of both theory and facts.

It is not difficult for a mathematician, having made certain assumptions, to develop, perhaps, an elegant theory, quite dazzling to the inexperienced; but the engineer requires that the hypotheses be proved correct, or at least nearly so, before he can use them in practice.

So far from enforcing an unproved hypothesis by confident assertions, the writer may have leaned too much the other way by too often employing the word "probable" where a stronger adjective would have better suited, especially in the parts referring to true curves of pressure. It is evident that the difficulty of locating the real pressure curve in a simple arch is increased for its combinations in a much greater ratio.

It has been pointed out, however, that the arch may satisfactorily be tested without its true position being known, from considerations of arches at the limit of stability. In fact, it is believed that this true position is closely approximated to by drawing a curve of press ures, within limits, approximately equidistant from the center line, corresponding to the maximum and minimum of the thrust (see art. 27).

This principle receives corroboration from certain constructions pertaining to the solid arch, which apply to the voussoir arch when the stones are cut perfectly, the mortar joints very thin, and no joints open, the spandrels not being supposed to exert any resistance.

In such cases, the pressure curve for the solid arch is identical with that for the voussoir arch.

This view will be clearly exposed in a paper that will follow this, on "Solid Arches," to which the reader is referred.

THE AMERICAN IRON TRADE IN 1878.

From "The Bulletin."

We are again placed under obligations to our special correspondents for information which enables us to give our usual annual summary of the condition of our iron and steel industries at the beginning of a new year. The information thus obtained has been analyzed with care, and so much of it as relates to pig iron has been embraced in a table, which is herewith presented, in connection with comparative figures for 1877, which are taken from our last annual report. As more detailed information than has yet reached us will be received in due time from individual producers, it is to be expected that the figures of production, stocks, etc., which will appear in our next annual report will be slightly different from those of the table; but the general results in districts and States, and in the country at large, will not be materially altered. The figures now given in the table and in the remainder of this summary are partly estimates, but they are estimates made not by us but by practical ironmasters, each one of whom testifies only concerning his own district and of his own knowledge. We are thus particular in giving all the facts which affect the tabulated and other statements which are here submitted, that the trade may know what degree of credibility to attach to these statements. We think we are justified in claiming for them substantial accuracy in all essential particulars.

Pig Iron.—The production of pig iron in the United States in 1878 was about 70,000 net tons (2,000 pounds) in excess of the production of 1877. In 1877 there were produced 2,314,585 tons, and in 1878 the production was 2,382,000 tons. A reference to our table will show which States increased and which decreased their production during the past year. Pennsylvania shows an increase of over 100,000 tons, while Ohio shows a decrease of over 30,000 tons. In 1878 Pennsylvania made more than 50 per cent. of the total production of pig iron in the United States.

The pig iron produced in 1878 we classify as follows according to the kind of fuel used, and give comparative figures for preceding years:

Kind of Fuel.	1878.	1877.	1876.	1875.
Anthracite	1,039,000	934,797	794,578	908,046
Bituminous	1,093,000	1,061,945	990,009	947,545
Charcoal	250,000	317,843	308,649	410,990
Total	2,382,000	2,314,585	2,093,236	2,266,581

At the close of 1877 there were in the United States 716 blast furnaces, of which 270 were in blast and 446 were out of blast. At the close of 1878 there were 700 furnaces, of which 260 were in blast and 440 were out of blast. These figures, taken in connection with those of production above given, indicate an increased average production of the active furnaces in 1878 over 1877. During 1878 there were 18 furnaces torn down, burned down, or otherwise taken out of the active list, and there were two new furnaces erected—one in Ohio and one in Tennessee, showing a net decrease in the year of 16 furnaces.

The stocks of pig iron on hand and *unsold* at the close of 1877 amounted to 642,351 net tons. At the close of 1878 they were very much less, about 516,000 tons. At the close of 1876 stocks amounted to 686,798 tons. These figures show a decrease in stocks of 44,447 tons from 1876 to 1877, and of 126,351 tons from 1877 to 1878. In the whole of Pennsylvania there was a decrease in 1878 of about 30,000 tons, although in the Lehigh Valley there was an increase of about 13,000 tons. There was a decrease in New York of about 24,000 tons; in Ohio of about 17,000 tons; and a marked decrease in Michigan, Missouri

and some other States. The shrinkage in stocks was remarkably uniform in all iron-producing States, and it is very significant of the caution which characterized this branch of the iron trade throughout the year. No State materially increased its stocks in 1878.

The pig iron on hand and unsold at the close of the year, and at the close of other recent years, was made with fuel as follows:

Kind of Fuel.	1878.	1877.	1876.	1875.
Anthracite...........	192,000	239,493	268,122	274,743
Bituminous..........	150,000	156,818	174,302	165,482
Charcoal...........	174,000	246,040	244,374	320,683
Total..............	516,000	642,351	686,798	760,908

The consumption of pig iron in 1878 was apparently greater than in 1877. Production was greater and the reduction of stocks was also greater. There are unknown quantities that elude the grasp of the statistician in attempting an estimate of the consumption of pig iron in any given year, so that an exact statement is never possible; but with the two most important elements of the problem given—production and stocks—an approximation to a correct result is not difficult. In 1878 production was increased about 70,000 tons and stocks were decreased about 126,000 tons. As our imports of pig iron in 1878 did not vary greatly from the imports of 1877, and as our exports of pig iron were less, and as we have failed to detect any speculative movements in 1878 that would withdraw large blocks of pig iron from the market, we think it entirely safe to assume that we increased our consumption of pig iron in 1878 over 1877 about 195,000 tons.

Manufactured Iron and Steel.—It follows from what has just been said that the rolling mills of the country were more steadily employed in 1878 than in 1877. But of this there is abundant and positive evidence additional to that which is based upon the increased production of pig iron and the decrease in pig iron stocks. Pittsburgh rolling mills and steel works were more generally employed in 1878 than in 1877. Iron shipbuilding is known to have been more active in 1878 than in 1877. The elevated railroads of New York have required a large quantity of finished iron. Iron bridge building, on home and foreign account, has shown an improvement over 1877. Government and other public buildings have been pushed toward completion, and have required many thousand tons of iron. There was a large increase during the year in the number of locomotives and railroad cars manufactured. There were more miles of railroad constructed in 1878 than in 1877. Ten out of eleven of our Bessemer steel works were busy during the whole year in the production of steel rails, and the manufacture of iron rails during the year was certainly as active as in 1877. Other steel works are known to have been busy. The consumption of old iron rails in 1878 is known to have been quite large, both in iron rail mills and in other rolling mills. The prosperity of our farmers, who have had two good consecutive crops, for which they have found a market, and the general revival of business throughout the country in the latter half of 1878, were influences which favorably affected the iron trade of the country. The railroads were generally well employed throughout the year.

Iron and Steel Rails.—We have above referred to the improvement in this branch of the iron trade during 1878, but we now add more specific information concerning the manufacture of both iron and steel rails in that year. In 1877 the production of iron rails amounted to 332,540 net tons, which was a great reduction from the product of 1876, which was 467,168 tons. In 1878 this decline was wholly arrested, the production during the year being fully as great as in 1877, and probably a few thousand tons greater. In 1877 the production of Bessemer steel ingots was 560,587 net tons, and the production of Bessemer rails was 432,169 tons. In 1878 the

production of ingots was about 730,000 net tons, and the weight of Bessemer rails produced was about 600,000 net tons. For these statements we have the best authority. Putting the iron and steel rail products of the year together, we have in round numbers a total of 930,000 net tons as the rail product of the year. This product has only once been exceeded in our history, in 1872, when the product reached 1,000,000 net tons. In 1879 we will probably equal even that immense product.

Production of Lake Superior Iron Ore.—The production of iron ore in this celebrated mining field was much greater in 1878 than in 1877. The shipments in 1878 amounted to about 1,125,000 gross tons, against 960,982 tons in 1877. This large production has only once been equaled in the history of the district, in 1873, when the shipments amounted to 1,167,379 tons.

Railroad Construction.—The mileage of new railroads constructed in the United States in 1878 was very much greater than in 1877, and greater than in any other year since 1873. The *Railroad Gazette* publishes detailed statistics for 1878, which show that in that year there were built and track laid upon 2,688 miles of new railroad, against 2,177 miles in 1877, 2,657 miles in 1876, 1,758 miles in 1875, 2,305 miles in 1874, and 4,069 miles in 1873. The recovery in 1878 from the depression in railroad building which existed in 1877 was marked and decided, and we need scarcely add forms one of the most encouraging signs of the day.

Prices.—The only discouraging feature of the iron trade of 1878 is that which relates to prices. In both iron and steel rails there was an improvement, but in pig iron and bar iron there was a decline from the exceptionally low prices of 1877. The following table shows the range of prices throughout the year:

MONTHS.	No. 1 Anthracite Foundry Pig Iron in Phila.	Iron Rails at Works in Pa.	Bessemer Steel Rails at Works in Pa.	Best Refined Bar Iron in Phila.
January	$18 50	$32 50	$41 00	$44 80
February	18 50	32 50	41 50	44 80
March	18 50	32 50	41 50	44 80
April	18 50	32 50	42 00	44 80
May	18 00	32 50	43 50	44 80
June	17 25	32 50	43 00	44 80
July	17 25	33 00	43 50	44 80
August	17 50	33 00	42 50	44 80
September	17 50	33 00	42 50	44 80
October	17 00	33 00	42 50	42 56
November	16 50	33 00	42 00	42 56
December	17 00	33 00	41 00	42 56
Average	$17 67	$32 75	$42 20	$44 24

The decline in the price of pig iron during the year was $1.50 a ton, and on bar iron it was one-tenth of a cent per pound, or $2.24 a ton. Iron and steel rails sold during the year at average prices which were higher than quotations in January.

Conclusion.—The old year, take it all in all, was a more active and a more prosperous year for the American iron trade than either 1876 or 1877. There was an improvement in the demand for all iron and steel products and prices, although not satisfactory, were well maintained except in the case of pig iron. This branch of the trade has had a hard struggle, and many furnaces have been run without profit. The new year opens with the promise of a still more active and more prosperous business for our iron and steel manufacturers than the old year gave to them. Business is in fewer hands and home competition cannot be so desperate as it has been. Foreign competition is for the present not to be dreaded. Prices it is hoped are at last at the lowest point to which they can possibly fall, while the unmistakable and undeniable revival of general prosperity throughout the country gives every assurance of a continuance of the increased demand for iron and steel which characterized the old year.

THE MISSISSIPPI AS A SILT BEARER.

By ROBERT E. McMATH, C. E.

Written for VAN NOSTRAND'S MAGAZINE.

AMONGST the topics engaging the interest of the public, perhaps no one is more worthy the attention of the engineering profession than the discussion of matters relating to the Mississippi River, the improvement of its navigation, the restriction of its floods, the fixation of its bed, and the drainage of its border lands.

These topics are important because of many considerations of public economy, but aside from these, the whole subject is of peculiar interest to the Civil Engineer as being a field, which, in the near future, will afford opportunity for the exercise of the highest order of talent, and for the execution of many very important works.

The profession at large is therefore interested, and should carefully examine all facts, theories and plans advanced; for to engineers the nation must look for that dispassionate discussion and formation of opinion which ought to precede the adoption of any scheme. Schemes looking to this field are already in existence, and, however diverse they may be in origin and ostensible purpose, are alike in this, that they essentially are efforts to monopolize the field for private, or corporate, glory and profit.

To facilitate examination of the various schemes which have been, and plans which may be presented, it will be profitable to inquire into the resources of reliable information upon the subject of the great river.

In a professional, or scientific point of view the information is meager; for the reason that the subject is beyond the reach of private investigation, and those made by the General Government have not been systematically kept up, because Congress never has realized the importance of such studies, and has almost invariably refused to grant the necessary appropriations.

In the way of general information much is in existence scattered through the public documents and periodicals, but mostly unavailable, because few have access to complete sets of public documents, and still fewer to files of the numerous periodicals in which fugitive articles have been published. The reports, of Lee, Long, Cram, Kearney, and Shreve, are well worthy of collection and publication for the experiences related, the facts recorded, and the opinions given. Material of like value is scattered through reports made by engineers and others connected with the Levee works executed under State or local organizations; but this too is practically unavailable. Of formal reports available to the ordinary professional reader we have, Ellet on the Ohio and Mississippi Rivers, Humphreys and Abbot's Report on the Mississippi River, and the reports of U. S. Engineers at various dates, since the resumption of civil works at the close of the war of the rebellion, in Reports of Chief of Engineers and other official documents.

None of this material is formally scientific except the report of Humphreys and Abbot, and some of the reports of the inspecting engineer of the jetty works at the mouth of the Mississippi; scientific, I mean in the sense, that in addition to statements of conclusions and reasonings, the data upon which they are founded are also given.

Quite an extensive literature is extant, composed of essays and arguments for and against certain projects of improvement. These arguments are, professedly at least, based on actual observation, and personal acquaintance with the river and the problems involved in a proposition for its improvement.

Personal knowledge, however accurate, cannot furnish a foundation upon which to build a demonstration of any general fact. Its province is limited to practical work, either of improvement or investigation.

The discussions referred to have not therefore added appreciably to our knowledge of the subject, because consisting chiefly of assertions, assumptions, and speculations.

Facts in dry detail, recorded as observed without reduction, interpolation, or generalization, are greatly needed that

every man may scrutinize for himself the foundation of opinions, assertions and assumptions.

In this way only may we hope to establish certain general principles upon which practical schemes may be profitably discussed.

Such statements of fact, when accompanied by an exposition of the mode and means of observation, particularly when there is any novelty in either mode or means, are entitled to the respect and confidence given an honest and capable witness. But beyond the range of statements of fact the weight of personal authority ceases, and the man who is unwilling to submit his every opinion and conclusion to the test of unsparing criticism has no business to enter the field of scientific discussion.

It is not the purpose in this paper to attack, or defend, any man's work, theory, or statement of fact, but rather, to bring prominently forward facts which have unaccountably been left unnoticed hitherto, to point out in what respects our present knowledge is deficient, to suggest means by which the deficiency may be remedied, and to draw, so far as the facts presented will warrant, practical conclusions to guide practical conduct, until further investigation shall show a truer and better way.

Without further preface, I introduce a tabulated statement of the areas of a considerable number of cross-sections of the Mississippi, between the Missouri and Ohio Rivers, showing the stage of water, locality and date of section, the actual area found, the width at the date and stages and the mean depth of section. The division of the table headed "Low water," contains the areas, widths, and mean depths that would result from reducing the data of the preceding columns to the low water of 1863, in all sections above and including Horsetail No. 2, and to the low water of 1872 in all sections below Horsetail.

The low water of 1872 reads 1'.87 above the low water of 1863 upon the present iron gauge at the foot of Walnut Street, St. Louis, (as it is commonly described, though really it is near midway between Market and Walnut) and 2,50 upon the gauge at the St. Louis Elevator, which was the one used by the Signal Service Observer in 1872.

TABLE I.

Date.	Locality.	Stage.	Actual.			Low Water.				Remarks.
			Areas.	Width.	Mean Depth.	Areas.	Width.	Mean Depth.	Year.	
			sq. ft.	ft.	ft.	sq. ft.	ft.	ft.		
July 6, 1878	Cahokia Chute....	22.0	39,346	1798	21.88	4,598	902	5.09	1863	by Wm. Popp.
"	Arsenal...........	22.0	38,158	1915	19.92	3 875	830	4.67	"	"
"	Sum of both Chutes	22.0	77,504	3713	20.87	8,473	1732	4.89	"	"
May 17, 1873	Cliff Cave.........	21.8	73,664	3440	21.40	9,016	2570	3.51	1872	by J.D. McKown
May 28, 1872	Carondelet.........	20.0	63,311	2806	22.56	14,839	1480	10.26	1863	by Capt. Allen.
July 23, 1873	Brickey's Mill.....	19.6	54,152	1850	29.30	21,595	1482	14.57	1872	by J.D. McKown
May 6, 1873	Horse Tail Bar, 1..	18.5	61,725	4800	12.86	0	0	0	1863	"
"	" " 2..	18.5	55,211	4250	12.75	157	90	1.74	"	"
Oct. —, 1837	Bissell's Point.....	16.8	53,253	1835	29.02	31,380	1145	27.44	"	by Lt. R. E. Lee.
April 9, 1872	Horse Tail Bar, 1..	16.5	74,536	4800	15.53	3,694	2110	1.75	"	by S. E. McGregory
"	" " 2..	16.5	66,948	4250	15.51	5,429	1890	2.87	"	"
July 13, 1874	*1 Mile above Ohio	14.54	39,508	2475	16.3	7,078	1520	4.67	1872	by J.D. McKown
June 5, 1874	*Philadelphia Point	11.75	42,187	3740	11.20	12,807	2002	6.39	"	"
Nov. 25, 1876	Cahokia Chute....	10 5	24,492	1580	15.50	11,332	1062	10.67	1863	by Wm. Popp.
"	Arsenal...........	10.5	21,515	1898	11.33	4,078	835	4.88	"	"
"	Sum of both Chutes	10.5	46,007	3488	13.19	15,410	1897	8.12	"	"
Aug. 23, 1873	*Above Chester ...	10.25	26,912	1740	15.5	14,986	850	17.0	1872	by J.D. McKown
Oct. —, 1843	Bissell's Point.....	10.06	33,320	1980	16.77	15,324	1500	10.21	1863	by Capt. Cram.
"	Venice Ferry.....	10.06	34,287	2366	14.50	13,091	1848	7.08	"	"
"	Bloody Island, East	10.06	16,071	1373	11.70	5,609	707	7.91	"	"
"	" " West	10.06	25,241	1721	14.66	9,538	1400	6.81	"	"
"	Sum of both Chutes	10.06	41,312	3094	13.35	15,147	2107	7.19	"	"
Sept 10, 1872	Penitentiary Point.	9.08	47,395	2465	19.23	25,552	1784	14.32	1872	by Clem'nt Smith

TABLE I—(*continued*).

Date.	...er.		Remarks.
	Depth.	Year.	
Sept. 10, 1872	.46	1872	byClem'ntSmith
"	.08	"	"
Sept. 11, 1872	.35	"	"
"	.58	"	"
Sept. 26, 1872	.75	"	"
Sept. 27, 1872	.90	"	"
—— 1843	.63	1863	by Capt. Cram.
—— "	.21	"	"
—— "	.08	"	"
Oct. —, 1846	.45	"	byHenry Kayser
"	.72	"	"
"	.90	"	"
Sept. —, 1847	13	"	"
"	94	"	"
"	.68	"	"
Oct. 16, 1872	.65	1872	byClem'ntSmith
Oct. 17, 1872	.54	"	"
"	.37	"	"
"	.24	"	"
Dec. 9, 1870	.90	1863	byS.E.McGregory
Oct. 16, 1877	.53	"	by Wm. Popp.
Dec. —, 1844	.64	"	by Capt. Cram.
"	.19	"	"
"	.84	"	"
Oct. 23, 1873	.01	1872	byJ D McKown
Sept. 30, 1838	81	1863	byCapt. R. E. Lee
"	.34	"	"
Nov. 14, 1872	.87	1872	byJ.D. McKown
Oct. 31, 1872	.49	"	"
Nov. 20, 1872	.14	"	"
—— 1842	.58	1863	by Henry Kayser
Dec. 4, 1874	.56	1863	byJ.D. McKown
Dec. 5, 1874	.10	"	"
"	.25	"	"
Sept. —, 1839	.66	"	byCapt. R. E. Lee
"	.06	"	"
"	.92	"	"
Jan. 21, 1861	55	"	byC'y Eng'r St. Louis
"	.12	"	"
"	86	"	"
"	93	"	"
"	.33	"	"
"	.50	"	"
"	.95	"	"
Feb. 9, 1861	.35	"	"
"	17	"	"
Feb. 7, 1861	.17	"	"
"	.66	"	"
"	.56	"	"
"	.85	"	"
—— 1865	.64	"	"
—— "	.21	"	"
—— "	31	"	"
—— "	.80	"	"
—— "	83	"	"
—— "	.52	"	"
—— "	.19	"	"
—— "	.35	"	"
—— "	.70	"	"
—— "	00	"	"
—— "	.67	"	"
—— "	.61	"	"

The above table is arranged, in order, according to the stage of water when the section was measured. An arrangement which is in this case allowable, for the contributions by tributaries, except the Maramec and Kaskaskia, are too small to affect the areas, and these excepted tributaries would not noticeably influence the areas when at a low stage. All the sections which are reduced to the low water of 1863 are located above both these main tributaries, also the Cliff Cave section, and those marked with a * are below the Kaskaskia and therefore include both.

A careful examination of the column of actual areas, after making allowance for the ordinary causes of variation, will suggest that the area does not diminish, when the river approaches a low stage, so rapidly as would naturally be expected.

See that part of the table covering stages ranging below 6½ feet and compare with areas at stages between 6½ and 12 feet.

With our suspicions awakened by the study of the actual areas, we turn to the column of reduced areas at low water, and comparing these, as deduced from high and low stages, it is immediately apparent, that the areas deduced from low stage sections are much larger than those obtained from high stage sections. The suspicion then becomes certainty that important changes occur at the bottom of the river, which in some way accompany the change of stage.

While a general study of the whole list leads to the above conclusion, a particular comparison of the columns of areas and widths of the low water sections shows that the areas also increase, within limits, as the width diminishes. This suggestion is chiefly based upon the sections made at St. Louis in 1861 and 1865, and is confirmed by the sections at Carondelet, Brickey's Mill, and others made at the higher stages. It will be noticed that, in nearly every case in which the deduced low water area exceeds 15000 square feet the low water width is 1600 feet or less, or else that the differences of width at the higher and extreme low stage are not great.

Restating our conclusions in other terms we have learned, that while the bed of the river ordinarily rises and falls in some unknown relation to the rise and decline of stage, there is an adjustment of widths and form of section at high and low stages by which this change can be reduced to a minimum, if not prevented altogether.

At this point it is advisable to introduce another tabular statement, derived from the preceding, by selecting those sections which have for any reason been repeated, and comparing the areas found by these repeated measurements. In this table the geographical order of arrangement is followed.

(*See Table on following page.*)

At a first glance, this table may seem inconsistent with the conclusion stated above, but if the sections at Arsenal and Cahokia chutes, Horsetail and Brickeys, which embrace shorter periods than most of the others, are examined, they will be found to abundantly confirm it. The other cases only point to the fact that we have not yet taken notice of all the elements of the problem; in other words, our statement is incomplete.

Strictly speaking, our hypothesis requires comparison of sections taken at short intervals, so that each low water area may be compared with those deduced from the preceding and succeeding high waters; indeed, the effect of each wave of rise and fall should be studied.

Obviously, the rise and fall of the bottom, which we have noted, is connected with the fact that our river is a silt-bearer, which is excessively turbid in times of flood, and only moderately so at low stages. We may, therefore, suspect that these changes at the bottom depend upon the variations in the amount of silt carried by the river at different stages and seasons. Hence we infer: *First*, that after a flood the removal, by the less turbid waters of a declining stage, of the deposit made in the bed during the flood, is a work in which time is an important element; consequently, that a rapid decline from a high stage is unfavorable to the clearing of the river bed. *Second*, that in floods of equal height, coming from different sources, or from the same source at different seasons, the amount of deposit will differ, if the tributaries bring in dissimilar quantities or qualities of material.

Since the Missouri water was found, by the engineers of the St. Louis water-

Table II.

Date.	Locality.	Stage.	Actual.			Low Water.		
			Areas.	Width	Mean depth.	Areas.	Width	Mean depth.
Oct. —, 1837.	Bissell's Point...........	16.8	53,253	1835	29.02	31,308	1145	27.34
" , 1843.	"	10.06	33,220	1980	16.67	15,324	1500	10.21
Dec. 9, 1870.	Bischoff's Dike..........	7.3	29,940	1430	20.94	19,520	1310	14.90
Oct. 16, 1877.	"	7.0	18,909	1510	12.52	7,677	1388	5.53
Oct. —, 1843.	Venice Ferry.......... ..	10.6	34,287	2366	14.50	13,091	1848	7.08
Dec. —, 1844.	"	7.0	44,447	2376	18.71	27,815	2200	12.64
Sept. 30, 1838.	"	6.8	28,910	2587	11.17	15,630	1320	11.81
——— 1842.	"	6.0	32,790	2684	12.22	17,016	2244	7.58
Dec. —, 1844.	Kerr's Island to Gingras..	7.0	33,072	2794	11.81	13,822	2662	5.19
Sept. —, 1839.	" " ..	4.8	36,732	3260	11.27	22,635	2614	8.66
Oct. —, 1843.	Vine Street	10.06	41,312	3094	13.35	15,147	2107	7.19
——— , 1865.	"	0.0	—	—	—	29,285	1850	15.83
Dec. —, 1844.	Market Street...........	7.0	53,820	3740	14.39	29,180	3300	8.84
Sept. 30, 1838.	"	6.8	30,600	3511	8.71	9,078	2719	3.34
Jan. 21, 1861.	Lombard Street	3.25	25,205	1975	12.76	19,137	1750	10.93
——— 1865.	"	0.0	—	—	—	20,677	1590	13.00
Feb. 7, 1861.	Miller Street	2.19	20,726	1680	12.34	17,371	1370	12.66
——— 1865.	"	0.0	—	—	—	20,845	1645	12.67
July 6, 1878.	Cahokia Chute..........	22.0	39,346	1798	21.88	4,598	902	5.09
Nov. 25, 1876.	"	10.5	24,492	1580	15.50	11,332	1062	10.67
July 6, 1878.	Arsenal Chute...........	22.0	38,158	1915	19.92	3,875	830	4.67
Nov. 25, 1876.	"	10.5	21,515	1898	11.33	4,078	835	4.88
July 6, 1878.	Both Chutes	22.0	77,504	3713	20.87	8,473	1732	4.89
Nov. 25, 1876.	"	10.5	46,007	3488	13.19	15,410	1897	8.12
May 6, 1873.	Horsetail Bar, No. 1.....	18.5	61,725	4800	12.86	0	0	0
April 9, 1872.	" "	16.5	74,536	4800	15.53	3,694	2110	1.75
May 6, 1873.	Horsetail Bar, No. 2.....	18.5	55,211	4250	12.75	157	90	1.74
April 9, 1872.	" "	16.5	66,948	4250	15.51	5,429	1890	2.87
July 23, 1873.	Brickey's Mill...........	19.6	54,152	1850	29.3	21,595	1482	14.57
Sept. 10, 1872.	"	9.08	37,565	1683	22.32	23,157	1356	17.08

works, in 1867, to carry in suspension nearly 2½ times the amount of sediment carried by the water of the Upper Mississippi, when the waters of the two streams were running side by side, with a common velocity, past Bissell's Point, we may safely conclude that the section of river below the confluence of these two streams is well calculated to test the inferences stated above, the test needing only reliable gauge-records for the months preceding and including the time when the sections were measured. Taking the sections at Bischoff's Dike, which present comparative areas that are not explainable by difference of stage, we will seek in the gauge record at St. Louis for the years 1870 and 1877 an explanation of the great difference in the areas at stages so nearly alike. A difference which affected not only this section, but in like degree a stretch of river at least two miles in length, or the whole extent covered by the special survey to which I am indebted for these facts.

In 1870 the highest water (26′.21) was of the date of April 16th. From that date the river declined slowly, arriving at 6.17 September 3d; then, rising slowly, it reached 17.83 November 5th, and fell to 7.3 December 9th, the date when the section was measured in 1870. In 1877 the river was high during the latter half of April, and through May, June and July. The highest water of the season was July 4th (26.55), whence it rapidly declined to 7.00 early in October, the section being remeasured October 16th.

In the former case, 7 months and 23 days passed between the day of highest water and that of sounding the section, in the latter but 3 months and 12 days.

The suggestion that time is an important element is verified in this case, and probably the other suggestion also, for the absence of a June rise in 1870 is strongly indicative of small contributions by the Missouri for that year. ×

The Carrollton sections of 1851 were made at high and low stages, and conse-

× Basic paragraph p 226

– [illegible] follow

quently ought to show more decided changes of areas than the other sections, which were made at mean stages. (For reasons see earlier pages of Chapter 4 of Mississippi report.)

Without entering into details, a study of the table will confirm the conclusions at which we have arrived from other evidence, but no such conclusion could have been reached from a study of the Humphreys and Abbot data alone, for there are well marked exceptions to the general conformity to the rule for which no explanation can even now be given.

The table proves that the bottom changes attending variation of stage, which we have found to be a prominent physical fact in the section of river between the Missouri and Ohio rivers, have a more obscure but real existence throughout the Lower Mississippi. And the well-known fact that the depths upon the bars, at the mouths of the several passes, is greater when the river is at a low stage than during floods demonstrates that the selfsame phenomenon extends to the very mouth of the river.

Interrupting for a time our prescribed course, the attention of those who have been interested in the discussion as to the cause of reported shoalings below Bonnet Carre crevasse and below Cubit's Pass and the Jump, is called to the possibility that the facts upon which the discussion is based could as readily be found, if sought for at the proper time of year, above as below the crevasses or outlets which, by some, are charged with having produced the reported shoalings. Until it is conclusively proven, by soundings repeated at various times, and at different stages of water, particularly low waters, that the reputed shoalings are permanent, and not those attendant upon a high river, arguments based thereon can have no legitimate place in a discussion of the merits of outlets.

Fallacies arising from honest misinterpretation of facts are as misleading as those which have a purely speculative, or even feigned origin. Prof. Forshey, in the paper previously quoted, says (page 25): "It has 180,875 square feet of cross-section, being 2735 feet wide at high water and 2152 feet at low water. It has increased 13,646 feet in section in 23 years." The latter sentence certainly contains one of these honest misinterpretations which might lead to serious results if carried into the category of unquestioned facts.

Returning to our special topic, we may receive as proven by the testimony of a considerable number of sections taken, out of a list of nearly three hundred, so as to cover:

First. The river passing out of a narrow gorge into a wide expanse; at Bissell's Point, Venice Ferry and Kerr's Island to Gingras. (The conditions just mentioned existed when the sections were made, but not now.)

Second. An extended narrow channel of nearly uniform width; in front of the city of St. Louis.

Third. An extremely wide flat reach; Horsetail Bar.

Fourth. A straight reach of moderate width; near Brickey's Mill.

Fifth. A wide, straight reach; below Vancil's Landing.

Sixth. Sundry sections where the river is divided by islands.

Seventh. A few scattered sections in that part of the river which is subject to the influence of back water from the Ohio.

Eighth. The river passing around a sharp bend; as near Carrollton, La.

These various conditions fairly represent the variations to which the river is subject. And a phenomenon which is found to occur under all these conditions may safely be received as a general fact, resulting from an ever-acting cause and governed by determinate law.

Knowing the existence of the general fact, we now may consider how to pass from this general to detailed knowledge of how the results are brought about, and thus to the law of the changes noted.

I intend to remark chiefly upon modes of fluid operations. From what has been said of the value of hydrography and hydrometry, as usually understood, when applied to silt-bearing streams, it is obvious that improved methods must commence with the field surveys.

Since the facts we seek are general, they can be studied to as good advantage at localities of limited extent as by extended surveys, provided a sufficient number of localities be chosen for special study, and the selection be made so as to cover diverse conditions of flow. Also, since we have learned that time, stage,

and source of waters are elements in the variations of the bottom phenomena, frequent repetitions of some parts of the field work will be of prime importance.

It will be readily admitted, that none of the following observations and determinations can properly be omitted, if the purpose be a thorough study of a silt-bearing stream, and doubtless it will be found necessary to add to the list:

First. Stage of water in the main river and important tributaries observed simultaneously. The relative elevations and distances between gauges should be known.

Second. Shore line changes, whether of main banks or bar outlines.

Third. Repeated located soundings upon fixed lines.

Fourth. Cross sections plotted and areas calculated.

Fifth. Lateral movement of cross section determined.

(*a*) With reference to center of figure;

(*b*) With reference to center of moment of stream.

Sixth. Determination of bottom velocities.

Seventh. Determination of mean velocities.

Eighth. Determination of volumes at all stages.

Ninth. Determination of quantity and quality of material in suspension at various depths.

Tenth. Determination of quantity and quality of material drifted along the bottom.

The above observations should be continued so as to include two low water periods and the intermediate high water, in other words, about fifteen months; beyond that time more or less of the list could be dropped.

Concerning practical methods, it is probably not necessary to say anything so far as the first five items of the list are concerned, for the methods in ordinary use will suffice until improved.

Satisfactory methods of velocity measurement, and for the determination of the movement of drifted matter on the bottom are yet to be invented. It is, therefore, well to develop the conditions of the problem in each of these cases.

So far as discharge measurements are concerned, it is now conceded that the velocity at a depth, somewhere between mid and two-thirds depth, bears a fixed ratio to the mean velocity in a vertical plane, and that between these limits the velocity varies but slightly; therefore, since for the purpose we now have in view absolute discharges are not necessary, if we secure proportional accuracy, it will suffice, if double floats are used, (observing the requisites of good double floats as stated by Gen. Ellis, See part 2, Report of Chief of Engineers U. S. A. for 1875, page 306), to allow a length of suspending cord equal to two-thirds of the depth, and to rest upon the probability that, whatever may be the uncertainty as to the exact position of the lower float, it will not be far from the mean of the several depths given by the different authorities, as the proper position of such a float.

At times, when the depths are varying rapidly by changes at bottom and surface, as is the case when the river is rising or falling rapidly, it is evident that an approximate discharge, determined by soundings and floats observed within the space of a few hours, may be much more accurate than a discharge obtained by a much better method, but by observations occupying a considerable length of time. For our discharges then we may desire, but do not need, better results than can be obtained by double floats.

The controverted question concerning long and short bases for float observations need not be considered with reference to determination of positions. The length of base should be decided with reference to uniformity in cross section, both in area and form, and parallelism of currents, on one hand, and the errors of time interval, as affecting the observed velocity, on the other; the first calling for a short, the latter for a long base.

To determine the position of floats the following method is suggested, and claimed to have the following merits when applied to large and rapid rivers:

1. As great accuracy in position as is practicable by theodolite angles measured from a short base; and a truer determination of the direction and consequent length of the float's path.

2. It simplifies the observations, and lessens the duties of observers, and thus removes all excuses for missed floats, or approximations.

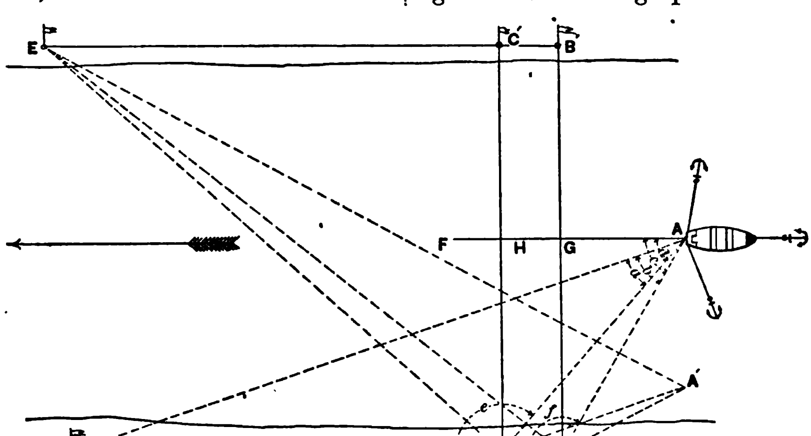
Referring to the accompanying diagram, B, C is the base of observation, which may be taken of any desired length. B and C are each occupied by an instrument. B′ and C′ are range flags; D and E are flags at known points; A is an anchored boat, and AF is the path of any float. An observer is stationed in the stern of the boat with a sextant. From angles *a* and *b*, measured by the sextant, and angles *e* and *f*, measured at C and B, the position of A is approximately fixed, the uncertainty arising from the swing of the boat at its anchor. This being done, B and C fix their cross hairs on B′ and C′, after which a trial float should be run. B and C are to find the float in the vertical range of their telescopes and clamp them, so as to make the path of the trial float central to their field of view. B and C, from this on, have only to watch for the passage of floats, note and record the time of passage, and to let a hand signal fall at the instant the float passes their cross hairs.

A has supervision of starting the floats and watches, through his sextant, by direct vision, the float, and keeps B, by reflection, coincident with it. When B's signal falls (seen in the mirror), A reads angle *c*; then, bringing C into reflection, he watches for C's signal, and when he sees it reads angle *d*. By intersections which may be very readily plotted, we have G and H, giving, with A, three points in the path of the float. If, by any mishap, either angle *c* or *d* is missed, the path may, without serious error, be assumed to be a straight line through A and the position determined. The only error of this system arises from the motion of A, which may be limited at will by using additional anchors.

For positions of A nearer the base than mid-river, as A′, angles for its position may be measured from E instead of D, as indicated.

By this system, but one observer is required to be specially skillful in handling instruments, and his angles are within the range of ordinary tangent screws, and he is therefore little liable to lose an angle. The manual dexterity required to handle the sextant is not difficult of acquirement, and should indeed be part of a hydrographer's regular education.

A time-keeper should be stationed with B or C unless a system of electric signals with chronograph is used.



The determination of bottom velocities is one of the unsolved problems of hydraulics, whose solution, while of special importance because of its relation to the subject of the transport of sediment, is also important to the general subject of hydraulics because involving the discovery of all the now unknown relations existing between velocities at different depths.

The degree of perfection attained in chronographic records by electricity, and in the means of measuring currents by meters, justifies one in saying that the facilities for the solution of the problem of the vertical curve of velocities are now such as to demand further effort.

It may properly be assumed that the determination of bottom velocity depends upon the possibility of determin-

ing the vertical curve of velocities; that is (since it cannot be directly observed in a muddy stream), if the law of the vertical curve is known, the bottom velocity would be ascertained by completing the curve when three or more points in it are known. Previous experimenters have entertained this view, and many efforts to determine the law of the curve have been made, but the law has remained undiscovered.

If we now consider the Horsetail Bar sections Nos. 1 and 2, which were 1930 feet apart (and the same facts extended still further), we note the entire disappearance of the area when reduced to low water, and, in fact, the deepest sounding in No. 1 was 17 feet at the 18′.5 stage of May 6th, 1873, and the same in No. 2, except a single depth of 22 feet, close to the west shore, near the face of a rock ledge, which sounding allows a small remnant of the section to appear in the list.

These sections, made in May, cannot be supposed to represent the maximum filling, for this maximum would naturally occur at the June rise, which that year was well marked, lasting some six weeks. Indeed, we know that, at this locality, there has been as little as six feet of water in the main channel when the gauge readings showed the river at a 14-foot stage. The soundings of May, 1873, were made after a rise to 25.45, culminating April 11th, which rise probably was due to the Upper Mississippi, or to local rains. The sections of April 9th, 1872, were made after a short and quick rise, beginning March 29th and culminating, April 3d, at 17.65, which is almost conclusive proof that it was caused by local rains affecting only the nearer tributaries, conditions favorable to a moderate filling of the low-water channel.

The conclusions of our analysis are then borne out by the facts, so far as they are on record. It would be more satisfactory if we could trace the exact source of the several waves, but the data are not at hand, and will not be until gauge records are kept at points on every important tributary to the Mississippi, far enough from the main stream to be beyond the influence of back-water, and below the main affluents of the tributary.

The facts and analysis are given to lay before those not familiar with the river itself proof of one cardinal fact of great practical importance, which either has altogether escaped the attention, or has not been duly appreciated by those who have undertaken to explain the physics of this river. Persons familiar with the river will possibly recognize in the statements which have been made little, if anything, more than is implied in the maxims, current among boatmen, about the channel flattening when the river rises and cutting out as it falls; also, that a rapid decline from a high stage is followed by a season of bad navigation through shifting channels; also, that a spurt of a rise, coming after the cutting-out process has developed, is considered undesirable; and, again, such statements as the following: "A rise of a foot, coming from the Missouri, adds nothing to the channel depth out to Cairo, but a rise coming out of the Illinois will add materially to the channel depth, even though it does not increase the gauge readings at St. Louis." These results of the observation of practical men confirm the deductions from a study of the cross sections, which we now restate.

A deposit of silt in the bed of the river is made when the turbidity of the stream passes a certain limit, and is removed when the turbidity falls below that limit. These conditions usually attend—the one a rising and the other a falling river. The amount of deposit depends also upon the sources from which the flood waters come and the subsequent removal upon the time occupied by the decline.

I would include in the term turbidity all material conveyed by the stream, whether borne in suspension, swept along the bottom, or moving as a semi-fluid mass along the bed with its own motion as a semi-fluid.

Without here indicating an opinion as to the actual existence of all these modes of transport of sedimentary matter, and particularly the latter, I will introduce the testimony of witnesses as follows:

The late General R. E. Lee was, in 1838, in charge of the improvement of the Des Moines Rapids, by excavating a channel. In his report of October 24th, 1838, he says: "Unless it (the rock loosened by blasts) is removed, the effect of the second course of blasts is much diminished, and the current of

sand, constantly drifting over the bed of the river, soon fills up the crevices and renders reblasting necessary." The context shows that the operations were limited to low stages affording wading depths; therefore the comparative transparency of the water, at that locality and stage, favors the supposition that Lee's statement of a "current of sand constantly drifting over the bed of the river" was founded upon direct observation. If the current of sand existed at low water, it certainly did at high.

It is related, upon the authority of the chief engineer of the bridge over the Missouri at St. Charles, that during the sinking of one of the caissons certain fender piles, which had been driven above the caisson, disappeared, and when the caisson reached the bed rock the lost piles were found lying on the rock under the caisson. What depth of sand overlaid the rock was not stated, but it was considerable.

Again, in the report of the chief engineer of the Illinois and St. Louis Bridge of June, 1868, page 21, it is said: "I had occasion to examine the bottom of the Mississippi below Cairo during the flood of 1851, and at 65 feet below the surface I found the bed of the river, for at least three feet in depth, a moving mass, and so unstable that, in endeavoring to find footing on it beneath the bell, my feet penetrated through it until I could feel, although standing erect, the sand rushing past my hands, driven by a current apparently as rapid as that at the surface. I could discover the sand in motion at least two feet below the surface of the bottom, and moving with a velocity diminishing in proportion to the depth at which I thrust my hands into it." In this very explicit and circumstantially detailed statement we are assured that the bottom was moving in time of flood, and under water of very considerable depth—65 feet.

Professor C. G. Forshey, in a paper entitled "The Physics of the Gulf of Mexico, and of its chief affluent the Mississippi River" read at the Nashville meeting of the American Association for the Advancement of Science in August, 1877, Page 18 of pamphlet copy says: "It was ascertained early, say within the first month of observation, that there was some other undiscovered method of explaining the amount thrown out on the sand-bars, in the crevasses of the levees and banks, and carried to the bars of the river at its mouths.

"A single experiment made by the writer of these pages brought to light the secret.

"The matter was borne along the bottom of the river, too heavy to be held in suspension. By far the greater amount passed off thus invisible. The amount could not be measured. The experiments were continued. Samples were taken during the whole of two years and the drift matter never was found absent beneath a running current. From the greatest depth on the line of observation and from the gently sloping shore of the bar on the other side, the parcels were taken.

"After much reflection and desire to make the estimate reliable, I have concluded to estimate the amount of drift matter, rolled or pushed along the bottom of the river, at three-fourths of the whole quantity by weight."

We have now from four independent sources direct testimony, based on actual observation, all agreeing as to the one thing under investigation, namely, that very important movements are in progress at the bottom of the river, and we may add, at all stages of water, at all depths, and at all parts of the river, for our evidence is drawn from both of the great rivers whose union makes the true Mississippi, from the main stream after receiving its other great tributary, the Ohio, and from the deep and comparatively stable section at its approach to the Gulf. A fact, which is invariably found under such varying conditions, must be a general one. Of the detail of this general fact no one can pretend to have an adequate knowledge, and yet it must be admitted that such accurate knowledge is of the highest importance, both in a scientific and practical point of view.

Regarded from the scientific standpoint, since we have proven that the flow of sand is inseparable from the flow of water (an idea which is in fact involved in the most ordinary conception of a silt-bearing stream) it must follow, that the hydraulics of such a stream are more complicated than of rivers of clear water traversing stable beds.

TABLE III.

Locality.	No. of Section, H and A.	High Water. Areas. Sq. ft.	Difference. Sq. ft.	Per Cent. of Change.	r or Hyd. Mean Depth.	Difference. ft.	Width.	Difference.	Center of Moment about base.	Low Water. Areas. Sq. ft.	Difference.
Columbus.	1	161,419	—	+.018	69.94	—	2260	—	1061.5	64,274	—
	2	164,350	+2931	—	71.61	+1.67	2260	0	1086.4	65,585	+ 1311
	3	164,568	—	+.066	69.26	—	2320	—	1087.0	64,286	—
	4	175,345	+10777	—	74.61	+5.35	2320	0	1119.8	72,604	+ 8318
Vicksburg.	13	178,648	—	—.001	58.15	—	3056	—	1776.9	55,399	—
	14	178,450	— 198	—	57.93	—0.22	3050	—6	1737.2	55,800	+ 401
	15	177,392	—	+.025	57.60	—	3040	—	1732.9	55,003	—
	16	181,800	+4408	—	59.23	+1.63	3034	—6	1725.7	59,865	+ 4862
Carrollton, La.	48	215,643	—	+.0008	70.12	—	3035	—	1561.5	172,330	—
	49	215,806	+ 163	—	70.52	+0.40	3035	0	1556.3	172,493	+ 163
	55	235,450	—	—.039	87.69	—	2605	—	1004.0	197.837	—
	56	226,267	—9183	—	84.90	—2.79	2605	0	936.5	188,805	— 9032
	56	233,193	—2257	—	—	—	—	—	—	—	—
	57	219,381	—	+.031	87.05	—	2460	—	923.6	184,731	—
	58	227,458	+8077	—	90.62	+3.57	2460	0	941.5	192,808	+ 8077
	58	226,877	+7496	—	—	—	—	—	—	—	—
	62	218,393	—	+.081	73.78	—	2930	—	1181.3	178,980	—
	63	236,069	17676	—	79.75	+5.97	2930	0	1221.1	196,656	+17676
	64	208,141	—	+.063	74.60	—	2760	—	1080.5	170,041	—
	65	222,036	+13895	—	79.30	+4.70	2760	0	1039.8	183,673	13632
	68	190,080	—	+.101	72.55	—	2575	—	984.9	154,630	—
	69	209,241	+19161	—	80.01	+7.46	2575	0	1052.6	172,941	+ 8311
	69	229,325	—	—	—	—	—	—	—	—	—
	70	190,538	—	+.110	73.28	—	2550	—	959.6	154,990	—
	71	211,530	+20992	—	81.67	+8.39	2550	0	1015.6	175,305	+20315
	72	191,320	—	+.102	74.01	—	2530	—	952.4	156,410	—
	73	210,937	+19617	—	81.61	+7.60	2530	0	1027 2	176,012	+19602
	74	185,312	—	+.087	71.24	—	2525	—	968.5	149,361	+
	75	201,446	+16134	—	78.70	+6.46	2525	0	998.2	165,495	+16134
	76	185,338	—	—	—	—	2705	—	—	150,030	—
	77	171,278	—	+.104	66.77	—	2530	—	994.5	135,327	—
	78	189,128	+17850	—	73.59	+6.82	2530	0	1014.5	153,177	+17850
	78	194,945	+23667	—	—	—		—	—	—	—
	79	173,014	—	—	—	—	2670	—	—	138,200	—
	80	170,096	—	+.095	66.05	—	2540	—	1010.5	135,000	—
	81	186,333	+16237	—	72.32	+6.27	2540	0	1028.3	150,520	+14520
	81	188,160	+18064	—	—	—	—	—	—	—	—
	83	192,746	—	—.040	72.87	—	2600	0	995.1	156,083	—
	84	185,092	—7654	—	69.94	—2.93	2600	0	971.1	148,342	— 7691
	85	204,627	—	—.001	75.93	—	2650	0	1036.9	167,164	—
	86	204,348	— 279	—	75.97	+0.04	2650	0	1046.8	166,885	— 279
	88	184,574	—	—	—	—	2925	—	—	145,200	—
	88	198,302	—	—	—	—	—	—	—	—	—
	89	189,268	—	—	—	—	2950	—	—	149,010	—
	89	197,914	—	—	—	—	—	—	—	—	—
	91	220,246	—	—	—	—	2780	—	—	180,000	—
	91	188,075	—	—	—	—	—	—	—	—	—
	92	202,228	—	—	—	—	2600	—	—	164,115	—
	92	205,117	—	—	—	—	—	—	—	—	—

TABLE III—(*continued*).

Water.									
Per Cent. of Change.	r or Hyd. Mean Depth.	Diff.	Width.	Diff.	Center of Moment about base.	No. of Soundings in Section.	Month.	Year.	Remarks.
+0.02	33.47	—	1905	—	1017.07	46	Dec.	1857	Rising Rapidly.
—	31.37	—2.10	1955	+50	1059.60	73	Oct.	1858	Falling.
+.129	31.60	—	2020	—	1039.1	41	Dec.	1857	Rising Rapidly.
—	35.76	+4.16	2025	+ 5	1104.2	58	Oct.	1858	Falling.
+ .007	23.59	—	2340	—	1978.2	28	Feb.	1858	" mean stage.
—	23.27	—9.32	2385	+45	1923.6	48	Sept.	1858	" " lower.
+ .088	23.45	—	2335	—	1907.0	36	Feb.	1858	" "
—	24.94	+1.49	2395	+60	1919.7	36	Sept.	1858	" " "
+.0009	62.10	—	2740	—	1554.0	31	June	1851	High stage.
—	62.50	+0.40	2740	0	1563.3	23	Nov	1851	Low "
— .046	79.61	—	2410	—	948.8	24	June	1851	High "
—	77.06	— 255	2390	20	875.1	23	Nov.	1851	Low "
—	—	—	—	—	—	—	—	1872	———
+ .044	83.21	—	2160	—	901.8	23	June	1851	High "
—	87.24	+4.03	2160	0	897.3	24	Nov.	1851	Low "
—	—	—	—	—	—	—	—	1872	———
+ .098	75.20	—	2325	—	1125.8	21	June	1851	High "
—	84.04	+8.84	2325	0	1214 0	21	Nov.	1851	Low "
+ 080	72.51	—	2320	—	1036.4	21	June	1851	High "
—	76.85	+4.34	2355	+35	1053.5	22	Nov.	1851	Low "
+ .118	67.67	—	2245	—	915.0	48	June	1851	High "
—	75.08	+7.36	2265	+20	1003.9	36	Nov.	1851	Low "
—	—	—	—	—	—	—	—	1872	———
+ .131	66 95	—	2275	—	902.6	43	Feb.	1851	Rising after fall.
—	75.56	+8.61	2280	+ 5	971.8	38	Sept.	1851	Falling.
+ .125	69.66	—	2240	—	884.8	45	Feb.	1851	Rising after fall.
—	76.03	+6.37	2260	+20	990.1	37	Sept.	1851	Falling.
+ .108	63.42	—	2223	—	903.0	45	Feb.	1851	Rising after fall.
—	72.91	+9.49	2238	+ 15	953.4	31	Sept.	1851	Falling.
—	—	—	2260	—	—	—	Feb.	1859	Mean stage.
+ .132	60.41	—	2219	—	938.2	44	Feb.	1851	Rising after fall.
—	66.75	+6.34	2259	+ 40	974.7	80	Sept.	1851	Falling.
—	—	—		—	—	—	—	1872	———
—	—	—	2220	—	—	—	Feb.	1859	Mean stage.
+ .107	59.60	—	2230	—	949.2	44	Feb.	1851	Rising after fall.
—	65.86	+6.26	2235	+ 5	988.7	34	Sept.	1851	Falling.
—	—	—	—	—	—	—	—	1872	———
— .049	66.82	—	2295	—	950.7	27	June	1851	High stage falling.
—	63.40	—3.42	2300	+ 5	902.2	24	Dec.	1851	Low "
— .002	69.94	—	2345	—	975.0	23	June	1851	High " "
—	69.97	+0.03	2345	0	98.19	24	Dec.	1851	Low "
—	—	—	2500	—	—	—	June	1851	High " "
—	—	—	—	—	—	—	—	1872	———
—	—	—	2595	—	—	—	June	1851	High " "
—	—	—	—	—	—	—	—	1872	———
—	—	—	2460	—	—	—	June	1851	High " "
—	—	—	—	—	—	—	—	1872	———
—	—	—	2355	—	—	—	June	1851	High " "
—	—	—	—	—	—	—	—	1872	———

Therefore, observations of a special character are required to detect the complex laws of a sediment-bearing stream. Take so simple a matter as a hydrographic survey, for example: these bottom changes forbid—*First:* An attempt at reduction of soundings to a standard low-water stage, as is customary in hydrographic work. *Second.:* They render it impracticable to join surveys made at different dates. *Third:* No direct comparisons can be made between different surveys covering the same ground, if such surveys are made at different stages of water, or at the same stage after a diverse succession of stages. *Fourth:* It is impossible to distinguish permanent changes of contour from the fluting ones due to the silt movement. No argument can be required to establish these statements. Every one must see that they are inevitable consequences of the fact of bottom changes attending fluctuations of stage.

Passing from hydrography to hydrometry it is equally obvious that the important element of depth is subject to variations from below as well as at the surface when the river rises and falls. Therefore, a continued series of discharge measurements will be subject to great errors, in respect to areas, velocities, and volume, if the ordinary assumption of unchanged bottom be indulged, and the same remark applies to attempted studies of the vertical curve of velocities.

This seems to have escaped the notice alike of hydraulic authors and critics, though ample facts to show the existence of the error have been in print for years.

I have introduced a tabulated statement (pp. 228–9) from Humphreys and Abbot's Appendix C, of remeasured cross-sections, adding thereto a remeasurement in certain cases made in 1872, the latter on the authority of Prof. Forshey's paper already quoted, and to be found on the last page of the pamphlet copies.

The order of arrangement is geographical, and the table shows the same facts as the preceding ones, except that the hydraulic mean depths are substituted for mean depths, in order to specially direct attention to the influence of bottom changes upon the elements usually entering into hydraulic formulas. The columns headed "center of moment about base" were made out partly with the hope of being able to show a measure of the lateral vibration of the river section, between high and low stages, in passing around a well-defined bend, as at Carrollton; and, in other cases, to show the extent of lateral movement, in order that it might be seen that no marked shifting of the bed occurred between the dates of measurement, which might account for the change of areas. The first purpose failed, because the sections are not real—that is, actually measured at high and low stages—consequently the demonstration of a very interesting fact must be deferred until real data are available.

The absence of the specific dates when the measurements were made detracts from the value of the facts here recorded. The general state of the gauge during the month in which the measurements were made is given in the column of remarks.

Since none of the areas given were actually measured, but, as they stand in the table, are really predicted areas based upon data, obtained at some intermediate but not definitely-known stage, therefore areas predicted from sections measured at comparatively high stages will be smaller than those predicted from measurements at low stages, if the theory advanced in the foregoing pages is correct.

If the reader has access to the work of Humphreys and Abbot, it will be profitable to turn to the gauge records of Appendix B for localities and years named when examining the foregoing table. *

Although several writers have brought forward formulas derived from the data in their possession the result is nothing but discord, because the points on the curves were not determined by simultaneous observations.*

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* Since the paragraph, giving credit for the legitimate derivation of formulas from the data by various writers on hydraulics, was written, grave doubt has arisen as to the legitimacy of instituting comparisons of vertical curves, taken at various localities and stages, by dividing the observed curves into tenths of the total depth and plotting to a uniform scale the co-ordinates thus obtained; and the further reduction to a mean velocity of unity by dividing all the ordinates in each curve by its mean velocity. These steps are founded upon an assumption, which may or may not be true, that larger and smaller streams are proportionally affected as to their several parts or elements. Taking an extreme case for example, a rivulet flowing under certain conditions is transformed by heavy rainfall into a torrent; can it be supposed that the conditions of flow have simply expanded symmetrically? and yet this is really the assumption when the procedure criticized is applied to observations at various stages of water to bring them into condition for a comparison of things which, for aught we know, may be essentially unlike.

* Insert here the [illegible] portion beginning 2d line [illegible] column, p. 222.

Taking the existence of pulsations or irregularities of currents as an established fact, it will be readily understood, that observations at the several depths at different times, and probably for irregular periods, cannot be expected to give a true mean curve. If this point be admitted, and I think it must be, the anomalous results of all previous observations are accounted for, and the remedy suggested. For, by the use of current meters and a chronographic register, it would be possible to make simultaneous observations at as many points of the vertical as we choose to provide meters. And if the revolutions of the several meters are recorded upon one chronograph, data in convenient and unmistakable form will be available for the determination of the vertical curve at any instant; or, by taking means, by groups, of obtaining a series of true mean curves, which, when extended to cut the bottom, will furnish the only possible solution of the problem of bottom velocity.

Looking at this suggestion from a practical point of view, it is probable that it would require but little more trouble and time, to run a series of meters when the vertical wire required for one is in place than to run that one singly, and less if the single meter be run successively at as many different depths as the series. If I am right in this, the cost of a complete series of simultaneous observations would not be much more than an incomplete system would cost, aside from the first expense for meters and electrical apparatus.

The foregoing suggestions it is hoped may prepare the way for the solution of some of the unsettled questions concerning the flow of streams. I, for one, do not believe that the presence of grit in the water presents any insuperable obstacle to the use of properly constructed meters in the Mississippi, though patient and protracted experiment may be required before that proper construction is realized.

My statement of the deficiency of practical methods included the determination of the quantity and quality of drifted material.

The collection and determination of suspended matter is merely a matter of patient careful detail; and the same may be said of material drifting along a stable bottom.

A trap or dredge similar to to the slip water bottle used on the Challenger expedition can certainly be devised, which will bring the desired specimens to the surface for examination.

But if there is a stream of sand running beneath that of water, the question of detecting and measuring the movements of this under-current is important. It will be remembered that, when speaking of the various modes of transport of sediment, I reserved expression of opinion concerning the weight to be given certain statements which I then quoted.

The statements of Gen. Lee and Prof. Forshey imply no more than the movement of sand and other bodies along and in close proximity to the bottom; but when that bottom is itself composed of sand or other material, readily moved by any disturbing cause, it will be readily understood, that no definite line of division can be drawn between material in motion and that at rest. But the observations incidentally made by means of the caisson and diving bell seem to testify to an extensive movement at the bottom, which in quantity moved would render the movement in suspension trifling in comparison, presenting a question concerning which, we cannot afford to remain in doubt. The evidence quoted is suggestive, but not conclusive; for the presence of so large a body as a caisson or diving bell, would of itself tend to produce the facts observed, by creating a violent disturbance of the local fluid currents, which would react upon the unstable material of the bottom. Some more reliable method of observation must be adopted. Possibly the presence of a diver in armor would not create disturbance enough to vitiate observations made directly as to the principal facts, but exact measurement of the depth and velocity of movement would present more difficulty; which difficulty need not be attacked until the demonstration of the fact makes its solution necessary.

The thorough investigation, suggested in the preceding pages, into the subject of silt movements, is not simply in the interest of the Mississippi, but would contribute to the science of hydraulics in respect to other silt-bearing rivers. Unquestionably every general fact brought

to light will add to the ability of engineers to deal successfully with such streams. But it is not necessary that the application of the very imperfect knowledge we now have to practical experiments should await the result of scientific investigations, for really the two are so intimately related that it would be useless to attempt to separate them Theory must be tested by practice, and practice should continually improve, as theories are perfected.

The general fact, whose existence and influence it is the chief design of this paper to trace, may be summed, in language intended, so far as practicable, to avoid expressing theory as to how or why the phenomena exists, thus—

First. The lower part of the bed of a silt-bearing stream is filled with sedimentary material at high stages, and emptied at low in a manner analogous to that in which the upper part of the same bed is filled and emptied of water.

Second. Unusually narrow parts of the river either receive less than an equal proportion of the deposit, compared with equal areas of wide parts, or else the process of removal of the deposit is more speedy in the one case than in the other, with the probabilities in favor of the latter supposition.

Third. The amount of deposit depends upon the source from which a flood comes and the degree of its subsequent removal upon the time occupied by the decline.

Another class of deposits due to material derived from neighboring caving banks is not included in the above statement; for the period of greatest caving corresponds in most cases with the falling stage. This class of deposits probably has much to do with the condition of navigation, but must be considered separate from those continuous movements which constitute the river a silt-bearer. Caving banks and resulting bars are found in clear streams as well as in the most turbid. These would be local, but the feature we are tracing is found, with rare exceptions, wherever the test is made.

The practical influence of the above-stated facts is very important, for, after a June rise, to a stage of 25 feet or upwards, the natural river does not furnish a continuous navigable channel above the Ohio, 12 feet in depth, at any time after the stage has fallen to 20 feet—that is, the low water channel of the preceding year is entirely obliterated, and, in addition, a fill of eight feet or more in depth, above the plane of low water, is made upon one or more of the bars between the Missouri and Ohio.

It will readily be understood that an improvement, which, following clear-stream precedents, looks only to furnishing a required depth at the lowest stages, may fail to secure even that depth at mean stages, because works designed with a view to low water exclusively will not have the height requisite to secure active influence at higher stages. It has been found by experience that, to secure a depth of 8 feet or more, at all stages, it is necessary that the works should be so designed as to be operative at all stages below 20 feet, which is satisfied by giving dikes and dams an elevation of 13 or 14 feet.

It has been found practicable to build dikes and other works upon these sand foundations by giving them a broad base of brush. Furthermore, it is found that, in the interest of economy, it is desirable to lay these foundations as rapidly as possible, immediately after the culmination of the mid-summer rise, in order to retain the deposits when at their maximum elevation. Practically, operations should commence as soon as the cessation of drift renders it possible to work.

The greater depth at that time is not wholly inimical to success, for, while adding to the difficulty of placing foundations, it affords an area of cross-section, which is so slightly diminished by the foundation courses that no serious increase of current or scour is excited. A dam is now in progress across Cahokia chute, the foundation for which was laid upon a section not materially changed from that given in Table 1, as of the date of July 6th, 1878, and stage 22 feet.

The writer suggested, several years since (in 1868), that, in cases where the reclamation of ground from the river was desired, or the narrowing of the channel to improve the navigation, advantage might be taken of the high water deposits, and their permanence secured by rapidly-constructed barriers against their removal. The same suggestion, with the

addition of the possibility of utilizing, dredging or scraping as a means of aiding navigation, by taking advantage of the high state of the bars early in the fall, is made in the Report of Col. James H. Simpson, on Part of Third Subdivision of the Mississippi Transportation Route, Appendix CC 4 to Report of Chief of Engineers U. S. A. for 1875, page 490.

The fact under consideration may then be treated as a helpful agency in the work of regulating the bed of the stream and improving its navigability; the practical problem being to avail ourselves of the help and avoid the hindrances, which the silt-bearing character of the river renders possible.

Study of the sections given in the tables will certainly justify the position maintained by the writer for several years, that the floods of the Mississippi are not simply the consequence of an extraordinary volume of water, but that they, in no small degree, depend upon the order and intervals at which the rises from the several great tributaries follow each other. The facts that have been presented show how considerable a part of the ordinary bed is occupied by deposited material at ordinary high stages, and strongly suggest the probability of vastly greater deposits in times of great floods.

If the maximum of deposit precedes the maximum of water discharge (as it will if a flood from a silt-bearing tributary, arriving when the main river is at a comparatively low stage, is quickly followed by a flood in the main stream) it is very certain that the flood level will rise much higher for a given volume, than when the clearer tributaries send forth their floods first, and the silt-bearers following drop their burden in a deepened and enlarged bed. So far as the floods are influenced by this cause their control by levees must ever remain uncertain and hazardous. For it is within the range of probabilities that one or more of the lower silt-bearing tributaries will send out a burdened discharge to fill the bed of the main river, and that a flood from other sources following immediately after, or overlapping, the first wave will exceed the ability of the obstructed channel to discharge within banks or levees of any practicable dimensions.

In like manner no certain relief from danger of overflow can be expected to follow from any possible system of outlets. No possible depletion of the lower part of the river can counteract a cause acting along its entire length, which, while most potent near the entrance of the silt-bearing tributaries, at so great a distance as that from the mouth of Red river to Carrollton, is able to cause a variation in a single season of one-ninth of the high water area, as was the case in 1851, at Sections 70 and 71, shown in Table 3. Take another and more striking example. The history of Horsetail bar, a few miles below St. Louis, shows that, in 1876, nearly one-third of the area found between banks, at a thirty foot stage, was filled with sand during the moderate flood of that year.

The point of these statements is, that gauge readings of themselves afford no indication whatever of the volume of the river at either high or low stages. Consequently conclusions reached from a study of gauge records alone will be fallacious, whether the purpose of such study be to establish a grade line for a levee system, or to prove the beneficial effect of certain crevasses, and thence draw an argument in favor of opening artificial outlets.

The facts here presented are an emphatic condemnation of both levees and outlets as anything more than modes of alleviating the evil locally. They do not and cannot fully meet it, and therefore should have no place in plans embracing the whole field and presented or urged as a national work.

In the last volume of this Magazine—September number, 1878, page 223—a plan is suggested, by one who, like the writer, condemns levees and outlets as solutions of the Mississippi problem, which consists in "simply constructing light willow or brush dams during low water on the shoals which are then dry, or nearly so, at the various wide places in the river where the bars always exist."

The files of the office of the Chief of Engineers contain an unpublished paper by the writer, written in 1868, which proposed the same plan. I would now preface a statement, like the above quotation, by asserting that first of all it is neces-

sary in wide places and caving bends to provide by artificial protection a bank, able to resist the abrading and eroding power of the river, against which the concentrated volume of the narrowed river may be safely thrown. For the wide places exist, chiefly, because the banks afford less resistance to erosion than the bed to scour. Clearly the only remedy is to increase the resisting power of the bank. When this is done, permanency of the bank favors permanence in the position of the channel, which in time will be scoured to ample depth for navigation, and the shoal areas may, in many cases, be left to themselves.

Until the progress of the country in population and wealth justifies the radical improvement of the tributaries, the practical problem is to assist the Mississippi to convey to the sea its necessary burden of water and silt. To the professional reader it is not necessary to prove how much traffic is impeded by stoppages and unloading and reloading by the way.

Caving and yielding banks are, in effect, like throwing upon an already overcrowded thoroughfare an immense amount of local traffic. Protection of banks would cut off these local contributions, and the deepened localized channel would have increased ability to carry without interruption its burden of material.

Considered in its broadest aspect there can be little room for dispute that the improvement and maintainance of the navigation, the restriction of floods, the protection against erosions, and the drainage of the bordering lands are so inseparably linked together that the thought of separating them is folly.

Viewed as a whole, it requires no argument to show that the fixation of the bed is the work upon which the successful realization of each and all these desired benefits must depend. It therefore stands logically in the first place in a scheme of improvement, and, in the opinion of the writer, it includes also the realization of all the desired results in no small degree. For the fixation of the bed will lessen the quantity and regulate the movement of sediments to a very considerable extent, and, I have shown, that to these navigation and protection against floods are closely related.

PROGRESS IN PURIFYING SEWAGE BY PRECIPITATION.

From "The Builder."

The object of this communication is to lay before the Congress certain practical and economic results which the writer has arrived at in regard to perfecting the purification of sewage by chemical treatment, and in facilitating the disposal of the sludge derived therefrom. He desires to acknowledge the valuable cooperation he has received from Professor Wanklyn and Mr. J. C. Mellis, C. E.

In considering every system for dealing with sewage, it is desirable that the question should be regarded as a means of enabling sanitary authorities to overcome the great and increasing difficulty in the disposal of their sewage refuse at the least cost, rather than as one for making money out of sewage, which has been too much aimed at, and has thereby thrown back sanitary improvement considerably.

In the course of the writer's experience in advising with reference to precipitation works and systems, he has seen more and more clearly that sulphate of alumina aided by milk of lime proves the most efficient agent to comply with the beforementioned conditions for purifying sewage. This he has now advanced upon, inasmuch as he has discovered that the efficiency of sulphate of alumina is greatly increased by the presence of some protosulphate of iron, which apparently produces results *altogether out of proportion to its chemical equivalent* (the salts of iron being known to chemists as purifiers of foul fluids), the proportions varying with the composition of the sewage, but their combination having a marked sanitary and economic advantage.

* By Mr. Henry Robinson, C. E. Read at the Sanitary Congress, Stafford.

Practical data on a large scale have already been obtained at the sewage precipitation works of the Rivers Purification Association (to whom the writer is engineer) at Coventry and Hertford, where the improved system of working has been adopted with complete financial and sanitary success. The process previously employed at these places was (as is well known with reference to Coventry) crude sulphate of alumina and milk of lime, which gave results admittedly of a highly satisfactory kind. The addition of the salts of iron has, however, led to the production of an excellent effluent, and at a cost even less than previously.

The employment of this combination has also resulted in the tanks and the sludge being deprived of the slight smell hitherto noticeable at times. The economic results attending this compound system of precipitation will be recognized when the writer states that, at the Coventry Sewage Works the purification of the very foul sewage (containing between six and seven grains of chlorine per gallon, and between twelve and thirteen parts per million of albuminoid ammonia, with a large amount of dye from the manufactories) is now accomplished for £1 1*s.* 6*d.* per million gallons.

At Hertford the improved system of working has been applied with equally satisfactory results.

The way to dispose of the sludge from precipitation works has long engaged the attention of the writer. Artificial heat will accomplish it, and several ingenious appliances have been devised for this purpose. The cost, however, is heavy, and the value of the resulting manure is not yet sufficiently established to justify its employment on commercial grounds.

The writer has now adopted the system of removing the water from the sludge by means of filter-presses, and he has devised a simple application to economically effect this.

This press, which is founded on an old and well-known Belgian apparatus for filtering impure liquids, by means of a combination of discs with filtering-cloths on their surface, has been arranged in order to economize labor and time in working, so that the discs open and shut automatically, by which the pressed sludge is removable quickly and without disturbing the filtering-cloths. The fluid sludge is thus converted rapidly and economically into the form of cakes, and the drying can be further carried on to any desired length, either by exposure to the air or otherwise. The difficulty hitherto experienced in disposing of the sludge to farmers for utilization upon land is thus got over, as the writer has found that there is no difficulty in selling the sludge from the process he is referring to, provided it is made portable. The theoretical value assigned to sludge from the writer's process is, according to chemists, 16*s.* 9*d.* a ton if it contains about 50 per cent. of moisture, or 27*s.* a ton if dried down to 15 per cent. of moisture.

IT is very well known to those who have traveled in the Alps, that the inhabitants believe that avalanches rarely fall when the sky is overcast, but that they do so, frequently, when the sky grows clear. In winter the monks of St. Bernard always urge travelers not to leave the monastery when the sky is clearing, and many times those who have neglected that advice have fallen victims to their imprudence. M. Dufour, in a paper read before the Paris Academy of Sciences, endeavors to explain the phenomenon by reference to the contraction and decrease of strength of snow and ice under decrease of temperature. "In cold weather," he says, "when the sky clears off the temperature falls, especially just before sunrise, and then the filaments of ice which retain the snow on the slopes of the mountains contract and snap, the mass begins to slide, and draws others in its train; for the slightest cause of movement, a shout or the smallest shock may cause the fall of enormous avalanches." A circumstance of which M. Dufour was a witness confirmed him in his views. A meadow of several acres in extent had been prepared at Morges for skaters by covering it with water, which froze while the heavens were covered. One night the sky cleared off, and M. Dufour noticed a sensible fall in the thermometer. Immediately afterwards he heard crackings in all directions, due to the contraction of the ice from the increased cold, and numerous splits were observable. That phenomenon is precisely analogous to what occurs when the heavens clear up and cause the fall of avalanches.

THERMODYNAMICS.

By HENRY T. EDDY, C. E., Ph. D., University of Cincinnati.

Written for VAN NOSTRAND'S MAGAZINE.

I.

1. INTRODUCTORY.—In 1824 Carnot * laid the foundation of the modern theory of Thermodynamics in the principle that when a heat engine performs work continuously in such a manner that the working substance returns periodically to its initial condition as to temperature, pressure, etc., then during each of these periods or cycles, the working substance must receive heat at a higher and reject it at a lower temperature; and that the work done in the cycle depends in no way upon the working substance, but only upon the temperatures at which the heat is received and rejected.

He further showed that the cycle of maximum efficiency, between two given temperatures, is that in which all the heat received by the working substance is received at the higher temperature, and all the heat rejected, is rejected at the lower temperature. And hence, such a cycle is termed a cycle of Carnot.

Carnot provisionally accepted the hypothesis that heat or caloric is an indestructible material substance, but he evidently regarded this as a doubtful hypothesis.

This error vitiated his reasoning in several regards; in particular, he supposed the amounts of heat received and rejected to be equal, as they must be, if heat is a substance; but the points above enumerated stand to-day as a part of accepted scientific truth.

In 1834, Clapeyron † pointed out the fact, that Watt's diagram of energy is a geometrical expression of the quantities involved in the cycles treated by Carnot. But it was not possible to treat the cycles of Carnot with entire correctness, until the nature of heat itself had been discovered.

As early as 1798, Rumford ‡ had announced, as the result of his experiments in boring cannon, that heat cannot be a substance, and that it was quite impossible for him to conceive it to be anything except motion; but it was not until 1840, or later, that it was proved with entire completeness, by the laborious and elaborate series of experiments of Joule, that heat is energy.

In 1850, Clausius* first made use of Carnot's investigations, as the point of departure for a correct theory of thermodynamics, in accordance with the law of the equivalence of heat and energy.

Clausius † has stated Carnot's principle, thus corrected, substantially as follows:

Whenever a quantity of heat is converted into work, and the working substance is finally found in its initial condition, another quantity of heat must, during the cycle, be transferred from a hotter body or source, to a colder body or refrigerator; and the amount of the heat so transferred, depends only on the temperatures of the two bodies between which the transfer is effected, and not on the nature of the body through which the transfer is effected.

This is Carnot's principle, and is a direct consequence of the second law or principle of thermodynamics, which will be subsequently demonstrated. It is often incorrectly spoken of as being itself the second principle.

In 1849, however, Rankine had derived the general equation of the mechanical action of heat from mechanical considerations based on his "hypothesis of molecular vortices," as to the constitution of matter.

The theory of heat was, during the next few years, very fully worked out and applied by the independent investigations of Rankine, Clausius and Thomson, each of whom has sought to deduce Carnot's cycle (and hence the general equation of thermodynamics) from some physical axiom or hypothesis. We shall discuss these hereafter, and wish merely to state in this connection, that in 1853,

* Reflexions sur la puissance du feu, et sur les machines propres à développer cette puissance, par Sadi Carnot. Paris, 1824.

† Journal de l'Ecole Polytechnique, tome XIV.

‡ Philosophical Transactions, 1798.

* Poggendorff's Annalen, 1850.

† Pogg. Ann., 1854, Vol. XCIII.

Rankine* announced a general law of energy, which seems to afford a better basis for the theory of thermodynamics than any other axiom or hypothesis heretofore proposed.

But being apparently much absorbed in his molecular hypotheses, he has neglected to sufficiently enforce and explain this most valuable conception. As a consequence, the more abstruse and less useful, though more fully explained, axioms of Clausius and of Thomson, furnish the basis of every published treatment of this subject, while if Rankine is referred to at all, only his very words are quoted, showing that their scope is not fully grasped.

We propose in the following treatment to use Rankine's ideas as the basis of reasoning, but to use the analytic forms of such investigators as may appear convenient, among whom may be mentioned, besides Rankine, Clausius and Thomson. the more recent investigators Maxwell, Zeuner, Hirn and Boltzmann.

The reader will find a minute historical statement of the progress of the dynamical theory of heat in Tait's "Sketch of Thermodynamics," and other historical matter in Clausius' *Mémoire, in Poggendorff's Annalen*, Nov., 1863, Upon an Axiom in the Mechanical Theory of Heat.

2. Work, Energy, Heat.—A unit of work is the amount of work performed in raising a unit of weight through a unit of height against gravity.

Energy is capacity for performing work, and is measured in units of work.

An absolute unit of heat is the quantity of heat required to raise a unit of weight of water at its maximum density (39°.1F.) through one degree of temperature.

3. The First Law of Thermodynamics.—Heat is energy, and has capacity for performing work, so that the number of units of work which can be performed by given quantity of heat is proportional to a the number of units of heat in that quantity.

The words "can be performed" in this statement refer to the fact, stated in Carnot's principle, that we find insurmountable difficulties in attempting to convert *all* of a given quantity of heat into work, but the part of any given quantity of heat which we do succeed in converting into work is so converted in as real and exact a sense as that in which, water, for example, is converted into steam.

*A Manual of the Steam Engine and other Prime Movers, art. 244.

However, there is no such intrinsic difficulty in converting work into heat, and the results of Joule's final experiments he has stated thus:

1°. The quantity of heat produced by the friction of bodies, whether solid or liquid, is always proportional to the quantity of work expended.

2°. The quantity of heat capable of increasing the temperature of a pound of water (weighed in vacuo and taken at between 55° and 60°F.) by 1°F. requires for its evolution the expenditure of a mechanical force (*i.e.* an amount of work) represented by the fall of 772 lbs. through the space of one foot.

The two experimental truths which underlie all modern physics are the indestructibility of matter and of energy; which may be otherwise stated by saying, that the total mass of the universe is constant, and the total energy of the universe, actual and potential, is also constant.

The experimental verification of the first law of thermodynamics was a principal step in proving the indestructibility of energy.

From this first law we see that heat may be measured in units of work as well as in absolute units.

4. The Second Law of Thermodynamics.—Any effect whatever which is caused by heat, and which is measured in units of work, is proportional to the number of units of heat producing that effect.

The second law is made by Rankine to depend upon the first law in substantially the following manner:

Heat is energy, but energy is a thing of such a nature that its parts are like the whole, and it is unchanged in any particular, except in magnitude, by subdivision or by multiplication; hence, heat also possesses the same characteristics, and any effect caused by heat is proportional only to the amount of heat acting, pro-

vided the effect is measured in units of work.

This is otherwise stated by saying that the effect, so measured, of any given quantity of heat is the sum of the separate effects of any parts into which that quantity of heat may be supposed to be divided.

The most natural conception of heat, and one which assists the thought, is to regard heat as the energy of molecular or atomic motion.

Various attempts* have been made to derive the fundamental equation of thermodynamics resulting from the second law from the first law, by known mechanical principles.

But Tait says,† respecting these attempts, that they virtually assume, in course of the demonstration, a consequence of the law to be proved, and hence are inconclusive.

Clausius incidentally referred to this matter in his address at the 41st meeting of German Naturalists and Physicists at Frankfort in words to this effect:

"Besides, there is a second principle, which is not yet contained in the first, but requires a special demonstration."

Rankine however states‡ that "Carnot's principle (*i.e.*, the second principle) is not an independent principle in the theory of heat, but is deducible as a consequence from the equations of the mutual conversion of heat and expansive power" (*i.e.*, from the first principle). The demonstration of this which he has given rests upon his hypothesis of molecular vortices.

So far as can be now seen the demonstration of the second law above given, which is dependent only upon the known properties of energy, and which is, simply, one case of a like proposition, enunciated by Rankine, respecting energy in general, is valid and complete, and obviates the necessity of any intricate analytic processes which must of necessity assume the molecular constitution of matter in some general way at least.

* The second proposition of the Mechanical Theory of Heat deduced from the First, by C. Szily. Philosophical Magazine, Jan., 1876.
On the Second Law of Thermodynamics in Connection with the Kinetic Theory of Gases, by H. S. Burbury. Phil. Mag. Jan., 1876.
Boltzmann, Sitzungsberichte der Weiner Akad. Vol. LXIII.
† Sketch of Thermodynamics, 2d Edition, Edinburgh, 1877, p. 38.
‡ Phil. Mag. Series IV, Vol. VII.

It is to be further noted in this connection that universal experience attests the fact that heat, from whatever source, is identical in its general nature and properties, just as energy is, which fact is confirmatory of the above demonstration.

It is to be understood that the necessity for the existence of the second law is to afford a basis for Carnot's principle, previously stated, and that any truth which affords such a basis is called the second law.

5. Other Forms of the Second Law.—It may be useful to consider briefly two other statements of the second law which are proposed as physical axioms and are intended to command our assent from their self evident truth.

1°. "Heat cannot of itself pass out of a colder into a hotter body." (Clausius, 1850). This axiom does not contradict Prevost's Theory of Exchanges, but simply states that in the exchange of heat between two bodies the colder body will receive more than the hotter body. Clausius has, subsequently, restated it thus: "It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature."

2°. "It is impossible, by means of inanimate material agency, to derive mechanical effect from any portion of matters by cooling it below the temperature of the coldest of the surrounding objects. (Thomson 1851).

All that can be said respecting these proposed axioms is, that they are not self evident, and so are not axioms at all. Indeed, Clausius* has devoted a long memoir to showing that the action of lenses in concentrating rays of heat will not probably vitiate his axiom.

Besides these axioms, various molecular hypotheses have been advanced, from which Carnot's principle has been derived.

1°. Hypothesis of Molecular Vortices. (Rankine, 1849).

2°. Hypothesis of Circulating Streams of any figure whatever. (Rankine, 1851 and 1855)†.

3°. Hypothesis of Periodic Motions. (Boltzmann, 1866.‡)

4°. Hypothesis of Quasi-Periodic Mo-

* Pogg. Ann., January, 1864. Vol. CXXI.
† Phil. Mag. series IV, Vol. XXX.
* Sitzumgsberichte der Weiner Akademie, Vol. LIII.

tions. (Clausius, 1871† and more recently).

THE THIRD LAW OF THERMODYNAMICS.

6. The quantity of heat existing in a unit of weight of a given substance depends only on the temperature of the substance, and this quantity of heat is proportional to the absolute temperature of the substance; *i. e.*, the quantity of heat is proportional to the difference between its temperature in its present state, and its temperature when in a state devoid of heat-energy.

This law may be regarded either as an axiom, or as defining the method to be adopted in measuring temperature.

That there is a condition in which any substance is devoid of energy in the form of heat must be admitted, unless we can regard the heat-energy of a unit of any substance whatever as inexhaustible.

All the difficulties which have heretofore attended the demonstration of the second law have equally beset the definition of temperature.

Rankine † would restrict the third law by making it read, "the quantity of heat existing in a unit of weight of a given substance, while it remains in the same condition, solid, liquid or gaseous, depends only on its temperature," etc., etc; but Clausius ‡ has shown that Rankine, from his own principles, would be obliged to remove that restriction, and agree that the quantity of sensible heat in a body does not depend upon the state of aggregation, solid, liquid, or gaseous in which it is.

Clausius ‡ has also shown, that this third law is a consequence of Carnot's principle, and to this demonstration we may recur after we have discussed that principle.

Indeed, so called axioms have been proposed as the basis of Carnot's principle, which include at once both the second law and the third also; viz:—

1°. "The mechanical work which heat can perform in any change of arrangement of the parts of a body, is proportional to the absolute temperature at which the change takes place." (Clausius.*)

By "change of arrangement of the parts of a body" is meant expansion, fusion or vaporization, or the reverse of either of these, as well as chemical decomposition; and the words "can perform" refer to the case in which such variations of state are effected in a reversible manner, *i.e.* there is no radiation or conduction or the like.

2°. "If the temperature be infinitesimal, the quantity of energy (in the form of heat) converted into work by an isothermal transformation must likewise be infinitesimal." (Belpaire.†)

Thompson,‡ however, evades this difficulty by proposing a definition of temperature dependent upon Carnot's principle, and it appears that this highly philosophic basis for the scale of temperatures would hardly differ perceptibly from that derived from the third law; but the practical graduation of an instrument on the basis of Carnot's principle has not been yet accomplished by reason of the physical difficulties encountered in the process.

The foregoing remarks convey no adequate idea of the extent, originality and importance of the labors of Thomson and of Clausius, but as no physicist is ignorant of their great discoveries in this and kindred fields, it is not necessary to deal further with the subject in this connection.

7. TRANSFORMATIONS OF HEAT.—When heat is imparted to a body, there are in general three ways in which the energy of the heat is expended.

1°. Temperature is augmented, and the part of the heat so expended still exists in the substance as sensible heat.

This is explained on the molecular theory of matter as actual energy of motion of molecules.

2°. Internal work is performed during the expansion or contraction accompanying the change of temperature.

On the molecular theory this is regarded as work performed against atomic or molecular forces in causing fusion, vaporization, dissociation or the like, or in preparing the body for such changes.

Work so expended is no longer heat,

* Pogg. Ann. Vol. CXLII.
† Steam Engine, p. 307, 7th Ed.
‡ Pogg. Ann., November, 1863, Vol. CXX.

* Pogg. Ann., 1862, Vol. CXVI.
† Bulletin de Belg., 1872.
‡ Phil. Mag., 1848.

and does not affect the thermometer, and hence is called potential or latent heat. Although latent heat, not being heat at all, is a misnomer, it is nevertheless convenient to retain this term which expresses a part of the exploded hypothesis of the materiality of caloric.

3° External work is performed by the heat during expansion or contraction, under the action of external forces such as pressure. The heat which is thus converted into work is stored up as potential energy of the external bodies which exert pressure upon the substance to which heat is imparted.

The first two forms of energy together constitute the increment of the internal energy, actual and potential, in contrast with the last which is the external work.

The last two forms of energy together constitute the increment of the so-called latent heat or potential energy, in contrast with the first which is the increment of the actual (sensible) heat or actual energy.

In order to express these statements as equations, let the following quantities be expressed in units of work, as they evidently can be by the first law.

Let dh = the total increment of heat imparted to a unit of a given substance.
" ds = the part of dh which increases its sensible heat or temperature.
" dm = the part of dh expended in doing internal work against molecular forces.
" dw = the part of dh expended in doing external work.
" di = the increment of internal energy.
" dl = the increment of latent heat.

Then,

$$di = ds + dm \quad . \quad . \quad . \quad . \quad (1)$$

$$dl = dm + dw \quad . \quad . \quad . \quad . \quad (2)$$

$$dh = ds + dm + dw \quad . \quad . \quad . \quad (3)$$

$$dh = di + dw \quad . \quad . \quad . \quad . \quad (4)$$

$$dh = ds + dl \quad . \quad . \quad . \quad . \quad (5)$$

These equations evidently refer to a unit of *any* substance, homogeneous or otherwise, or to any mixture whether undergoing fusion, solidification, vaporization, condensation, dissociation, or any change, chemical or otherwise.

8. Specific Heat.—The total quantity of heat which must be imparted to a unit of weight of a given substance in order to augment its temperature by 1°, or the total quantity rejected in decreasing its temperature by 1° is its specific heat. This is usually expressed in absolute units, but for convenience we shall suppose it expressed in units of work according to the first law.

Calorimetric determinations of this quantity are readily made for any given substance under given conditions of pressure and temperature. If equation (3) be considered for the moment to apply to the case of a unit of a given substance which is increased 1° in temperature, then the only one of the four quantities in that equation which is fixed by that statement is ds, for the others will depend upon the change of volume and the pressure, but by the third law ds is invariable and independent of any consideration except temperature and the kind of substance. Hence this part of the specific heat is called the real specific heat, or the true calorific capacity of the substance.

Let s = the total sensible heat in a unit of weight of a given substance.
" k = its real specific heat.
" t = its absolute temperature.

As before stated, absolute temperature is counted from such an origin as zero, that its temperature vanishes with its sensible heat.

By the third law, $$\frac{s}{t} = k \quad . \quad . \quad . \quad (6)$$

$$\therefore \frac{ds}{dt} = k, \qquad \therefore \frac{ds}{s} = \frac{dt}{t} \quad . \quad . \quad . \quad . \quad (7)$$

Equations (7) may also be written in the following forms:

$$\frac{s_2 - s_1}{t_2 - t_1} = k, \qquad \frac{s_2 - s_1}{s_1} = \frac{t_2 - t_1}{t_1} \quad . \quad . \quad . \quad (8)$$

in which the subscripts 1 and 2 refer to any two states of the substance. Equations (7) and (8), which do not contain k, the specific heat of the given substance, are general and refer to any substance whatever.

It is now evident that the third law may also be stated in this form:

Equal increments of sensible heat imparted to any given substance cause equal increments of temperature.

9. Thermometers.—It is well known that the mercurial and other solid and fluid thermometers are inaccurate, because the expansions by which they measure variations of temperature are not uniform; at least, they are inaccurate on the hypothesis that equal expansions are caused by equal variations of temperature, for, on that hypothesis, thermometers made of different substances give discordant results. But with air thermometers, the case is different. Air is very nearly a perfect gas. The definition of a perfect gas is one that fulfills the Law of Gay Lussac, a law expressed by the equation

$$\frac{pv}{t}=\frac{p_0v_0}{t_0} \quad . \; . \; . \; . \qquad (9)$$

in which p, v, t, are respectively the pressure, volume and temperature of the gas at any state, and p_0, v_0, t_0, at same assumed initial state.

In an air thermometer we read, by a scale, the expansion of a confined mass of air, which expansion is caused by change of temperature, and is effected against a constant external pressure. It is easily seen that under all circumstances, we have the general equation

$$dw=pdv \quad . \; . \; . \; . \qquad (10)$$

in which dv = the expansion or increment of volume.

By (9), $dw=pdv=\frac{p_0v_0}{t_0}\,dt$ . . . (11)

$$\therefore \frac{dv}{dt}=\frac{p_0v_0}{pt_0}=\text{a constant} \ldots (12)$$

Hence, in this thermometer, since p is constant, equal increments of temperature, as defined by the third law, cause equal expansions.

10. Isothermal and Adiabatic Lines.—Let Fig. 1 be a diagram in which the

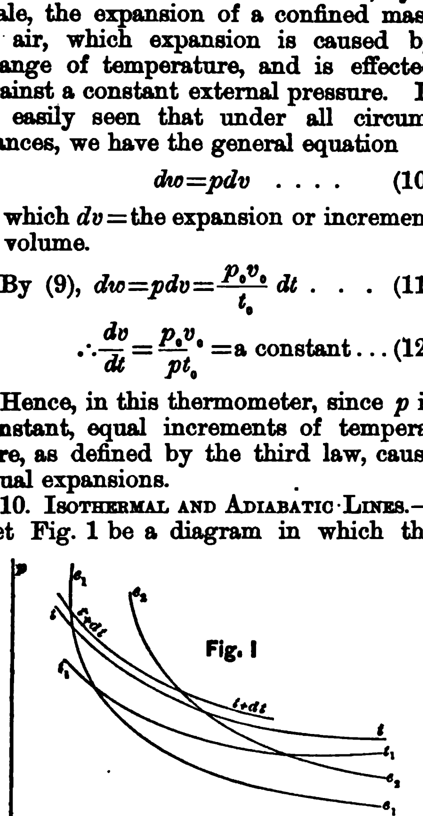


Fig. I

different states of a unit of weight of a given substance are represented graphically by its volume v and pressure p, just as is done by the Indicator Diagram of the steam engine. This is a matter so commonly understood that it is not necessary here to explain it in detail. Let t_1t_1 be the isothermal of the given substance at the temperature t_1, *i.e.* t_1t_1 is a line such that when a point moves along it, the co-ordinates, v and p of the point, correctly represent the magnitudes of volume and pressure of the substance, in case the temperature t_1 is unchanged.

Similarly let tt be any other isothermal. Again, let e_1e_1 be an adiabatic, *i.e.*, a line representing in the same manner the v and p of successive states of the substance, on condition (not that temperature is constant but) that no heat is imparted to the substance or taken from it. Similarly let e_2e_2 be any other adiabatic.

The forms of the isothermals and the adiabatics for any given substance can, in general, be found from experimental data only, but we shall show subsequently how to make the equations of these curves depend upon a very small number of such data.

If a substance expand along e_1e_1, then

by (5) $dh=0$, $\therefore$ $dl=-ds=dm+dw$. (13)

Hence during such expansion sensible heat is withdrawn from the substance and expended in doing work external and internal. Now suppose successive isothermals to be drawn at a distance of 1° from each other, it then appears from (8) that the same quantity of heat disappears, or is withdrawn from the total sensible heat of the substance every time it passes from one isothermal to the next lower isothermal, however that passage be effected, whether along an adiabatic or not.

If a substance is made to pass along an adiabatic no heat is imparted to or withdrawn from the substance by surrounding bodies, but if the passage occurs along any other line, then heat is also imparted to or rejected from the working substance in the form of heat. Suppose, as a most important example, that the substance expands along an isothermal, then heat must be supplied to it in order to preserve the temperature t unchanged. Now since t is constant, in the variation supposed, the total sensible heat is constant also, by the third

law, as expressed in (8). Hence in this case

$$ds=0, \therefore dh=dl=dm+dw \quad . \ . \ (14)$$

Hence it appears that the heat imparted along an isothermal is all expended in performing work, external and internal.

But h, the total heat imparted to the substance during its expansion along t from the adiabatic e_1 to e_2, is measured in units of work by the area included between the curves $e_1 \, t \, e_2$, as appears from the fact that if the substance, by help of the heat imparted, be carried along t from e_1 to e_2 and then left to expand along e_2 until devoid of heat-energy, it will, during these two operations, perform an amount of external work greater by the area included between $e_1 \, t \, e_2$ than if left to expand along e_1 from its initial state at the intersection of t_1 and e_1 until devoid of energy.

In general, the amount of heat imparted to a substance in order to carry it along any route x from the state 1 to the state 2 is measured in units of work by the area included between x and the two adiabatics e_1 and e_2 infinitely extended, which pass through 1 and 2 respectively.

Now suppose two isothermals drawn very near together, the first at a temperature t, the second at a temperature $t+dt$, then the amount of heat h required to carry the substance along t from the adiabatic e_1 to another adiabatic e_2 at some finite distance from e_1, is not so great as the amount of heat $h+dh$ required to carry it along $t+dt$ from e_1 to e_2 by an amount represented in units of work by the area of the quadrilateral included between the lines t, e_1, $t+dt$ and e_2.

The cause requiring a larger supply of heat along $t+dt$ than along t can only be found in the higher temperature (*i. e.*, the greater amount of sensible heat) existing in the substance along $t+dt$. Hence this is a case to which the second law applies

$$\therefore h+dh : h :: s+ds : s \quad . \ . \ . \ (15)$$

$$\therefore \frac{dh}{h}=\frac{ds}{s}, \therefore \text{by (7)}, \frac{dh}{h}=\frac{dt}{t} \quad . \ . \ (16)$$

$$\therefore \frac{h_2-h_1}{h_1}=\frac{t_2-t_1}{t_1}, \text{ or } \frac{h_2}{h_1}=\frac{t_2}{t_1} \quad . \ . \ (17)$$

From these equations it appears that the amount of heat required in order to carry a substance along an isothermal from one adiabatic to another depends upon which isothermal it is, and the amount required is proportional to the absolute temperature.

11. Definition of Entropy.—During any small variation in the state of a unit of a given substance, *i.e.*, during its passage along any short distance of any route whatever:

Let t=the mean absolute temperature.
" dh=the heat imparted.
" de=the entropy imparted.

Then $$dh=tde \quad . \ . \ . \ . \ . \ (18)$$

is the equation which defines entropy. Entropy, which may be roughly defined as "heat divided by temperature," plays a very important role in thermodynamics, which fully justifies its introduction as one of the variables to be used in expressing the state of a substance. Entropy has been used by Tait, Thomson, Maxwell and others, with an entirely different signification, but the term is here used in the sense originally given it by its inventor Clausius*. It is identical in meaning with Rankine's "thermodynamic function" and Zeuner's "thermic weight."

12.—The Law of the Variation of Entropy.—The amount of entropy which must be imparted to a unit of weight of a given substance in order to make it pass from one given state to another (of volume and pressure) is independent of the route by which the passage is effected, and depends only on the initial and final states of the substance.

For let two points be selected on a diagram of volumes and pressures which shall represent the two given states, and let them be designated by 1 and 2. Also let there be two routes from 1 to 2 which are represented by any two curves x and y. Moreover let the curves x and y be cut by a series of isothermals and of adiabatics drawn at any equal or unequal small distances from each other. Then the variations of state which occur along the route x can be represented as exactly as we please, by a series of infinitesimal

* Pogg. Ann. 1865, Vol. CXXV.

variations in a zigzag manner along these isothermals and adiabatics, and the same is true of the variations of state along y. Let us call that zigzag route x', which nearly coincides with x, and similarly that one y' which nearly coincides with y.

Now during any variations along adiabatics either on x' or y', we have

$$dh=0, \qquad \therefore de=\frac{dh}{t}=0, \quad . \quad . \quad . \quad (19)$$

hence, during such variations, entropy is constant. But during any variation along an isothermal in the route x' between a pair of successive adiabatics, we know by (17) that dh_x, the heat imparted is proportional to t_x the absolute temperature, and during the variation along y' between the same pair of adiabatics dh_y, the heat imparted, has the same ratio to t_y its absolute temperature.

$$\therefore de=\frac{dh_x}{t_x}=\frac{dh_y}{t_y},$$

$$\therefore e_2-e_1=\int_1^2 \frac{dh_x}{t_x}=\int_1^2 \frac{dh_y}{t_y}$$

$$\therefore e_2-e_1=\int_1^2 \frac{dh}{t} \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

which expression depends only on the initial and final states of the substance.

EQUI-DISTANT ADIABATICS AND ISOTHERMALS.

13. By art. 10, $\dfrac{h_2-h_1}{h_1}=\dfrac{t_2-t_1}{t_1}$ . . (17)

Let the difference of temperature (or the distance) between successive isothermals be one degree

$$\therefore t_2-t_1=1 \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Also, let the excess of heat required in order to make the substance pass along t_2 from e_1 to e_2 above that required in order to make it pass along t_1 from e_1 to e_2 be one unit of work,

$$\therefore h_2-h_1=1 \quad . \quad . \quad . \quad . \quad (22)$$

$\therefore$ by (17), $h_1=t_1, h_2=t_2$, and $h=t$ . . (23)

Hence, the successive members of a system of adiabatics may be fixed in such a way by (21) and (22), and at such a distance apart, that the number of units of heat which must be imparted to the working substance in order to make it pass along a given isothermal from any adiabatic to the next, is numerically the same as the absolute temperature.

From (22) it appears that the quadrilateral area included between any pair of successive adiabatics and any pair of successive isothermals, is one unit of work.

By (23), $h=t, \ \therefore h\div t=1$ . . . . (24)

Hence, the entropy imparted in passing along any isothermal whatever, from one adiabatic of this system to the next, is unity. But the passage need not be restricted to an isothermal, for along an adiabatic

$$dh=0 \ \therefore \frac{dh}{t}=0 \quad . \quad . \quad . \quad . \quad (19)$$

Hence, entropy does not vary along an adiabatic, hence the adiabatics are isentropics.

The successive isothermals should be numbered so that temperature vanishes when the substance is without sensible heat. And the successive adiabatics should be so numbered that entropy vanishes when the substance has no energy actual or potential.

14. INTEGRATING FACTOR.—One of the factors which will render integrable every differential expression for the heat imparted, such as (3), (4) or (5), is t^{-1}. For suppose, as in art. 12, that there are two routes x or x' and y or y' leading from 1 to 2; then, as shown in art. 10, the heat which must be imparted to make the substance pass from 1 to 2 along x, is the area included between the curves $e_1 \ x \ e_2$, and the heat imparted along y is the area between the curves $e_1 \ y \ e_2$. These areas are not in general equal, consequently $\quad h_2-h_1=\int_1^2 dh$ is not in general integrable until the route from 1 to 2 is given, for its value is dependent upon that route. But we have shown that $e_2-e_1=\int_1^2 t^{-1}\,dh$ . . (20) is not dependent upon the route from 1 to 2.

But any differential expression which is a function of several variables and their differential coefficients, and which, when integrated between assigned limits, can have but a single value, and this value dependent alone upon those limits, is therefore a differential expression which can be integrated (*i.e.*, summed) between those limits. Hence, such an expression fulfills all the requirements of integrability. But these requirements

are expressed algebraically by the so called "equation of condition of integrability," which holds in case of a total or exact differential. Hence, (20) is an integrable expression, and $t^{-1}\,dh$ is an exact differential which fulfills all the conditions of integrability, algebraic, or otherwise which can exist.

It is possible by the help of the second law to show algebraically, that t^{-1} is an integrating factor, and this is done subsequently in certain cases; but it appears more in accordance with physical reasoning, to show why t^{-1} is necessarily an integrating factor, than to divert the attention to the forms employed in the algebraic process for proving the same thing. The foregoing proof is that usually given to show that the forces which act towards the fixed centers (as do the forces of nature) have always a definite potential, or force function, for every point of space, and that the work done against such forces in moving a particle from 1 to 2, is not dependent upon the path. This matter is treated in detail by Clausius.*

* Dingler's Polytechnic Journal, Vol. CL.

TURBINE WHEELS.

ON THE INAPPLICABILITY OF THE THEORETICAL INVESTIGATIONS OF THE TURBINE WHEEL, AS GIVEN BY RANKINE, WEISBACH, BRESSE AND OTHERS, TO THE MODERN CONSTRUCTIONS INTRODUCED BY BOYDEN AND FRANCIS.

By Prof. W. P. TROWBRIDGE, Columbia College.

Written for Van Nostrand's Magazine.

The increasing importance of the Turbine wheel as a motor, gives especial interest to everything relating to the history of its development; and although the improvements which have been introduced from time to time have been brought about by practical men in the first instance, yet the professional engineer is often called upon to investigate the merits of special designs, or to decide upon the theoretical performance of particular wheels.

To enable him to do so, it is essential for this as for all other motors, that the scientific principles of construction should be determined and introduced into text books of instruction for general information. We accordingly find full, and often tediously minute, mathematical investigations following the introduction of the Reaction Turbine of Fourneyron, in France years ago.

The improvements in construction and design made by our own countrymen, Uriah A. Boyden and James B. Francis, marked a second period in the useful applications of the Turbine, no less important and radical in some respects than those which characterized the invention of Fourneyron. For the services thus rendered to industry, the names of Boyden and Francis deserve more universal recognition than they have yet received. My object is not, however, to assert the claims to which they are entitled, but to show, that the mathematical investigations which have been alluded to as following the introduction of the Fourneyron wheel and which are still given in text-books as the accepted expositions of the subject of Turbines, are insufficient and inapplicable to the modern constructions which are founded upon the principles introduced by Boyden and Francis.

In other words, while the Turbine wheel has been improved and its efficiency greatly increased, we look in vain for suggestions which might lead to such improvements, in the treatises on Turbines, by Rankine, Weisbach, Bresse and others, which have been and are still most frequently studied for information on the subject.

Mr. James B. Francis remarks in his well known work entitled "Lowell Hydraulic Experiments" that "the turbine has been an object of deep interest to many learned mathematicians, but up to this time the results of their investigations, so far as they have been published,

have afforded but little aid to Hydraulic Engineers."

How far this statement may be literally true or not, it is unnecessary to discuss: but interpreted as the conclusion of Mr. Francis, in regard to the improvements which it was his object to describe in the Lowell Hydraulic Experiments, the statement seems undeniable; because it may be shown, I think, that if Boyden and Francis had followed strictly the rules of construction, laid down in the works which have been alluded to, they would have failed in their efforts to construct Turbines giving any considerable increase of efficiency over the old Fourneyron and Fontaine or Jonval wheels of European design and construction.

Writers on Turbines had, up to that time, insisted upon a mechanical axiom or principle of construction which, although exemplified in the Fourneyron Reaction wheel, was directly violated by Boyden and Francis in their constructions.

Their success was a sufficient demonstration of the correctness of their practice. Thus while their improvements have been universally accepted and foled, at least in this country, the student and the engineer are continually confronted with the old mathematical theories; and it is not surprising that others besides Francis have been puzzled in their efforts to apply old formulas to the newer constructions.

The theorem or axiom insisted upon by these writers was that the "water must enter the wheel *without shock*," and hence, the mathematical condition that the *tangential velocity of the wheel, where it receives the water, and the corresponding component of the velocity of the entering water* must be equal; the effect of which is to prevent all *impulsive* effects of the entering water.

Taking the old Fourneyron wheel as an example, the buckets or floats were given a radial direction at the point where they received the water, and the relative velocities were made such by the mechanical conditions assumed, that the water entered the wheel *radially*, with a tangential component equal to the velocity of the bucket at this point. Its effects in producing *mechanical work* were thus made to depend solely upon the subsequent deviation which it experienced in passing through the wheel. This device was to avoid the *shock* of the entering water.

It is to be noted that both Rankine and Weisbach, in discussing the impulse and reaction of jets of water upon moving *vanes*, make no reservation in regard to the *shock* due to impulse, but demonstrate that water may impinge at any angle and with any relative velocity upon vanes and, by a suitable arrangement of curvature and velocities, may have all the energy destroyed; and a perfect efficiency may be obtained.

It is difficult to understand why in their discussions of the *Turbine* wheel, they insist on a different principle, and lay down a mechanical axiom at variance with these demonstrations.

The following discussion is intended to apply to the constructions introduced by Boyden and Francis, but, as will be seen in the development, the general formulas deduced are applicable to nearly all water-wheels, by making simple and proper suppositions in regard to the quantities which enter into them.

The Turbine wheel consists essentially of two rims or crowns firmly attached to an axis, numerous curved vanes or *buckets* being fixed between the crowns, and the main stream of water which passes through the wheel is divided by guide blades, not attached to the axis, into numerous jets or smaller streams, which impinge upon these buckets simultaneously at all points of the circumference, and produce motion in the wheel.

It is sufficient, in discussing the action of the water on the wheel, to take one of these jets separately with its guide blade and bucket, inasmuch as all the jets act in the same manner.

The problem then, is to analyze the action of a single jet upon a curved bucket, and to find the mechanical work performed and the conditions of maximum efficiency.

To do this, it is proper to enumerate the axioms or theorems of the mechanics of fluids, which, are applicable to the subject, and which require no special proof. The proof of these theorems being derived from general observation and experience.

These theorems are as follows:

1*st*. When a surface moves in a given

direction under given pressures, the component pressures in all directions, except that of the motion, are neutralized, either by reciprocal actions, or by the fixed surfaces which guide the moving surface.

Therefore, in considering mechanical work done by given pressures acting upon moving surfaces, it is necessary to take into account those components only of the pressures which act in the direction of the motion, friction being neglected.

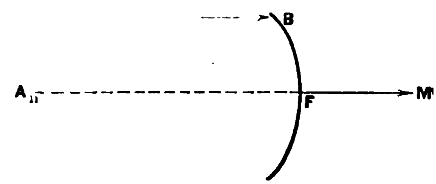
2nd. In whatever direction a surface be moving with reference to the earth, if a fluid moves along this surface in a direction opposite to the motion of the surface, and with a relative velocity equal to the velocity of the surface with reference to the earth, the fluid will be at rest with reference to the earth.

3rd. A fluid stream on striking a smooth surface, at any angle whatever, is not reflected like a solid, but flows along the surface. If the surface be fixed, and the stream be confined in a channel of uniform dimensions before and after striking the surface, the velocity of the stream will remain unaltered, friction not being considered.

If the surface be moving, the velocity under the same conditions after striking the surface will be the *relative* velocity of the surface and fluid before impact. If, for instance, a fluid jet impinge perpendicularly upon a plane surface moving with any velocity in the same direction, the *relative* velocity will be the difference of the two velocities and this will be the velocity with which the stream will flow along the surface.

This initial difference of velocities gives rise to a pressure due to *impulse*, the direction of the pressure being always normal to the surface at the point of impact.

4th. If the surface be curved, as in the following sketch, the same phenomena occur except that the relative velocity of the particle of the stream is not entirely destroyed until it reaches a point at which it is moving at right angles to the direction of the motion of the surface. For example, if the stream AB impinge upon the curved surface represented in the sketch, the surface moving in the direction FM, the difference of velocities of the surface and any particle will be wholly destroyed when the particle reaches the point F, the whole effect up to this point is one of *impulse* and is the same as if the stream impinged directly upon the surface at F and was deviated instantaneously at right angles to the motion FM.



5th. After the particle passes the point F it flows along the curved surface without any further change of relative velocity; a change of curvature, having no effect to change the velocity of *flow relatively to the surface.*

A change of curvature, however, causes a change of *direction* of the motion of the fluid with reference to the motion of the surface and the reaction of the fluid or resistance which it offers to this change, produces a pressure, the pressure due to *reaction.*

If a fluid *vein*, having a fixed direction and velocity with reference to the earth, impinge upon a surface which also has a motion with reference to the earth, the energy imparted to the surface may thus be separated into two parts: that due to the *impulse* and that due to *reaction.* The effect of impulse may be represented by the expression $\overline{M.x.u.}$, in which M is the mass of fluid striking the surface in one second, x the relative velocity in the direction of the motion of the surface, *i.e.*, the difference between the velocity of the surface and the component of the velocity of the fluid in the same direction.

The second effect, due to reaction, results from the deviations which the particles of the fluid vein undergo while in contact with the surface, after they have attained the velocity of the surface. This effect is measured by an expression precisely similar to the first, viz., Mxu, in which M is the mass, as before, x, the relative velocity *imparted* to the fluid in a direction opposite to the motion, and u the velocity of the surface. The total effect being

$$W = M(xu + X_1 U)$$

This being the energy in foot-pounds imparted to the surface.

If all the energy of the vein is destroyed so that it comes to rest with reference to the earth, it is evident that x_1 must then be equal in amount, and directly opposite to u, and the efficiency of such an arrangement will be unity; that is, all the energy of the fluid stream will be transferred to the surface, and we shall have

$$W=M\ (xu+X_1U)=\frac{Mv_1^2}{2}$$

$$E=1$$

v_1 being the velocity of the fluid with reference to the earth before striking the surface.

If there is no effect from impulse, the relative velocity of the particles of the fluid vein and the surface in the direction of the motion of the surface must be zero—or $x=O$.

The whole effect produced must then be from reaction, and it is evident that for an absolute maximum of effect, $M\ X_1\ U$ must be equal to

$$\frac{Mv_1^2}{2} \text{ or } X_1\ U=\frac{v_1^2}{2}.$$

If the surface be a plane surface, so that there is no deviation of the vein after it strikes the surface, the whole effect will be due to impulse, and $W=M.x.u$, will be the total work performed, or the total energy imparted to the surface.

These three general cases,

$W=M(ux+X_1U)$. impulse and reaction,

$W=MX_1U$ . . . reaction,

$W=Mxu$ . . . impulse,

explain the whole theory of the action of a stream of water, or of multiple streams in producing motion in water-wheels; not including, however, those cases in which water acts by its weight. In these cases, the first expression may be applied by substituting for the expression representing reaction, $M\ X_1\ U$, a form of expression representing the action of weight falling through a given distance, and the expression

$$W=M(xu+Wh)$$

will be the general expression for the energy of Overshot and Breast wheels, W being the weight of water entering the wheel in one second, $M\,x\,u$ being the work of impulse of the water entering the buckets and h the height of fall.

Nearly all modern turbine wheels are constructed after one of three types, or of some combination of these types. They are illustrated in the following sketches, and usually receive the designations of

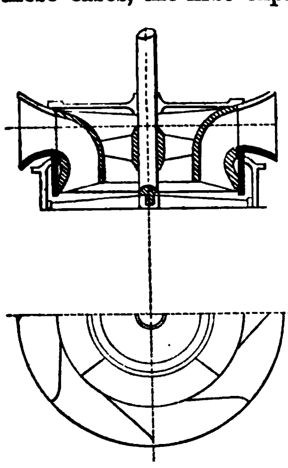
Outward flow wheels,
Inward flow wheels, or center vent wheels.
Parallel flow wheels.

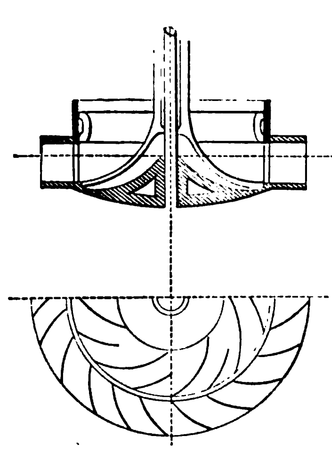
In the outward flow wheel of which the Fourneyron reaction wheel is the earliest type, the water flows usually downward through a tube or conduit, and is diverted by fixed guide blades in an outward direction, or from the axis of motion; the form of the fixed guides giving the water a tangential whirl as it enters the wheel.

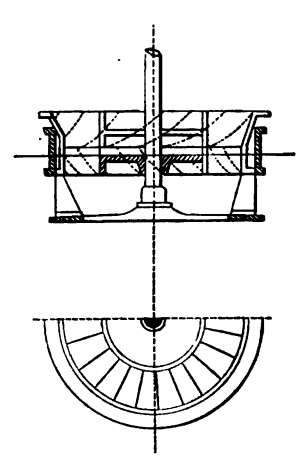
In the inward flow, the water flows first in the direction of the axis, usually downward, and is then diverted by fixed guide blades inwardly, or toward the axis of motion, the same fixed guides giving the water, as before, a tangential whirl as it enters the wheel.

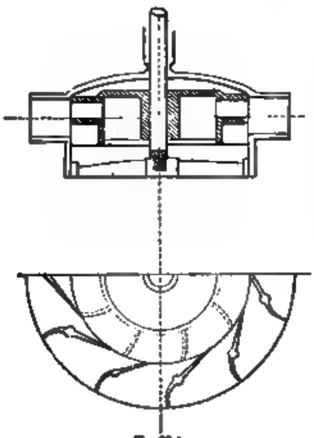
In the parallel flow wheel, the water moves parallel to the axis of motion, before and after it passes through the wheel; the fixed guide blades, as before, giving the water a tangential whirl.

The principle types of some of the best wheels in use are represented in the following cuts:









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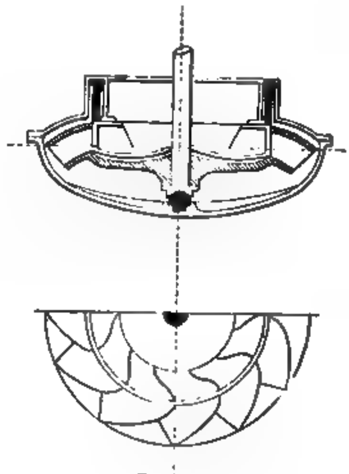
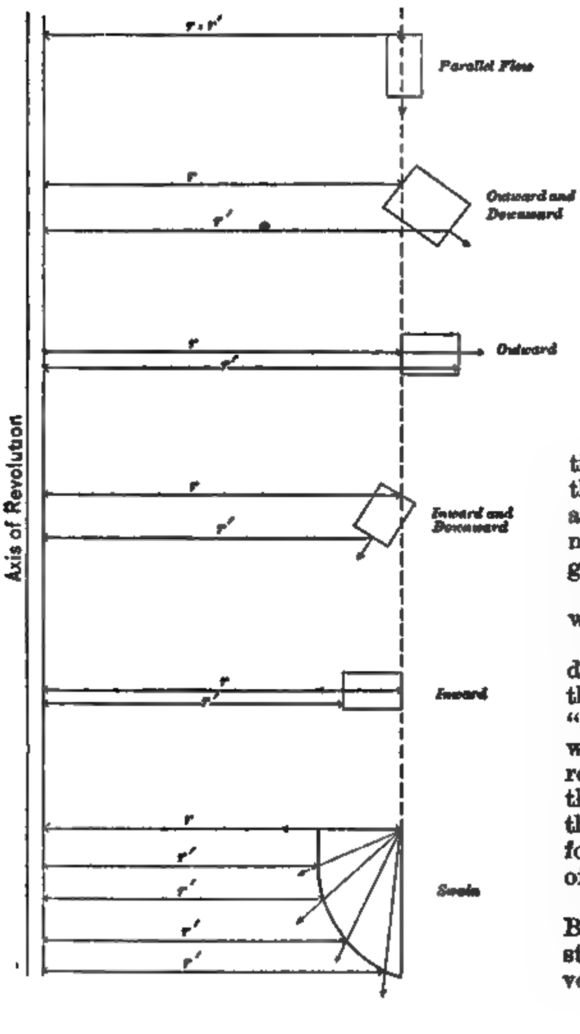
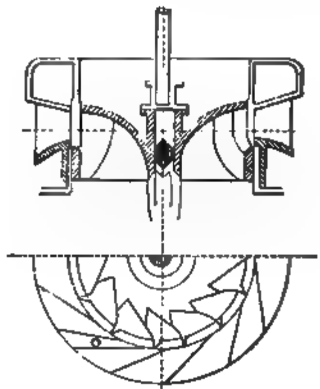


Excelsior





Swain

The position of the buckets, the direction of the water entering the wheels and the relative values of r and r', the outer and inner radii of the wheels, may be seen more clearly by reference to the diagram given opposite.

To investigate the problems connected with these wheels more minutely:—

Suppose a jet or stream, having a given direction and velocity with reference to the earth, to strike a surface, a vane or "float," all points of which have a motion with a given direction and velocity with reference to the earth; the component of the velocity of the jet in the direction of the motion of the surface may be easily found by the theorem of the parallelogram of velocities.

Let ABC (Fig. 1) be the direction, and BC$=v$, the velocity of a jet of water striking the surface BE, which has a velocity BD$=u$.

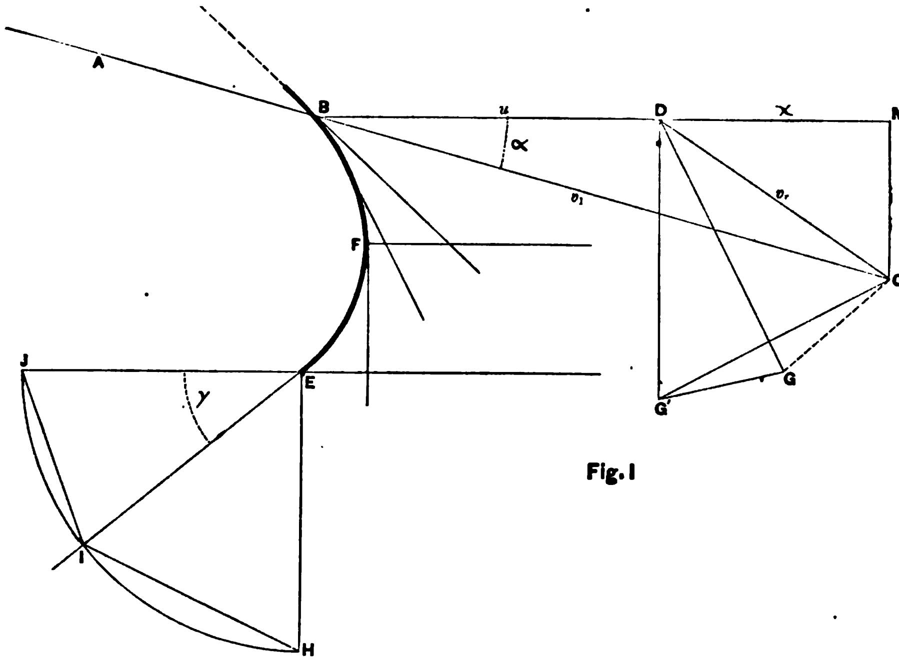


Fig. I

Let the angle DBC$=a$ be the angle made by the direction of the jet with the direction of the motion of the surface.

Then $v_1 \cos. a$ will be the component of the velocity of the stream in the direction BD of the motion of the surface; DM$=x$ will be the difference of velocity in the direction of the motion of the jet and surface and DC the total relative velocity of the jet and surface, in magnitude and direction.

If we draw a tangent to the surface at B and draw the line DG, parallel to this tangent, making DG equal to DC, CG will be the instantaneous deviation due to the impact.

When the jet reaches the point F of the surface, a point at which its direction is perpendicular to the motion of the surface, if we draw the line DG′ parallel to the tangent at F or perpendicular to the motion of the surface, the total deviation of the jet will be CG′ and its projection on BDM, or its component in the direction of the motion BD, will be

$$X=(v_1 \cos. a-u)$$

The relative velocity x_1 of the jet *in the direction of the motion* is not entirely destroyed until it reaches the point F at which its direction is perpendicular to the direction of the surface BD, and hence we may call the total effect up to this point the impulse of the fluid.

If M be the mass of fluid which passes in a unit of time, the momentum of the fluid with reference to the direction of motion of the surface will be

$$Mx=M(v_1 \cos. a-u) \text{ and}$$

the mechanical work performed by this momentum will be

$$Mxu$$

After the jet passes the point F, the only pressure that it can exert on the surface will be due to a change of direction or deviation, the *reaction* being the pressure exerted.

If the fluid passes off the surface at E, in the direction of the tangent, at E the amount of the additional deviation may be found by drawing EI in the direction of the tangent, and EH in the direction of the jet at F, making EH$=$EI$=$DC.

Then HI will be direction, and will represent the pressure of the jet due to *reaction*, and the projection of this line on the direction of motion, JE, will be the component of this pressure in the direction of the motion.

Representing by γ the angle of departure, made by the tangent IE with the direction of motion, this component will be

$$\overline{IE} \cos. \gamma$$

or calling $IE=DC$ the *relative* velocity, V_r this component will be

$$V_r \cos. \gamma,$$

and the mechanical work performed per second will be the component pressure $\overline{MV_r \cos. \gamma}$ multiplied by U, the velocity of the surface, or

$$\overline{MV_r \cos. \gamma}\, U.$$

The total work performed by the jet upon the surface due to *impulse* and *reaction*, will be

$$W=M(XU+\overline{V_r \cos. \gamma\, u})$$

the sum of the two effects.

If $\gamma=o$, the work of reaction will be $M V_r U$, and the total work

$$W=M(XU+V_r U).$$

Since V_r is a function of u, we may find under what circumstances this work will be a maximum.

The problem resolves itself into this: in the known triangles BCD, and BCM required to find the position of the line DC such that $W=u(x+v_r)$ shall be a maximum.

By applying the methods of Calculus it will be found that the expression will be a maximum when $u=v_r$

That is for a maximum,

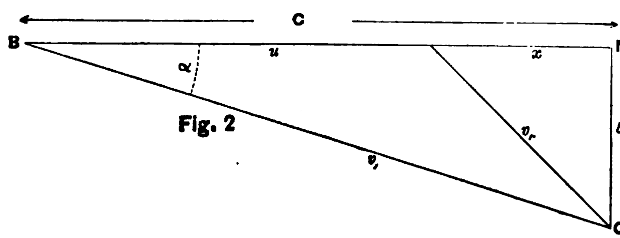


Fig. 2

Let $BM=C$ $CM=b$

then

$$x=\frac{c^2-b^2}{2c} \quad v_r=u=\frac{c^2+b^2}{2c}$$

and

$$xu+v_r\, u=xu+u^2=\frac{v_1^2}{2}$$

The same may be demonstrated by geometry. When $\gamma=o$ and $v_r=u$ the fluid in leaving the vane or surface has a motion in a direction opposite to that of the vane and with a relative velocity equal to that of the vane; hence it will be at rest with reference to the *earth:* and all its energy will have been imparted to the vane.

Since v_1 is the original velocity of the fluid with reference to the earth, its energy is $M\frac{v_1^2}{2}$; but we have under the conditions of the maximum just found

$$(W)_{\text{max.}}=M(xu+u^2)=M\frac{v_1^2}{2}$$

or the maximum work is the whole energy of the fluid, and the efficiency

$$E=\frac{\frac{Mv_1^2}{2}}{\frac{Mv_1^2}{2}}=1$$

If the angle γ is not zero, then the work performed cannot be an absolute maximum, or the efficiency cannot be unity, because the fluid will not leave the vane with a direction and velocity equal and opposite to the direction and velocity of the vane. In other words, the fluid will, after leaving the vane have a motion with reference to the earth, the energy due to which will be lost.

The value of this lost work is easily found. In practice the angle γ is always very small.

If a parallelogram (Fig. 3) be drawn, having for sides, the relative velocity $EI=v_r$ and $ET=u$ the diagonal ER for very small angles will be practically $\overline{v_r \, sin. \, \gamma}$ which will be the velocity of the stream after it leaves the vane at E. The energy due to this velocity will be $\frac{Mv_r^2 sin.^2\gamma}{2}$ and the general value of the work will be

$$W=M\left(xu+u^2-V_r^2\frac{sin.^2\gamma}{2}\right)$$

$$W=M\left(\frac{v_1^2}{2}-\frac{v_r^2\, sin.^2\gamma}{2}\right)$$

and the efficiency

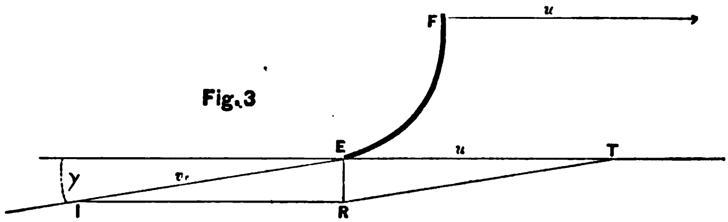


Fig. 3

$$E=\frac{M\left(\frac{v_1^2}{2}-\frac{v_r^2 sin.^2\gamma}{2}\right)}{\frac{Mv_1^2}{2}}$$

or

$$E=1-\frac{v_r^2}{v_1^2} sin.^2\gamma$$

Assuming now that the velocity u was that which gave the absolute maximum when $\gamma=o$ or $u=v_r=\frac{v_1}{2 \cos. a}$ (see Fig. 2) we have by substituting this value of v_r in the expression for the efficiency

$$E=1-\frac{1}{4}\frac{\sin.^2\gamma}{\cos.^2 a}.$$

From this it is apparent that the efficiency is increased by making a and γ small.

If $a=o$ then the stream impinges at right angles to the vane or in the direction of its motion, in which case $x=v_1-u$ and $u=\frac{v_1}{2}$ for the maximum efficiency, and the expressions for the efficiency become for $\gamma=o$

$$E=I$$

and for γ a small angle

$$E=I-\tfrac{1}{4}\sin.^2\gamma$$

Variations from maximum work for variations of u.

If we describe a circle (Fig. 4) on $BM=v_1 \cos. a$ and lay off x and u in such a way that $OC=v_r=u$, we see that the

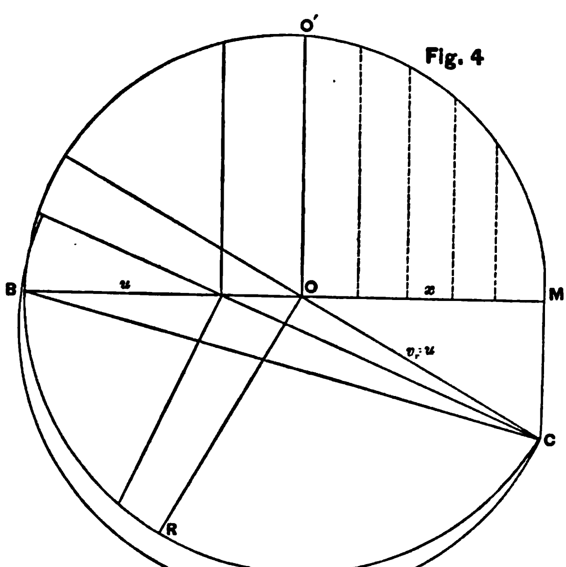


Fig. 4

two terms of the expression $xu+v_r u$ will vary as the squares of the ordinates (of the circles) OO′ and OR respectively, and the positions of these ordinates are such that the variations of their squares, *i. e.*, $\overline{OO''}^2=xu$ and $\overline{OR}^2=v_r u$ will be a minimum, or in other words, small variations of u from the value giving the absolute maximum, may occur without affecting greatly, the value of the work.

Variations from maximum efficiency for variations of the angles a and γ.

In the expression for the efficiency

$$E=1-\frac{1}{4}\frac{\sin.^2\gamma}{\cos.^2\alpha}$$

found on the supposition that γ is small. If α is also small, the efficiency is not greatly affected by very small changes in these angles.

α being a fixed angle by the conditions of the problem; if γ be variable within small limits, the variations will not affect greatly, the work performed.

Method of finding the work performed and the conditions of absolute maximum when the vane or surface revolves around a fixed axis.

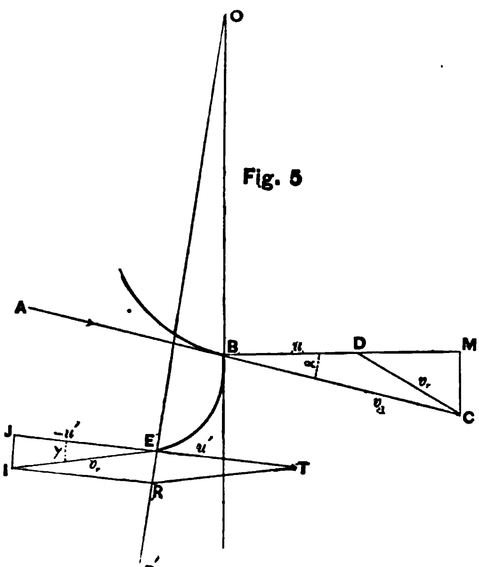


Fig. 5

Let ABC (Fig. 5) be the direction of the jet with reference to the earth; BD $=u$ the velocity of the point B at which the tangent to the vane coincides with the radius OB of the circle, around whose center the motion takes place: EJ the direction perpendicular to the radius OE, at the point at which the jet leaves the surface or vane, x, u, v_r, α and γ designating the same quantities as before and u' the velocity of the vane at E.

Then, according to the conditions of maximum effect, the jet should leave the van at E, in a direction opposite to u', the direction and velocity of the point of departure E, around the center, and with a velocity relatively to the vane $(-u')$.

If $BC=v_1$ is the original velocity of the jet with reference to the earth, then the absolute maximum work will be

$$M\frac{v_1^2}{2}.$$

Since the velocity of the vane at B is $u=ar$, and the velocity at E is $u'=ar'$: a being the angular velocity of the motion; and, since for the absolute maximum work, $v_r=DC$ must be equal to u'; in the triangle BMC, the distance $BD=u$ and $DC=v_r=u'$ must be to each other as r to r'; r being the radius OB and r'— the radius OE.

The work of impulse will be, as before, Mxu, since the relative velocity in the direction of the motion (*i. e.*, perpendicular to the radius) is all destroyed at B.

The work of *reaction* of the jet in moving from B to E and passing off at E in a direction opposite to u', will be $Mv_r u''$; u'' being a velocity of motion due to a length of radius between r and r', and the total work, supposing at first, $\gamma=o$ will be

$$W=M(xu+v_r u'').$$

For the absolute maximum or

$$W=M\frac{v_1^2}{2}$$

and $E=1$ we must have from the mechanical conditions $v_r=u'$ and this expression becomes

$$W=M(xu+u'u'')=M\frac{v_1^2}{2}.$$

The value of $u'u''$ which gives the maximum may be found from the following considerations. If the vane had a velocity at all points equal to u, then this term $u'u''$ would be u^2 as in the case first discussed, and if its velocity at all points was u', this term would be u'^2. The true value when the vane revolves around an axis O, will be a mean between u^2 and u'^2,

or $\frac{u^2+u'^2}{2}$ and

$$W_{\max.}=M\left(xu+\frac{u^2+u'^2}{2}\right)=M\frac{v_1^2}{2}$$

and

$$E=\frac{\frac{Mv_1^2}{2}}{\frac{Mv_1^2}{2}}=1.$$

Another method of proof is as follows:

In the triangle BMC.

$$(x+u)^2+v_r^2-x^2=v_1^2$$

or $$(x+u)^2+u'^2-x^2=v_1^2$$

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$$xu+\frac{u^2+u'^2}{2}=\frac{v_1^2}{2}$$

Since $u=ar$

" $u'=ar'$ ∴ $u=u'\frac{r}{r'}$

and

$$\frac{u^2+u'^2}{2}=\frac{u'^2\frac{r^2}{r'^2}+u'^2}{2}$$

$$=\frac{u'^2}{2}\left(1+\frac{r^2}{r'^2}\right)$$

$$=u'^2\left(\frac{r'^2+r^2}{2r'^2}\right)$$

$$=u'ar'\left(\frac{r'^2+r^2}{2r'^2}\right)$$

$$=u'a\frac{r'^2+r^2}{2r'}$$

Hence $$u''=u'a.\frac{r'^2+r^2}{2r'}$$

and $\frac{r'^2+r^2}{2r'}$ is the radius or lever arm at which the velocity is u''.

The value of u which gives this maximum will be

$$u=u'\frac{r}{r'}$$

The formula for the absolute maximum work then becomes

$$W_{max.}=M\left(xu+\frac{u^2+u'^2}{2}\right)=M\frac{v_1^2}{2}$$

and the efficiency

$$E=1$$

This formula may, from what precedes be put under the form

$$W_{max.}=M\left\{xu+\frac{u'^2}{2}\left(1+\frac{r^2}{r'^2}\right)\right\}=M\frac{v_1^2}{2}$$

In Fig. (5) when the center of motion is O, the stream flows outwardly from the center and r' is greater than r.

If the center of motion were at O′, the stream would flow inwardly, or towards the center, and r would be greater than r'.

If $r=r'$, then $u=u'$ and the formula becomes

$$W_{max.}=M\left(xu+u^2\right)=M\frac{v_1^2}{2}$$

as before found.

If the angle γ is not zero; but the relative velocities such as would otherwise give the absolute maximum of work, the real work performed and the efficiency may be found as in the first case.

The lost work will be, as before $M\frac{v_r^2\sin.^2\gamma}{2}$ and the total work

$$W=M\left(xu+\frac{u^2+u'^2}{2}-\frac{V_r^2\sin.^2\gamma}{2}\right)$$

$$=M\left(\frac{v_1^2}{2}-\frac{v_r^2\sin.^2\gamma}{2}\right)$$

and for the efficiency

$$E=\frac{M\left(\frac{v_1^2}{2}-\frac{v_r^2\sin^2\gamma}{2}\right)}{M\frac{v_1^2}{2}}=1-\frac{v_r^2 sin.^2\gamma}{v_1^2}$$

The relation between v_r and v_1 may be found from the triangle BMC, Fig. 6.

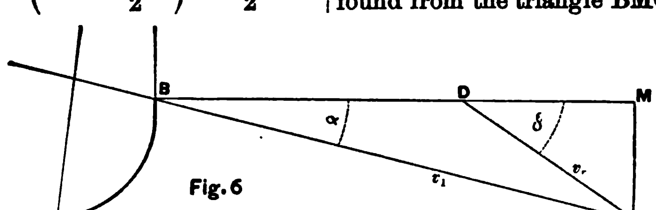


Fig. 6

If the angle CDM be designated by δ, then

$$v_1\sin.a=v_r\sin.\delta$$

and $$\frac{v_r}{v_1}=\frac{\sin.\ a}{\sin.\ \delta}$$

then $$E=1-\frac{\sin.^2a}{\sin.^2\delta}\sin.^2\gamma$$

From this expression it appears as before that the efficiency is increased by making a and γ small.

Variations of work for small variations of u and v_r.

It is evident from the conditions of this problem that a variation of the angular velocity of motion, will cause

simultaneous and proportional changes in the velocities u and u' and in the expression for work

$$W=M(xu+v_r\,u) \quad . \quad . \quad . \quad (\gamma=o)$$

The work will vary as the sum of the squares of the ordinates DO and DO′ (or as $\overline{DO}^2+\overline{DO'}^2$) of the same circles described on the diameters $\overline{x+u}$ or BM and $\overline{v_r+u''}$: u'' being a quantity whose value lies between u and v_r.

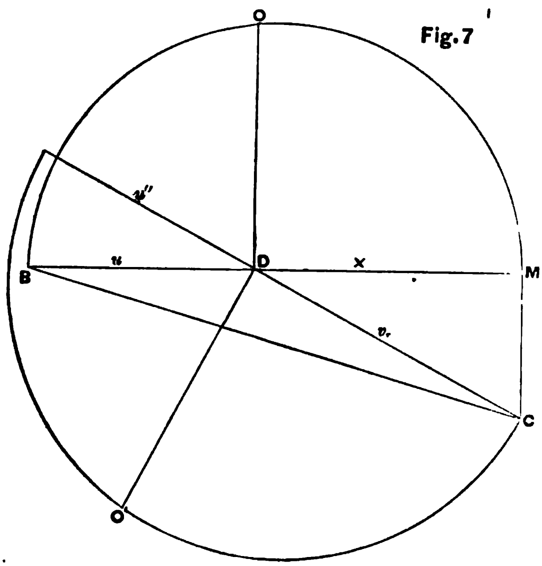


Fig. 7

The squares of these ordinates will vary very slowly near the values, which make the work an absolute maximum, since the ordinates in this case lie near the centers of the semicircles.

The expression

$$W_{max.}=M\left(xu+\frac{u^2+u'^2}{2}\right)$$

or

$$W_{max.}=M\left(xu+\frac{u'^2}{2}\ 1+\frac{r^2}{r'^2}\right)$$

is applicable to the three principle types of turbine wheels, by giving to r and r' their proper *relative* values.

For outward flow wheels $r<r'$
For inward flow wheels $r>r'$
For parallel flow wheels $r=r'$

The diagram (Fig. 8) shows that since the ratio of r to r' is the same as that of u to u', that for the same angle a, the velocity of maximum efficiency (BD′$=u$) will be greater for the inward flow than for the outward flow (BD$=u$) and that for each type of wheel with the same angle of incidence, the velocity of maximum efficiency will have a particular value, viz: for parallel flow u will equal v_r the relative velocity, $\left(=\frac{v_1}{2\cos. a}\right)$ for outward flow wheels, u will be less than v_r and for inward flow, u will be greater than v_r

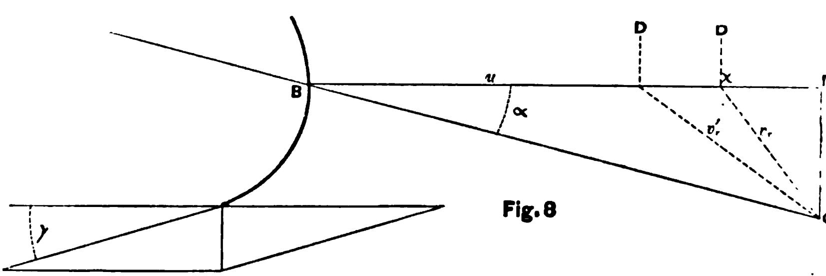


Fig. 8

We have now to discuss particular cases of the general expression for work

$$W=M(xu+v_r\,u')$$

as applied to different classes or types of water-wheels. Supposing in all cases $\gamma=o$.

IMPACT WHEELS.

This class includes all water-wheels in which the fluid stream acts by impulse, the blades or floats being plane and no reaction taking place from the curvature of the blades after impact.

The axes of such wheels may be vertical, with a single stream or several streams impinging upon plane floats; or the axis may be horizontal as in the case of the old undershot wheel.

To apply the formula to this class, it is only necessary to make the second term zero. The formula will then become $W=Mxu$, and since $x=v_1\cos. a-u$

$$W=M(v_1\cos. a-u)u.$$

This expression is a maximum when $v_r\cos. a-u=u$ or

$$u=\frac{v_1 \cos. a}{2}$$

This is the velocity of greatest efficiency, and the efficiency is in all cases

$$E=\frac{W}{\frac{Mv_1^2}{2}}=\frac{Mu^2}{\frac{Mv_1^2}{2}}=\frac{M\frac{v_1^2\cos.^2 a}{4}}{M\frac{v_1^2}{2}}$$

or

$$E=\frac{\cos.^2 a}{2}.$$

If a is zero, then $u=\frac{v_1}{2}$ is the velocity of greatest efficiency, and $E=\frac{1}{2}$. The usual efficiency of these wheels in practice, is only about $\frac{1}{3}$.

REACTION WHEELS.

A reaction wheel is one in which there is no effect from impulse; or in the general expression

$$W=M(xu+v_r u')$$

it is necessary to make $x=o$. The expression then becomes

$$W=Mv_r u''$$

In these wheels, the deviation of the stream caused by the curvature of the blades is the cause of the energy exerted and several distinct cases may arise.

First, suppose the stream of flow in a direction parallel to the axis of revolution as in the Jonval Wheel; that is, suppose it to be a parallel flow reaction wheel.

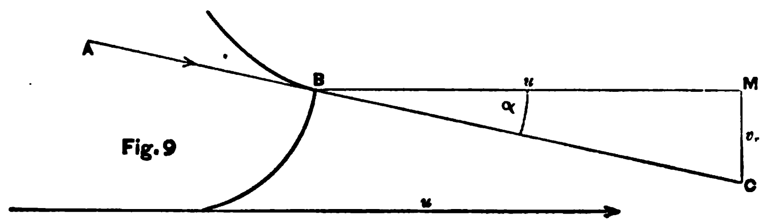


Fig. 9

Then since $x=o$ (Fig. 9) $u=v_1 \cos. x$ and the expression becomes

$$W=Mv_r v_1 \cos. a$$

But the same construction shows that $v_r=MC=v_1\sin.a$ and the value of W becomes

$$W=Mv_1^2\sin.a \cos. a$$

For this expression there is no absolute maximum except in one particular case, *i.e.*, for $a=45°$ in which case $v_1\sin.a=v_1 \cos. a$ and $v_1^2 \sin. a\cos. a=v_1^2 \sin.^2 a=\frac{v_1^2}{2}$ and $W=M\frac{v_1^2}{2}$

$$E=1$$

It follows from this that if in the parallel flow wheel, the principle is insisted on that the water must enter without shock, or in the language of Bresse, if "it is necessary that at the point B, (Fig. 8) the water shall be directed tangentially to the floats" the latter being radial at the point of entrance of the water, the wheel becomes simply a reaction wheel and there is no theoretical absolute maximum except for the particular angle $a=45°$.

Bresse, in discussing the problem for the Fourneyron and parallel flow turbines, deduces nine equations involving sixteen unknown quantities, which involve the necessary conditions of a perfect wheel. A discussion applicable only upon the particular conditions which he presents, but not applicable to American Turbine wheels as now constructed and employed. Nevertheless his discussions which have been translated for use in this country are entitled *Turbine Wheels*, and there is no indication given that they are not applicable to all Turbines.

Second.—If under the supposition that there is no impulsive action of the water, we suppose the stream to flow outwardly from the axis. The work performed will be as before for $x=o$

$$W=Mv_r u''$$

or putting for $v_r u''$ in the general case $\frac{u'^2}{2}\left(1+\frac{r^2}{r'^2}\right)$ we have

$$W=M\frac{u'^2}{2}\left(1+\frac{r^2}{r'^2}\right).$$

This applies to the case where the water enters tangentially to the floats, which are radial in direction as at B Fig. 7 or 8, and for which the value of $u=v_1 \cos. a$.

The condition is thus fulfilled as expressed by Rankine, that "the whirling velocity of the water, when it enters the wheel, must be equal to the tangential velocity of the wheel," a condition appli-

cable, according to this author, to all turbines.

But in this case it is evident from the expression

$$W=M\left\{\frac{u'^2}{2}\left(1+\frac{r^2}{r'^2}\right)\right\}$$

that there is no theoretical maximum and the wheel becomes simply a reaction wheel. For an absolute maximum or an efficiency equal to unity, we must have

$$W_{max.}=M\frac{u'^2}{2}\left(1+\frac{r^2}{r'^2}\right)=M\frac{v_1^2}{2}$$

which can only be the case when $r=r'$ and $u'^2=\frac{v_1^2}{2}$ which takes us back to the particular case of the parallel flow wheel, with an angle $a=45.°$

In the outward and inward flow reaction wheels, *i.e.*, those wheels in which the buckets are radial at the point of entrance of the water, and in which the velocity of the wheel at this point is the same as the component velocity of the water in the same direction, there is no work of impulse, and the wheels become reaction wheels, for which the efficiency cannot be unity, but there will always be a loss of energy due to the ratio $\frac{r}{r'}$.

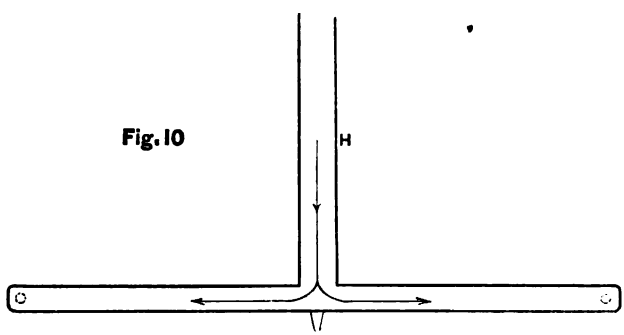
Application of the general formula for work

$$W=M(xu+v_r\,u'')$$

to the wheel known as Barker's Mill.

In this wheel the water descends in a column H, Fig. 10, which is at the same

Fig. 10



time the axis of motion, and flows outwardly to the ends of two tubular arms where it issues from two orifices on opposite sides of the arms.

As the water enters the arms without impulse, the term Mxu in this formula will be zero and the expression for work will be

$$W=Mv_r\,u''$$

u'' will be the circular velocity of the end of the arm at the point where the water issues, v_r being the relative velocity of the arm and fluid at the same point, may be found thus. In the plan of the wheel, Fig. 11, let v_1 be the velocity due to the head. A particle at the end of the arm is subjected to two velocities, one v_1 due to the head and another u' due to the rotation of the arm. Its actual velocity with reference to the earth, will be represented by the diagonal of the rectangular parallelogram constructed on these

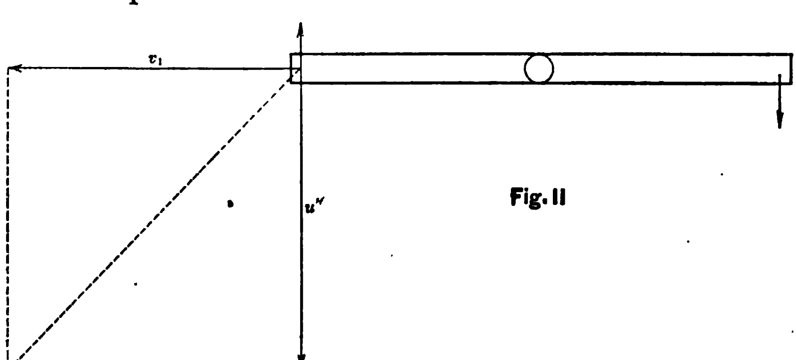


Fig. 11

two lines, and will be expressed analytically by

$$\sqrt{v_1^2+u''^2}$$

This will be the velocity with reference to the earth, with which the jet will issue from the arm in a direction opposite to its motion; but the arm has a velocity $u''=ar$ (a being the angular velocity and r the radius of the arm) and the relative velocity with reference to the arm will be

$$v_r = \sqrt{v_1^2+u''^2}-u''$$

and the work performed will be found by multiplying this by u' and by M, the mass of water which issues in a unit of time or

$$W=Mv_r\, u''=Mu''(\sqrt{v_1^2+u''^2}-u'')$$

an expression which has no maximum.

In the foregoing discussion, no account has been taken of the resistances which arise from friction, or loss of energy from contractions, bends and other causes which influence the flow through narrow passages, like the spaces between the guide blades and the wheel buckets or floats. These influences will, of course, affect the efficiency of all wheels, and also the velocity of maximum efficiency and, of course, in no actual case can an efficiency of unity be attained. The general theory is not, however, altered on this account; and inasmuch as it is impossible to separate the influences referred to from each other, or to ascertain the values of co-efficients which might be applied, the only means of ascertaining how near the performance of any wheel approaches the maximum efficiency is by a practical test.

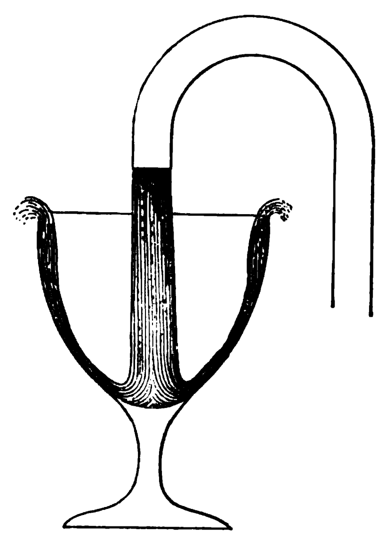
From Francis' experiments it appears that the velocity of maximum efficiency is somewhat greater than that which would be given by theory when friction is not considered.

It has been shown that a considerable change in velocity of a wheel when it runs at a speed giving its maximum efficiency, influences but little, the work performed.

A change in velocity in the Tremont turbine from .48 to .68 of the velocity due to the fall, affected the efficiency by only two per cent., and Francis recommends a velocity of .56 of the velocity due to the fall, as the best in practice, which is in excess of the theoretical velocity of greatest efficiency, when friction is not considered.

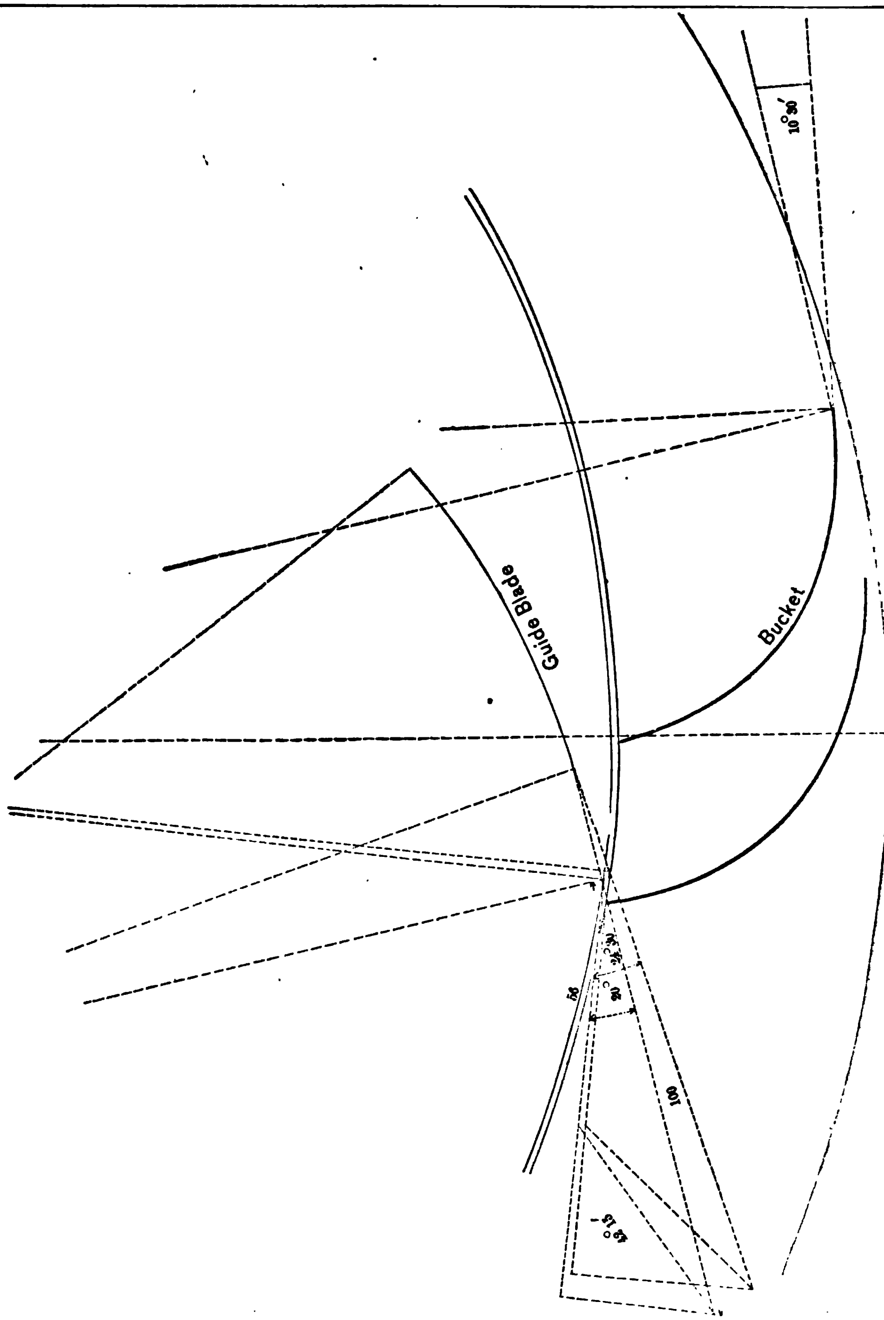
The following diagram of the Tremont turbine and Francis' center-vent, or inward flow turbine, exhibit the particulars of these wheels: 1st, in the small ratio $\frac{r}{r'}$, of the outer and inner radii and the small angles of incidence and departure of the water; and when the velocity of the wheels which give the maximum efficiency is considered, it appears that the velocities of the wheels, at the point where they receive the entering water, is much less than the tangential component of the entering water, thus causing a large part of the work to be done by impulse.

An unnecessary importance seems to have been attached to the idea that a stream of water, to produce its best effect upon a vane, or float, must glide upon the latter in a tangential direction. The following simple experiment may be made by any one who wishes to be convinced of this.



Place a goblet under an ordinary goose neck spout, from which a clear and transparent stream issues, impinging on the bottom of the goblet. If the velocity of the stream is sufficient, the water will rise up and flow over the edge of the goblet at all points in a clear, unbroken sheet, the liquid threads being continuous and unbroken, and the whole remaining as transparent as the glass itself. A more satisfactory experiment may, per-

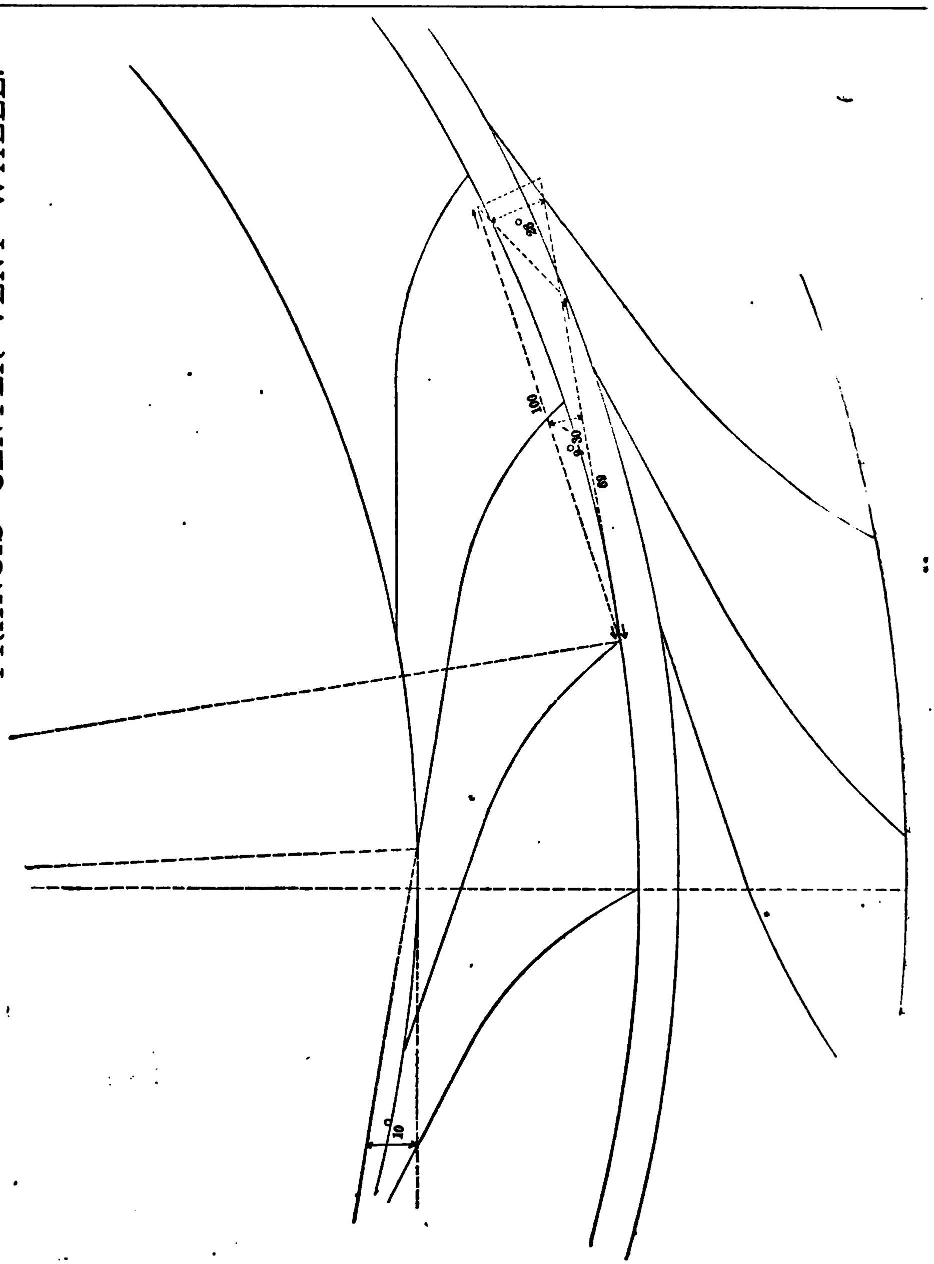
TREMONT TURBINE.



haps, be made by inverting the goblet and allowing the stream to enter from below; more satisfactory, because considerable head is required in the first case. This experiment shows that the disturbances in the stream, from impact, which have been considered of so much importance, do not exist in this, the most unfavorable case, where the impact is normal.

The Poncelet wheel, a vertical water wheel, the construction of which is familiar to all engineers and students, is one in which a principal virtue is supposed to

FRANCIS' CENTER VENT WHEEL.



exist in a curved form of the buckets—such that the water enters the wheel "without shock."

Weisbach states that "to obtain as great an effect as possible with one of Poncelet's wheels, it is necessary that the water should enter the wheel without shock." And it has been considered that one object of the peculiar construction and arrangement of Poncelet was to attain this particular end; whereas a little consideration shows that the curvature of the blade and the peculiarities of construction were designed for quite another purpose.

In fact, the analysis would be quite as

complete for this wheel as far as impulse is concerned, if the water impinged directly upon a plane float.

It is only to obtain the effect of reaction by the returning water that the curvature is necessary.

In order to find the actual value of the efficiency as deduced from the formula

$$E=1-\frac{\sin.^2\alpha}{\sin.^2\delta}\sin.^2\gamma$$

we have only to substitute values for α, δ and γ. Taking, for example, $\alpha=22°$, $\gamma=11°$ and $\delta=46°$ the values employed in the Tremont Turbine, substituting and solving we have

$$E=.9902$$

For Francis' Center Vent Wheel $\alpha=9°30'$ $\gamma=10°$ and $\delta=28°$ which give for the efficiency

$$E=.9962$$

These results show a theoretical efficiency practically equal to unity, when friction and other disturbing resistances are not considered. The angles of incidence and departure being so small as to have but a slight influence in lessening the efficiency when the proper theoretical velocity is given to the wheels.

The angles of incidence of the Swain wheel are about 12° only, and the angle of departure of the inward flow wheels now made is not much greater. It is not surprising, therefore, that a common working efficiency of 84 per cent. should be attained.

The efficiency of an impulse and reaction turbine, properly consructed and run with the proper velocity, being thus practically unity, or 99 per cent., when loss of energy from friction, and other causes of loss of energy of a similar character, are not considered the greatest possible efficiency in practice.

Estimating the co-efficient of velocity of discharge from the guide blades at ,97 the loss of energy due to this loss of velocity will be, $[(\frac{1}{97})^2-1]\frac{Mv_1^2}{2}$ equal $06\frac{Mv_1^2}{2}$; and supposing the friction in the wheel to consume the same amount of energy, the total loss will be $,12\frac{Mv_1^2}{2}$.

If we suppose the friction of the pivots on which the wheel runs, and the collars of the axis to be in the most favorable case $,03\frac{Mv_1^2}{2}$ we have a total of $,15\frac{Mv_1^2}{2}$ which with the loss of .01 due to the obliquity of the angles α and γ will give a total of 16 per cent. of the energy due to the fall, leaving a practical efficiency of 84 per cent. which is the efficiency found by Francis for an ordinary Swain wheel.

RULES FOR DESIGNING WHEELS.

Let D be the diameter of the wheel.

Let α be the angle of incidence of the water with the tangent to the circumference.

Let H be the depth between the crowns at the first circumference or point of entrance.

C a co-efficient of discharge.

A the area of section of entrance of water in the direction of the radius, then the volume of flow will be

$$V=\pi D.H.v_1 \sin.\alpha.C.$$

Assuming a diameter D. a co-efficient of discharge C and an angle of incidence α the depth H will be

$$H=\frac{V}{C.\pi D.v_1 \sin.\alpha}$$

By experiments on the Tremont turbine in which D=8.25308 feet=sum of the widths of the orifices of discharge across the guide blades, and for the heights of the orifices of discharge H, or depth of wheel=.9314 feet. Francis found C=.624.

Hence his general rule, making $H'=\frac{1}{10}$ of the *outer diameter* D',

$$V=H'.D'.C v_1=0.10\ D'^2 e\sqrt{2gh_1}$$
$$=0.5\ D'^2\sqrt{h_1}$$

In which H_1 is the *depth of the wheel at the outer extremities of the buckets* and $H'=\frac{1}{10}D'$.

The simplicity of Francis' formula for the flow

$$V=0.5D'^2\sqrt{h_1}$$

is one of the results of his valuable experiments, giving by a very simple calculation the necessary diameter of an outward flow turbine for a given head h_1 and a given volume of flow V. For a given horse power, $\overline{HP}$ in foot pounds per second, with an efficiency of 75,

$$\overline{HP}=\frac{0.75\times 62.33}{550}\times Vh_1$$

and since $V=0.5\,D'^2\sqrt{h_1}$

$$D'=4.85\sqrt{\frac{P}{h_1\sqrt{h_1}}}$$

Formulas which are applicable to outward flow wheels, within the limits assigned by Francis.

To make the formula more general, we have

$$V=.62\ D.H.\pi.v_1\ \sin.\ \alpha$$
$$=f.D^2\ v_1\ \sin.\ \alpha\ \pi.\ ,62$$

f being ratio of D to H.

Supposing V to be the actual volume of flow per second, and the wheel to utilize .75% of the fall h, multiplied by the weight, then we have the horse-power per second,

$$\overline{\overline{HP}}=\frac{0.75\ .(62\ .43)}{550}\ Vh_1\ \ldots\ldots\ (a)$$

and substituting for V its value

$$V=f.D^2.v_1\ \sin.\ \alpha\ .62\ \pi$$
$$=f.D^2.v_1\ \sin.\ \alpha\ 1.95$$

we have

$$\overline{HP}=\left(\frac{0.75\times 62.43}{550}\times 1.95\right)f.D^2.v_1\sin.\ \alpha h_1$$

or

$$\overline{HP}=\left(\frac{0.75\times 62.43\times 1.956}{550}\right)$$
$$f.D^2.\ \sqrt{2gh_1}\ \sin.\ \alpha\ h_1$$

$$\overline{\overline{HP}}=\left(\frac{0.75\times 62.43\times 1.956\times 8\ .03}{550}\right)$$
$$f.D.^2\sin.\ \alpha h_1\ \sqrt{h_1}$$

$$\overline{\overline{HP}}=1.33\ f.D^2.\ \sin.\ \alpha\ h_1\ \sqrt{h_1}$$

$$D=\sqrt{\frac{\overline{HP}}{1.33\,f.\ \sin.\ \alpha\ h_1\ \sqrt{h_1}}}\ \ \ldots\ldots\ (b)$$

f being the ratio of diameter to depth, H.

Having found the diameter D (the diameter of that circumference on which the water first impinges) whether for outward or inward flow, the second diameter may be found from the relation of the radii shown in the diagrams which have been given.

The depth $H=f.D.$ of the wheel should be somewhat greater at the discharging diameter D′, in order to make allowance for the greater space occupied on the circumference by the oblique positions of the extremities of the buckets.

The angular velocity of the wheel being $\overline{2\pi N}$ in which N is the number of revolutions per second. The tangential velocity will be

$$u=2\pi Nv$$

For parallel flow wheels

$$V=C.A.v_1\ \sin.\ \alpha$$
$$=C.\pi(r'^2-r^2)\ \sin.\alpha$$

C being a co-efficient of discharge and A the area of the annular space through which the water flows, equal to $\pi(r'^2-r^2)$.

If D be the mean diameter and f the ratio of D to $(r'-r)$ or $f=\frac{r'-r}{D}$ then the formula (b) will apply to parallel flow wheels.

FORM OF BLADES.

For the three typical wheels, the *outward flow*, the *inward flow*, and the *parallel flow*, the path of a particle of water continues in one plane. In the outward and inward flow wheels, the planes of motion of the particles are perpendicular to the axis. In the parallel flow, the planes of the paths are parallel to the axis.

The lips of the buckets at the points of entrance to the wheel are generally straight edges, and for these three wheels are either perpendicular or parallel to the axis of motion.

The discharging edges are lines parallel to the receiving edges and the buckets at the discharging edges have a constant angle γ, along the edge, with the circumference.

In the outward and inward flow wheels the edges are elements of the cylindrical surfaces of the outer and inner circumferences of the wheel, and in the parallel flow, the receiving and discharging edge of the buckets are radial.

Combinations of these simple wheels, however, present different conditions; the *inward* and *downward* flow wheels represented by the Swain and Risdon, for example, and the *outward* and *downward* flow represented by the Excelsior wheel.

The general principles to be kept in view for all buckets, are that the channels between the buckets shall not have abrupt changes in direction; that they shall be as short as possible; that the curvature of the buckets shall be continuous; that the discharging edges of the buckets shall have a uniform discharging angle, and that the cross-section of the

channels between the buckets shall be uniform throughout. It is especially important that the water should leave the guide blades and enter the wheel in clear, transparent streams, without contraction, in order that that these streams may continue unbroken through the wheel to the point of discharge.

SEWAGE AND IRRIGATION WORKS IN GERMANY.

From "The Builder."

The question of sewage irrigation, after having been allowed to recede somewhat into the background for some time, is again attracting public attention to a more than ordinary degree. It is a subject which has repeatedly given rise to lively controversy in England. On that account, a reference to what other countries have been doing, and are still doing, in the matter will be of interest. The steps taken in Germany, with that view especially, will throw additional light on the knotty point whether sewage irrigation "pays." Authorities at home disagree on the financial aspect of such undertakings, but it has been proved that better crops can be grown with sewage alone than could be grown under tillage, costing sometimes as much as from 2*l.* to 3*l.* per acre. It has also been shown that if the sewage can be delivered upon land by gravitation close to the sewage works, this mode of disposing of it is more profitable, as well as more consonant with the objects of sewering towns, than discharging it into rivers at some distance below them. Others, again, will have it that the most economical plan to dispose of town sewage is to carry it, if possible bodily, far enough into the open sea that there is no chance of its being brought back again by the tide. Whichever system may be preferred, or whichever may be the most practicable under varying conditions, we think it will be sufficient to show that if irrigation by sewage tends to decrease the expense of removing it from towns, it will be the more economical as well as the safer way of disposing of it. As we have said, and we might cite numerous cases in this country, confirmatory of our view, so much has been proved. For this reason we need not dwell any longer on the question as it affects England, but proceed at once to examine its aspects in Germany. And here, again, it will not be uninteresting to note that the Germans have taken a leaf out of our book: that they are introducing, or have introduced, a system of sewerage into some of their larger cities at least, partly by the help of British engineering and British capital; which is eminently true in the case of Danzig, at one time one of the most unhealthy cities of the Continent, and a city, the purifying of which, was a far more Herculean task than the celebrated cleansing of the Augean stable.

Although the Danzig contract itself was based upon the extravagant reports prevalent in England (about 1865 to 1870) as to financial results of sewage farming, we have it upon the best authority that experience there is such, that similar undertakings, if properly arranged and conducted, may be confidently recommended for safe and satisfactory investments. Indeed, the Danzig undertaking has turned out so well, as we shall presently be able to show, that the municipal authorities of another large city, viz., Breslau, formerly also notorious for its insalubrious condition, with a population of 270,000 souls, have recently concluded a contract with Messrs. J. & A. Aird & Mare, the eminent Berlin firm of engineers, who completed the Danzig works, and have been working the sewage farm there for some time, for laying out the land and undertaking the management of the works and farming for twelve years. Moreover, Berlin, with over a million inhabitants, after having for several years made a most exhaustive series of trials of all the known or projected systems, has finally adopted the irrigation system. The chief engineer of the Berlin municipality, Herr Baurath Hobrecht, states the total cost of the irrigation arrangements, including purchase of the land, buildings, cost of mains from the pumping stations, laying

out the lands, roads, etc., to be 260 thalers per morgen, or about 60*l.* per acre, the morgen being about six-tenths of an acre. The Berlin authorities are completely satisfied with the financial results.

After this general preliminary statement, let us consider the case of Danzig; and as the supply of water is closely connected with the removal of sewage—indeed the latter could not be thought of without, a few remarks on that point will be necessary. The municipality of Danzig resolved as early as 1863 to provide the city with pure drinking-water and a complete system of sewerage. The Danzig waterworks, if a system of wooden pipes liable to pollution from external causes may be called by that name, dated from the time of the German Knights. They only supplied water for general use, however, and drinking water was brought into the city on wagons, and was sold at a high rate. However, with the opening of the new waterworks in 1869, by which pure water was brought a distance of about fifteen miles from reservoirs *circa* 350 ft. above sea level, all this was changed. The daily supply, since the day of their opening, has varied between 245,000 and 400,000 cubic feet. The total cost for executing all the works connected with the undertaking has been not quite 550,000 thalers (£82,000).

The draining system of the city was so defective that it would have been extremely injudicious to discharge into it an increased quantity of used and polluted water. The highly dangerous accumulation of filth of all descriptions in houses and courts, street gutters and cesspools, as well as in the public water reservoirs, called for urgent reform, by introducing a new system of drainage. When, therefore, the question of water supply was once determined, the carrying out of drainage works could be seriously thought of. A contract was concluded with Messrs. Aird for executing them, and detailed plans, on the project as submitted by Herr Barauth Wiebe, of Berlin, were prepared by Mr. Baldwin Latham, C. E. The works were begun in August, 1869, and completed by the end of 1871, at a cost of 700,000 thalers (£105,000).

There remains to consider the more important part of our subject, the disposal of the sewage for irrigation purposes in Danzig. In the absence of a more conveniently situated and better site, the dune district on the coast of the Baltic, between Weichselmünde and Heubude, the property of the city of Danzig, has been selected as the ground for purifying the sewage and utilizing it for agricultural purposes. The land in question was partly covered with firs; about 200 morgens were let for pasturage, at the low rent of 50 thalers.

By the contract of September 13, 1869, Mr. A. Aird acquired the right to use the sewage of Danzig, as well as a grant of 2,000 morgens for irrigation and farming purposes, for a term of thirty years, for which Mr. Aird undertook, for a like period, to keep in perfect repair and working order all the sewers and drains, and the pumping-station. The expenses for this undertaking of Mr. Aird's may be estimated at from 8,000 to 10,000 thalers (£1,500) per annum. At the end of the term, Mr. Aird has to give up possession of the land, with all improvements and plant, without claiming compensation for the capital laid out in the cultivation of the land. Any buildings erected by him he may take down and remove, unless the city of Danzig choose to purchase the same at a valuation. The contract was a risky undertaking, yet, as will be shown by and by, Mr. Aird has rendered by it a great service not only to Danzig but also to other German cities which have undertaken similar works.

The plant at the pumping station includes two steam engines, each sixty-horse power, of which one is at work, as a rule, only from fourteen to eighteen hours a day. During heavy rains, and from May to September, one engine is always at work. In the former case, from 350,000 to 390,000 cubic feet, in the latter, from 525,000 to 550,000 cubic feet,—of which about 175,000 cubic feet are water let in from the river Mottlau, as then the sewage alone would not be sufficient—are daily taken to the area of irrigation. The channel into which the delivery-pipe empties itself lies at the highest point of the dune, feeding from here all the principal irrigation trenches. The latter branch off on both slopes, and give off again into smaller channels. The ground is leveled in accordance with the gradient, the irrigation being affected according as it is wanted, either

by damming up or through furrows. Should there be any surplus sewage, this flows into the sea.

A few words as to the nature of the ground. The dune district in question, it should be stated, is, by its nature, very little suited for agricultural purposes, and for purifying sewage. The sterility of the dune sand is still further increased by being impregnated nearly everywhere with a fine red sand, so-called *Fuchssand* (a kind of red garden gravel). The hilly configuration of the ground, moreover, made the works of leveling rather expensive, the cost of the area brought under cultivation being £11 5*s*. per morgen (£15 15*s*. per acre), but will be much less in the future. In consequence of the capillarity of the sand, the level of the ground water is very high, and this increases the difficulty of draining. The growth of the shooting and young plants is not unfrequently jeopardised by sand-drifts.

Although dune sand at first permits sewage water to run through it more quickly than is desirable, its power of absorption is still great enough to allow of the development, even in the first year, of a luxurious vegetation of the most exacting plants. In consequence of the property inherent in roots of plants, not only of appropriating substances contained in solution in the ground water, but also of dissolving and absorbing matter coming into contact with the finest root-fibres, luxuriousness of vegetation contributes no less to purifying of sewage water than does the power of absorption of the soil. But the latter also steadily and speedily increases, consequent upon the rapidly progressing formation of humus, to which the organic substances suspended in sewage water, and being deposited in the sand, as well as the decomposing remains of vegetable substances produced, contribute equally largely.

The filtrated water, if it does not come into contact with the red sand, and is not colored reddish by it, is quite clear, free from smell, and tasteless. The purifying process has been complete. The filtrated water has been tested in vain by chemical analysis for the complicated chemical elements of a urinary or fœcal nature. They have, as far as they have not been absorbed by the soil or assimilated by the roots of plants, experienced a lasting change of substance, and have been oxydized to carbonic acid, nitric acid, ulmic acid, butyric acid, etc., by the oxygen of the air contained in the soil and continually replaced.

Of the dune area intended for agricultural purposes, there were in cultivation, in 1872, 30 morgens; in 1873, 120 morgens; in 1874, 250 morgens. These operations had, to some extent, the character of experiments, which were to form sound foundations for future cultivation. There not being a market for fresh-cut grass, and haymaking interfering with the development of the after-growth, the cultivation of grass is considerably confined, and that of beet-root, turnips, maize, oil-seeds, cereals, vegetables, also tobacco, carried on. In 1876, 500 morgens were leveled and prepared for cultivation and tilled as follows:—90 morgens with beets, 34 morgens with colza, 60 morgens (on which that year for the first time oats were grown) with rape-seed, 50 morgens with buckwheat, 12 morgens with barley, 12 morgens with tobacco and maize, 15 morgens (on which rape-seed and flax had stood) with late turnips, 1½ morgen with hemp, 1 morgen with caraway-seed, 50 morgens with Timothy grass and clover (sown end of June and beginning of August), 5 morgens with vegetables; 170 morgens of freshly leveled land were reserved the same year (1876) for planting next spring with beets and other summer crops.

The irrigation, as already stated, was begun on a small scale in 1872 after—at the end of 1871—the sewerage system had been completed. A beginning was made, as in England, with Italian rye-grass. An area of eight morgens, first laid out, was sown on May 1, with rye-grass; the first cut was taken on June 12, of a length of 18½ in.; the second on July 5, of 19½ in.; the third, on July 30, of 20 in.; the fourth, on September 5, of 20½ in.; the fifth, on November 1, of 28½ in. The second area, of 6¼ morgens, was sown on June 7, and gave four harvests—on July 8, August 7, September 23, and November 3—of similar lengths. The third area, of 4 morgens, was sown on August 14, and gave in October, a cut of 24 in. long. In 1872, the grass grew at the rate of 1 in. in twenty-four hours.

Those few figures show the extraordinary effect of sewage irrigation upon sterile sand; indeed, the growth of grass the following year was still greater. Although rye-grass is the best for irrigation, because it not only withdraws, by its fine roots, most surely the organic matter contained in sewage (according to recent observations even animal substances direct, like insectivorous plants), but also favors, by the intertwining of its roots, the filtration of water, it was found necessary to attempt also other products of the field, as no buyers could be found for the great quantity of grass obtained, and there was not a large stock of cattle in the neighborhood. Mr. Aird had to decide whether he was to acquire a large farming stock and become a cattle-breeder, or whether he was to proceed to utilize the irrigated are a in another way. Beets, wheat, rye, rapeseed, mustard, and tobacco, were grown; next, vegetables were attempted; and, finally, also flowers—the latter for their seeds. Besides the area under cultivation by him direct in 1876, 36 morgens were devoted to vegetables and flowers, and 244 morgens let out on leases. The farmers grew, especially Swedes, summer corn, and tobacco (18 morgens). The cultivation of the latter is said to be very remunerative, and the tobacco, if not a Havana leaf, to be certainly of a good flavor.

But not only the dimensions of the area cultivated have increased, the yield has become better paying, even handsome. A proof of this is, that already half of the irrigation area is let. In 1876, from 15 to 20 thalers were paid per morgen; in 1877, the offers went up as high as 24 thalers per morgen, or about £5 per acre, and this for soil formerly dune sand. Those rents are explained by looking over the yields as given in the table below:—

Cereals, &c.	Seed sown.	Yield of Corn.	Straw.
	Scheffel.	Scheffel.	Cwt.
Winter wheat.....	1.20	15¼	22¼
Winter rye........	1.10	12	29
Summer rye.......	1.35	9	12
Barley............	1.40	11¼	12
Oats..............	2.00	21¼	16
Winter colza......	1.40	13¼	—
Winter rapeseed...	1.10	14	—
Summer rapeseed..	1.60	9¼	—

In 1876, from 200 cwt. to 320 cwt. of swedes, according to the age of the irrigated field, were obtained per morgen. Tobacco yielded about 13¼ cwt. The cereals grown were good, partly excellent (rye, oats, summer rape), and sold readily at market rates, as well as the straw, which was very healthy, Swedes fetched in 1876, from 10*d.* to 11*d.* per cwt.; tobacco £1, 11*s.* 6*d.* per cwt. The yield of hay was in the second year (1873) in the area on which Timothy and clover were grown from 26 cwt. to 27 cwt. per morgen, and sold at 3*s.* 9*d.* to 4*s.* per cwt. The area planted with vegetables, and flowers for seed, yielded 110 thaler brutto per morgen. Mustard gave 5½ cwt. per morgen.

An important question arose as to how the irrigation system would work in winter. Danzig is much exposed to the tempering effect of the sea, but still the winters are very severe. But since the beginning of the irrigation, the latter has been carried on throughout the winter without difficulty. The sewage arrives on the field even if the cold weather is continuous; at most a thin ice crust is formed, below which it runs and sinks into the ground. As the growth is then at a standstill, even of rye-grass, it will not be frozen out. Newly prepared ground is principally irrigated in winter, so that it receives a thorough manuring then.

The consumption of sewage-water in summer is already so large that additional water is let into the drains from the river Mottlau. The large quantities of water which are thus daily brought on to the irrigation area must, of course, find a natural outflow. This is, in the first place, the ground-water, the level of which is from 3 ft. to 5 ft. under the surface. The filtrated water collects in small ditches, which lead into larger channels, emptying directly or indirectly into the Vistula.

Of course, we might give detailed statistics of the indirect benefit conferred by the the drainage works on the health of Danzig. But this not being within the scope of the present inquiry, though an important consideration in all sewerage undertakings, we need only remark that they are most striking. To mention only one instance, frequently the cause of great mortality, that of typhus fever, gen-

erally the result of bad drainage. In 1868 the deaths from that disease were eighty-nine; in 1876, they were only twelve.

The above statement will give our readers a fair idea of the Danzig works, and the prospects of similar undertakings in Germany. We may, however, point to the case of Danzig as one in which the change from the formerly prevailing abominable system—system is almost too good a word for a state of matters where everything was worse than chaos,—to a well-regulated arrangement of sewerage and sewage irrigation, coupled with a bountiful supply of pure water, has been most beneficial.

Altogether, the results have been astonishing. On the one hand, a desolate moving dune district has been changed, as if by the wand of an enchanter, into fertile fields and garden land. On the other, a city formerly the permanent stronghold of filth and disease, the soil of which was reeking with unclean accumulations, to which additions were daily being made—its inhabitants breathing the noxious effluvia with such a combination of nastiness produces, and drinking poisonous water,—has been transformed into a healthy city. What is more remarkable, the change has been effected at a comparatively moderate cost. Indeed, the expense must be looked upon as small when we consider what an immense amount of good has been done.

We now proceed to consider the case of the German capital. The drainage of Berlin has been a most difficult and complicated subject for the engineer. Until 1873,—when the necessity for doing something in the matter of the general drainage of their city was recognized by the corporation, and the work of canalization commenced,—the capital was without any properly regulated system of drains and sewers. It is true a number of schemes had been proposed, and for various reasons, some of a technical, others of a financial nature, rejected; the earliest proposal dating from 1816.

About twelve years ago a plan for the drainage of Berlin was submitted to the authorities by Herr Geheim-Oberbaurath Wiebe. In this, one system of sewers for the whole city was projected, with a main outlet in the river Spree near Charlottenburg. Berlin being very flat, and only a few feet above the level of the river, the execution of this project,—a main sewer necessarily falling from end to end of the city (with provision for future extension in all directions),—the subsoil being running sand, permitting water to percolate in large quantities, would have been attended with great difficulties and enormous expense, and was consequently also rejected.

The project of Herr Baurath Hobrecht, adopted in 1873, and now being carried out, however, is based upon what is called in Germany the "radial" system—a system eminently suited to the special circumstances of Berlin, besides preventing the pollution of the river, the sewage being ultimately used for irrigation purposes. There are five systems of sewers, each with a pumping-station, and each starting from the centre of the city and running in a different direction.

The many advantages of this plan are at once apparent. The main sewers are, as a great depth is in no case necessary, comparatively easy to execute; the branch sewers in proportion. As any extensions can only be necessary in the periphery, the works in the central (built-upon) area have an unusually permanent character, and are never likely to be subject to alteration. The extensions necessary can easily be connected without any interruption to the trunk systems. The height the sewage has to be pumped is much reduced. It is undoubtedly also an advantage that the authorities are in no way bound to any one particular site or direction as to irrigation areas. Besides these advantages, the dangers of serious flooding, accidents, disturbances, etc., are greatly lessened by this division and distribution.

The works in Berlin are approachiug completion. Already two radial systems are working, so far as pumping operations are concerned, and already for two seasons irrigation has been carried on, and with most satisfactory results. It is to be hoped that the Metropolitan Board of Works will one day take courage, and address to the municipal authorities of Berlin an official inquiry on the subject.

The irrigation area for the three systems on the south side of the Spree is distant from the various pumping-stations about eight miles. In order to reach the fields by gravitation, the sewage has to

be lifted at the stations nearly 65 ft. In the middle of this year the amount of sewage passed through the pumping-station of the radial system No. III. was nearly 240,000,000 gallons per month, which is only about the sixth part of the maximum capacity of that station.

The irrigation farm of Osdorf comprises, in round numbers, about 2,000 acres, of which at present about 500 acres have been adapted for irrigation. Of this area, 225 acres are grass land, 150 acres are used for market-gardening, while 125 acres are covered by shallow reservoirs, serving for storing provisionally the sewage during winter, should severe weather involve a stoppage of the irrigation. The irrigation ground is undulating, rising in some instances 40 ft. The delivery-pipe is carried to nearly all higher-lying points, where there are outflow-holes with ordinary slides. The steeper slopes are covered with Italian rye-grass, while the more level ground is used for vegetable-growing, and for this purpose formed into terraces, with narrow raised beds, divided by trenches, from which the water penetrates laterally to the roots. The paths between the fields are planted with fruit-trees, the luxurious growth of which, combined with the even more than luxurious look of the fields, imparts a most satisfactory impression.

The irrigation area at Osdorf has been acquired at an expense, including the laying out of the farm, of £125,000 (over £60 an acre). The total cost of the draining and sewerage of Berlin is estimated at £2,000,000. There will be two sewage-farms on the north side of the river Spree, when the drainage of that part of the city is once in working order.

Strange to say, the project was at first looked upon with most unfavorable eyes by the rich as well as the poor of the German capital; but what is stranger still, the municipal authorities of Berlin as those of other German cities where sewage and irrigation works have been projected, met with but scant encouragement from the Government. However, the beneficial effects are already appearing, in a direction, too, in which the poorer classes of Berlin have hitherto been great sufferers. Leaving out of the question the improvement in the sanitary condition of the city,—which, of course, is by itself a highly important factor,—the Berliners are already reaping substantial advantages. The success of market-gardening on the communal irrigation farm, the excellent quality of the vegetables of all descriptions grown there, has sent down the prices of those indispensable commodities wonderfully. "When once the works are a few years older, when a little more experience has put them more in working order, especially when gardeners practised in the systematic cultivation of market-gardens have been engaged," writes us a correspondent intimately acquainted with Berlin, "the Berliners will not be long in turning enthusiastic defenders of the welcome little stranger, formerly very much ill used, which has brought them clean streets without open gutters, pure air, salubrious water-closets and courts, and good vegetables and milk, which will now also be within the reach of the poor man."

A few words in conclusion as regards sewage and irrigation works, with which waterworks are closely connected, in Germany. People in that country are gradually becoming alive to the fact that the health of towns may be improved by the supply of pure air and pure water. Witness the numerous works undertaken with this view, not only in the larger cities, but also in smaller towns, the mere mention of which would make up a very respectable list. Of the more populous cities, besides the two which form the subject of this article, the cases of Breslau and Munich may be cited. As regards Breslau, the sewer system, on which Messrs. Aird, of Berlin, have been engaged during the last few years, will be completed in 7879. Sewage irrigation is to come into operation there on the completion of the works. The lands are already being prepared for irrigation.

With respect to Munich, plans for the sewerage have been prepared by J. Gordon, C. E. (formerly of Carlisle). The execution of those plans involves an estimated expenditure of £734,000, and they are now being revised by Herr Geheimrath Wiebe. A deputation of the Munich magistracy recently spent some days in Danzig, inspecting the works there and visiting the farm. Departing, they expressed the most thorough approval of the system and the arrangements.

REPORTS OF ENGINEERING SOCIETIES.

American Society of Civil Engineers.—The last numbers of Transactions are well filled with important matter. The following papers are published with abundant illustrations:

No. 166. Reminiscences and Experiences of Early Engineering Operations on Railroads, with especial reference to steep inclines. By W. Milnor Roberts.

Discussions on Inclined Planes for Railroads. By O. Chanute and by Wm. H. Paine.

No. 167. Distribution of Rain Fall, October 3d and 4th, 1869 By James B. Francis.

No. 168. The Gauging of Streams, By Clemens Herschel.

No. 169. Dangers Threatening the Navigation of the Mississippi River, and the Reclamation of Alluvial Lands. By B. M. Harrod.

No. 179. Brick Arches for Large Sewers. By R. Hering.

No. 180. Fall of the Western Arched Approach to South Street Bridge in Philadelphia. By D. McN. Stauffer.

Discussions on above Papers. By E. S. Chesborough, W. Milnor Roberts, F. Collingwood, and R. Hering.

Engineers' Club of Philadelphia.—At the meeting of the Club held January 18th, Mr. D. McN. Stauffer read a paper on "Conical Arches." The eastern approach to the South Street bridge in this city is made up, in part, of a somewhat peculiar piece of arched masonry, which contains certain novel features of useful application at other points.

The center lines of South Street and of the bridge proper intersect at an angle of 33° 25′, necessitating a curve in the approach from the east. To conform in design with the "late lamented" western arched approach, this curve was pierced by three arches. By adopting the conical arch much money was saved in the foundations, in the arch piers themselves, and in the haunching above them. The improvement in appearance, brought about by substituting the light arch for the heavy thickened pier ends, sometimes adopted in similar cases, is certainly very great.

A full and detailed description of the arch was given and its peculiar features explained.

Mr. R. Hering mentioned having seen the arch-work on 42d Street, New York, which fell during last fall, and also brick arch-work in the Brooklyn approach to the East River bridge, which showed very decided signs of giving way. He thought if the bond in the brick work of these arches had been properly broken, as described in Mr. Stauffer's paper, that they would not have failed.

Mr. Ashburner contributed interesting facts in regard to the recently completed "Cleveland Viaduct." The pivot span of this bridge is the heaviest in this country.

Mr. W. B. Ross exhibited a series of cards which he has designed for the use of engineers. They are intended to facilitate and expedite the calculation of quantities and areas. They are particularly convenient in calculating excavations and embankments. Quantities can be ascertained in less than one-third the time usually taken. The error is always a percentage of the quantity, and is about constant, being one-twentieth of one per cent.

Mr. Howard Murphy read some interesting notes upon the early waterworks of Philadelphia. It has been shown that Franklin's proposition, to supply the city with water from the Wissahickon Creek, would have given the required quantity until 1868.

Chas. E. Billin, *Secretary.*

At a meeting of the Manchester Scientific and Mechanical Society held on the 12th ult., a paper on the methods of communication between passengers and guards on railway trains was read by Mr. W. H. Bailey. The writer, after condemning the two methods at present in use, viz., the cord system and electricity, proceeded to describe a system invented by Mr. Hy. Morris, of Manchester. By this system a simple apparatus, attached to each carriage, put it in the power of any passenger to attract the attention of the guard by exploding, in rapid succession, two waterproof detonators or fog signals that were attached to the end of each carriage, and at the same time showing a red disc or semaphore at right angles to each side of the carriage, one short pull of the handle in any compartment being all that was necessary to give the signal. In the discussion which followed some slight improvements in the details of the apparatus were suggested, but generally it was highly commended as a simple and effective appliance; and very much preferable to the cord system, which had long since shown its inefficiency.

Liverpool Engineering Society.—This society held the first meeting of the present year on Wednesday evening, the 15th ult., at the Royal Institution. The chief business of the evening was the reading and discussion of the paper by Mr. J. S. Brodie on "The Disposal of Town Refuse." The author considered that in spite of the attention that has been paid to this all-importantant subject, and the sums of money that have been spent by various companies with greater or less results, the question as to how best to get rid of such refuse as that from dwellings, factories and slaughter-houses remains *in statu quo*. With a view to further elucidation and discussion the present paper was written, and it gives a clear and concise description of the more important schemes that have been tried for the treatment of sewage. Up to thirty years ago town refuse was either sent into the nearest watercourse or deposited on waste-land with the effect of either poisoning the stream or polluting the air. The introduction of gravitation waterworks and the w. c. system even aggravated the former evil, and a committee was appointed in 1866, which, after ten years' labor, made a report, the result of which was the Rivers Pollution Act. Natural watercourses being barred as receptacles for sewage, recourse was had to other methods of disposing of it, and Mr. Brodie considered those which had been tried as (1) Precipitation, (2) Filtration, (3) Irrigation. Under the first head Scott's lime and cement process was described and a specimen of the cement was exhibited; Whitthead's and the celebrated A.

B. C. were also described. Weare's and Bailey Denton's mode of filtration came next. Passing over irrigation as an agricultural question, the author next described the Rochdale tub system and earth closets. In conclusion, Mr. Brodie believed that unless something altogether unexpected came forward, towns would have to contrive to pay to get rid of their refuse just as they have to pay for their water supply.

IRON AND STEEL NOTES.

THE AGE OF STEEL.—Since the Admiralty approved of steel for shipbuilding, the use of iron for this purpose has rapidly diminished. The iron trade, indeed, is in a most deplorable state from this and other causes. Not only has it to cope with the prevailing depression, but it has fallen into disfavor for many of the purposes to which it was at one time exclusively applied. For torpedo launches it is altogether discarded; and there are indications that the war-ship of the future will be not an iron-clad, but a steel-clad. For the splendid vessels of the Cunard and other lines iron propellers are now a thing of the past. Only last Wednesday there was launched from Messrs. J. and G. Thomson's yard, at Glasgow, the largest Cunard steamer yet built, the Gallia, which has been fitted with four steel propeller plates, each of which weighs 65 cwt. These are the largest propeller plates ever made. They were manufactured by Messrs. John Brown & Co., Atlas Steel and Iron Works, Sheffield. Of course, these are not the first propeller plates made in steel—another Sheffield firm having been engaged in this speciality for some time. Steel is pushing iron out of the field in other departments, such as boiler-plates, bridge-plates, and even girders for house-building. For some time now the Dutch Government have employed nothing but steel in the construction of their bridges; while boiler-makers are rarely calling for iron-plates; and the difficulty of meeting Belgian competition in iron girders is being obviated by the use of steel, the carrying power of which is greatly superior to the ordinary form of girders. Altogether, the iron trade, which was at one time called "the backbone of England's commercial supremacy," is simply becoming a servant to steel, which is swiftly sweeping the ruder metal out of many markets where King Iron once reigned supreme.

NEW LIGHT ON STEEL-MAKING.—It would seem that the presence of more than one or two-tenths per cent. of phosphorus in pig-iron is no longer to be considered, as heretofore, an insuperable obstacle to its conversion into ingot steel. It was established by the Terre Noire Company, some two years ago, that as much as 0.32 per cent. of phosphorus can be tolerated in very mild steel, and, as is well known, large quantities of Martin steel made from old iron rails and pure pig have, by the aid of ferro-manganese, been manufactured on this principle. The difference between the cost of changing old iron rails, and that of using pure materials, is, however, in most localities not sufficient to cover the extra expense of using ferro-manganese. It remained, however, an axiom with steel makers, that no removal of phosphorus could be hoped for in any direct steel process, till it was announced from the Blaenavon Ironworks that there were means by which phosphorus could be removed with certainty and economy, and that intensity of temperature was no obstacle to its removal. In confirmation of the Blaenavon experiments, we learn that very important results have been obtained in Belgium with M. Ponsard's fornoconvertisseur lined with one of the Blaenavon basic preparations. The maintenance of the necessary highly basic slag was effected by the addition of lime and a certain amount of ore, as prescribed by Mr. Thomas, the patentee of the process, who assisted at the operations. In the first cast of four tons, notwithstanding that the operations were conducted under very unfavorable circumstances, an analysis of the steel showed that 90 per cent. of the phosphorus contained in the pig had been removed. An examination of samples taken at intervals shows a progressive decrease of phosphorus in the bath and its transference to the slag; the amount of silica in the latter being kept at about 22 per cent. A somewhat more basic slag is, however, generally preferred. The second cast gave very similar results. As the Ponsard apparatus is able to deal with pig very low in silicon there appears to be now no class of pig which may not be considered as available for the manufacture of steel. The only impurity which is not removed almost completely is sulphur, though this is eliminated to a considerable extent; fortunately, however, sulphur is readily removed in the blast-furnace. We understand it is now in contemplation to regularly work the Ponsard converter in combination with the new basic process on the highly phosphoretic pig of Belgium and Germany. This will give an economy of from 30 to 50 francs a ton over the use of Bessemer pig, and give a fresh life to the drooping fortunes of the manufacturers.

RAILWAY NOTES.

AT the meeting of the North-Eastern Railway Company, the chairman stated that he thought they had "broken the neck" of the signaling difficulty. They had spent in the half year on new works about £14,000, but still the cost of maintaining the signaling of the line was something like £1,000 less than a year ago, and they had now interlocked the line to nearly 2,750 places in four years and a half. In the permanent way they were now deriving the benefit of the steel rails, which were such a burden to them for a long time, and some idea of what had been in this direction may be gleaned from the fact that the company has laid something like 1,600 miles of single line, or 800 of double line wholly with steel rails, including nearly the whole extent of the main lines. The chairman added that "though he hoped they would continue to feel the advantage of steel rails, he was afraid of some process coming into play which would have a similar effect on steel as the hot blast had on the manufacture of iron, since which no good iron has been made in the country." It is curious that we have done so well with bad iron for so many years.

RAILWAY WORK IN JAPAN.—At the meeting on Tuesday, the 10th of December, at the Institution of Civil Engineers, Mr. Bateman, President, in the chair, a paper was read on "Railway Work in Japan," by Mr. W. Furniss Potter, M. Inst. C. E.

The author stated that there were, at present, 66½ miles of railway in Japan, 142¾ miles laid out, with working plans, sections, and estimates completed, and 455 miles projected, the general route only having been examined and decided upon. The earthworks of the existing lines had been made for a double way, and the bridges for a single way. The permanent way was of double-headed 60 lb. rails on the Yeddo-Yokohama and Kobe-Osaka lines; but on the Osaka-Kioto line, 60 lb. flat-bottomed rails on cross sleepers were used. The superstructure of the smaller bridges was originally of timber, but had been renewed with iron. The larger bridges were all of the Warren girder type, and as a rule of 100 ft. spans. The foundations were on brick wells 12 ft. in diameter, and on an average about 60 ft. deep. Native examples of engineering were chiefly remarkable for their temporary character. The usual foundation for the largest buildings was only a few stones on the surface of the ground. The natives were very clever in making artesian borings for water, and a detailed description of the *modus operandi* was given. The workmen were extremely intelligent and industrious, especially the carpenters, who were by far the most numerous and skillful. The wages of first-class carpenters were 1*s.* 8*d.* per day; of blacksmiths, 1*s.* 6*d.*; of bricklayers and masons, 1*s.* 5*d.*, and of coolies, 11*d.* Materials found in the country for construction were not very good, except timber, which was abundant. No limestone possessing hydraulic properties had been found. It was impossible to furnish any reliable information as to the cost of the works, as the Japanese officials avoided giving particulars on this point to the foreign staff. The chief engineering difficulty in Japan was the treatment of the water-shed. The beds of the rivers were nearly all higher than the surrounding country, varying from a few feet to 40 ft., or more. In some instances the railway had been taken under the rivers by tunneling, and an example of this was given. As a rule, however, the rivers were bridged over, and approached by steep gradients, and high embankments. The flood-waters were confined in the rivers by huge banks which were gradually built up by the natives, as the beds of the river became silted up, and were frequently formidable works. The general character of the country was a series of highly-cultivated and well-watered plains, bounded by ranges of hills of the metamorphic formation. Where these hills had to be crossed there would be some heavy works. These features were described in detail. The traffic on the railways already constructed was considerable, and it was estimated that on future railways the passenger traffic alone would pay a dividend of seven per cent. Not much had been done in goods traffic, as the existing lines were in competition with the water communications. In the future development of railway work in Japan, two essential points were necessary, greater economy of construction, and the introduction of English capital and enterprise. These could be obtained if the principle of surface lines were adopted, and the natural jealousy of the government, of foreign interference, were abolished.

ENGINEERING STRUCTURES.

THE GREAT HUNGARIAN TUNNEL.—On the 21st October the great Josef adit at Schemnitz in Hungary was opened. The works have been carried on since 1872, the Hungarian Government granting £10,000 a year toward them. The adit is over ten miles long, being some 50 yards longer than Mont Cenis Tunnel. The total cost of the undertaking was £459,900; it was carried out entirely by Hungarian enterprise, and partly with Hungarian machinery.

RECORD OF A FLOATING DERRICK.—The 100-ton floating derrick City of New York, designed by Engineer Isaac Newton, formerly the principal assistant of Gen. McClellan, has just been taken up. This structure has a remarkable record. It was launched over seven years ago, and has not been raised from the water since. During this period it has been used almost daily raising weights of 60 or 70 tons. At one portion of the river wall in 1875 it transported and laid 1,780 cubic yards (3,560 tons) of Beton blocks, laying this great quantity of masonry in less than 18 days in 14 feet of water. Before it was launched, its computed weight was 1,142,518 pounds; and its actual displacement on launching was found to be 1,142,644 pounds, showing a remarkably close calculation.

The derrick has, on several occasions, lifted considerably over 100 tons. On one occasion it raised a tug-boat, which had filled and sunk, from the bottom of the river and placed it on the bulkhead. Its estimated weight was 130 tons.

It lately launched Ericsson's torpedo boat Destroyer, by raising it from the ground and placing it in the water. Notwithstanding the severe constant use to which the machine has been subjected no accident has occurred, and the repairs have consisted chiefly of two renewals of the wire rope of the main fall.

ORDNANCE AND NAVAL.

IMPROVEMENTS IN STEEL ARMOR PLATES.—Certain improvements in the manufacture of steel armor plates have just been patented by Mr. Thomas Hampton, of the Phœnix Bessemer Steel Works, Sheffield.

The inventor informs us that steel plates produced under this patent will not be more costly in manufacture than the ordinary wrought-iron plates; that his method will necessitate no alteration in the existing plant or machinery, while puddling furnaces are entirely dispensed with. A large armor plate will shortly be forwarded to the Admiralty to undergo the usual tests with shot and shell. The smaller plates heretofore experimented upon have proved so satisfactory that the result of the approaching trial is awaited with confidence.

The invention consists in the combination of two distinct processes; the first being the method of building up or construction of the plate for the purpose of obtaining a quality of steel possessing great toughness, the second process consisting in case-hardening or recarbonizing the surface or part of the surface of plates so constructed.

Steel is employed which may be made by either of the processes known as the Bessemer, Siemens, Siemens-Martin, or crucible, and the ingots so made may be either rolled direct into suitable slabs, plates, or sheets, or they may be first hammered or squeezed, and afterwards rolled. The slabs, etc., so reduced, are piled in any desired number, and placed in a reberberatory or other heating furnace, and brought to a proper heat for welding together, which may be effected in the usual manner by rolling, hammering, squeezing, or other equivalents. If desired, such plates may be piled, heated and welded together, until the requisite thickness or strength is attained. The plate being brought to the desired dimensions, is now submitted to a process, or processes, of case-hardening, or recarbonizing, on either a portion or the whole of its surface, as may be required, for the purpose of imparting to it a degree of hardness which shall tend to break up and destroy any projectile striking it at the moment of impact as well as to increase its resisting power to penetration by shot or other missile.

In order to further increase the density of the steel, the plates, after being recarbonized, may be again submitted to the action of rolling, hammering, or squeezing.

A contract has been concluded by M. Sibiriakoff, of Irkutsk, in Siberia, for the building of a steamer of 350 tons burden for the purpose of going to the assistance of the Vega. It is expected that the steamer will be ready soon enough to start fully equipped with provisions in time to reach Behring's Straits by way of the Suez Canal next August, in order to assist Professor Nordenskjold and his companions. The vessel will afterwards trade to the Lena, and, if possible, even to the Yenisei. The vessel has been designed with a special view to her future service. Inside she is almost like an ordinary iron vessel covered with wood. The propeller and rudder are so constructed that they can be lifted out of the water.

BOOK NOTICES.

The Strength of Materials. By Wm. Kent, M. E. Van Nostrand's Science Series No. 41. New York : D. Van Nostrand. Price 50 cts.

The author of this little treatise sets forth in a very clear manner the present state of our knowledge on the strength of such materials as are used in engineering constructions. He especially insists, and proves by quotations from accepted authorities, that our present tables are inexact and the formulas conflicting.

A thorough acquaintance with the best literature of the subject, and a complete familiarity with the methods and results of tests, are evident throughout the book.

The aim of the essay is to urge engineers and constructors to insist upon a thorough knowledge of the strength of all materials employed in important structures, and to obtain this knowledge by aid of tests more carefully made than those upon which many of our present tables are based.

Practical Theory of Voussoir Arches Applied to Stone Bridges, Tunnels, Domes, and Groined Arches. By Wm. Cain, C. E. Van Nostrand's Science Series No. 42. New York : D. Van Nostrand. Price 50 cts.

Scarcely any branch of practical engineering offers so many difficulties to the young engineer as the construction of the voussoir arch. The theoretical examples of the books fail to meet in his mind the practical requirements. So he often follows precedent, and works from copy or by rule of thumb, and completes the work in ignorance whether such failure as there may be lies on the side of clumsiness or insecurity, with a lingering fear that, taken as a whole, the structure possesses both of these qualities.

Prof. Cain has previously given us so complete a discussion of the principles involved in the practical construction of arches (Science Series No. 12) that experienced engineers have employed it as a guide in important works. Taken with the present work, a treatise is afforded whose completenss in a practical point of view is scarcely equaled by anything else in the English language.

Text-Book on the Steam Engine. By T. M. Goodeve, M. A. New York : D. Van Nostrand. Price $2.00.

As a text book this is the most satisfactory treatise we have seen.

It is full without being voluminous, and accurately scientific without the forbidding array of analytical formulas that so often abound in similar books for students.

The plan of the work is simple and comprehensive. First is presented a historical sketch of the steam engine as it was known before Watts' time. Then an elucidation of the principles of thermodynamics and their application in the use of steam.

A chapter on mechanism follows and then a discussion of the principles of expansion—the theory of valve motion and the Indicator. Boilers and the consumption of fuel receive their share of attention ; compound engines, the injector, link work, and miscellaneous details close the work.

Examination questions are furnished as a guide to both teacher and student.

The book is abundantly illustrated.

Prof. DuBois's Graphical Statics.

In our issue for November appeared a portion of a review of the above American book, translated from Wochenschrift des Vereins deutscher Ingenieure, in which the critic charged that Prof. Dubois had used the work of Prof. Weyrauch without proper acknowledgement.

To this Prof. Dubois replied at considerable length in the December issue of this magazine, strongly asserting that there was no

proper ground for the charge, at the same time complaining that the translation was of a part of the original article only.

In the January magazine to satisfy the demands of our contributor of the translation, urged in consequence of the complaint of Prof. Dubois, we gave the entire article.

We are now offered plenty of material upon both sides of the controversy, which, at present, we decline to publish.

It is due, however, to Prof. Dubois to say that a reply from Prof. Weyrauch, has appeared in the journal which contained the original criticism, in which Prof. W. says he has not received the impression that the extracts from his work were so imperfectly acknowledged that the American reader would be misled in regard to the extent of the obligation to the German original, and expresses satisfaction with the efforts of Prof. Dubois to bring about "mutual good understanding and relations between German and American students."

Examples of Steam, Air, and Gas Engines of the Most Recent Approved Types, as employed in Mines, Factories, Steam Navigation, Railways and Agriculture, practically described; with an account of all the principal projects for the production of Motive Power from Heat, which have been propounded in Different Times and Countries. By John Bourne, C. E. London: Longmans. For sale by D. Van Nostrand. Price $30.00.

The publication of the present work, which was begun in 1868, was continued in monthly parts till 1870, when the author intimated that, as he was desirous of embodying in it some new information, which he could not at that moment communicate to the public, he proposed to suspend the publication for a short time to enable this addition to be made, in order that the work might be rendered more complete than would otherwise be possible. The author was at that time engaged in a series of elaborate experiments to determine in what way coal-dust could be best utilized in the generation of motive power.

In the present work the author has recapitulated the best proportions for Compound Engines and of the boilers suitable for working them. He has also recapitulated the leading features of all the principle Furnaces, Engines, or other expedients for the production of power from heat, and in some cases, as in the Air Engine of Stirling and the Caloric Engine of Ericsson, the author has described the structure in detail. Examples have been given of modern steam engines and boilers of every class; and in the appendix a variety of useful information, which could not be conveniently embodied in the text, has been introduced. While, therefore, the engineer will here find examples of the most modern and approved forms of engine construction, he will also find such a recapitulation of ingenious but unfruitful projects as may aid the progress of invention, prevent future waste of effort, and at all events satisfy an intelligent curiosity. The author believes that he has omitted to notice no project of the least importance for the production of motive power, and in most cases he has given his opinion as to the character of its pretensions.

MISCELLANEOUS.

Prof. Stanley Jevons still believes in his theory of the sunspots and commercial crises, and he requests our "so-called practical men to give up the idea that theory is all nonsense." But where is this particular theory, and what is it, and when we have it shall we be able to prevent commercial crises without preventing the spots on the sun's face?

Considering the unfavorable economical results hitherto obtained from steam turbines, Herr Muller, of Cologne, has lately patented a machine in which, instead of one turbine wheel several are placed together in a common case divided into a corresponding number of chambers of increasing size. The steam (or water under pressure) entering at the smaller end, finds an ever increasing cross-section of passage, and gives impulse to the successively larger turbine wheels on the common axis.

Gilding on Glass.—A new process by M. Dodon, is thus given by the *Moniteur de la Céramique*: Gold, chemically pure, is dissolved in aqua regia (1 part nitric and 3 parts hydrochloric acid). The solution effected, the excess of acids is evaporated on a water-bath till crystallization of the chloride of gold takes place; it is then taken off and diluted with distilled water of such quantity as to make a solution containing 1 gram of gold to 200 cubic centimeters of liquid; a solution of caustic soda is then added until the liquid exhibits an alkaline reaction. The solution of gold is now ready for reduction. As a reducing agent, an alcoholic solution of common illuminating gas is used. This is prepared by simply attaching a rubber tube to a gas-jet and passing the current of gas for about an hour through a quart of alcohol. This liquid (which should be kept in a closed vessel) is added in quantities of from two to three cubic centimeters to 200 cubic centimeters of the alkaline solution of gold before mentioned; the liquid soon begins to turn to a dark green color, and at length produces the metallic layer of gold of known reflecting power.

As an improvement on the process, as well as for convenience in executing it, there may be added to the alcoholic solution of gas an equal quantity of glycerine (28° to 30° Baumé) previously diluted with its own volume of distilled water.

If the gold employed is an alloy, the foreign metals must, in all cases, be first removed; and especially the least traces of silver, because the very smallest quantity of this metal totally prevents the regular and uniform deposition of the gold.

The bath thus once prepared, it is proposed as a method of gilding mirrors, and also for all the articles of various branches of industry, where this process of gilding could be used with success and to advantage, such, for instance, as boxes, necklace beads, candlesticks, glass ornaments, frames of table mirrors, cups, saucers, spoons, lanterns, and reflectors, and for objects generally in glass or crystal that are capable of being completely gilded.

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ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION.

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III.

These considerations now enable us to determine p wholly in terms of the initial co-ordinates. To do this it is only necessary to write $x=x_0+u$, $z=z_0+w$, then from what has just been said we have at once

$$\sigma x=\sigma x_0,$$
$$\sigma z=\sigma z_0,$$

Substituting these in the expression for φ and this again in the equation giving the pressure and we obtain for the latter

$$p=\rho\left\{\left(\frac{2\pi}{\tau}\right)^2\left(a_1\varepsilon^{\sigma z_0}+a_2\varepsilon^{-\sigma z_0}\right)\right.$$
$$\left.\sin.\left(\frac{2\pi}{\tau}t+\sigma x_0\right)+gz_0+gw\right\}+p_0$$

or as it may be written substituting for w its value

$$p=\rho\left\{\left(\frac{2\pi}{\tau}\right)^2\left(a_1\varepsilon^{\sigma z_0}+a_2\varepsilon^{-\sigma z_0}\right)\right.$$
$$\left.+g\sigma\left(a_1\varepsilon^{\sigma z_0}-a_2\varepsilon^{-\sigma z_0}\right)\right\}\sin.\left(\frac{2\pi}{\tau}t+\sigma x\right)$$
$$+p_0+\rho g z_0$$

Now for the determination of the constants, we observe first: that particles of the fluid originally on the bottom of the canal, must necessarily remain there during the motion; second, for particles of the fluid whose z co-ordidates are equal to zero, *i.e.*, for particles on the surface of the fluid at rest, p must be a constant $=p_0$. Let h denote the depth of the canal and the first of these conditions is evidently reached by making $w=o$ for those particles for which $z=h$. This gives us then

$$w=\sigma\left(a_1\varepsilon^{\sigma h}-a_2\varepsilon^{-\sigma h}\right)\sin.\left(\frac{2\pi}{\tau}t+\sigma x\right)=o$$

and consequently

$$a_1\varepsilon^{\sigma h}-a_2\varepsilon^{-\sigma h}=o$$

from which follows,

$$\frac{a_1}{a_2}=\frac{\varepsilon^{-\sigma h}}{\varepsilon^{\sigma h}}$$

Again, make $z=o$ and $p=p_0$; this gives obviously,

$$\left(\frac{2\pi}{\tau}\right)^2(a_1+a_2)+g\sigma(a_1-a_2)=o$$

from which

$$\left(\frac{2\pi}{\tau}\right)^2=-g\sigma\frac{a_1-a_2}{a_1+a_2}=g\sigma\frac{\varepsilon^{\sigma h}-\varepsilon^{-\sigma h}}{\varepsilon^{\sigma h}+\varepsilon^{-\sigma h}}$$

Now making for brevity

$$\sigma a_2 \varepsilon^{-\sigma h} = -a$$

And our expressions for u and w become,

$$u = -a\left(\varepsilon^{-\sigma(h-z)} + \varepsilon^{\sigma(h-z)}\right) \cos.\left(\frac{2\pi}{\tau}t + \sigma x\right)$$

$$w = -a\left(\varepsilon^{-\sigma(h-z)} - \varepsilon^{\sigma(h-z)}\right) \sin.\left(\frac{2\pi}{\tau}t + \sigma x\right)$$

By introducing the wave length now we can determine the constant σ. Reverting to the expression previously given for φ, and for convenience retain the *two* constants a_1 and a_2—this was

$$\varphi = \left(a_1 \varepsilon^{\sigma z} + a_2 \varepsilon^{-\sigma z}\right) \sin.\left(\frac{2\pi}{\tau}t + \sigma x\right).$$

Let l denote the wave length; we know that φ will remain unchanged by writing for x the quantity $x+l$, as this simply has the effect of transferring the origin of co-ordinates from one end of the wave to the other. This substitution gives

$$\varphi = \left(a_1 \varepsilon^{\sigma z} + a_2 \varepsilon^{-\sigma z}\right) \sin.\left(\frac{2\pi}{\tau}t + \sigma[x+l]\right).$$

In order that φ remain unchanged, we must clearly have

$$\sigma = \frac{2\pi}{l}.$$

If we call ω the velocity of each particle, we have also

$$l = \tau\omega, \text{ or } \tau = \frac{l}{\omega}$$

Substituting those in the expression obtained above for p, and it becomes

$$p = p_0 + \rho g z_0 + \rho\left\{g\sigma \frac{\varepsilon^{\sigma h} - \varepsilon^{-\sigma h}}{\varepsilon^{\sigma h} + \varepsilon^{-\sigma h}} \left(a_2 \frac{\varepsilon^{-\sigma h}}{\varepsilon^{\sigma h}} \varepsilon^{\sigma z_0} + a_2 \varepsilon^{-\sigma z_0}\right) + g\sigma\left(a_2 \frac{\varepsilon^{-\sigma h}}{\varepsilon^{\sigma h}} \varepsilon^{\sigma z_0} - a_2 \varepsilon^{-\sigma z_0}\right)\right\} \sin. \frac{2\pi}{\rho}(\omega t + x_0)$$

which reduces easily to

$$p = p_0 + \rho g z^0 - 2\rho g a \left\{\frac{\varepsilon^{\frac{2\pi}{l}z_0} - \varepsilon^{-\frac{2\pi}{l}z_0}}{\varepsilon^{\frac{2\pi}{l}h} + \varepsilon^{-\frac{2\pi}{l}h}}\right\} \sin. \frac{2\pi}{l}\left(\omega t + \varepsilon\right)$$

The same substitutions give us for φ the value

$$\varphi = -\frac{al}{2\pi}\left\{\varepsilon^{\frac{2\pi}{l}(h-z)} + \varepsilon^{-\frac{2\pi}{l}(h-z)}\right\} \sin. \frac{2\pi}{l}(\omega t + x)$$

and in like manner we can obtain for u and w the values

$$u = -a\left\{\varepsilon^{-\frac{2\pi}{l}(h-z)} + \varepsilon^{\frac{2\pi}{l}(h-z)}\right\} \cos. \frac{2\pi}{l}(\omega t + x),$$

$$w = -a\left\{\varepsilon^{-\frac{2\pi}{l}(h-z)} - \varepsilon^{\frac{2\pi}{l}(h-z)}\right\} \sin. \frac{2\pi}{l}(\omega t + x).$$

The value of ω is easily obtained.

$$\left(\frac{2\pi}{\tau}\right)^2 = \left(\frac{2\pi\omega}{l}\right)^2 = g\sigma \frac{\varepsilon^{\sigma h} - \varepsilon^{-\sigma h}}{\varepsilon^{\sigma h} + \varepsilon^{-\sigma h}} = g\frac{2\pi}{l} \frac{\varepsilon^{\frac{2\pi}{l}h} - \varepsilon^{-\frac{2\pi}{l}h}}{\varepsilon^{\frac{2\pi}{l}h} + \varepsilon^{-\frac{2\pi}{l}h}}$$

from this is readily obtained

$$\omega = \sqrt{\frac{gl}{2\pi} \cdot \frac{\varepsilon^{\frac{2\pi}{l}h} - \varepsilon^{-\frac{2\pi}{l}h}}{\varepsilon^{\frac{2\pi}{l}h} + \varepsilon^{-\frac{2\pi}{l}h}}}$$

In the discussion of these values for φ, u, w and p lies the whole theory of the motion of plane waves in a perfect fluid. We will now proceed to an examination of these quantities. Denote by z' the vertical ordinate at the time t of a particle on the wave surface whose other co-ordinates are x and y.

$$w = \frac{dz'}{dt} = -a\left\{\varepsilon^{-\frac{2\pi}{l}(h-z)} - \varepsilon^{\frac{2\pi}{l}(h-z)}\right\} \sin. \frac{2\pi}{l}(\omega t + x)$$

integrating

$$z' = \frac{a\rho}{2\pi\omega}\left\{\varepsilon^{\frac{2\pi}{l}(h-z)} - \varepsilon^{\frac{2\pi}{l}(h-z)}\right\} \cos. \frac{2\pi}{l}(\omega t + x).$$

Differentiating this expression with respect to x and we obtain.

$$\frac{dz'}{dx}=-\frac{a}{\omega}\left\{\varepsilon^{-\frac{z\pi}{l}(h-z)}-\varepsilon^{\frac{z\pi}{l}(h-z)}\right\}\sin.\frac{2\pi}{l}(\omega t+x)$$

This vanishes for the values

$$\omega t+x=o$$
$$\omega t+x=\tfrac{l}{2}$$
$$\omega t+x=l,$$

which are the values of $\omega t+x$ and for the points of maxima and minima of the longitudinal section of the wave. Differentiating again

$$\frac{d^2z'}{dx^2}=-\frac{a\rho}{2\pi\omega}\left\{\varepsilon^{-\frac{2\pi}{l}(h-z)}-\varepsilon^{\frac{2\pi}{l}(h-z)}\right\}\cos.\frac{2\pi}{l}(\omega t+x)$$

This vanishes for $\omega t+x=\frac{l}{4}$, and $\frac{3l}{4}$.

Consequently there are points of contra-flexure at $\frac{1}{4}$ and $\frac{3}{4}$ of the wave length. It is obvious, from these considerations, what the curve is.

It has before been remarked that u and w satisfy the equation of an ellipse; this is now

$$\frac{u^2}{a^2\left\{\varepsilon^{-\frac{2\pi}{l}(h-z)}+\varepsilon^{\frac{2\pi}{l}(h-z)}\right\}^2}+\frac{u^2}{a^2\left\{\varepsilon^{-\frac{2\pi}{l}(h-z)}-\varepsilon^{\frac{2\pi}{l}(h-z)}\right\}^2}=1$$

The plane of the ellipse is vertical and its longer axis is in the direction of the motion of the wave. Suppose now the particle under consideration to lie very near the surface of the wave—that is, z is very small as compared with h; then the terms containing $\varepsilon^{-\frac{2\pi}{l}(h-z)}$ may obviously be discarded, and the only other terms which remain in the expressions for the semi-axes of the ellipse will depend on

$$\varepsilon^{\frac{2\pi}{l}(h-z)}=\varepsilon^{\frac{2\pi}{l}h}\cdot\varepsilon^{-\frac{2\pi}{l}z}=A\varepsilon^{-\frac{2\pi'}{l}z}.$$

The equation of the ellipse thus becomes

$$\frac{u^2}{A^2\varepsilon^{-\frac{4\pi}{l}z}}+\frac{v^2}{A^2\varepsilon^{-\frac{4\pi}{l}z}}=1$$

The equation of a circle whose radius is

$$A\varepsilon^{\frac{2\pi}{l}z}=R$$

Of course the same result would be obtained by supposing the depth of the fluid infinite. Thus for particles near the surface of a body of water of finite depth—or for particles anywhere within the mass of a body of water of infinite depth—the motion is in a vertical circle whose radius is given above. Suppose again that the wave has an appreciable length—say $l=h$; then for particles very near the surface the semi-axes become very nearly.

$$\varepsilon^{2\pi}+\varepsilon^{-2\pi},\ \text{and}\ \varepsilon^{2\pi}-\varepsilon^{-2\pi}$$

or the path of the particle is nearly circular—the ratio between these quantities being nearly 1.000,007.

The lengths of the axis continuously decrease as z increases. This is obvious in the case of the vertical axis given by

$$2a\left\{\varepsilon^{\frac{2\pi}{l}(h-z)}-\varepsilon^{-\frac{2\pi}{l}(h-z)}\right\}$$

for as z becomes larger the exponents in this quantity become smaller, thus causing the first term in the brackets to diminish as the second increases, and consequently making the total value of the quantity diminish rapidly. Take now the horizontal axis denoted by

$$2a\left\{\varepsilon^{\frac{2\pi}{l}(h-z)}+\varepsilon^{-\frac{2\pi}{l}(h-z)}\right\}=\beta$$

differentiating this with respect to z and we have

$$\frac{d\beta}{dz}=2a\frac{2\pi}{l}\left\{\varepsilon^{-\frac{2\pi}{l}(h-z)}-\varepsilon^{\frac{2\pi}{l}(h-z)}\right\}$$

For $z<h$ the second of these terms is always greater than the first and, consequently, $\frac{d\beta}{dz}$ is negative; but the increment dz is supposed positive, consequently $d\beta$ is negative, or the axis decreases as z increases, *i.e.*, as we pass from the surface of the fluid. For $z=h$ this axis becomes $=2a$, and the vertical axis vanishes. That is, for particles of

water at the bottom of the canal there is only a motion of translation backwards and forwards in lines of length$=2a$. For particles at the surface of the fluid and for $h=l$ the horizontal axis is nearly $=535.5a$, or the ratio between the lengths of the horizontal axis at top and bottom of the fluid is nearly 267.7.

Referring now to the values of u and w near the surface, we have

$$\begin{cases} u=-\mathrm{A}\varepsilon^{-\frac{2\pi}{l}z}\cos.\frac{2\pi}{l}(\omega t+x) \\ w=\mathrm{A}\varepsilon^{-\frac{2\pi}{l}z}\sin.\frac{2\pi}{l}(\omega t+x)\end{cases}$$

Differentiating these for t, squaring and adding the results, and we obtain the expression

$$\left(\frac{du}{dt}\right)^2+\left(\frac{dw}{dt}\right)^2=\left(\frac{2\pi\omega}{l}\mathrm{R}\right)^2$$

The quantity on the left hand side of this equation gives the square of the velocity of the fluid particle in its circular path; this, as we see from the second member of the equation, is independent of the time, and is directly proportional to the radius of the circle; but the radius depends upon the quantity $\varepsilon^{-\frac{2\pi}{l}z}$ for its value, and this increases as z decreases—therefore, for particles near the surface of shallow water, we have the velocity x varies inversely as their depth. From this it is evident that the water at the top of the wave moves most rapidly forward, while that at the bottom moves most rapidly backward. In the expression for ω discarding the terms $\varepsilon^{-\frac{2\pi}{l}h}$ we have for the velocity of translation of particles near the surface of water of finite depth, or anywhere within the mass of a body of water of infinite depth

$$\omega=\sqrt{\frac{gl}{2\pi}}.$$

Substituting for ω its value of $\frac{l}{\tau}$ and we find for τ the value

$$\sqrt{\frac{2\pi l}{g}}.$$

Calling t' the time of oscillation of a simple pendulum of length l, we have

$$\tau=\sqrt{\frac{2}{\pi}}t'.$$

From the above value of ω we see that the velocity of transmission of the wave varies as the square root of the length. In all cases, indeed, the velocity is nearly as the square root of the length, for the factor

$$\frac{\varepsilon^{\frac{2\pi}{l}h}-\varepsilon^{-\frac{2\pi}{l}h}}{\varepsilon^{\frac{2\pi}{l}h}+\varepsilon^{-\frac{2\pi}{l}h}}$$

is nearly equal to unity. In the case of very shallow water, the velocity diminishes considerably—as the quantity just written decreases rapidly with h—vanishing as is obvious for $h=o$.

So far we have confined ourselves to a single wave, that is, to a single value of φ satisfying the equation $\Delta^2\varphi=o$. But we have seen that if there are several values of φ each satisfying this equation, that collectively they satisfy the equation

$$\Delta^2\Sigma\varphi=o.$$

In the case when the wave lengths are the same but the phases different, we can easily find the result of adding together the waves given by the functions φ, φ_1, . . . φ_i.

The value that we have already obtained for φ may be written

$$-\frac{a_0}{\sigma}\left\{\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right\}\sin.\sigma(\omega t+x+a_0)$$

where it is to be understood that $a_0=o$ and is merely introduced for future symmetry. Any other function φ_i which satisfies the equation $\Delta^2\varphi=o$, may be written under the above conditions

$$\varphi_i=-\frac{a_i}{\sigma}\left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right)\sin.\sigma(\omega t+x+a_i)$$

And it is not difficult to see that a summation of these functions will give us

$$\Sigma\varphi=-\frac{\mathrm{A}}{\sigma}\left\{\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right\}\sin.\sigma(\omega t+x+\Psi)$$

When

$$A^2 = \sum_{i=o} a_i^2 + 2\sum_{j=o} a_j \sum_{k=j+1} a_k \cos.\sigma\,(a_{k-1}-a_k)$$

and

$$\tan.\Psi = \frac{\sum_{i=1} a_i \sin.\ \sigma a_i}{\sum_{j=o} a_i \cos.\ \sigma a_j}$$

or inversely

$$\Psi = \frac{1}{\sigma}\tan.^{-1}\frac{\sum_{i=1} a_i \sin.\ \sigma a_i}{\sum_{j=0} a_j \cos.\ \sigma a_j}$$

If any of the quantities σa, &c.,$=\pi$ or $(2n+1)\pi$ a change takes place in the summation. Suppose $\sigma a_i = \pi$ then φ_i becomes $-\varphi_i$ and is subtracted instead of added to the other functions; but if $\sigma a_i = (2n+1)\pi$ we have since $\sigma = \frac{2\pi}{l}$,

$$a_i = (2n+1)\frac{l}{2}.$$

That is if the difference of phase is an odd multiple of half the wave length, the corresponding wave function is to be subtracted instead of added to the others in finding the resultant of the system.

Suppose now that we have two waves of the same length and amplitude, but of different phases and moving in opposite directions; the wave functions are obviously

$$\varphi = -\frac{a}{\sigma}\left\{\varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)}\right\}\sin.\ \sigma\ (\omega t + x)$$

$$\varphi_1 = -\frac{a}{\sigma}\left\{\varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)}\right\}\sin.\ \sigma\ (\omega t - x + a)$$

adding we have

$$\varphi + \varphi_1 = -\frac{a}{\sigma}\left\{\varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)}\right\}\sin.\ \sigma\ (\omega t + x) + \sin.\ \sigma\ (\omega t - x + a)$$

expanding the trigonometric factor and reducing by aid of the relations

$$\cos.\ a = 2\cos.^2\frac{a}{2} - 1,\ \sin.\ a = l\sin.\frac{a}{2}\cos.\frac{a}{2},$$

we readily find this expression to become

$$\varphi + \varphi_1 = -\frac{2a}{\sigma}\left\{\varepsilon^{(\sigma h-z)} + \varepsilon^{-\sigma(h-z)}\right\}\sin.\ \sigma\ (wt + \frac{a}{2})\cos.\ \sigma\ (x - \frac{a}{2}).$$

From this we obtain by differentiation the values of the displacements

$$u = 2a\left\{\varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)}\right\}\sin.\ \sigma\left(x - \frac{a}{2}\right)\sin.\ \sigma\left(wt + \frac{a}{2}\right)$$

$$w = 2a\left\{\varepsilon^{\sigma(h-z)} - \varepsilon^{-\sigma(h-z)}\right\}\sin.\ \sigma\left(wt + \frac{a}{2}\right)\cos.\ \sigma\left(x - \frac{a}{2}\right).$$

Dividing the first of these by the second we have

$$\frac{u}{w} = \frac{\varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)}}{\varepsilon^{\sigma(h-z)} - \varepsilon^{-\sigma(h-z)}}\tan.\sigma\left\{x - \frac{a}{2}\right\};$$

this ratio is independent of the time, consequently each particle moves in a straight line the inclination of which varies with x and z. Since also this ratio is the same at any given point, no matter what be the time, the wave is a standing wave or has no progressive motion.

Hence if there exist in the liquid two waves having the same length and amplitude but moving in opposite directions the result is a single standing wave in which the particles move constantly in right lines whose inclinations to the axes vary with x and z.

Reverting now to our values of u and w, suppose that $u=o$, that is, that there be no horizontal motion, this gives us

$$\cos.\ \frac{2\pi}{l}(\omega t +) = o$$

or $$\omega t + x = \frac{l}{4} \text{ or } \frac{3l}{4} \text{ or } \frac{(2n+1)l}{4}.$$

That is, there is no horizontal motion at the nodes of the wave. The greatest horizontal motion evidently corresponds to

$$\omega t + x = o, \frac{l}{2}, \text{ or } l$$

or the greatest horizontal motion is at the highest point of the crest and the lowest point of the trough of the wave—and evidently the motions at these points are in opposite directions—which we have seen before from other considerations.

In like manner by making $w=o$ we find that at the top and bottom of the wave in there is no vertical motion. Also, that the greatest vertical motion is at the nodes of the wave.

Similar results are obtained by exam-

ining the equations a. There is no horizontal motion at the points when $x-\frac{a}{2}=\frac{l}{4}$ or $\frac{3l}{4}$ but there is a maximum of vertical motion; also, there is no vertical motion at the points where $x-\frac{a}{2}=o$, $\frac{l}{2}$, or l, but there is the greatest horizontal motion.

Airy has shown in his treatise on "Tides and Waves," that if the channel is of variable depth or width, that waves of the nature just described, that is, waves caused by the simple oscillation of the particles, could not exist by themselves, but require for their existence the action of certain exterior forces into the nature of which it is not necessary here to go. Without going into a mathematical discussion of the reflection of waves, I will merely state that after impinging upon a wall the particles of the wave move up and down the surface through a distance equal to twice their previous vertical displacement, and the same with particles at a distance of half a wave length from the wall; particles at a distance from the wall of one quarter a wave length, merely vibrate in a horizontal direction. When a series of waves enters shallow water, the period remains the same, but the velocity and wave length diminish; the front of the wave becomes steeper than the back, and continues to become more and more abrupt until the top of the wave curls over the front and the wave breaks in surf on the beach.

§ 4.

CYLINDRICAL WAVES.

If we throw a pebble into a body of water, or if we simply bring a solid body in contact with the water at one point we know that a series of waves is generated which are circular in form, concentric and having their center at the point where the disturbance takes place. The waves thus generated are called cylindrical waves, and the line passing through the center of these circles and normal to the surface of the fluid is called the wave-axis, and evidently is the geometrical axis of the concentric cylinders.

In the case of such waves as this it is evidently not admissible to assume the displacement in any direction as equal to zero, there will clearly be motion in the direction of all these axes. Our axis of z will be assumed as having the same direction as in the foregoing section, and the axes of X and Y will lie in the surface of the fluid at rest. Our equation of continuity will have the general form

$$\frac{d^2\varphi}{dx^2}+\frac{d^2\varphi}{dy^2}+\frac{d^2\varphi}{dz^2}=o$$

the displacements being of course given by

$$u=\frac{d\varphi}{dx},\quad v=\frac{d\varphi}{dy},\quad w=\frac{d\varphi}{dz}.$$

The same remarks that were previously made concerning the form of φ will hold here, the waves being supposed to emanate from the wave axis, oz we can write for φ the equation

$$\varphi=\varepsilon^{\pm\sigma z}\,f\,(x,\,y)\,\begin{matrix}\sin\\ \cos\end{matrix}\left\{\frac{2\pi}{\tau}\,t\right\}$$

when τ as before denotes the periodic time. If the wave axis be taken as the axis of z we have, r denoting the distance from this axis to any point in a plane parallel to the plane of $x\,y$,

$$r^2=x^2+y^2$$

and we may with φ in the form,

$$\varphi=\varepsilon^{\pm\sigma z}\,f\,(r)\,\begin{matrix}\sin\\ \cos\end{matrix}\left\{\frac{2\pi}{\tau}\,t\right\}$$

We must now, as before, determine the form of f. Substitute this value of φ in the equation of continuity, and it is easily found to reduce to the form

$$\frac{d^2f}{dr^2}+\frac{1}{r}\frac{df}{dr}+\sigma^2f=o$$

Transforming this by the substitution

$$r=\frac{s}{\sigma}$$

we obtain a known form

$$\frac{d^2f}{ds^2}+\frac{1}{s}\frac{df}{ds}+f=o.$$

This is a particular case of the more general equation,

$$\frac{d^2f}{ds^2}+\frac{1}{s}\frac{df}{ds}+\left(1-\frac{n^2}{s^2}\right)f=o$$

of which a particular solution is the Bessel's function $J_n\,(s)$ given by

$$J_n(s)=\frac{s_n}{2^n n!}\left\{1-\frac{s^n}{2(2n+2)}+\frac{s^4}{2.4(2n+2)(2n+4)}\right.$$
$$\left.\frac{s^6}{2.4.6(2n+2)(2n+4)(2n+6)}+\dots\right\}$$

For our case $n=o$, and the function $J_0(s)$ is a particular solution, viz.:

$$J_0(s)=\left\{1-\frac{s^2}{2^2}+\frac{s^4}{2^2.4^2}-\frac{s^6}{2.^24.^26^2}+\dots\right\}$$

This is easily obtained directly, calling f_0 the particular solution sought assume

$$f_0=a+a_1s+a_2s^2+a_3s^3+a_4s^4+\dots$$

Substituting in the differential equation, and we have

$$o=a_1s^{-1}+(a+4a_2)+(a_1+9a_3)s+(a_2+16a_4)s^2+\dots$$

from which we have

$$a_1=a_3=a_5=\dots\dots a_{2i+1}=o$$

$$a_2=-\frac{a}{4},\ a_4=\frac{a}{64}=\frac{a}{2^2.4.^2},\ \&c.\dots$$

when i is of course any positive integral. This gives us then for our particular solution

$$f_0=J_0(s)=a\left\{1-\frac{s^2}{2^2}-\frac{s^4}{2^2.4^2}+\frac{s^6}{2^2.4^2.6^2}+\dots\right\}$$

Designate now by $Y_0(s)$ the other particular solution of the differential equation, and for brevity write simply Y_0 and J_0 instead of $Y_0(s)$ and $J_0(s)$. Let now Σ denote a function of s, then it is well known that we can write

$$Y_0=J_0\Sigma$$

Substituting this in the differential equation and it becomes

$$\left(\frac{1}{s}+\frac{2}{J_0}\frac{dJ_0}{ds}\right)\frac{d\Sigma}{ds}+\frac{d^2\Sigma}{ds^2}=o$$

Dividing this by $\frac{d\Sigma}{ds}$ and integrating, gives

$$\log. s+2\log. J_0+\log.\frac{d\Sigma}{ds}=\text{const.}$$

or assuming the const. $=o$, and passing to exponentials

$$sJ_0^2\frac{d\Sigma}{ds}=1$$

from which

$$\frac{d\Sigma}{ds}=\frac{1}{sJ_0^2}$$

and

$$\Sigma=\int\frac{ds}{sJ_0^2}$$

and, by substitution in the equation giving Y_0,

$$Y_0=J_0\int\frac{ds}{sJ_0^2}.$$

Now from the value of J_0 it is clear that the expansion of $\frac{1}{J_0^2}$ can only contain even powers of s and we can thus write

$$\frac{1}{sJ_0^2}=\frac{1}{s}\left\{1+As^2+Bs^4+Cs^6+\right\};$$

multiplying by ds and integrating gives

$$Y_0=J_0\log. s+J_0\left\{A\frac{s^2}{2}+B\frac{s^4}{4}+C\frac{s^6}{6}+\right\}$$

or as it may be written for brevity

$$Y_0=J_0\log. s+E_0$$

The quantity E_0 is the product of two infinite series each of which contains only positive integral powers of s and, consequently, according to a principle in the theory of the Bessel's functions can be developed in a series of these functions and, moreover, as all the powers in J_0 and the other factor of E_0 are even, only the even Bessel's functions will appear in the development thus

$$Y_0=J_0\log. s+aJ^0+bJ^2+cJ^4+\dots$$

The co-efficients a, b, c have to be determined.

Take again the differential equation

$$\frac{d^2f}{ds^2}+\frac{1}{s}\frac{df}{ds}+f=o$$

and perform the operation

$$\frac{d^2}{ds^2}+\frac{1}{s}\frac{d}{ds}+1$$

on the quantity $J_0\log. s$ and we find

$$\left\{\frac{d^2}{ds^2}+\frac{1}{s}\frac{d}{ds}+1\right\}J_0\log. s=\frac{2}{s}\frac{dJ_0}{ds}.$$

Represent the operator for brevity by Δ, then this is

$$\Delta J_0\log. s=\frac{2}{s}\frac{dJ^0}{ds}$$

We have now from the general differential equation affording Bessel's functions

$$\Delta J_n=\frac{n^2}{s^2}J_n$$

and for $n=o$

$$\Delta J_o=o$$

and for $n>o$

$$\Delta J_n = \frac{n}{2s}\left\{ J_{n-1} + J_{n+1} \right\}$$

according to a known relation connecting these three consecutive functions. And so we have, finally

$$\Delta J_0 \log. s = -\frac{2}{s} J_1$$

Now from the above value of Y_0 we have

$$\Delta Y_0 = \Delta(J_0 \log. s) + a\Delta J_0 + b\Delta J_2 + c\Delta J_4 + \ldots$$

And by aid of the transformations just given

$$\Delta Y_0 = -\frac{2}{s} J_1 + \frac{b}{s}(J_1 + J_3) + \frac{2C}{s}(J_3 + J_5) + \frac{3d}{s}(J_5 + J_7) \times \ldots$$

Now Y_0 being a particular solution of the differential equation $\Delta f = o$, we must have $\Delta Y_0 = o$; this enables us to find

$$b = -2c = 3d = -4c = 5g = \&c. \ldots\ldots = 2$$

and by substitution

$$Y_0 = J_0 \log. s + aJ_0 + 2[J_2 - \tfrac{1}{2}J_4 + \tfrac{1}{3}J_6 - \tfrac{1}{4}J_8 + \ldots]$$

The complete solution of the differential equation $\Delta f = o$, *i.e.*,

$$\frac{d^2 f}{ds^2} + \frac{1}{s}\frac{df}{ds} + f = o$$

is now given by

$$f = AJ_0 + BY_0$$

or

$$= (A + B \log. s)\, J_0 + BE_0$$

It may be verified without much difficulty that the quantity E_0 is of the form

$$E_0 = \left\{ \frac{s^2}{2^2} - \frac{1+\frac{1}{2}}{2^2.4^2} s^4 + \frac{1+\frac{1}{2}+\frac{1}{3}}{2^2.4^2.6^2} s^6 + \ldots\ldots \right\}$$

The quantities J_0 and Y_0 expressed in the form of definite integrals are—vide Boole's Diff. Equas.

$$J_0 = \frac{1}{\pi}\int_0^\pi \cos(s \sin \omega)\, d\omega$$

$$Y_0 = \frac{1}{\pi}\int_0^\pi \cos(s \sin \omega) \log(4s \cos^2\omega)\, d\omega$$

and we have for f by substitution

$$f = \frac{1}{\pi}\int_0^\pi \cos(s \sin \omega)\,(A + B \log(4s \cos^2 \omega))\, d\omega$$

or as this may be written

$$f = \int_0^\pi \cos(s \sin \omega)\,(C + D \log(s \cos^2\omega))\, d\omega$$

when

$$C = \frac{A + B \log. 4}{\pi},\quad D = \frac{B}{\pi}.$$

Before going on to the application of these results to the problem in hand, we will investigate the change produced in the quantities J_0 and Y_0 by allowing s to become very great.

Instead of f in the differential equation $\Delta f = o$ write $f\sqrt{s}$ this equation thus becomes

$$\frac{d^2(f\sqrt{s})}{ds^2} + \left(1 + \frac{1}{4s^2}\right) f\sqrt{s} = o$$

and this for s very large is simply

$$\frac{d^2(f\sqrt{s})}{ds^2} + f\sqrt{s} = o.$$

This equation gives on integration

$$f\sqrt{s} = a \cos s + b \sin s$$

or

$$f = \frac{a \cos s + b \sin s}{\sqrt{s}}$$

when a and b are of course constants. We have then obviously from this

$$J_0 = \frac{\alpha \cos s + \beta \sin s}{\sqrt{s}}$$

$$Y_0 = \frac{\alpha' \cos s + \beta' \sin s}{\sqrt{s}}$$

from which we can see that for infinitely great values of s the functions J_0 and Y_0 will vanish.

Now by substituting for s its value of σr we can, by taking as the argument of the functions thus obtained the quantity $-\frac{s^2}{2^2}$, or $-\frac{\sigma^2 r^2}{4}$ with f in the form

$$f = \left\{ 1 + \sum_{i=1}^{i=s} \frac{\theta^i}{(i!)^2} \right\} (A + B \log. 2\sqrt{-\theta}) - B \sum_{i=1}^{i=\infty} \frac{\theta^i}{(i!)^2} \cdot \sum_{j=1}^{j=i} \frac{1}{j}$$

For convenience of reference hereafter we shall write this in the form,

$$f = A\Pi(\theta) + B\Omega(\theta).$$

Substituting this value of f in the expression for φ we obtain,

$$\varphi = \varepsilon^{\pm \sigma z}[A\Pi(\theta) + B\Omega(\theta)]\left\{ \sin \frac{2\pi}{\tau} t + \cos \frac{2\pi}{\tau} t \right\}$$

or expanding this and writing instead of

A and B, the quantities a_1, b_1, a_2, b_2, α_1, β_1, α_2, β_2, we have

$$\varphi=\pi\sin\frac{2\pi}{\tau}t\left(a_1\varepsilon^{\sigma z}+\alpha_1\varepsilon^{-\sigma z}\right)+\pi\cos\frac{2\pi}{\tau}t\left(a_2\varepsilon^{\sigma z}+\alpha_2\varepsilon^{-\sigma z}\right)$$
$$+\Omega\sin\frac{2\pi}{\tau}t\left(b_1\varepsilon^{\sigma z}+\beta_1\varepsilon^{-\sigma z}\right)+\Omega\cos\frac{2\pi}{\tau}t\left(b_2\varepsilon^{\sigma z}+\beta_2\varepsilon^{-\sigma z}\right)$$

This gives us for u, v, w the following values;

$$u=\left\{\sin\frac{2\pi}{\tau}t\left(a_1\varepsilon^{\sigma z}+\alpha_1\varepsilon^{-\sigma z}\right)+\cos\frac{2\pi}{\tau}t\left(a^2\varepsilon^{\sigma z}+\alpha_2\varepsilon^{-\sigma z}\right)\right\}\frac{d\pi}{dx}$$
$$+\left\{\sin\frac{2\pi}{\tau}t\left(b_2\varepsilon^{\sigma z}+\beta_2\varepsilon^{-\sigma z}\right)+\cos\frac{2\pi}{\tau}t\left(b_2\varepsilon^{\sigma z}+\beta_2\varepsilon^{-\sigma z}\right)\right\}\frac{d\Omega}{dx},$$

$$v=\left\{\sin\frac{2\pi}{\tau}t\left(a_1\varepsilon^{\sigma z}+\alpha_1\varepsilon^{-\sigma z}\right)+\cos\frac{2\pi}{\tau}t\left(a_2\varepsilon^{\sigma z}+\alpha_2\varepsilon^{-\sigma z}\right)\right\}\frac{d\pi}{dy}$$
$$+\left\{\sin\frac{2\pi}{\tau}t\left(b_1\varepsilon^{\sigma z}+\beta_1\varepsilon^{-\sigma z}\right)+\cos\frac{2\pi}{\tau}t\left(b_2\varepsilon^{\sigma z}+\theta_2\varepsilon^{-\sigma z}\right)\right\}\frac{d\Omega}{dy},$$

$$w=\pi\sin\frac{2\pi}{\tau}t\,\sigma\left(a_1\varepsilon^{\sigma z}-\alpha_1\varepsilon^{-\sigma z}\right)+\pi\cos\frac{2\pi}{\tau}t\,\sigma\left(a_2\varepsilon^{\sigma z}-\alpha_2\varepsilon^{-\sigma z}\right)$$
$$+\Omega\sin\frac{2\pi}{\tau}t\,\sigma\left(b_1\varepsilon^{\sigma z}-\beta_1\varepsilon^{-\sigma z}\right)+\Omega\cos\frac{2\pi}{\tau}t\,\sigma\left(b_2\varepsilon^{\sigma z}-\beta_2\varepsilon^{-\sigma z}\right)$$

The expression for the fluid pressure is obtained here in the same manner as in the case of plane waves, and is

$$p=\rho\left\{\left(\frac{2\pi}{\tau}\right)^2\varphi+gz\right\}+\text{const.}$$

Writing as before $z=z_0+w$, remembering that σ and w are quantities of the first order of magnitude, and so discarding terms containing σw or higher orders, we have

$$p=p_0+\rho g z_0+\rho\left\{\pi\sin\frac{2\pi}{\tau}t\left\{\left(\frac{2\pi}{\tau}\right)^2\left(a_1\varepsilon^{\sigma z_0}+\alpha\varepsilon^{-\sigma z_0}\right)+g\sigma\left(a_1\varepsilon^{\sigma z_0}-\alpha_1\varepsilon^{-\sigma z_0}\right)\right\}\right.$$
$$+\pi\cos\frac{2\pi}{\tau}t\left\{\left(\frac{2\pi}{\tau}\right)^2\left(a_2\varepsilon^{\sigma z_0}+\alpha\varepsilon^{-\sigma z_0}\right)+g\sigma\left(a_2\varepsilon^{\sigma z}-\alpha_2\varepsilon^{-\sigma z_0}\right)\right\}$$
$$+\Omega\sin\frac{2\pi}{\tau}\tau\left\{\left(\frac{2\pi}{\tau}\right)^2\left(b_1\varepsilon^{\sigma z_0}+\beta_1\varepsilon^{-\sigma z_0}\right)+g\sigma\left(b_1\varepsilon^{\sigma z_0}-\beta_1\varepsilon^{-\sigma z_0}\right)\right\}$$
$$\left.+\Omega\cos\frac{2\pi}{\tau}t\left\{\left(\frac{2\pi}{\tau}\right)^2\left(b_1\varepsilon^{\sigma z_0}+\beta_2\varepsilon^{\sigma-z_0}\right)+g\sigma\left(b_2\varepsilon^{\sigma z_0}-\beta_2\varepsilon^{\sigma-z_0}\right)\right\}\right\}$$

For particles on the surface of the fluid at rest we have, of course, $z_0=o$ and $p=p_0$. This gives us

$$\pi\sin.\frac{2\pi}{\tau}t\left\{\left(\frac{2\pi}{\tau}\right)^2(a_1+\alpha_1)+g\sigma(a_1-\alpha_1)\right\}+\pi\cos\frac{2\pi}{\tau}t\left\{\left(\frac{2\pi}{\tau}\right)^2(a_2+\alpha_2)+g\sigma(a_2-\alpha_2)\right\}$$
$$+\Omega\sin\frac{2\pi}{\tau}t\left\{\left(\frac{2\pi}{\tau}\right)^2[b_1+\beta_1]+g\sigma\left(b_1-\beta_1\right)\right\}+\Omega\cos\frac{2\pi}{\tau}t\left\{\left(\frac{2\pi}{\tau}\right)^2\{b_2+\beta_2\}+g\sigma\left(b_2-\beta_2\right)\right\}=o$$

In order that this may be satisfied we must have obviously

$$\frac{a_1-\alpha_1}{a_1+\alpha_1}=\frac{a_2-\alpha_2}{a_2+\alpha_2}=\frac{b_1-\beta_1}{b_1+\beta_1}=\frac{b_2-\beta_2}{b_2+\beta_2}=\frac{\left(\frac{2\pi}{\tau}\right)^2}{g\sigma}$$

from which we obtain,

$$\frac{a_1}{\alpha_1}=\frac{a_2}{\alpha_2}=\frac{b_1}{\beta_1}=\frac{b_2}{\beta_2}$$

We can now write

$$w=\sigma\pi\sin\frac{2\pi}{\tau}t\left(a_1\varepsilon^{\sigma z}-\alpha_1\varepsilon^{-\sigma z}\right)+c_1\sigma\pi\cos\frac{2\pi}{\tau}t\left(a_1\varepsilon^{\sigma z}-\alpha_1\varepsilon^{-\sigma z}\right)$$
$$+c_2\sigma\Omega\sin\frac{2\pi}{\tau}t\left(a_1\varepsilon^{\sigma z}-\alpha_1\varepsilon^{-\sigma z}\right)+c_3\sigma\Omega\cos\frac{2\pi}{\tau}t\left(a_1\varepsilon^{+\sigma z}-\alpha_1\varepsilon^{-\sigma z}\right)$$

when the meaning of the constants

c_1, c_2, c_3 is obvious. Introducing now the condition that $w=o$ for $h=o$ we have as in plane waves

$$\frac{a_1}{a_1} = \frac{\varepsilon^{-\sigma h}}{\varepsilon^{\sigma h}};$$

This gives us again

$$\left(\frac{2\pi}{\tau}\right)^2 = g\sigma\frac{\varepsilon^{\sigma h}-\varepsilon^{-\sigma h}}{\varepsilon^{\sigma h}+\varepsilon^{-\sigma h}}.$$

We have now for φ the equation,

$$\begin{aligned}\varphi = {} & a_1\pi\sin\frac{2\pi}{\tau}t\left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right) \\ & + b_1\pi\cos\frac{2\pi}{\tau}t\left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right) \\ & + a_2\Omega\sin\frac{2\pi}{\tau}t\left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right) \\ & + b_2\Omega\cos\frac{2\pi}{\tau}t\left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right)\end{aligned}$$

when a_1, b_1, a_2, b_2, are new constants, whose values are respectively,

$$\frac{a_1}{\varepsilon^{\sigma h}},\ \frac{c_1 a_1}{\varepsilon^{\sigma h}},\ \frac{c_2 a_1}{\varepsilon^{\sigma h}},\ \frac{c_3 a_1}{\varepsilon^{\sigma h}},$$

or those multiplied by any arbitrary constant. This value of φ may be written in the form

$$\varphi = \left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right)\left[(a_1\pi+a_2\Omega)\sin\frac{2\pi}{\tau}t+(b_1\pi+b_2\Omega)\cos\frac{2\pi}{\tau}t\right]$$

from which we obtain

$$u = \left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right)\left\{\left(a_1\frac{d\pi}{dx}+a_2\frac{d\Omega}{dx}\right)\sin\frac{2\pi}{\tau}t+\left(b_1\frac{d\pi}{dx}+b_2\frac{d\Omega}{dx}\right)\cos\frac{2\pi}{\tau}t\right\}$$

$$v = \left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right)\left\{\left(a_2\frac{d\pi}{dy}+a_2\frac{d\Omega}{dy}\right)\sin\frac{2\pi}{\tau}t+\left(b_1\frac{d\pi}{dy}+b_2\frac{d\Omega}{dy}\right)\cos\frac{2\pi}{\tau}t\right\}$$

$$w = -\sigma\left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right)\left[(a_1\pi+a_2\Omega)\sin\frac{2\pi}{\tau}t+(b_1\pi+b_2\Omega)\cos\frac{2\pi}{\tau}t\right]$$

We have, however,

$$\frac{d\pi}{dx} = \frac{d\pi}{d\theta}\cdot\frac{d\theta}{dx} = \frac{d\pi}{d\theta}\cdot\frac{d}{dx}\left(-\frac{\sigma^2 r^2}{4}\right) = -\frac{\sigma^2 x}{2}\frac{d\pi}{d\theta},\ \&c.....$$

consequently,

$$u = -\frac{\sigma^2 x}{2}\left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right)\left\{\left(a_1\frac{d\pi}{d\theta}+a_2\frac{d\Omega}{d\theta}\right)\sin\frac{2\pi}{\tau}t+\left(b_1\frac{d\pi}{d\theta}+b_2\frac{d\Omega}{d\theta}\right)\cos\frac{2\pi}{\tau}t\right\}$$

$$v = -\frac{\sigma^2 y}{2}\left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right)\left\{\left(a_1\frac{d\pi}{d\theta}+a_2\frac{d\Omega}{d\theta}\right)\sin\frac{2\pi}{\tau}t+\left(b_1\frac{d\pi}{d\theta}+b_2\frac{d\Omega}{d\theta}\right)\cos\frac{2\pi}{\tau}t\right\}$$

From these equations we see that the path of the particle is always in a plane passing through itself and the axis of z.

The expression for p becomes now

$$p = p_0+\rho g z_0+2\rho g\sigma\left\{\left(a_1\pi+a_2\Omega\right)\sin\frac{2\pi}{\tau}t+(b_1\pi+b_2\Omega)\cos\frac{2\pi}{\tau}t\right\}\frac{\varepsilon^{\sigma z_0}-\varepsilon^{-\sigma z_0}}{\varepsilon^{\sigma h}+\varepsilon^{-\sigma h}}$$

The values which have been obtained for the displacements and the fluid pressure afford the complete solution of the problem under consideration.

The results obtained are, however, very much modified in the cases where the particles are removed to great distances from the axis. We have already seen the change produced in the function f in such a case, viz., this quantity becomes

$$f = \frac{a\cos s+b\sin s}{\sqrt{s}}$$

or since σ is a constant

$$f = \frac{A\cos\sigma r+B\sin\sigma r}{\sqrt{r}}.$$

We might have so transformed our first obtained value of f that the infinite series therein contained should have proceeded according to ascending powers of $\frac{1}{r^2}$ and thus obtained the same result; this, however, would have been a difficult process.

The quantities π, and Ω are now given by the equations,

$$\pi = \frac{\sin\sigma r}{\sqrt{r}},\ \Omega = \frac{\cos\sigma r}{\sqrt{r}}$$

Substituting these in our value for φ and we have,

$$\varphi = \frac{1}{\sqrt{r}}\left\{\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right\}$$

$$\left\{(a_1 \sin\sigma r + a_2 \cos\sigma r)\sin\frac{2\pi}{\tau}t + (b_1 \sin\sigma r + b_2 \cos\sigma r)\cos\frac{2\pi}{\tau}t\right\}$$

for the simplest case of waves we can make $a_1=b_2=o$ and $a_2=b_1=a$; thus

$$\varphi=\frac{a}{\sqrt{r}}\left\{\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right\}\sin\left(\frac{2\pi}{\tau}t+\sigma r\right)$$

Differentiating this for r gives us the radial velocity of a particle, representing this by η and we have

$$\eta=\frac{1}{\sqrt{r}}\left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right)\left\{\sigma a\cos\left(\frac{2\pi}{\tau}t+\sigma r\right)+\frac{a}{2r}\sin\left(\frac{2\pi}{\tau}t+\sigma r\right)\right\}$$

discarding term containing $\frac{1}{r}$,

$$\eta=\frac{\sigma a}{\sqrt{r}}\left(\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right)\cos\left(\frac{2\pi}{\tau}t+\sigma r\right)$$

also

$$w=-\frac{\sigma a}{\sqrt{r}}\left(\varepsilon^{\sigma(h-z)}-\varepsilon^{-\sigma(h-z)}\right)\sin\left(\frac{2\pi}{\tau}t+\sigma r\right)$$

The expression for p also becomes, after making the proper substitutions and reductions,

$$p=p_0+\rho g z_0+2\rho g\frac{\sigma a}{\sqrt{r}}\frac{\varepsilon^{\sigma z_0}-\varepsilon^{-\sigma z_0}}{\varepsilon^{\sigma h}+\varepsilon^{-\sigma h}},\quad \sin\left(\frac{2\pi}{\tau}t+\sigma r\right).$$

Introducing now the wave length l we have for great values of r

$$\frac{1}{\sqrt{r+l}}=\frac{1}{\sqrt{r}}$$

and also for a first approximation,

$$\frac{1}{\sqrt{r}}=\frac{1}{\sqrt{r_0+\eta}}=\frac{1}{\sqrt{r_0}}$$

we have, as before, $\sigma=\frac{2\pi}{l}$, and $l=\tau\,\omega$.

Thus φ becomes now,

$$\varphi=\frac{a}{\sqrt{r_0}}\left(\varepsilon^{\frac{2\pi}{l}(h-z)}+\varepsilon^{-\frac{2\pi}{l}(h-z)}\right)\sin\frac{2\pi}{l}(\omega t+r)$$

with as before,

$$\omega=\sqrt{\frac{lg}{2\pi}\cdot\frac{\varepsilon^{\sigma h}-\varepsilon^{-\sigma h}}{\varepsilon^{\sigma h}+\varepsilon^{-\sigma h}}}$$

which for great depths or for particles near the surface becomes,

$$\sqrt{\frac{lg}{2\pi}}.$$

The same deductions are to be made here as in the case of plane waves, viz., that for particles any where within the mass of a fluid of infinite depth or near the surface of a mass of finite depth the velocity varies as the square root of the wave length. Write now $a\sigma=-a$ and collect all of our expressions:

$$\varphi=-\frac{l}{2\pi}\frac{a}{\sqrt{r_0}}\left\{\varepsilon^{\frac{2\pi}{l}(h-z)}+\varepsilon^{-\frac{2\pi}{l}(h-z)}\right\}\sin\frac{2\pi}{l}(\omega t+r)$$

$$\eta=-\frac{a}{\sqrt{r_0}}\left\{\varepsilon^{\frac{2\pi}{l}(h-z)}+\varepsilon^{-\frac{2\pi}{l}(h-z)}\right\}\cos\frac{2\pi}{l}(\omega t+r)$$

$$w=\frac{a}{\sqrt{r_0}}\left\{\varepsilon^{\frac{2\pi}{l}(h-z)}-\varepsilon^{-\sigma(h-z)}\right\}\sin\frac{2\pi}{l}(wt+r).$$

$$p_2 p_0+\rho g z_0-2\rho g\frac{a}{\sqrt{r_0}}\frac{\varepsilon^{\frac{2\pi}{l}z_0}-\varepsilon^{-\frac{2\pi}{l}z_0}}{\varepsilon^{\frac{2\pi}{l}h}+\varepsilon^{-\frac{2\pi}{l}h}}\sin\frac{2\pi}{l}(wt+r)$$

$$\omega=\sqrt{\frac{lg}{2\pi}\cdot\frac{\varepsilon^{\frac{2\pi}{l}h}+\varepsilon^{-\frac{2\pi}{l}h}}{\varepsilon^{\frac{2\pi}{l}h}+\varepsilon^{\frac{2\pi}{l}h}}}$$

We see from these expressions that the amplitudes of cylindrical waves differ only from those of plane waves by the factor $\frac{1}{\sqrt{r_0}}$—or in cylindrical waves the

amplitudes varies inversely as $\sqrt{r_0}$—and for particles very remote from the axis the amplitudes will vanish; whereas in the case of plane waves we saw that the amplitudes were always the same for the same depth.

From the expressions for the displacements we have as before

$$\frac{\eta^2}{\frac{a_2}{r_0}\left\{\varepsilon^{\frac{2\pi}{l}(h-z)}+\varepsilon^{\frac{2\pi}{l}(h-z)}\right\}^2}+\frac{w^2}{\frac{a^2}{r_0}\left\{\varepsilon^{\frac{2\pi}{l}(h-z)}+\varepsilon^{\frac{2\pi}{l}(h-z)}\right\}^2}=1$$

Now if h be finite we have as before for particles within the mass of the fluid—except near the surface—that they move in ellipses whose plane is vertical and passing through the wave axis, and whose transverse axis is in the direction of r. Also the axes of the ellipse decrease as r_0 increases and for particles infinitely remote from the wave axis they vanish, or these particles are at rest. The axes also, as in the case of plane waves, continuously decrease as z increases, and for $z-h$ the transverse axis becomes $\frac{2a}{\sqrt{r^0}}$ and the vertical axis vanishes as it should do.

If z be very small as compared with h, the equation of our ellipse becomes

$$\frac{\eta^2}{A^2\varepsilon^{-\frac{4\pi}{l}z}}+\frac{w^2}{A^2\varepsilon^{-\frac{4\pi}{l}z}}=1$$

when

$$A=\frac{a}{\sqrt{r_0}}\varepsilon^{\frac{2\pi}{l}h}$$

This is the equation of a circle whose radius is

$$A\varepsilon^{-\frac{2\pi}{l}2}$$

That is, for particles near the surface of a mass of fluid of finite depth, or for particles any where within the mass of a fluid of infinite depth, the motion is in a circle. It is shown as in the case of plane waves that this circular motion is uniform. In fact, all the results that we have obtained for plane waves are transferable into the corresponding results for cylindrical waves by merely multiplying by the factor $\frac{1}{\sqrt{r_0}}$.

Suppose now that we have a series of n waves of the same wave length and amplitude but of different phases, starting from the same axis; let these waves be defined in the same manner as in the case of plane waves and we shall have for the resultant wave function

$$\Sigma\varphi=-\frac{A}{\sigma\sqrt{r_0}}\left\{\varepsilon^{\sigma(h-z)}+\varepsilon^{\sigma(h-z)}\right\}\sin\sigma(\omega t+r+\psi)$$

when,

$$A^2=\sum_{i=o}^{i=n}a_i^2+2\sum_{j=o}^{j=n}a_j\sum_{=j+1}^{k=n}a_k\cos\sigma\left(\begin{matrix}a & - & a\\ k-1 & & k\end{matrix}\right)$$

and

$$\Psi=\frac{1}{\sigma}\tan^{-1}\frac{\sum_{i=1}^{i=n}a_i\sin\sigma a_i}{\sum_{j=o}^{j=n}a_j\cos\sigma a_j}$$

If the wave lengths are the same, but the amplitudes different by reason of different initial values of r_0, the change in the form of these quantities is very slight; they become

$$A^2=\sum_{i=o}^{i=n}\frac{a_i^2}{\sqrt{r_i}}+2\sum_{j=o}^{j=n}\frac{a_j}{\sqrt{r_j}}\sum_{k=j+1}^{k=n}\frac{a_k}{\sqrt{r_k}}\cos\sigma\left(\begin{matrix}a & - & a\\ k-1 & & k\end{matrix}\right)$$

and

$$\Psi=\frac{1}{\sigma}\tan^{-1}\frac{\sum_{i=1}^{i=n}\frac{a_i}{\sqrt{r_i}}\sin\sigma a_i}{\sum^{i=n}\frac{a_i}{\sqrt{r_j}}\cos\sigma a_j}$$

Suppose now that we have two waves of the same wave lengths and amplitudes, but of different phases and going in opposite directions. The resultant wave function will be

$$\varphi+\varphi'=-\frac{2a}{\sigma\sqrt{r_0}}\left\{\varepsilon^{\sigma(h-z)}+\varepsilon^{-\sigma(h-z)}\right\}\sin\sigma\left(\omega t+\frac{a}{z}\right)\cos\sigma\left(r-\frac{a}{z}\right)$$

which corresponds to a standing wave. Differentiating for r and z we have for the displacement of η and w,

$$\eta = \frac{2a}{\sqrt{r_0}} \left\{ \varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)} \right\} \sin \sigma\left(\omega t + \frac{a}{z}\right) \sin \sigma\left(r - \frac{a}{z}\right)$$

$$w = \frac{2a}{\sqrt{r_0}} \left(\varepsilon^{\sigma(h-z)} - \varepsilon^{-\sigma(h-z)} \right) \sin \sigma\left(\omega t + \frac{a}{z}\right) \cos \sigma\left(r - \frac{a}{z}\right)$$

The ratio $\frac{\eta}{w}$ is independent of t, and we make the same deduction as before, that the particle move in right lines whose inclination to the axes varies with z and r.

Suppose that we have a series of parallel wave axes, and that waves proceed from them having the same length and the equal amplitudes but different phases. Let the wave functions be given as

$$\varphi_0 = -\frac{a_0}{\sigma\sqrt{r_0^{(0)}}} \left(\varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)} \right) \sin \sigma(\omega t + r^{(0)} + a_0)$$

$$\varphi_1 = -\frac{a_1}{\sigma\sqrt{r_0^{(1)}}} \left(\varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)} \right) \sin \sigma(\omega t + r^{(1)} + a_1$$

$$\vdots$$

$$\varphi_n = -\frac{a_n}{\sigma\sqrt{r_0^{(n)}}} \left\{ \varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)} \right\} \sin \sigma(\omega t + r^{(1)} + a_n)$$

adding these we have

$$\Sigma\phi = -\frac{1}{\sigma} \sum_{i=o}^{i=n} a_i \left\{ \varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)} \right\} \left\{ \frac{\sin \sigma(\omega t + r^{\circ} + a_0)}{\sqrt{r_0^{(0)}}} + \frac{\sin \sigma(\omega t + r^{(1)} + a_1)}{\sqrt{r_0^{(1)}}} \cdots + \frac{\sin \sigma(\omega t + r^{(n)} + a_n)}{\sqrt{r_0^{(n)}}} \right\}$$

from which

$$w = \sum_{i=o}^{i=n} a_i \left\{ \varepsilon^{\sigma(h-z)} - \varepsilon^{-\sigma(h-z)} \right\} \left\{ \frac{\sin \sigma(\omega t + r^{(0)} + a_1)}{\sqrt{r_0^{(0)}}} \cdots + \frac{\sin \sigma(\omega t + r^{(n)} + a_n)}{\sqrt{r_0^{(n)}}} \right\}$$

The waves may evidently so move that at certain points the vertical displacements shall be equal to zero. We can determine these points by placing the trigonometric factor of w equal to zero; thus

$$\frac{\sin \sigma\,(\omega t + r^{(0)} + a_0)}{\sqrt{r_0^{(0)}}} + \frac{\sin \sigma\,(\omega t + r^{(1)} + a_1)}{\sqrt{r_0^{(1)}}} + \cdots \frac{\sin \sigma\,(\omega t + r^{(n)} + a_n)}{\sqrt{r_0^{(n}}} = o$$

This expression can be divided into two parts, one of which shall have for a factor $\sin \sigma\, \omega\, t$, and the other $\cos \sigma\, \omega\, t$.

$$\sin \sigma\, \omega\, t \left\{ \frac{\cos \sigma\,(r^{(0)} + a_0)}{\sqrt{r_0^{(0)}}} + \frac{\cos \sigma\,(r^{(1)} + a_1)}{\sqrt{r_0^{(1)}}} + \cdots \frac{\cos \sigma\,(r^{(n)} + a_n)}{\sqrt{r_0^{(n)}}} \right\}$$
$$+ \cos \sigma\, \omega\, t \left\{ \frac{\sin \sigma\,(r^{(o)} + a_0)}{\sqrt{r_0^{(o)}}} + \frac{\sin \sigma\,(r^{(1)} + a_1}{\sqrt{r_0^{(1)}}} + \cdots \frac{\sin \sigma\,(r^{(n)} + a_n)}{\sqrt{r^n_{(o)}}} \right\} = o$$

Equate separately to zero the factors multiplying $\sin \sigma . \omega\, t$ and $\cos \sigma_0\, \omega\, t$, square and add the resulting equations and we have after some easy reductions,

$$\sum_{i=o}^{i=n} \frac{1}{r_i^{(i)}} + \sum_{j=o}^{j=n} \sum_{k=1}^{k=n} \cos \sigma \left\{ \frac{(r^{(j)} - a_j) - (r_k - a_k)}{\sqrt{r_0^j r_0^k}} \right\} = o$$

For the simple case of $n=2$ or two wave axes we have since $a_0 = o$

$$\frac{1}{r_0^{(o)}} + \frac{1}{r_0^{(1)}} + \frac{2}{\sqrt{r_0^{(o)} r_0^{(1)}}} \cos \sigma\,(r^{(o)} - r^{(1)} a\,) = o$$

If $r_0^{(o)} = r_0^{(1)}$ this becomes

$$\frac{2}{r_0} + \frac{2}{r_0} \cos \sigma\,(-a_1) = o$$

If now $-\sigma a_1 = \pm(2n+1)\pi$ this equation will be satisfied, *i.e.*, if

$$a = \mp(2n+1)\,\frac{l}{2}$$

Therefore if the difference of phase is an odd multiple of half the wave length the vertical displacement is zero—but only for the points for which $r_0^{(0)} = r(_0)^{(1)}$. The points defined by the equation

$$r_0^{(0)} = r_0^{(1)}$$

lie on a plane which from its relation to the waves may be called the plane of symmetry. We will now examine a little more closely the conditions at this plane of symmetry. We have

$$\varphi_0 = -\frac{a_0}{\sigma\sqrt{r_0}} \left\{ \varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)} \right\} \sin \sigma(wt + r)$$

$$\varphi' = -\frac{a_1}{\sigma\sqrt{r_0^{(1)}}}\left\{\varepsilon^{\sigma(h-z)} + \varepsilon^{-\sigma(h-z)}\right\}\sin\sigma(\omega t + r' + a_1)$$

writing, for convenience, r' for $r^{(1)}$. We have also

$$u = \frac{d\varphi}{dr}\frac{dr}{dx} + \frac{d\varphi'}{dr}\frac{dr'}{dx}$$

$$v = \frac{d\varphi}{dr}\frac{dr}{dy} + \frac{d\varphi'}{dx'}\frac{dr'}{dy}$$

when $r^2 = x^2 + y^2$ and $r'^2 = (x-2a)^2 + y^2$. Now since

$$\sigma a_1 = \pm(2n+1)\pi$$

we will have for $r = r'$

$$\frac{d\varphi}{dr} = -\frac{d\varphi'}{dr'}$$

At the plane of symmetry $n = a$ so that

$$\frac{dr}{dx} = -\left(\frac{dr'}{dx}\right), \text{ and } \frac{dr}{dy} = \frac{dr'}{dy};$$

Therefore for $r = r'$ we have

$$u = 2\frac{d\varphi}{dx}\frac{dr}{dx}, \quad v = o, \quad w = o.$$

That is, at the plane of symmetry the displacement perpendicular to it is twice as great as that due to either wave acting separately, and the displacements parallel to this plane and the vertical displacements are equal to zero. Suppose now, further, that $a_1 = o$: then we have for $r = r'$,

$$\frac{d\varphi}{dr} = \frac{d\varphi}{dr'};$$

therefore,

$$u = o, \quad v = 2\frac{d\varphi}{dr}\frac{dr}{dy}, \quad w = 2\frac{d\varphi}{dz},$$

From which we have—if there is no difference of phase between the waves from the parallel axes—at the plane of symmetry there is no displacement in the direction of the axis of X *i, e*, in the direction perpendicular to this plane; also that the displacement parallel to the plane and the vertical displacement are twice as great as they would be if there was but one wave.

The reader who is interested in the subject of wave motion will do well to read an article on the subject by Lord Rayleigh in the April number of the *Philosophical Magazine* for 1876. An article in the September number of the same publication for 1878, though not bearing directly upon the subject, will also be found to contain much that is of value and interest; the article referred to is entitled "Hydrodynamic Problems in reference to the Theory of Ocean Currents," by M. Zöppritz. The mathematical theory of wave motion remains pretty much as Airy left it when he completed his work on the subject—so no better reference can be given than to that work—for any one wishing to acquire a thorough knowledge of the subject.

UNSINKABLE RIVER STEAMERS.

From "The Nautical Magazine."

Ever since iron has been in use as a material for ship-building, naval architects have been alive to the fact that although iron ships, as compared with wooden ones, are exceptionally liable to serious local damage, there is, on the other hand, a possibility of so constructing them, that such local damage shall not imperil the vessel. Much misapprehension has, however, obtained as to the arrangement, desirability, and uses of water-tight bulkheads in iron vessels. This was forcibly exemplified by the absurd legislation in respect of them, which formed a part of the Merchant Shipping Act of 1854, and which was wisely repealed in 1862. The 300th section of that Act prescribes that every steamer should be divided as nearly as possible into three parts by watertight bulkheads. Of course such a division was practically of no use whatever, as if one of the end compartments were pierced, the trim of the ship would be so much altered that, even if the water did not get to her hatchways, she could not live long in a seaway. As if still further to increase the absurdity of the whole thing, it was settled that the bulkheads need not extend to the upper deck in a vessel of more

than two decks. In nine out of ten cases of three-decked vessels, if one-third of the ship's displacement were lost, the water would certainly be higher than the top of the bulkheads and would then run into the other compartments and sink the vessel. Since that time the collision bulkhead, as it has been called, has been insisted on by the Registries, and has commended itself to shipowners generally, and many ships and their crews have been saved simply through having this provision in case of the vessel being pierced forward. A common fault is to place the collision bulkhead too far forward, but we believe that this is done less frequently now than it was a few years ago. In screw-steamers also, the idea of the water-tight compartment at the after-end has met with almost general approval, as affording a security in the frequent cases of injury to the propeller shaft which may be the cause of leakage.

A third application of the same principle is in the case of the double bottom, which has the additional advantage of affording the means for the use of water ballast. The water ballast tank has, in the past, mostly been confined to that part of the vessel forming the cargo hold, but now frequently extends the whole length of the ship.

The admiralty have constructed not only ironclads, but all war vessels in compartments, and in many cases the partitions are very numerous and the partitioned spaces very small. Most vessels of recent build have athwartship bulkheads, water tight flats, and also fore-and-aft bulkheads. War vessels are, of course, designed to encounter special risks and this extreme sub-division is specially useful to them.

There is, however, another class of vessels which have to encounter special risks, and in which we think the water-tight compartment principle might be much more usefully and easily applied than in sea-going merchant steamers. We refer to passenger steamers employed in the navigation of rivers and other smooth waters. These vessels are mostly built and fitted with a view to the conveyance of large numbers of deck passengers, the number being so large as to cause it to be altogether out of the question to attempt to provide for emergencies by life-saving apparatus. In many cases, it is true, the apparatus might be carried, but it would be useless in face of the fact that the crew of the steamer is so very small, sometimes almost infinitesimal compared with the passengers, and that the passengers, in vessels of this class, are usually especially helpless. Further, in many vessels which depend for a large part of their earnings on the conveyance of cargo, it has often been urged that water-tight partitions are a source of much inconvenience, and rather than put up with them the owners would give up the passenger trade. We think that perhaps the inconvenience has much been exaggerated, but it does certainly exist, and in reference to this we may mention a fact bearing upon the case which has recently come under our notice. The Admiralty have been enquiring into the capabilities of our large merchant steamers for being converted into useful crusiers in the event of war with a maritime power. Three hundred was the estimated number of vessels which, from their size and speed, were likely to be useful; and detailed inspection has shown that about one hundred of them fulfill the bulkhead condition, that is, are so far divided, that when in fighting trim, which, of course, is much less than their usual load draught, a shot hole in any one compartment would not be the cause of such extra immersion as to be a source of serious danger. The special point to which we refer, however, is, that many vessels otherwise suitable have failed, because of their owners objecting to having a bulkhead which cuts off the engine room from the boiler space. In the case of river passenger steamers the bulkheading of the hold need not be objected to from possible inconvenience, as regards either the stowage of cargo or easy communication between engines and boilers. These latter vessels do not usually lay themselves out for cargo, and what they do carry is in the form of miscellaneous goods which could as well, and as conveniently; go into small compartments, as into an undivided hold.

One serious drawback to compartments in seagoing vessels is the necessity for water-tight doors. There must for instance be such a door between the engine and boiler compartments, since to have them completely parted by a

bulkhead extending to the upper deck would be an intolerable nuisance, and even in supposable cases, a source of danger. When there are doors, the question is, will they act when wanted? Some of the vessels in the Royal Navy have been able to make good use of their water-tight sub-divisions, but in other cases the arrangement has altogether failed of its purpose. Our readers will remember the case of the *Agincourt*, which was run upon the Pearl Rock and got off again safely. The *Bellerophon* also sustained damage in a collision which would have been fatal to a vessel of ordinary construction. On the other hand the *Vanguard* and the German iron-clad *Grosser Kurfürst*, which was sunk by her consort in the English Channel, are instances of failure. There is first the liability of water-tight doors getting out of order so as not to act when wanted, and, secondly, the danger that they sometimes cannot be shut promptly after a collision has taken place. If, however, water-tight bulkheads were a part of the construction of smooth-water vessels, there would be no great need for doors at all, and it would certainly be safest to dispense with them. To go on deck to pass from one compartment to another, in a steamer which was never in a rough sea, would be always practicable, and in shallow vessels it could hardly be considered a very serious inconvenience.

It must further be remembered that in smooth-water steamers a much smaller number of sub-divisions would be sufficient than in the case of sea-going ships, and this for two reasons, viz.: that it is necessary for an ocean steamer to have a certain minimum free board, and that for her to be only sub-divided to such an extent as to be merely unsinkable, would be of little service to her, except she were very near a port. On the other hand, all that is necessary for a smooth-water steamer is that she should be insured against sinking: the smallest margin of buoyancy would give ample time in most cases to rescue all her passengers if not to save the vessel herself. Secondly, it it is well known that most river steamers, and more especially those which carry very large numbers of passengers, have a considerable margin of buoyancy, amounting in many instances to more than their total load displacement. For these reasons we should suppose that five bulkheads would usually be sufficient to ensure safety, even in the case of most extreme risk, that is, when the vessel is struck so near a bulkhead as to fill two adjoining compartments. This would necessitate a division between the engine and boiler rooms, and that the usual partitions at the ends of those spaces should be made securely water-tight. This only leaves a further necessity for the division of the fore and after portions of the vessel, each into two parts, certainly an inconvenience, perhaps a serious inconvenience, but nothing when compared with the resulting gain.

Before leaving this subject we must make some reference to the question of strength. It would, of course, be useless to construct a vessel unsinkable when two of her compartments are filled, and yet otherwise so weak that in such an emergency she would tear in two. In the event of any two adjoining compartments being pierced, a considerable longitudinal strain would be brought upon that part of the structure of the ship bounding and adjacent to the compartment into which the water was admitted. In most vessels floating, even in still water, the different parts of the hull are subjected to strains caused by the unequal distribution of weight and buoyancy. In a sailing ship having no cargo on board there is obviously an excess of buoyancy in the midship body, and the weight of the extreme ends is partly supported by the buoyancy amidships, thus bringing a strain upon the vessel, the tendency of which is to cause her to lose her sheer or to become *hogged*.

The extreme case of straining after a collision would be when the vessel was so struck that water was admitted to both engine and boiler rooms. Under these circumstances the weight of the engines and boilers, and of the structure of the ship near them, would be borne by the increased immersion of the ends, and it is obvious that very severe strains would be brought upon the midship body of the ship, the tendency of the strain being to alter the shape of the vessel, so that the buoyant ends should rise and the middle sink.

A NEW RULE FOR CALCULATING THE CONTENTS OF LAND SURVEYS.

By J. WOODBRIDGE DAVIS, C. E.

Written for Van Nostrand's Magazine.

Land surveying occupies but a small portion of the province of an engineer. When such work does present itself, he can meet it with numerous ready devices and rules, numerical and graphical, for determining areas circumscribed in the field.

The simplest of exact rules, hitherto presented to the world for this purpose, is the familiar method of *Double Meridian Distances.* Although this is otherwise named the *Pennsylvania Method*, it appears to have originated about a century ago in the mind of Thomas Burgh, of Ireland, of whom it is stated in preface to *Gibson's Surveying, New York*, 1834, that he received from the Irish Parliament twenty thousand pounds sterling for his discovery.

It is, however, on account of the infrequency of such work in the engineer's practice, that he often makes use of some instantly conceived rule for ascertaining areas, though it may be liable to mechanical errors, or may require much numerical labor, rather than commit to mind again the method for finding double meridian distances, although each several part of that excellent rule is as simple as arithmetic can be. With a desire to furnish a more easily remembered rule to the engineer, and at the same time to those who make land surveying a business, a method more vitally important in time-saving, the writer has endeavored, by applying to the case a principle he had already used in another branch of engineering, to substitute for the old a simpler and shorter rule, governing a process requiring, likewise, less labor, with what degree of success he leaves it to the judgment of those interested to decide.

To determine the area of a polygon surveyed, after finding the latitude and departure of each course as for ordinary method, proceed according to the following rule, which for convenience of reference shall be called C.

Rule C.

*Multiply the total latitude of each station by the sum of the departures of the two adjacent courses. The algebraic half sum of these products is the area.**

Take as an example that worked out on page 185 of *Gillespie's Land Surveying.*

The two methods are identical so far as the construction of the first seven columns of each. The work necessary to find double meridian distances for old method is not shown. No such outside work is required for the new method. To find the *total latitude* of each station, add to total latitude of preceding station the latitude of preceding course. If the latitude of last station, found in this way, be equal to the latitude of last course with reversed sign, the work is correct. However, the latitude of first station is

* This rule was used to the exclusion of the old method last Fall, at Columbia College, in both the School of Mines and the School of Arts.

Example of Old Method of Calculating Contents of Land Surveys.

Station	Bearing.	Distance.	Latitude.		Departure.		Double Mer. Distances.	Double Areas.	
			N+	S−	E+	W−		+	−
1	N 35° 00′ E	2.70	2.21	—	1.55	—	1.55	3.4255	—
2	N 83° 30′ E	1.29	.15	—	1.28	—	4.38	0.6570	—
3	S 57° 00′ E	2.22	—	1.21	1.86	—	7.52	—	9.0992
4	S 34° 15′ W	3.55	—	2.93	—	2.00	7.38	—	21.6234
5	N 56° 30′ W	3.23	1.78	—	—	2.69	2.69	4.7882	—
								2)21.8519	
							Square Chains	10.9259	

EXAMPLE OF NEW METHOD OF CALCULATING CONTENTS OF LAND SURVEYS.

Station	Bearing.	Distance.	Latitude.		Departure.		Total Latitude	Adjacent Departures	Double Areas.
			N+	S−	E+	W−			
1	N 85° 00′ E	2.70	2.21	—	1.55	—	—	—	—
2	N 83° 30′ E	1.29	.15	—	1.28	—	2.21	2.83	6.2543
3	S 57° 00′ E	2 22	—	1.21	1.86	—	2.36	3.14	7.4104
4	S 34° 15′ W	3.55	—	2.93	—	2.00	1.15	−0.14	−0.1610
5	N 56° 30′ W	3.23	1.78	—	—	2.69	−1.78	−4.69	8.3482
									2)21.8519
								Square Chains	10.9259

always zero; of last station it is always the latitude of last course with reversed sign; and of second station it is always the latitude of first course. To find the *adjacent departures*, add the departures of the two courses, one on each side of station.

The advantages of this plan of calculation over the old method, are the following:

1°. *The rule governing the operation is brief, simple and easily remembered.*

2°. *The labor of computing double meridian distances is not needed. To find each total latitude or pair of adjacent departures, used instead of double meridian distances, only the sum of, or difference between, two numbers already in the table must be found. This work is done directly in table.*

3°. *Whereas by the old method a double area product must be found for every side of the polygon, by the new method the number of such products is always one less than the number of sides.* If one or more stations have same latitude as initial station, an equal number of other products disappear. If two alternate stations have same longitude, another product disappears.

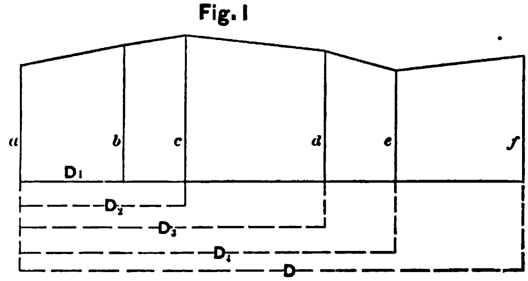
4°. *The factors, used in the method here described, for finding double area products, are almost always smaller than those used in the old method, sometimes containing fewer digits. The multiplications are, therefore, generally easier in this method than in the old.* The arithmetical aggregate of products of old method, for this example, is nearly 40. Of the present method this aggregate is 22. The average size of product by first rule is 7.9187. By second rule it is 5.5435.

Any station may be taken as the initial station, conveniently, the beginning point of survey. It is unnecessary to make a plot to determine most westerly or most easterly point, or for any other purpose connected with calculation of area. The most advantageous station, however, to which to refer the others, is that whose latitude is most nearly an average of the latitudes of all. This position insures the smallest double area factors.

The principle, upon which the foregoing rule may be made to depend, shall now be demonstrated, and a few of its applications shown. Although some of these have been already published at length by the writer, it seems proper to record them briefly, at least, in this connection, as the general rule appears to be new.

Consider any series of five trapezoids, lying consecutive with their bases on one straight line. The area of series, as found by ordinary method, the symbols of Fig. 1 used, is

Fig. I



$$\tfrac{1}{2}[D_1(a+b)+(D_2-D_1)(b+c)+(D_3-D_2)(c+d)+(D_4-D_3)(d+e)+(D-D_4)(e+f)]. \quad (1)$$

This may be transformed into

$$\tfrac{1}{2}[D_1(a-c)+D_2(b-d)+D_3(c-e)+D_4(d-f)+D(e+f)]. \quad (2)$$

From this is derived the following rule, which is true, at least, for any series of five trapezoids arranged as in Fig. 1. For convenience let this be called Rule A.

RULE A.

To find the area bounded partly by any broken line, determined from a base line by means of rectangular ordinates, and otherwise bounded by the base line and terminal ordinates.

Multiply the distance of each intermediate ordinate from first end by the difference between the two adjacent ordinates, always subtracting the one more distant from the one less distant, as measured on broken line. Also, multiply distance of last ordinate from first by the sum of last two ordinates. Divide the sum of these products by 2.

If this rule apply to a series of n trapezoids, its area is

$$\tfrac{1}{2}[D_1(a-c)+D_2(b-d)+\text{etc.}+D_{n-1}(i-k)+D_n(j+k)]. \quad (3).$$

Add another trapezoid to the latter end of this series. Its area is

$$\tfrac{1}{2}(D_{n+1}-D_n)(k+l).$$

Add this to (3). The result is the area of a series of $n+1$ trapezoids, and is expressed by the following:

$$\tfrac{1}{2}[D_1(a-c)+\text{etc.}+D_{n-1}(i-k)+D_n(j-l)+D_{n+1}(k+l)].$$

Therefore, the rule applies to a series of $n+1$ trapezoids.

If from the series of n trapezoids the last trapezoid, whose area is

$$\tfrac{1}{2}(D_n-D_{n-1})(j+k),$$

be subtracted, the area of the remaining series of $n-1$ trapezoids is the following:

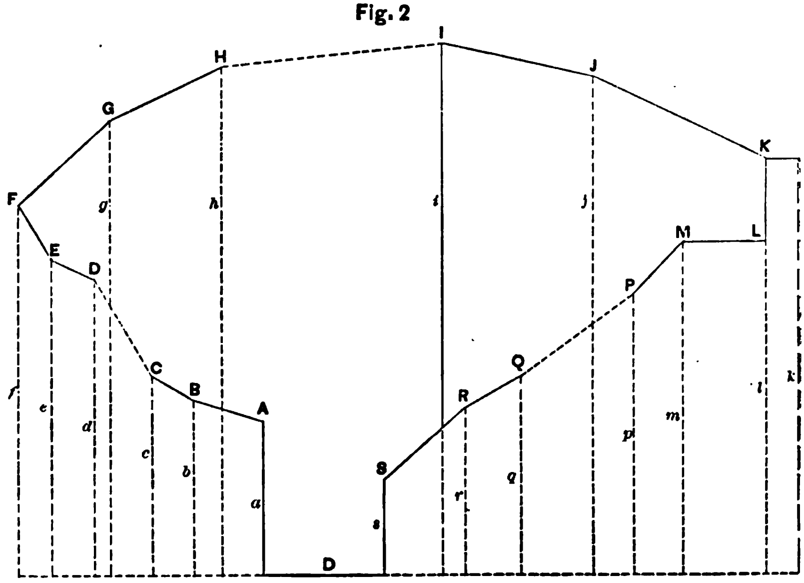
$$\tfrac{1}{2}[D_1(a-c)+D_2(b-d)+\text{etc.}+D_{n-1}(i+j)].$$

Therefore, the rule applies to a series of $n-1$ trapezoids.

It is certain that the rule applies to a series of five trapezoids. Let $n=5$. Then $n+1=6$, and $n-1=4$. Consequently, the rule applies to a series of six trapezoids, and to a series of four. For this reason it applies to series of three and seven trapezoids, and so on. Because a series containing any number of trapezoids may be found by continually adding, or subtracting, one trapezoid to, or from, a series of five, and because the application of the rule is not affected by each such operation, it follows that Rule A applies to a series consisting of any number of trapezoids.

To prove that the rule is perfectly general, whatever the *position* of trapezoids, let us consider a series in which

Fig. 2



portions of the broken line are retrogressive, or, in which some of the trapezoids are subtractive, as in Fig. 2, where the broken line is ABC . . . DEFGH . . . IJKLMP . . . QRS, and the terminal ordinates are a, s. The dotted lines between C and D, H and I, P and Q, represent omitted parts of broken line, which may be filled with any number of lines.

For convenience, let the distances from the ordinate a to the other ordinates, as b, j, be denoted by the same letters, respectively, with primes, as b', j'. Distances measured leftward from the initial ordinate, a, considered negative, the area between FGH . . . IJK, the ordinates, f, k, and the base line, is, by rule, whatever the number of trapezoids,

$$\tfrac{1}{2}[(g'-f')(f-h)+\text{etc.}+(j'-f')(i-k)+(k'-f')(j+k)].$$

This may be separated into two parts. One is

$$\tfrac{1}{2}[g'(f-h)+\text{etc.}+j'(i-k)+k'(j+k)]. \quad (4)$$

The other is, evidently, the product of $-\frac{1}{2}f'$ and the sum of all the ordinates from F to K, diminished by the product of $-\frac{1}{2}f'$ and the sum of all the same ordinates except the first two, f, g. This part is, therefore, equal to

$$-\tfrac{1}{2}f'(f+g). \quad (5)$$

The area included by FED . . . CBA and base line, between ordinates, f, a, is, by same rule, whatever the number of trapezoids,

$$\tfrac{1}{2}[e'-f')(f-d)+\text{etc.}+(b'-f')(c-a)+(-f')(b+a)],$$

which, as before, is equivalent with

$$\tfrac{1}{2}[e'(f-d)+\text{etc.}+b'(c-a)-f'(f+e)]. \quad (6)$$

The area between SRQ . . . PML and base line, terminated by ordinates, s, l, is, similarly,

$$\tfrac{1}{2}[(r'-\text{D})(s-q)+\text{etc.}+(m'-\text{D})(p-l)+(l'-\text{D})(m+l)],$$

or, $\tfrac{1}{2}[r'(s-q)+\text{etc.}+m'(p-l)+l'(m+l)-\text{D}(s+r)]. \quad (7)$

To determine the entire area as described by rule, the sum of (6) and (7) must be subtracted from the sum of (4) and (5). This performed, the expression for area can be arranged thus:

$$\tfrac{1}{2}\left\{\begin{matrix} b'(a-c)+\text{etc.}+e'(d-f)+f'(e-g) \\ +g'(f-h)+\text{etc.}+j'(i-k) \\ +k'(j-l)+l'(k-m)+m'(l-p) \\ +\text{etc.}+r'(q-s)+\text{D}(r+s) \end{matrix}\right\} \quad (8)$$

From this it is seen that the rule applies to the series of trapezoids. This example explains the intention of the phrase, *as measured on broken line*, which occurs in the rule.

The broken line can be continued from S in any direction, through any kind of complication, forming the upper sides of new trapezoids. To all cases the same rule applies; because the portions of broken line can have no direction not considered in this example, and because, if the rule apply to a series of n trapezoids, it applies also to series of $n+1$ and of $n-1$ trapezoids.

There is one case, however, which on account of its frequency in practice, and because the formula for this case is reduced, deserves a special rule. This is the case where the end of the line, as S, coincides with the beginning, as A. Then, the broken line forms the perimeter of a polygon, and the result of the rule is, the area enclosed; while, because D becomes zero, the only term, $\text{D}(r+s)$, in the formula for area, (8), which has for a factor the sum of two ordinates, vanishes.

To treat this case in the most general way, suppose B, C, D, E, F (Fig. 3) to be points anywhere situated in a plane and referred in position by rectangular co-ordinates to the point A in same plane; and suppose the points to be connected in order named by a broken line, ending again at B. Let the ordinates of the points be denoted by a, b, c, d, e, f, and the abscissæ by a', b', c', d', e', f'. Connect the origin by a straight line with any of the points, as B. Then the area of the polygon ABCDEFBA, which is equal to the area of the polygon BCD EFB, is by rule,

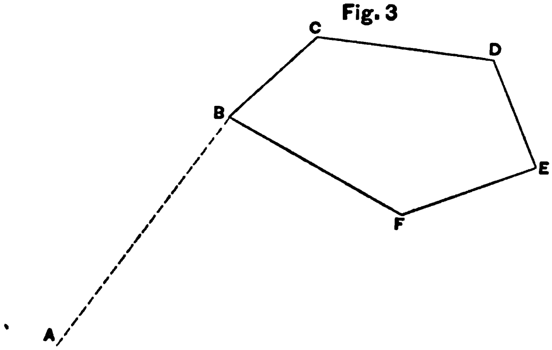
$$\tfrac{1}{2}[b'(a-c)+c'(b-d)+d'(c-e)+e'(d-f)+f'(e-b)+b'(f-a)] \quad (9)$$

$$=\tfrac{1}{2}[b'(f-c)+c'(b-d)+d'(c-e)+e'(d-b)+f'(e-b)] \quad (10)$$

$$=-\tfrac{1}{2}[b(f'-c')+c(b'-d')+d(c'-e')+e(d'-f')+f(e'-b')] \quad (11)$$

From expressions (10), (11) can be framed the following rule:

Fig. 3



RULE B.

To find the area of any polygon whose vertices are fixed by rectangular co-ordinate measurement.

Multiply the abscissa of each vertex by the difference between the ordinates of the two adjacent vertices; or, multiply the ordinate of each vertex by the difference between the abscissæ of the two adjacent vertices; always making the subtraction in the same direction around the polygon. Half the sum of these products is the area.

For the sides CD, DE, EF of polygon can be substituted any number of sides. This causes expression (9) to contain more or fewer terms like its middle terms, but does not affect the end terms. Therefore, the reduction to forms of expressions (10), (11) can still be made. Consequently, the rule is true whatever the number of sides the polygon possesses.

Since the rule for a series of trapezoids is independent of shape of broken line, the rule for areas of polygons, which is identical with the former, is independent of the shape of perimeters.

Expressions (10), (11) show that the rule is independent of position of origin of co-ordinates.

The rule is independent of direction of co-ordinate axes, because the demonstration is independent of this.

It follows that the rule is perfectly general.

If the origin be placed at a vertex, one term vanishes whichever way the rule be used. If one of the co-ordinate axes be passed through two vertices, two terms vanish, when the rule is used one way.

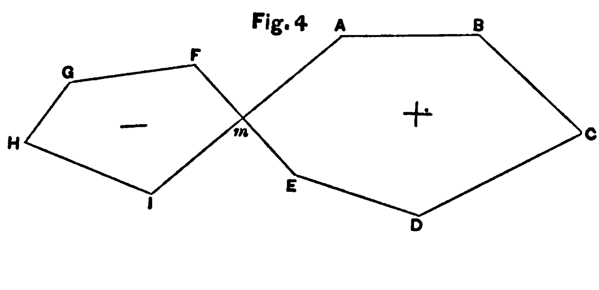
It is evident that Rule B is simply the case of Rule A when, the origin being at a vertex, the last and most difficult term vanishes; or, when the origin being not at a vertex, the last term vanishes, and of the remaining the two end terms reduce to one.

Sometimes the series of trapezoids, or the polygon, contains negative spaces. This always occurs when the boundary crosses itself. If, in Fig. 4, the broken line begin at A and follow the capital letters in alphabetical order, returning to A again, the area enclosed, as found by rule, is the area ABCDE m A, diminished by the area mIHGFm. But, if the broken line be considered to be ABCDEm IHGFmA, the area, by rule, is equal to the arithmetical sum of the areas of the two parts.

This can be explained as follows:

If a line enclose a space, the space is on one side only of the line; because, by definition, the line is a boundary. But, if the line cross itself, a portion of the line encloses new space on the other side of itself; that is, this portion not only does not enclose space of original description, but encloses space of directly opposite description. Hence, in this case, the

Fig. 4



amount of space of one description enclosed, is the amount enclosed on the proper side of the boundary less the amount enclosed on the other side of the boundary.

On the supposition that the line takes the course E*m*I, etc., the space is always on one side of the line, and is, consequently, all positive or all negative, accordingly as the line be supposed to follow the course around in the direction of the hands of a watch or in reverse direction. To accord with latter supposition, the polygon of Fig. 4 has eleven sides: to accord with first supposition, it is a polygon of nine sides.

The most general case of a series of trapezoids being considered, the series always contains negative spaces, when the broken line crosses itself, either end ordinate or the base line between end ordinates.

This case is met with in practice. A very practical example shall be given shortly. The following is an illustration from analytical geometry. The area between the axis of X and the line, $y=a-bx$, between the limits, $x=o$, $x=2a\div b$, is

$$\int_{x=o}^{x=2a\div b}(a-bx)dx=o.$$

But, arithmetically, the area is $a^2\div b$.

It is now seen that rule A, which always includes rule B, applies generally to all cases where areas are determined by dividing them into trapezoids and triangles, additive and subtractive, lying consecutive with their bases on one straight line. One general advantage of the rule is, therefore, that it reduces the solution of the most difficult cases to mere mechanical computation, saving us the labor of making plans and placing positive and negative trapezoids. Another advantage is that it makes the mechanical computation less; first, because it uses the differences between ordinates as factors, instead of sums of ordinates, and, second, because it often requires fewer terms than ordinary methods. These advantages appear more clearly, however, when illustrated by examples.

The case of such a series of trapezoids, as is shown in Fig. 1, is of frequent occurrence in all branches of engineering and in many other professions. Here rule A is invariably simpler than the ordinary method, except when the trapezoids are all of equal length. Let us apply the rule in the calculation of cubical contents of the borrow-pit, whose surface notes, the datum plane being fixed at 20 feet below elevation of station zero on base line, are the following:

STA.							
0	$\frac{20}{0}$	$\frac{15}{10}$	$\frac{10}{30}$	$\frac{15}{50}$		$\frac{25}{75}$	$\frac{20}{100}$
1	$\frac{15}{0}$	$\frac{10}{15}$	$\frac{10}{30}$	$\frac{15}{45}$		$\frac{20}{70}$	$\frac{20}{100}$
2	$\frac{10}{0}$	$\frac{10}{15}$	$\frac{12}{40}$		$\frac{20}{60}$		$\frac{25}{100}$
3	$\frac{15}{0}$	$\frac{18}{20}$	$\frac{20}{35}$			$\frac{26}{75}$	$\frac{25}{100}$
4	$\frac{20}{0}$	$\frac{23}{20}$	$\frac{27}{30}$			$\frac{20}{80}$	$\frac{27}{100}$

25′ | 25′ | 25′ | 25′

The line on left is base line. The cross-sections are taken at right angles to this, 25 feet apart, as indicated. Each pair of numbers represent the elevation and distance from base line, of a point.

To estimate the mass of earth between this surface and datum plane, arrange distances in one column, as shown, and the differences of adjacent elevations in

TABLE OF OPERATIONS.

Sta.	Dist.	$h-h'$	Prods.	Pris. Cor.
0	10	10	50.	
	30	0	0	
	50	−15	− 375.	
	75	− 5	− 187 5	
	100	45	2250.	25
1	15	5	75.	0
	30	− 5	− 150.	25
	45	−10	− 450.	0
	70	− 5	− 350.	0
	100	40	4000.	0
2	15	− 2	− 30.	50
	40	−10	− 400.	− 30
For Sta. 1. $\begin{Bmatrix}60\times-8\\60\times-5\end{Bmatrix}$	60	−13	− 780.	0
	100	45	4500.	0
3	20	− 5	− 100.	15
	35	− 8	− 280.	10
	75	− 5	− 375.	−120
	100	51	5100.	0
4	20	− 7	− 70.	0
	30	− 8	− 120.	0
	80	0	0	− 25
	100	57	2850.	0
			15157.5	− 50

Pris. Cor.÷6= −8.33

4)15149.17

9)378729.25

6)42081.03

7013.5 cu. yds.

the next, except for each last distance, 100, opposite which place the sum of last two elevations. From these calculate the column of products, using half the products of first and last stations. Multiply the sum by the distance between stations, 25 feet, and divide by 54.

As this example is copied directly from a book on earthwork calculation—*Formulæ for R.R. Earthwork, Quantities and Average Haul*—it contains a little more matter than has been explained, namely, that exhibited in last column, and the equation, Pris. Cor. $\div 6 = -8.33$, which should be omitted here.

The advantage of this plan over that usually employed for this work is apparent at sight, and time shall not be spent here to analyze it. Especially to be noticed, however, is the fact that the factors of this method are more easily handled than those of other methods, since the ones in column of differences are nearly all composed of a single digit each (two in this example are zero), thus making unnecessary the extra work of separate multiplication, while the larger ones, opposite last distances, require for the operation, in present example, merely a shifting of decimal point. Where the ground is more level than this—the commoner case—this feature is more marked, and is independent of depth of pit.

Although it comes not within the scope of this article to explain the nature of the prismoidal correction, it may be mentioned, as an indirect advantage of this method, that the correction can be applied with great ease, furnishing a result exactly the same as if to each separate solid had been applied the prismoidal formula with the immense labor which has driven calculators, generally, from exact work. To do this, simply find difference between each distance belonging to each station, and the corresponding distance belonging to the next, and multiply this by the difference, taken inversely, between the two numbers opposite in column headed $h-h'$; divide sum of these products by six and add to column of *products*. Thus,

$(15-10)(10-5)=25;\ (30-30)[0-(-5)]=0;$ etc.

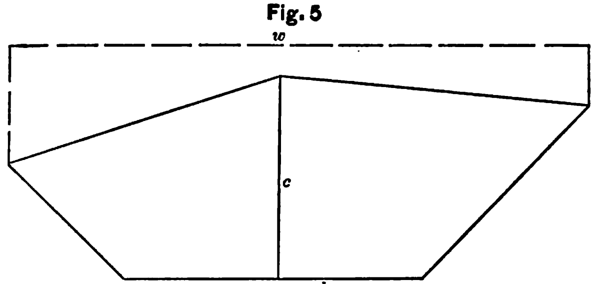
In this example, the new rule for summing trapezoids is applied in one direction only. If the cross sections were unequally distant from each other, the same rule could be advantageously applied in the direction of base line, too, as if the cross sections themselves were merely ordinates.

Because cross-sections of earthwork are generally calculated by means of trapezoids, the measurements in the field are commonly made from point to point, not from base line to each point, as shown in borrow-pit notes above. The latter method is, however, more convenient and less liable to errors. Often it is used by the tape-man to save time and trouble, even when he intends to use ordinary method of calculation. He stands on base line and pays out the tape to rodman, noting or announcing each distance, until the length of his tape or the change of level compels him to take a new position. With a hundred feet tape, in measuring above borrow-pit, he need never leave base line.

Rule B is useful in earthwork calculations. Let the following illustrate. It is well known, since the formula occurs in several late works, that the area of the ordinary "three-level" cross-section of railroad cut or fill, such as shown in Fig. 5, is

$$\tfrac{1}{2}w(c+sb)-sb^2, \qquad (12)$$

Fig. 5

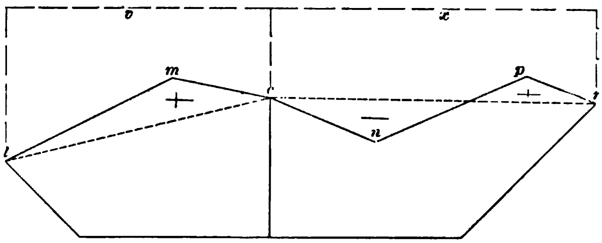


when w is the width of section between slope-stakes, c is the center cut or fill, b, the half-width of roadway, and s, the ratio of slope, equivalent to *vertical* ÷

horizontal. But no algebraic formula, except the one to be derived here, has ever been given for the case of a cross-section, such as represented in Fig. 6, whose surface line has more than one change of direction. Merely the ordinary rule, directing to divide the area into trapezoids and triangles, additive and subtractive, has been employed. But a formula for all such cases can be very easily constructed by means of rule B.

Thus, let m, n, p represent the dis-

Fig. 6.



tances of the "breaks" from grade, m', n', p', the distances of same points from center line, r, l, the elevations of slope-stakes referred to grade, x, v, the side-widths, and c, b, s, as before. Draw lines, cr, cl. These represent surface line of a three level section, having, otherwise, same measurements as the irregular section. If, now, proceeding in the direction of the hands of a watch, with origin of measurements at grade in middle point of roadway, and considering the surface line of irregular section to be *above* the lines, lc, cr, we apply the rule to the polygon, $c\ n\ p\ r\ c\ l\ m\ c$, the result will be its area *above* the surface of three-level section, or the algebraic sum of all the positive and negative portions. If this be added to area of three-level section, the sum is the required area. Therefore, to expression (12) add

$$\tfrac{1}{2}[n'(c-p)+p'(n-r)+x(r-c) \\ -v(c-m)-m'(l-c)]. \quad (13)$$

This sum is, since $w=v+x$,

$$\tfrac{1}{2}[v(m+sb)+x(p+sb)+m'(c-l)+ \\ n'(c-p)+p'(n-r)]-sb^2. \quad (14).$$

From this comes the following

Rule to find the area of an irregular cross section:—

Multiply each side-width by the height of last break on that side added to $s\ b$. To the sum add the product of the distance of each break from center and the difference in elevation of the breaks next one on each side, always subtracting the one farther from center from the one nearer. Divide by 2, and subtract sb^2.

No terms in (13), whose coefficients are distances from center to breaks, are disturbed by the addition. Therefore, however many breaks there be, expression (14) maintains its shape. Hence the rule is perfectly general.

The advantages of this rule are seen in the following statements:

The number of terms in parenthesis in expression (14) is always equal to the number of trapezoids between surface line and grade, into which it is necessary to divide the area, by ordinary method of calculation; but the labor involved is less.

Instead of computing the areas of the two outside triangles and subtracting them, as by common method, the calculator need only deduct the quantity sb^2, which is the same for all the sections.

No diagrams are required to aid in the work. The field notes, kept according to this plan, are sufficient.

Rule A applies, more or less approximately, to a series of solids. The following illustrates its use.

Cross-sections on a railroad line are taken, as much as possible, at a constant distance apart, generally 100 feet. But at intervals these 100 feet solids require further subdivision. Let Fig. 1 represent a side view of one of these solids. D is its length, 100 feet, D_1, D_2, etc., are the distances of intermediate stations from first end, and a, b, c, etc., are the areas of cross-sections. Expression (2) now represents, approximately, the content of the whole 100 feet solid. This content is identical with that obtained by common rule, which directs to multiply sum of end areas by half distance between for content of each component solid.

The advantages of formula (2) are

here, as before, *first*, that differences between cross-sectional areas are much smaller factors than sums of these; *second*, that in calculating together the areas of each pair of cross-sections, by means of expressions (12), (14), the common term, $-sb^2$ vanishes, and one or more terms in each contain a common factor sb; and, *third*, that because one term of (2) has for a coefficient D, the full length of solid, this term may be used alone to represent the content of whole solid, which may then be calculated in one series with all the other 100 feet solids, and, afterwards, the remaining terms may be used as a correction.

The last is the chief advantage. Its value cannot, however, be appreciated without an example. It enables us to construct a single formula for the content of any possible entire bank or cutting, whatever the variety in lengths of component solids, and however irregular the cross-sections may be. This combines the advantages of rule A in both directions, and eliminates every constant term and factor. As in case of borrow-pits, the prismoidal correction can be easily applied.

Rule A has many other applications in earthwork calculations. In fact, it was devised for that class of computations; and this is the writer's excuse for exhibiting three of its examples in this article. Let us, however, now turn to a new field.

If for words, *abscissa, ordinate, vertex*, in rule B, be substituted *longitude, latitude, station*, that rule applies technically to the case of land surveys.

Frequently, perhaps, the rule in this shape will be serviceable in surveying. The writer was recently engaged on the survey of a large tract of coal land, containing about one-hundred-and-fifty square miles, which belonged mostly to one company, but was owned here and there by individual settlers. The latitudes and longitudes of all the corners were determined by lines run in all directions through the woods, and were tabulated. Thus, any parcel of land could be easily plotted, when wanted, in any of the geological or other maps; while merely a glance at the table served to indicate its position in the district When the area of such a piece was required, it was only necessary to proceed as in the following example:

Station.	Total latitude.	Total longitude.	Difference between alternate longitudes.	Double areas.
a	7087	94851	+1201	8,511,487
b	10020	97403	—6518	—65,310,360
c	8181	101369	—5752	—47,057,112
d	5012	103155	+2765	13,858,180
e	2873	98604	+8304	23,857,392
			Area in sq. ft. =	66,140,413

If the latitudes were larger than longitudes, differences of latitudes should be made the factors with total longitudes.

When the determination of area is the only object, we may place the origin in most advantageous position, namely, at a station, thus getting rid of one term; and, instead of calculating the longitudes of all stations and finding differences between alternate ones, we may obtain the same result in simpler manner by adding the departures of each pair of adjacent courses. Modified in this way, the operation becomes governed by rule C, whose advantages have been already discussed.

Determinations of areas from diagrams, instead of numerical data, are frequently used. The common case is the polygon. Here rule B applies. Of course, to reduce work as much as possible, one axis should be passed through two vertices—conveniently, through one side. The importance of this case, warrants us in framing another rule by alteration of rule B.

Rule D.

To determine the area of a polygon in diagram:—

Perpendicular to one side prolonged, as base, draw ordinates from the vertices. Multiply each ordinate by the distance that the next following ordinate, as on perimeter, is to the right of next preceding ordinate. The algebraic half sum of these products is the area.

This requires one less distance to be measured, and contains one less term than the ordinary method of division into trapezoids. It is simpler also than the method of division into interior triangles, which is, perhaps, most commonly used. As an instance, consider the pentagon. By rule D, lay a straight edge on one side, and, sliding a triangle along this, draw three perpendiculars. By method of triangles, draw

two diagonals and three perpendiculars, all in different directions. Each method requires the same number of terms.

The writer has noticed several isolated instances wherein the formula for area of polygons has been demonstrated and used for special cases. The common rule for area of triangle is the case of formula (11) where the polygon has three vertices, and one axis passes through two. For the case of a triangle, one one of whose vertices is at origin, Julius Weisbach derived in section 112 of *Theoretical Mechanics*, an expression similar to formula (11). This enabled him to frame a very concise method for determining centers of gravity of polygons. Dr. Weisbach, in preface to first edition, makes particular mention of this section, together with a few others, as containing new matter peculiar to himself. In Gillespie's *Roads and Railroads*, p. 366, exists a formula for area of trapezium all of whose vertices are determined by measurements parallel with, and perpendicular to, one side. This formula contains two terms only, and is the case of formula (11), or more strictly of rule D, when one axis passes through two vertices. This formula was devised by E. M. Jenkins, C. E., now Registrar of Union College.

We are indebted to George Salmon for a demonstration of formulæ (10) (11) for all polygons. This occurs on some of the earlier pages of his *Conic Sections*.

The following is remarkable. Let us apply rule A, or its equivalent, formula (2), to a series of trapezoids of infinitesimal lengths—the differentials of any plane area. The trapezoids may vary in length to any extent. Thus, the abscissa may vary as any function of x, as u. Then the lengths of trapezoids are, according to position, the variable values of du. Let the corresponding ordinate vary as any other function of x, as y. Now, all the terms of formula (2), except the last, are represented by

$$\tfrac{1}{2}\int u[(y-dy)-(y+dy)]=-\int udy. \quad (15)$$

If we ascribe limits to this, the expression will represent the value of those terms for a definite series. The last term is, now,

$$\tfrac{1}{2}u[(y-dy)+y]=uy. \quad (16)$$

To this the same limits should be ascribed. The area is the sum of (15), (16).

But it is also $\int ydu$. Hence we obtain

$$\int ydu=uy-\int udy,$$

the well known *formula for integration by parts*, upon which so many general and special rules for integration depend.

A CONTRIBUTION TO THE HISTORY OF ELECTRIC LIGHTING.

By W. MATTIEU WILLIAMS, F.C.S., F.R.A.S.

From "The Journal of Science."

As the subject of lighting by electricity is occupying so much public attention, and the merits of various inventors and inventions are so keenly discussed, the following facts may have some historical interest in connection with it.

In October, 1845, I was consulted by some American gentlemen concerning the construction of a large voltaic battery for experimenting upon an invention, afterwards described and published in the specification of "King's Patent Electric Light" (Letters Patent granted for Scotland, November 26, 1845; enrolled March 25, 1846).

Mr. King was not the inventor, but he and Mr. Door supplied capital, and Mr. Snyder also held a share, which was afterwards transferred to myself. The inventor was Mr. Starr, a young man about 25 years of age, and one of the ablest experimental investigators with whom I have ever had the privilege of near acquaintance.

He had been working for some years on the subject, commencing with the ordinary arc between charcoal points. His first efforts were directed to maintaining constancy, and he showed me, in January of 1846, an arrangement by

which he succeeded in effecting an automatic renewal of contact by means of an electro-magnet, the armature of which received the electric flow, when the arc was broken, and which thus magnetized brought the carbons together and then allowed them to be withdrawn to their required separation, when the flow returned. This device was almost identical with that subsequently re-invented and patented by Mr. Staite (quite independently I believe), and which, with modifications, has since been rather extensively used.

Although successful so far, he was not satisfied. He reasoned on the subject, and concluded that the electric spark between metals, the electric arc between the carbons, and other luminous electric phenomena are secondary efforts due to the heating and illumination of electric carriers; that the electric spark of the conductors of ordinary electrical machines is simply a transfer of incandescent particles of metal, which effect a kind of electric convection, known as the disruptive discharge; and that the more brilliant arc between the carbon points is simply due to the use of a substance which breaks up more readily, and gives a longer, broader, and more continuous stream of incandescent convection particles.

This is now readily accepted, but at that time was only dawning upon the understanding of electricians. I am satisfied that Mr. Starr worked out the principle quite originally. He therefore concluded that, the light being due to solid particles heated by electric disturbance, it would be more advantageous—as regards steadiness, economy, and simplicity—to place in the current a continuous solid barrier, which should present sufficient resistance to its passage to become bodily incandescent without disruption.

This was the essence of the invention specified in King's Patent as "a communication from abroad," which claims the use of continuous metallic and carbon conductors, intensely heated by the passage of a current of electricity, for the purpose of illumination.

The metal selected was platinum, which, as the specification states, "though not so fusible as iridium, has but little affinity for oxygen, and offers a great resistance to the passage of the current." The form of thin sheets known by the name of leaf-platinum is described as preferable. These to be rolled between sheets of copper in order to secure uniformity, and to be carefully cut in strips of equal width, and with a clean edge, in order that one part may not be fused before the other parts have obtained a sufficiently high temperature to produce a brilliant light. This strip to be suspended between forceps.

I need not describe the arrangement for regulating the distance between the forceps, for directing the current, &c., as we soon learned that this part of the invention was of no practical value, on account of the narrow margin between efficient incandescence and the fusion of the platinum. The experiments with the large battery that I made—consisting of 100 Daniell cells, with 2 square feet of working surface of each element in each cell, and the copper-plates about $\frac{3}{4}$ of an inch distant from the zinc—satisfied all concerned that neither platinum nor any available alloy of platinum and iridium could be relied upon; especially when the grand idea of subdividing the light by interposing several platinum strips in the same circuit, and working with a proportionally high power, was carried out.

This drove Mr. Starr to rely upon the second part of the specification, viz., that of using a small stick of carbon made incandescent in a Torricellian vacuum. He commenced with plumbago, and, after trying many other forms of carbon, found that which lines gas-retorts that have been long in use was the best.

The carbon stick of square section, about one-tenth of an inch thick and half an inch working length, was held vertically, by metallic forceps at each end, in a barometer tube, the upper part of which, containing the carbon, was enlarged to a sort of oblong bulb. A thick platinum wire from the upper forceps was sealed into the top of the tube and projected beyond; a similar wire passed downwards from the lower forceps, and dipped into the mercury of the tube, which was so long that when arranged as a barometer the enlarged end containing the carbon was vacuous.

Considerable difficulty was at first encountered in supporting this fragile

stick. Metallic supports were not available, on account of their expansion; and, finally, little cylinders of porcelain were used, one on each side of the carbon stick, and about three-eighths of an inch distant.

By connecting the mercury cup with one terminal of the battery, and the upper platinum-wire with the other, a brilliant and perfectly steady light was produced, not so intense as the ordinary disruption arc between carbons, but equally if not more effective, on account of the magnitude of brilliant radiating surface.

Some curious phenomena accompanied this illumination of the carbon. The mercury column fell to about half its barometric height, and presently the glass opposite the carbon stick became slightly dimmed by the deposition of a thin film of sooty deposit.

At first, the depression of the mercury was attributed to the formation of mercurial vapor, and is described accordingly in the specification; but further observation refuted this theory, for no return of the mercury took place when the tube was cooled. The depression was permanent. The formation of vaporous carbon was suggested by one of the capitalists; but neither Mr. Starr nor myself was satisfied with this, nor with any other surmise we were able to make during Mr. Starr's lifetime, nor up to the period of final abandonment of the enterprise.

When this occurred, the remaining apparatus was assigned to me, and I retained possession of the finally arranged tube and carbon for many years, and have shown it in action worked by a small Grove's battery in the Town Hall of Birmingham, and many times to my pupils at the Birmingham and Midland Institute.

These exhibitions suggested an explanation of the mysterious gaseous matter, which I believe to be the correct one, and also of the carbon deposit. It is this:—That the carbon contains occluded oxygen; that when the carbon is heated, some of this oxygen combines with the carbon, forming carbonic oxide and carbonic acid, and a little smoke. I proved the presence of carbonic acid by the usual tests, but did not quantitatively determine its proportion of the total atmosphere.

If I were fitting up another tube on this principle, I should wash it with a strong solution of caustic potash before filling with mercury, and allow some of the potash solution to float on the mercury surface, by filling the tube while the glass remained moistened with the solution. My object would be to get rid of the carbonic acid as soon as formed, as the observations I have made, lead me to believe that—when the carbonic stick is incandescent in an atmosphere of carbonic acid or carbonic oxide—a certain degree of dissociation and re-combination is continually occurring, which weakens and would ultimately break up the carbon stick, and increases the sooty deposit.

The large battery above described was arranged for intensity, but even then it was found that the quantity (I use the old-fashioned terms) of electricity was excessive, and that it worked more advantageously when the cells were but partially filled with acid and sulphate. A larger stick of carbon might have been used with the whole surface in full action.

After working the battery in various ways, and duly considering the merits of the other forms of battery then in use, Mr. Starr was driven to the conclusion that for the purposes of practical illumination the voltaic battery was a hopeless source of power, and that magneto-electric machinery driven by steam-power must be used. I fully concurred with him in this conclusion, so did Mr. King, Mr. Dorr, and all concerned.

Mr. Starr then set to work to devise a suitable dynamo-electric machine, and, following his usual course of starting from first principles, concluded that all the armatures hitherto constructed were defective in one fundamental element of their arrangement. The thick copper-wire surrounding the soft iron core necessarily follows a spiral course, like that of a coarse screw thread; but the electric current or lines of force which it is designed to pick up and carry circulate at right angles to the axis of the core, and extend to some distance beyond its surface. The problem thus presented is to wind around the soft iron a conductor that shall be broad enough to grasp a

large proportion of this outspread force, and yet shall follow its course as nearly as possible by standing out at right angles to the axis of the armature. This he proposed to effect by using a core of square section, and winding round it a broad ribbon of sheet copper, insulated on both sides by cementing on its surfaces a layer of silk ribbon. This armature to be laid with one edge against one side of the core, and carried on thus to the angle; then turned over so that its opposite edge should be presented to the next side of the core; this side to be followed in like manner, the ribbon similarly turned again at the next corner, and so on till the core becomes fully enclosed or armed with the continuous ribbon, which would thus encircle the core with its edges outwards, and nearly at right angles to the axis, in spite of its width, which might be increased to any extent found by experiment to be desirable.

At this stage my direct co-operation and confidential communication with Mr. Starr ceased, as I remained in London while he went to Birmingham, in order to get his machinery constructed, and to apply it at the works of Messrs. Elkington, who had then recently introduced the principle of dynamo-electric motive-power, electro-plating, &c., and were, I believe, using Woolrich's apparatus, the patent for which was dated August 1, 1842, and enrolled February 1, 1843.

I am unable to state the results of his efforts in Birmingham. I only heard the murmurs of the capitalists, who loudly complained of expenditure without results. They had dreamed the same dream that Mr. Edison has recently re-dreamed, and has told the world so loudly. They supposed, that the mechanically-excited current might be carried along great lengths of wire, and the carbons interposed where required, and that the same electricity would flow on and do the duty of illumination over and over again, as a river may fall over a succession of weirs and turn water-wheels at each. Mr. Starr knew better; his scepticism was misinterpreted; he was taunted with failure and non-fulfillment of the anticipations he had raised, and with the fruitless expenditure of large sums of other people's money. He was high-minded, honorable, and a very sensitive man, suffering already from overworked brain before he went to Birmingham. There he worked again still harder, with further vexation and disappointment, until one morning he was found dead in his bed. Having, during my short acquaintance with him, enjoyed his full confidence in reference to all his investigations, both completed and incomplete, I have no hesitation in affirming that his early death cut short the career of one, who, otherwise, would have largely contributed to the progress of experimental science, and have done honor to his country. His martyrdom, for such it was, taught me an useful lesson I then much needed, viz., to abstain from entering upon a costly series of physical investigations without being well assured of the means of completing them, and, above all, of being able to afford to fail.

There are many others who sorely need to be impressed with the same lesson, especially at this moment and in connection with this subject.

The warning is the most applicable to those who are now misled by a plausible but false analogy. They look at the progress made in other things, the mighty achievements of modern Science, and therefore infer that the electric light —even though unsuccessful hitherto— may be improved up to practical success, as other things have been. A great falacy is hidden here. As a matter of fact, the progress made in electric lighting since Mr. Starr's death, thirty-one years ago, has been very small indeed. As regards the lamp itself, no progress whatever has been made. I am satisfied that Starr's continuous carbon stick, properly managed in a true vacuum, or an atmosphere free from oxygen, carbonic oxide, carbonic acid, or other oxygen compound, is the best that has yet been placed before the public for all purposes where exceptionally intense illumination (as in lighthouses) is not demanded. It is the steadiest, the cheapest, and least glaring in proportion to the amount of light it radiates. It has not been "pushed" like other devices, simply because it is nobody's exclusive property.

Comparing electric with gas lighting, the hopeful believer in progressive improvement appears to forget that gas making and gas lighting are as susceptible of further improvement as electric

lighting, and that, as a matter of fact, its practical progress during the last forty years is incomparably greater than that of the electric light. I refer more particularly to the practical and crucial question of economy. The by-products, the ammoniacal salts, the liquid hydrocarbons, and their derivatives, have been developed into so many useful forms by the achievements of modern chemistry, that these, with the coke, are of sufficient value to cover the whole cost of manufacture, and leave the gas itself as a volatile residuum that costs nothing. It would actually and practically cost nothing, and might be profitably delivered to the burners of gas consumers (of far better quality than now supplied in London) at one shilling per thousand cubic feet, if gas making were conducted on sound commercial principles,—that is, if it were not a corporate monopoly, and were subject to the wholesome stimulating influence of free competition and private enterprise. As it is, our gas and the price we pay for it are absurdities, and all calculations respecting the comparative cost of new methods of illumination should be based not on what we *do* pay per candle-power of gas-light, but what we *ought* to pay and *should* pay if the gas companies were subjected to desirable competition, or visited with the national confiscation I consider they deserve.

Having had considerable practical experience in the commercial distillation of coal for the sake of its liquid and solid hydrocarbons, I speak thus plainly and with full confidence.

There is yet another consideration, and one of vital importance, to be taken into account, viz., that—whether we use the electric light derived from a dynamo-electric source, or coal-gas—our primary source of illuminating power is coal, or rather the chemical energy derivable from the combination of its hydrogen and carbon with oxygen. Now this chemical energy is a limited quantity, and the progress of Science can no more increase this quantity than it can make a ton of coal weigh 21 cwts. by increasing the quantity of its gravitating energy.

The demonstrable limit of scientific possibilities is the economical application of this limited store of energy, by converting it into the demanded form of force without waste. The more indirect and roundabout the method of application, the greater must be the loss of power in the course of its transfer and conversion. In heating the boiler that sets the dynamo-electric machine to work, about one-half the energy of the coal is wasted, even with the best constructed furnaces. This merely as regards the quantity of water evaporated.

In converting the heat force into mechanical power—raising the piston, &c., of the steam-engine—this working half is again seriously reduced. In further converting this residuum of mechanical power into electrical energy, a further and considerable loss is suffered in originating and sustaining the motion of the dynamo-electric machine in the dissipation of the electric energy that the armature cannot pick up, and in overcoming the electrical resistances to its transfer.

I am unable to state the amount of this loss in trustworthy figures, but should be very much surprised to learn that, with the best arrangements now known, more than one-tenth of the original energy of the coal is made practically available. This small illuminating residuum may, and doubtless will, be increased by the progress of practical improvement; but, from the necessary nature of the problem, the power available for illumination at the end of the series must always be but a small portion of that employed at the beginning.

In burning the gas derived from coal, we obtain its illuminating power *directly*, and if we burn it properly we obtain nearly all. The coke residuum is also directly used as a source of heat. The chief waste of the original energy in the gas-works is represented by that portion of the coke that is burned under the retorts, and in obtaining the relatively small amount of steam-power demanded in the works. These are far more than paid for by the value of the liquid hydrocarbons and the ammonia salts, when they are properly utilized.

In concluding my narrative I may add, that, after Mr. Starr's death, the patentees offered to engage me on certain terms to carry on his work. I declined this, simply because I had seen enough

to convince me of the impossibility of any success at all corresponding to their anticipations. During the intervening thirty years, I have abstained from further meddling with the electric light, because all that I had seen then, and have heard of since, has convinced me that—although as a scientific achievement the electric light is a splendid success—its practical application to all purposes where cost is a matter of serious consideration is a complete and hopeless failure, and must of necessity continue to be so.

Whoever can afford to pay some shillings per hour for a single splendid light of solar completeness can have it without difficulty, but not so where the cost in pence per hour per burner have to be counted.

I should add that before the publication of King's specification, Mr. (now Sir William) Grove proposed the use of a helix or coil of platinum, made incandescent by electricity, as a light to be used for certain purposes. This was shown at the Royal Society on or about December 1st, 1845.

AN ULTRA-GASEOUS STATE OF MATTER.

From "The English Mechanic."

Probably the most remarkable contribution to modern science that the present year has witnessed is the paper "On the Illumination of Lines of Molecular Pressure, and the Trajectory of Molecules," by Mr. W. Crookes, F.R.S., read at the meeting of the Royal Society, on Dec. 5. It is not unknown to our readers that Mr. Crookes has been engaged for many years in the study of molecular physics, the radiometer being one of the results of that study, and, as it has turned out, a means to more important discoveries. Last year Pictet and Cailletet solidified what were known as the permanent gases, and now Mr. Crookes has demonstrated the fact that, under certain conditions, gases may become so far changed, both in physical constitution and properties, as to form a fourth state of matter. At least the distinguished assembly before whom he performed his experiments can have no doubt that, just as below the gaseous state there is the liquid and the solid, so above the liquid there is the gaseous and the ultra-gaseous, or ethereal. Mr. Crookes has not arrived at this important discovery *per saltum*, and he was not surprised when he found means of demonstrating the truth of an hypothesis he has more than once shadowed out in papers on the radiometer. In his latest paper he commences by referring to his examination of the dark space which appears round the negative pole of an ordinary vacuum tube when a spark from an induction coil is passed through. This space has been examined many times, under differing conditions; and Mr. Crookes has been able to arrive at several propositions. Thus, the setting up, by electrical means, of an intense molecular vibration in a disc of metal excites a molecular disturbance, which affects the surface of the disc and the surrounding gas. With a dense gas the disturbance extends a short distance only from the metal; but as rarefaction continues, the layer of molecular disturbance increases in thickness. In air, at a pressure of .078 mm., it extends for at least 18 mm. from the surface of the disc, and forms an oblate spheroid around it. The diameter of this dark space varies with the degree of exhaustion, the kind of gas, the temperature of the negative pole, and, to a slight extent, with the intensity of the spark. It is greatest in hydrogen, and least in carbonic acid, as compared with air under equal degrees of exhaustion. The shape and size of this dark space do not vary with the distance separating the poles, nor—except slightly—with alteration of battery power, or intensity of spark. When the power is great, the brilliancy of the other parts of the tube overpowers the dark space, rendering it difficult of observation. To determine whether this visible layer of molecular disturbance is identical with the invisible layer of molecular pressure or stress, Mr. Crookes has made many ex-

periments, one of which is accomplished by the aid of the electrical radiometer. An ordinary radiometer, with aluminium discs (coated on one side with a film of mica) for vanes is constructed in such a manner that an electrical current can be passed through the fly. Instead of a glass cup the fly is supported on a hard steel cup, and the needle-point on which it works is connected, by means of a wire, with a platinum terminal sealed into the glass. At the top of the bulb a second terminal is sealed in, and the radiometer can therefore be connected to an induction coil in such a manner that the movable fly becomes the negative pole. When connected with the coil a halo of velvety violet light is seen on the metallic side of the vanes, the mica side remaining dark. As the pressure increases a dark space begins to separate the violet halo from the metal, and at a pressure of .5mm. it extends to the glass, and positive rotation commences. On increasing the exhaustion the dark space widens out, and appears to flatten itself against the glass, the rotation becoming very rapid. Somewhat similar effects are witnessed when aluminium cups are used instead of discs for the vanes, the dark space retaining the shape of the cup almost exactly. The bright margin of the dark space becomes concentrated at the concave side of the cup to a luminous focus, and widens out at the convex side, touching the glass as the exhaustion is increased, when rotation commences, and becomes very rapid as the dark space further increases in size. This convergence of the lines of force to a focus attracted special attention, and another instrument, having the cup-shaped negative pole fixed, was devised for the examination of the phenomena, one of which was of an entirely novel and highly interesting nature. At very high exhaustions (those known as "Crookes' vacua," measured by a few millionths of an atmosphere) the dark space becomes so large that it fills the tube. The dark violet focus is still visible to the careful observer; but the rays diverging from this focus produce upon the part of the glass where they fall a spot of greenish-yellow light. If now a more perfect vacuum is obtained, the whole bulb becomes illuminated with a beautiful phosphorescent light, which is greenish-yellow because of the peculiar composition of the soft German glass used in the experiments. Other qualities of glass give other colors, the phosphorescence being due to the fact that the gaseous molecules are practically unimpeded in their movements, and impinge upon the glass with sufficient velocity to produce light, which is distinguished from the ordinary light of vacuum tubes by the following characteristics: The green focus cannot be seen in the space of the tube, but only where the projected beam strikes the glass. The position of the positive pole makes scarcely any difference to the direction and intensity of the lines of force producing the green light. The spectrum is a continuous one, most of the red and the higher blue rays being absent, while the spectrum of the light observed in the tube at lower exhaustions is characteristic of the residual gas. No difference can be detected in the green light by spectrum examination, whether the residual gas be nitrogen, hydrogen, or carbonic acid. The green phosphorescence commences, besides, at a different degree of exhaustion in different gases. The viscosity of a gas is almost as persistent a characteristic of its individuality as its spectrum; but Mr. Crookes finds that when the spectral and other characteristics of the gas begin to disappear, the viscosity also commences to decline, and at an exhaustion at which the green phosphorescence is most brilliant the viscosity has sunk to an insignificant amount. The rays exciting green phosphorescence will not turn a corner in the slightest degree, but radiate from the negative pole in straight lines, casting strong and sharply-defined shadows from objects which happen to be in their path. On the contrary, the ordinary luminescene of vacuum tubes will travel hither and thither along any number of curves and angles. By means of beautifully constructed specimens of ingenious apparatus Mr. Crookes exhibited the projection of molecular shadows, and demonstrated that they were molecular, not optical, although rendered apparent by optical appliances. From a consideration of the results of his experiments he advances the theory that the induction spark illuminates the lines of molecular pressure caused by the electrical excitement of the negative pole, and he gives

the following explanation: "The thickness of the dark space is the measure of the mean length of the path between successive collisions of the molecules. The extra velocity with which the molecules rebound from the excited negative pole keeps back the more slowly moving molecules which are advancing towards the pole. The fight occurs at the boundary of the dark space where the luminous margin bears witness to the energy of the collisions of the molecules. When the exhaustion is sufficiently high for the mean length of path between successive collisions to be greater than the distance between the electrode and the glass, the swiftly moving rebounding molecules spend their force, in part or in whole, on the sides of the vessel, and the production of light is the consequence of this sudden arrest of velocity."

We have not space to give an account, even in outline, of all the experiments and all the phenomena which Mr. Crookes describes in his paper, nor is it necessary, as it is sure to find its way into the hands of all students of physics. The theory propounded at the conclusion is, however, as we have said, one of the most remarkable discoveries of modern times, involving as it does the demonstration of a fourth state of matter. The modern idea of the gaseous state—the kinetic theory of gases—is that a given space contains millions of millions of molecules in rapid motion in all directions, each having millions of encounters in a second. In such a supposition the length of the mean free path of the molecules is exceedingly small as compared with the dimensions of the vessel, and the conditions are such that the constant collisions, which are the essential element of the gaseous state, can occur. But, under high degrees of exhaustion the free path is made so long that the number of collisions in a given time is small compared to the number of misses, and the average molecule being allowed to obey its own motions without interference, the properties constituting gaseity are reduced to a minimum, and the matter becomes exalted to an ultra-gaseous state, in which phenomena are seen, hitherto unknown and at present not comprehended. The phenomena discovered by Mr. Crookes in his exhausted tubes reveal to physical science a new world—a world where, for instance, the corpuscular theory of light holds good, and where light does not always move in straight lines, but where "we can never enter, and in which we must be content to observe and experiment from the outside." Perhaps the most astonishing of the many experiments performed by Mr. Crookes is that in which he showed that great heat is evolved when the concentrated focus of rays from an aluminium cup is deflected sideways to the walls of the glass tube by a magnet. The heat is sufficient to fuse platinum, but in the experiment shown at the Royal Society, to avoid destroying the apparatus, a luminous spot of incandescence was made to travel over the platinum foil by applying the magnet in different positions. This experiment was suggested by the discovery, while examining the green phosphorescent light produced by the focus of concentrated rays, that the glass tube at that spot became so hot as to burn the finger. Thus again at the end of the year we have to announce a great advance in physical science.

THE Belgian State Railroads, though increasing in mileage and in equipment, make smaller and smaller profits per train mile and per mile of road. The proportion of expenses to receipts is much greater than formerly. Some time ago several railroads were leased for fifty per cent. of their gross earnings, thinking that it was a fair bargain. Since then the expenses of its whole system, of which these leased roads are generally the less important parts, have one year been up to sixty-eighth per cent., and they seem unlikely again to be as low as sixty per cent. The Government roads seem to be extraordinarily well stocked, for there are 1066 locomotives and 33,395 cars of all kinds for the 1339 miles of road. It is hard to believe that there are not more than are necessary; for while the locomotives made an average mileage of 24,009 miles in 1872, they made but 18,588 in 1877, the number having increased in that time from 638 to 1066.

—*The Engineer*.

THE STABILITY OF DOCK WALLS.

By ROMILLY ALLEN, A. I. C. E.

Contributed to Van Nostrand's Magazine.

In a paper read before the Edinburgh and Leith Engineers' Society on the 5th of February, Mr. Romilly Allen, A.I.C.E., gave the following particulars of a new method of calculating the stresses on a Dock Wall:

The forces tending to displace a Dock Wall are:

Firstly. A Pressure of Water, acting normally to the face, and tending to push it backwards.

Secondly. A Thrust of Earth, acting in a direction parallel to the surface of the ground, tending to push it forwards.

Thirdly. The weight of the Masonry and of the Earth supported on the stepping behind, tending to pull it vertically downwards.

Since the wall is required to stand when the dock is empty, the pressure of the water will be neglected as being an element of safety when the dock is full.

The portion of the wall dealt with in the following investigation is a cross section of one foot in length.

The weight of the wall acts through its center of gravity, which is found thus:

Cut out the cross section in thick drawing paper of uniform texture. Mark a vertical line on a drawing-board placed upright, by means of a plumb bob. Hang up the section against the board by a pricking point stuck through one corner, and let it swing. When it has come to rest, mark where the vertical line cuts. Repeat the process and join the points so found with the respective points of suspension. The intersection of the lines will be the center of gravity required. With a section drawn to a scale of $\frac{1}{4}$ inch to the foot (which is a convenient size) the center of gravity, without the error of deviation from the true position exceeding one inch in magnitude. If the weight of the earth, which rests on the stepping behind the wall, is taken into account, its center of gravity must also be found in the same way and the center of gravity of the compound mass of earth and masonry be deduced therefrom.

The most convenient unit of weight to take is that of 1 cubic foot of masonry, as the number of square feet in the cross section will then represent the weight of the wall per foot forward. The earth thrust has now to be considered.

Its center of pressure is at a point $\frac{1}{3}$ of the height from the bottom. The amount of the earth thrust (cohesion being neglected) depends on the angle of repose of the soil, but the greatest value that it can ever attain is when the earth becomes saturated with water, and assumes a semi-fluid condition. The pressure, in this case, exerted against the back of the wall is equal to that which would be produced by a fluid of the same specific gravity as the earth, or

$$T=\frac{1}{2}w_1 h^2$$

where w_1 = weight of 1 cubic foot of earth

h = height of wall.

But the unit of weight adopted above, was 1 cubic foot of masonry, and if the ratio of the weight of earth to that of masonry be taken as 4 to 5, then

$$T=\frac{1}{2}\times\frac{4}{5}wh^2=\frac{2}{5}wh^2$$

where

w = weight 1 cubic foot of masonry.

Now this is a definite maximum value for the earth thrust which can under no circumstances be exceeded.

All other thrusts due to different angles of repose are therefore taken as fractions of the pressure produced by earth in a fluid condition, or what may be termed the Mud Thrust.

A scale, the use of which will be hereafter explained, should be constructed from the following table, in which the maximum thrust is taken at 1.000 and all others as fractions of it.

(*See Table on following page.*)

The different values for the thrusts of earth given in the table are calculated from the well known formula

Angle of Repose.	Thrust of Earth.
0°	1.000
5°	0.839
10°	0.704
15°	0.588
20°	0.490
25°	0.405
30°	0.333
35°	0.271
40°	0 217
45°	0.172
50°	0.132
60°	0.071
70°	0.031
80°	0.007
90°	0.000

$$T=\tfrac{1}{2}\,w_1\,h^2\tan^2.(45^\circ-\tfrac{1}{2}\varphi)$$

where φ=angle of repose.

The use of the scale, as applied to finding the stresses on a dock wall will now be explained.

The stability of the wall depends on the position of the point where the resultant of the earth thrust and the weight of the masonry cuts the base. This point is called the *Center of Resistance* of the base, and its position will vary according to the amount of the earth thrust.

The center of resistance will be furthest away from a line drawn vertically through the center of gravity of the wall, when the earth thrust is greatest, and having once ascertained the position of the center of resistance corresponding to the maximum earth thrust, the scale will enable us to find at once the position of the center of resistance due to a thrust of earth of any required angle of repose.

First then to find the center of resistance for the maximum earth thrust. The usual method is to represent the weight of the wall to some scale, chosen at random, on the line passing vertically through the center of gravity, measuring downwards from a point $\frac{1}{3}$ of the height up. This gives a point, either below or above the base, from which a horizontal line is drawn to represent the earth thrust and the end of it joined with the point $\frac{1}{3}$ of the height up; thus completing the triangle of forces, and the intersection of the resultant with the base is the required center of resistance. If, however, the weight of the wall, instead of being represented to a scale chosen at random, be represented to a scale, such that according to it the weight of the wall$=\frac{1}{3}$ of the height, then the side of the triangle of forces representing the earth thrust will coincide with the base, and if the length of this side, to the same scale be known, then the position of the center of resistance on the base will be found without any further lines of construction. . In order to find the required scale, let

A=sectional area of wall in square feet.

h=height of wall

Now by supposition

$$A=\tfrac{1}{3}h$$

for A represents the weight of masonry per foot run

$\therefore$ 1 cubic foot of masonry$=\frac{1}{3}.\frac{h}{A}=w$

But max. earth thrust$=\frac{2}{5}wh^2$

$\therefore$ Max. earth thrust$=\frac{2}{15}\times\frac{h^3}{A}$

which gives the following rule for finding the position of the center of resistance due to earth in a fluid state. Take the height in feet. Cube it and divide it by the number of square feet in the cross section. Multiply the quotient by $\frac{2}{15}$ which will give the distance in feet of the center of resistance from the point in the base lying vertically under the center of gravity of the section. The centers of resistance due to earth possessing different angles of repose are found by dividing up the distance obtained as above, in the same proportion as the divisions of the scale of earth thrusts, previously described. This is effected by parallel lines as shown on the diagram.

In conclusion the new method will be applied to a practical example designed by Mr. A. M. Rendel, M.I.C.E.—the wall of the Victoria Dock Extension, near London, perhaps the largest work in the world which has been constructed in concrete.

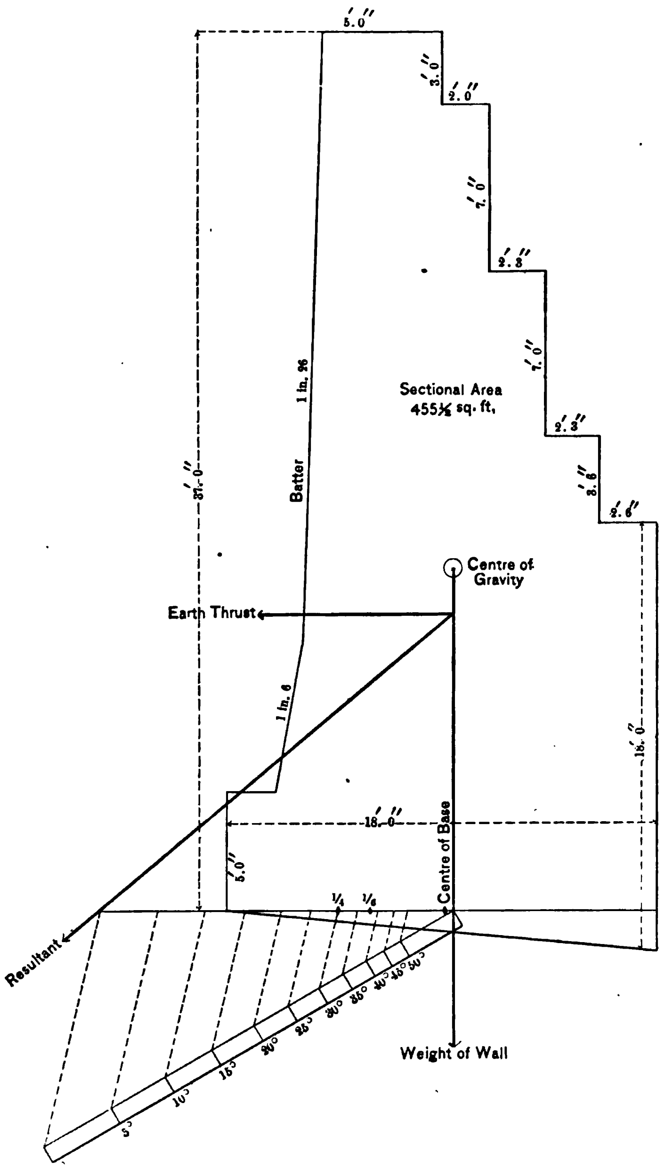
A section of this wall appeared in *Engineering*, March 29, 1878, and is here reproduced.

In this example in question

A=455.5 square feet h=37 feet.

$$\left.\begin{array}{c}\text{Max. earth thrust}\\ \text{measured along base}\\ \text{to required scale}\end{array}\right\}=\frac{2}{15}\times\frac{37^3}{455.5}$$

=14 feet 10 inches



Measure off the distance thus obtained along the base from a point lying vertically under the center of gravity of the section, which gives the position of the center of resistance due to earth in a fluid state.

To find the other centers of resistance for the different angles of repose, place the scale of earth thrusts with one end on the point in the base lying vertically under the center of gravity and inclined at any convenient angle to it.

Draw parallel lines as shown on the diagram.

The result of the investigation is as follows:

Earth in a fluid condition would more than overthrow the wall, and would cause the center of resistance to fall 5′.6″ beyond the toe.

Earth, whose angle of repose is 15° would just overthrow the wall.

Earth, whose angle of repose is 30° would keep the center of resistance with-

in the safe limit of deviation of $\frac{1}{4}$ the breadth of the base from the center of the base.

Earth, whose angle of repose is 40° would keep the center of resistance within the middle third of the breadth of the base, and there would be no tension in the masonry at any point.

MEANS ADOPTED FOR RANGING THE CENTER LINE OF THE ST. GOTHARD TUNNEL.

By C. DOLEZALEK, Section-Engineer of the St. Gothard Railway.

From "Zeitschrift des Architekten-und Ingenieur Vereins zu Hannover," published by Inst. of Civil Engineers.

The axis of the St. Gothard tunnel is a straight line about $9\frac{1}{4}$ miles long, with rising gradients of 1 in 172 and 1 in 1,000 respectively from both ends towards its center. At its extremities, viz. in Göschenen and Airolo, observatories were erected, distant 585 and 358 mèters respectively from the tunnel portals, in which were set up the transit instruments previously used in laying out the Mont Cenis tunnel.

The direction of the center line is given from the observatory at night by a lamp placed over that point in it, inside the tunnel, which can be accurately observed directly, its ranging being thence produced by a theodolite as far as the heading permits. A direct observation as far into the tunnel as possible is therefore of the greatest importance, and to obtain this as well as longer station lengths for the ranging in the interior of the tunnel, the Author devised the contrivances which form the subject of this Paper.

In 1875, to allow the signal to be shown at the right moment to the observer, telegraphic communication was established between the tunnel portal and the observatory, in both of which batteries with Morse's instruments were set up, while, in the unfinished tunnel itself, a wire was joined on by the use of portable field telegraphs.

As petroleum lamps with a bright flame proved far superior to common miners' lamps for signalling at long distances, the Author constructed one with the brilliant-burner ("Rundbrenner") of Schuster and Baer of Berlin, which gave on trial 1.8 time better illumination than the ordinary petroleum lamp. This burner has a double set of pinions moving two half wicks with the greatest regularity, and is screwed on to a large metal vessel having what is called a "double-vase ring." As this allows petroleum to be afterwards poured in without unscrewing the wick-holder, the centering of the lamp (over any station) is not thrown out during the whole period of its use, since the openings in the two rings can be made to coincide or not, at will. The vessel is now leveled on a movable bronze tripod, their centers being made accurately to coincide. This concentric position is in the first instance secured by the maker, but if thrown out at any time, the ring, on which the lamp rests, can be so set by small screws, moving in a circular slit, that the middle of the wick shall be concentric with the tripod, the ring in this case being eccentric to it. This adjustment, however, ought not to be necessary if the lamp is carefully handled. A cylindrical metal mirror is provided to intensify the brilliancy of the flame. This signal lamp surpassed all others in giving far longer station lengths under similar conditions; but it may even yet be advisable to devise apparatus for using the electric light in its place.

To diminish still more the delays and inaccuracies incident on such frequent settings-up of instrument and signal in the tunnel, the Author further constructed a stand applicable to either. It is in two parts, a top plate of metal resting on a large circular one of wood to which three legs are attached. This top plate is separate from the lower one, though capable of being centered accurately with it under or over any required point inside the tunnel, such point being denoted by a notch on an iron cramp,

which is driven into the ground. The weight (nearly 31 lbs.) of the metal plate ensures its steadiness, as its three pointed foot-screws work in small cups let into the wooden plate; by these it is levelled, and when the lamp is placed on it for use, it can be turned round and clamped in any direction.

Every station in the center line was fixed by the mean of eight distinct settings-up of the lamp, by which all level and collimation errors were eliminated from the observations. To deduce this mean readily, the metal plate consists of a bronze plate sliding in a cast-iron frame and provided with a clamp and tangent screw. The center of the bronze plate is given by a notch on either side, while to the two edges of the cast-iron frame strips of gummed paper are affixed, on which each observation is to be recorded by a pencil mark. To the mean of these marks the center of the bronze plate is now set by the notch, and in order that it may necessarily be coincident with that of either lamp or theodolite, as each is successively set up upon the plate, three small grooves radiate from it at angles of 120°, in which are secured the feet of either instrument of whatever size. At the next station the used paper-strips are scraped off, and fresh ones affixed. A plummet and line are attached to the stand for centering purposes.

The advantages claimed for this stand lie in the remarkable speed of "setting-up," in the elimination of all possible errors in the operation, and in the ready insertion of the lamp upon it on the center line; it is also easily carried about the tunnel packed in a chest. The wooden portion is only 1 mèter high besides the round wooden plate of 0.5 mèter outer, and 0.34 mèter inner diameter. Lead weights are attached to the lower parts of the legs to keep them steady if accidentally pushed. Weights above 20 kilogrammes (44 lbs.) should be made up from smaller ones to facilitate their manipulation; and since all the material, instruments, &c., are always forwarded from point to point in the tunnel on trollies, the transport of these lead weights offers no difficulty.

A light transit without vertical and horizontal circles, but with a powerful telescope magnifying thirty times, is advocated for ranging purposes inside the tunnel, and by its use greater rapidity in the work is anticipated.

GAS *VERSUS* ELECTRICITY.

By W. H. PREECE.

From "Nature."

The gas companies are at last awakening to the peculiarity of their position, and gas-shareholders are recovering their confidence in the stability of their property. It is interesting to observe how steadily the shares in all the great gas companies have during the last few weeks been rising, and unless any untoward event occurs there is no reason why in a short time they should not recover the position they so singularly lost in August of last year. Looking dispassionately upon the events that have occurred, it is difficult to understand how such a panic and scare could have arisen. Nothing of any sort or kind has been discovered either in the laws of electricity or in their application to electric lighting to account for it. We know no more of the electric light now than we did in 1862, when as great a display was made in our Exhibition of that year as was made in the French Exhibition of last year. There is no doubt, however, that the enterprise of our neighbors on the other side of the Channel in lighting up so brilliantly one of their grand new streets, produced a sensation that will not easily be forgotten. Englishmen never like to be beaten. We are accustomed to be startled by inventions from the other side of the Atlantic, but we are not accustomed to be beaten either in commercial enterprise or in inventive skill by our neighbors on this side of the Atlantic. Hence,

all of those, whose name is legion, who visited Paris last year, came back with exaggerated ideas of the effect of the electric light in the Avenue de l'Opera, and spread through England a profound opinion of the value of electricity as a means of illumination.

It seems to be forgotten that only three years ago a competitive trial of gas and electricity was made in the clock tower of the Houses of Parliament. Each of these lights was tried for several months, the electric light being a Serrin lamp lit by a Gramme machine; and that, after a very careful examination, gas was successful, was adopted, and is now used by the Office of Works.

Again, it seems to be forgotten that the Elder Brethren of the Trinity House have been experimenting upon this question ever since 1857, and that the results of their experiments have only led to the adoption of the electric light in three of their lighthouses. If the electric light had had the wonderful advantage over gas or oil that its projectors profess for it, surely the governors of such an institution as the Trinity House would have fitted up all the lighthouses upon our coasts with this wonderful light.

The recent experiments, however, have shown both the strength and weakness of the position of the gas companies. Their strength consists in their being in possession of the ground; their weakness consists in their producing only a poor light —and a very poor light—when compared with electricity. But is there any reason why this weakness should continue? Is there any reason why gas should remain such an indifferent light? There is none but that of expense, and expense will not deter people from having a better light if they can only get it. The Phœnix company has taken the question in hand, and has shown in the Waterloo Road what what can be done with gas when the question of expense is not considered. Indeed, it would almost seem, from the experiments that have been made, that the quantity of light to be produced by gas is only a question of the quantity of gas consumed in a given space. There are now burning in the Waterloo Road two brilliant gas lamps, giving a light of 500 candles; and this is greater, in point of fact, than the intensity of the light developed by any one of the electric lights that are now on trial in the thoroughfares of London. There is, however, a defect in gas light which remains to be eradicated, and that is, the color of the light. The one great advantage which the electric light has over gas is, that the electric light, owing to its very high temperature, produces rays of every degree of refrangibility, and therefore, as an illuminating power, it is equal to that of the sun. But gas light, owing to the lowness of its temperature, is deficient in blue rays, and is therefore not so effective in discriminating colors as the electric light.

A very marked advance towards perfection in this direction in gas lighting has been made in the albo-carbon process, by which the gas burnt is enriched with the vapor of naphthaline—a refuse of gas manufacture. This process is being introduced by Mr. Livesey, and, to judge by the experiments that have been shown, it is very promising indeed. The intensity of the light of a gas burner is improved at least five times, and in some experiments, witnessed by the writer, the improvement was as much as twenty times.

The tentative trials that are being made with the electric light in London cannot be said to be very successful.

That at Billingsgate was certainly a fiasco, that on the embankment is very brilliant, but we have yet to learn its cost, and there is no doubt whatever, that the efficiency of the light is very much less than that usually ascribed to the electric light. The trial on the Holborn Viaduct is not a success. The experiment seems to be conducted by some one who is not experienced in the working of electric circuits, for occasionally all the lamps are found extinguished, on other occasions only a portion of them are burning, and frequently they are very dull. It is quite difficult even at the distance of the Post Office, to distinguish the gas from the electric lamp. The same effect is observed on crossing Blackfriars Bridge and looking towards the Houses of Parliament, when there is the slighest mist in the air, and it is quite evident that the electric light has no more —if as much—penetrative power than gas.

A most complete and careful inquiry into the working of the electric light

has been made by Mr. Louis Schwendler for the East Indian Railway Company, and his results are extremely interesting. He has recommended the introduction of the light into certain railway stations where no gas exists, and the system he proposes to use is the Siemens dynamo-machine and one Serrin lamp, and thereby save that waste which the multiplication of the light unquestionably produces. He proposes to distribute this single light by diffusion on a plan originally suggested by the Duke of Sutherland. His investigation has been conducted in a thoroughly scientific spirit, and when his report is published it will be a very valuable addition to our knowledge of the theory of the electric light. It has been shown by the writer that the full effect of the current can only be obtained by one lamp on a short circuit, and that when adding to the lamps by inserting more of them on the same circuit, or on a circuit so that the current is subdivided, the light emitted by each lamp is diminished in the one case by the square, and in the other case by the cube of the number of lamps so inserted. Dr. Siemens maintains also the concentration of the power on one light, but other experimenters are endeavoring to partially multiply the light. For instance, Mr. Rapieff, in the *Times* office, very successfully distributes six lights about the office, and Ladd & Co., with the Wallace form of machine, also distribute six lights over the Liverpool Street Station. Although there is undoubtedly a loss of power in this distribution of the lamps, there may be an advantage in such distribution in cases like printing offices and railway stations. A successful experiment has been made by the British Electric Company, in lighting up some of the stations of the Metropolitan Railway Company, and the India Rubber and Gutta Percha Company have been successful in lighting up the London Bridge station of the London Brighton and South Coast Railway Company. In all these cases we have scarcely emerged from the sphere of experiment. The electric light has not yet been permanently introduced on any large scale. Many are trying it, many are captivated by the brilliancy of the light, and many in their eagerness to keep up with the spirit of the age, are introducing it, as, for instance, the London Stereoscopic Company, and the Messrs. Nichols, the clothiers in Regent Street, where, however, the light does not appear to give very great satisfaction through its fluctuation.

We were led to expect very much from the experiments of Mr. Werdermann, but his attempt to subdivide the light seems to have subsided, for we have heard nothing of it for some time past. Again, we have heard no more of M. Arnaud's discovery, and the accounts that reach us from America of the doings of the Sawyer-Mann light, and of the supposed discoveries of Mr. Edison, are unworthy of attention.

The present state of the electric light question may therefore be said to be a tentative one, and the gas companies are with much enterprise now giving their retort courteous by showing that they are in a position—if the people choose to pay for it—to give quite as powerful a light as the electric light; and, let us hope, before long that it will be quite as perfect. There can be no doubt that the use of electricity for the production of light is a very wasteful as well as a costly process, for the energy that is generated in the machine is not all consumed in the lamp, but is proportionately distributed over the whole circuit. It is, therefore, not utilized only in the place where it is wanted, as in the case of gas. If we are using a certain amount of energy in an electric lamp to light a street, we are wasting as much if not more energy in the street in maintaining the current to produce that light.

There are three points which all electric lights for general purposes should be required to attain. The first is a brilliancy far exceeding that of any known lamp: the second is a durability greater than that which would be required for night operations in England; and the third is absolute steadiness, to enable work to be conducted without affecting the eyes. There is no electric light that has yet been introduced which supplies us with these desiderata.

From the dust of its coal mines France makes annually 700,000 tons of excellent fuel, known here as patent fuel, and Belgium makes 500,000 tons.

RETAINING WALLS.

By Prof. WM. M. THORNTON.

Written for Van Nostrand's Magazine.

1. In the following article it is proposed to give a concise and simplified account of the theory (due to Mr. Lamé) of the Pressure of Earth, and its application to the designing of Retaining Walls by Rankine. Having been heretofore presented as part of a more general theory, it has been needlessly encumbered with mathematical difficulties, and has not been reduced to a form suited to purposes of computation. It is hoped that the mathematical processes here employed will be found intelligible to any one tolerably versed in Algebra, Plane Trigonometry and Elementary Mechanics; and that the results arrived at will prove useful in their application to the purposes of practical design.

2. STRESS.

It is obvious upon the least consideration that the forces between bodies or parts of bodies do not act at definite points but are distributed over surfaces (or volumes) of greater or less extent. When it becomes necessary to consider not the single resultant force but this distributed action we distinguish the latter by the word stress. This stress may vary in amount from point to point; but over a sufficiently small surface it may always be considered constant. The ratio of its total amount to the area of this surface is called the intensity of the stress. The resultant stress must be conceived as applied at the center of this area.

3. INTERNAL STRESS.

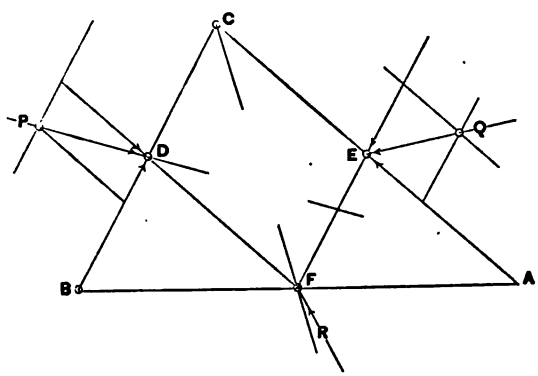
If a body in equilibrio under the action of external forces be anywhere divided by an ideal plane into two parts A,B each of these acts on the other with a certain stress at the plane of division. This is what is called the internal stress on this plane; the inclination of its resultant to the normal to the plane is called the obliquity of the stress; and the stress is a thrust, a shear or a pull as its obliquity is $<=>90°$. This force may, like any other, be resolved into components or compounded with others into resultants. In the questions which we shall have to consider the stresses are all parallel to one plane to which their planes are perpendicular. It will be necessary for us therefore to consider only the particles in a section parallel to this plane.

4. CONJUGATE STRESSES.

The intensities of shear on any two intersecting lines are equal; so that if the stress on a is parallel to b the stress on b is parallel to a.

Proof:—

Consider the elementary triangle ABC, its sides being so small that the stresses on them may be considered uniform, so that the resultants act at the middle points D,E,F. Resolve the stresses P, Q each into components parallel to a, b. The shears P_a, Q_b meet in C. The thrusts P_b, Q_a meet in F. Accordingly to balance R the resultant of P_a, Q_b must



act in CF. Hence $\frac{P_a}{a}=\frac{Q_b}{b}$ which was to be proved. For $CD=\frac{a}{2}$, $CE=\frac{b}{2}$.

The second part of the theorem is obvious. For if the stress on a is parallel to b the common intensity of shear is null. These are called conjugate stresses.

5. PRINCIPAL STRESSES.

Given the components of stress on two orthogonal axes it is required to determine the stress on a third axis through the same origin.

Let p, q denote the normal intensities of stress on Ox, Oy,

Let t denote the common intensity of shear on Ox, Oy,

Let θ denote the slope of the normal to the third axis,

Let σ denote the intensity of stress on it,

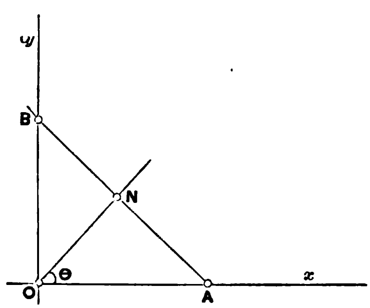
Let ω denote the obliquity of stress on it.

Then the axial components of the stress on AB are

$$X=q.\ OB+t.\ OA,$$
$$Y=p.\ OA+t.\ OB;$$

and the corresponding intensities are

$$x=q\cos.\theta+t\sin.\theta,$$
$$y=p\sin.\theta+t\cos.\theta.$$



The normal and tangential intensities therefore are

$$\nu=x\cos.\theta+y\sin.\theta,$$
$$\tau=x\sin.\theta-y\cos.\theta,$$
$$\therefore \nu=p\sin.^2\theta+2t\sin.\theta\cos.\theta+q\cos.^2\theta,$$
$$\tau=(q-p)\sin.\theta\cos.\theta-t\cos.2\theta.$$

The intensity of shear on the third axis will therefore be null if

$$\tan.2\theta=\frac{2t}{q-p}.$$

This equation determines two axes and only two at right angles to each other on which the stress is wholly normal. These are called the principal axes of stress at O. If they be chosen for co-ordinate axes, the components of stress on an axis whose normal has the slope θ are

$$\nu=p\sin.^2\theta+q\cos.^2\theta,$$
$$\tau=(q-p)\sin.\theta\cos.\theta.$$

The intensity σ and the obliquity ω of the stress on this axis are therefore determined by the equations

$$\sigma\cos.\omega=p\sin.^2\theta+q\cos.^2\theta,$$
$$\sigma\sin.\omega=(q-p)\sin.\theta\cos.\theta.$$

If we eliminate θ between these equations we obtain the quadratic

$$\sigma^2-(p+q)\sigma\cos.\omega+pq=0,$$

which determines the intensities of the two conjugate stresses at O, whose common obliquity is ω. By solving the quadratic we get

$$2\sigma=(p+q)\left\{\cos.\omega\pm\sqrt{\cos.^2\omega-\frac{4pq}{(p+q)^2}}\right\}$$

for the two values of the intensity.

6. MAXIMUM OBLIQUITY.

The expression under the radical in the last equation cannot be negative. Accordingly the maximum obliquity of two conjugate stresses at O is found from the formula

$$\cos.\Omega=\frac{2\sqrt{pq}}{p+q}$$

and the ratio of two conjugate stresses of obliquity ω is

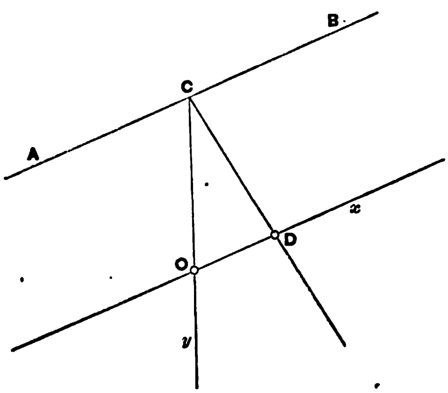
$$\rho=\frac{\cos.\omega-\sqrt{[\cos.^2\omega-\cos.^2\Omega]}}{\cos.\omega+\sqrt{[\cos.^2\omega-\cos.^2\Omega]}}$$

7. FRICTIONAL STABILITY OF LOOSE EARTH.

In such masses of earth as the engineer has to deal with, the only source of stability of which he can take account is the mutual friction of its grains. Adhesion increases this stability; but to an amount which varies so much from extraneous causes that no reliance can be placed on it. The obliquity of stress in such a mass must, therefore, never exceed the angle of repose ϕ; that is, Ω cannot be greater than ϕ, or the ratio of the less of two conjugate stresses to the greater cannot be less than

$$\rho=\frac{\cos.\omega-\sqrt{(\cos.^2\omega-\cos.^2\phi)}}{\cos.\omega+\sqrt{(\cos.^2\omega-\cos.^2\phi)}}$$

8. In a mass of earth with an indefinite plane for its upper surface, the condition of all sections by perpendicular vertical planes is alike. And at any point O in such a section the stress is vertical on a plane parallel to the upper surface, and equal to the weight of the column OC; that is, the intensity of the stress is $wh\cos.\omega$, where w is the weight of the earth per cubic foot, ω the slope of the upper surface and h the depth. The stress on Oy therefore acts along Ox, and is not less than $\rho wh\cos.\omega$. But this suffices to produce equilibrium at O. It is, therefore, in accordance with Moseley's principle of Least Resistance, the actual intensity of the conjugate stress.



9. PRESSURE OF EARTH ON A VERTICAL PLANE.

If therefore we make OD equal to ρh, the area COD will represent the total pressure on CO, which will be parallel to AB and pass through the center of gravity at COD. That is, the pressure on CO will be

$$P=\tfrac{1}{2}w\rho h^2\cos.\omega$$

and will be applied at two thirds of the depth below C.

10. UPRIGHT RECTANGULAR RETAINING WALL.

The relation between the height and thickness of an upright rectangular retaining wall may now be determined. Either of two methods may be pursued.

Let $t=nh$ denote the thickness of the wall,

Let mw denote the weight of the masonry per cubic foot,

Let s denote the coefficient of safety,

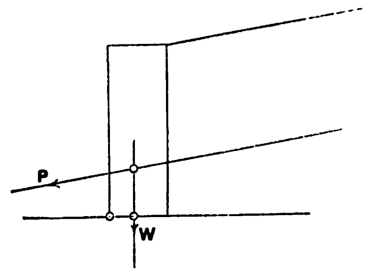
Let qt denote the deviation of center of resistance from center of base,

Let W denote the weight of the wall per foot.

1/. Take moments about the toe of the wall and employ the coefficient of safety. The forces and their lever arms are

$$P=\tfrac{1}{2}w\rho h^2\cos.\omega,\quad \tfrac{1}{3}h\cos.\omega-t\sin\omega;$$
$$W=mwht,\qquad \tfrac{1}{2}t;$$
$$\therefore\ \tfrac{1}{2}s\,mwht^2=\tfrac{1}{6}w\rho h^3\cos.^2\omega-\tfrac{1}{2}w\rho h^2t\sin.\omega\cos.\omega,$$

$$n^2+\frac{\rho s}{m}\sin.\omega\cos.\omega.n=\frac{\rho s}{3m}\cos.^2\omega.$$



2/. Take moments about the center of resistance. The forces and their lever arms are

$$P=\tfrac{1}{2}w\rho h^2\cos.\omega,\quad \tfrac{1}{3}h\cos.\omega-\tfrac{1}{2}(2q+1)t\sin.\omega;$$
$$W=mwht,\qquad qt;$$
$$\therefore\ n^2+\frac{2q+1}{4mq}\rho\sin.\omega\cos.\omega.n=\frac{\rho\cos.^2\omega.}{\sigma mq}$$

Of these two methods, the first is a very common practical device for ensuring stability; the second is to be preferred on theoretical grounds. Either of the two quadratics will afford the required value of a.

11. If in the formula which gives the value of ρ we put

$$\sin.x=\frac{\cos.\Phi}{\cos.\omega},$$

we find

$$\rho=\tan.^2\tfrac{1}{2}x;$$

and if in the quadratic

$$n^2+2bn=a^2$$

we put

$$b=a\cot.y,$$

we find

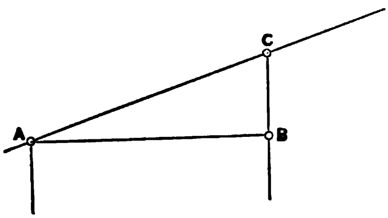
$$n=a\tan.\tfrac{1}{2}y.$$

Employing these substitutions we obtain the following alternative sets of formulæ for designing retaining walls;

1/
$$\begin{cases} \cot.\ y=\sqrt{\dfrac{3s}{4m}}.\tan.\ \dfrac{x}{2}\sin.\ \omega, \\ n=\sqrt{\dfrac{s}{3m}}.\ \tan\ \dfrac{x}{2}\tan.\dfrac{y}{2}\cos.\omega; \end{cases}$$

and

2/
$$\begin{cases} coty=\dfrac{2q+1}{8}\sqrt{\dfrac{6}{mq}}.\tan.\dfrac{x}{2}\sin.\omega, \\ n=\sqrt{\dfrac{1}{6mq}}.\ \tan\ \dfrac{x}{2}\tan.\dfrac{y}{2}\cos.\omega \end{cases}$$



12. EXAMPLE.

It is required to design an upright rectangular retaining wall to sustain a mass of earth of practically indefinite extent whose natural slope is 3:2, surface slope 12°30′, the weight of the masonry per cubit being $\frac{5}{4}$ the weight of the earth.

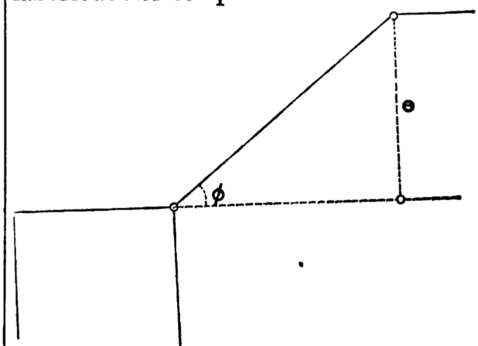
For Φ we have tan. $\Phi=\frac{2}{3}$, whence cos. $^2\Phi=\frac{9}{13}$ and therefore

L cos. Φ	9,92015
L cos. ω	9,98958
L sin. x	9,93057
x	58° 27′ 30″
$\frac{x}{2}$	29° 13′ 45″

In applying the other formulae we assume $s=2$ for the first set, $q=\frac{3}{8}$ for the second.

L tan. $\frac{x}{2}$	9,74784	9,74784
L sin. ω	9,33534	9,33534
	0,00959	$\bar{1}$,89356
L cot. y	9,12277	8,97674
y	82° 26′ 34″	84° 35′ 7″
$\frac{1}{2}y$	41° 13′ 17″	42° 17′ 34″
L tan. $\frac{x}{2}$	9,74784	9,7.784
L tan. $\frac{y}{2}$	9,94255	9,95890
L cos. ω	9,98958	9,98958
	$\bar{1}$,86350	$\bar{1}$,77546
Ln	$\bar{1}$,54347	$\bar{1}$,47178
n	0,34952	0,29633

That is, the first method gives $n=0.35$; the second $n=0,30$. The first value happens to be exactly that recommended by Trautwine [P.B. 331]; the second that recommended by Molesworth [P. B. 17]. The second is doubled sufficient. The calculations as will be seen are neither laborious nor complex.



13. The coefficient of safety of a given retaining wall is found from the relation (10.1) which gives

$$s=\frac{6mn^2}{\tan.^2\frac{1}{2}x\sin.2\omega\,[\cot.\omega-3n]}$$

Thus using the example of (12) we have

cot. ω	4,5107
3 n	0,9
	3,6107
	0,55759
2 L tan. $\frac{1}{2}x$	9,49568
L sin. 2 ω	9,62595
	$\bar{1}$,67922
L 6 m n^2	$\bar{1}$,82930
L s	0,15012

whence $s=1,413$. In this way the engineer can always judge whether the proposed value of n is sufficient.

14. The inclination i of the resultant to the vertical is given by the formula

$$\frac{\cos.(\omega+i)}{\sin.\ i}=\frac{W}{P}.$$

or

$$\cot.\ i=\tan.\ \omega+\frac{2mn}{\tan.^2\frac{1}{2}x\cos.^2\omega}$$

Thus using the same example we have

L tan. $\frac{1}{2}x$	9,74784
L cos. ω	9,98958
	$\bar{1}$,73742
	$\bar{1}$,47484
L 2 m n	$\bar{1}$,87506
	0,40022
	2,5132
tan. ω	0,2217
cot. i	2,7849

whence $i=20°$ which is less than the angle of repose of the masonry on its foundation. Should the result exceed

this angle, then the base of the wall must be tilted back at an angle greater than the excess.

15. RESUME.

Let Φ=the natural slope,
" ω " surface slope,
" m " density of masonry [earth=1],
" q " deviation of center of resistance,
" n " ratio of thickness to height,
" s " coefficient of safety,
" i " obliquity of pressure on base.

$$\sin. x = \frac{\cos. \Phi}{\cos. \omega}$$

$$\cot. y = \frac{2q+1}{8}\sqrt{\frac{6}{mq}} \tan. \tfrac{1}{2}x \sin. \omega$$

$$n = \sqrt{\frac{1}{6mq}} \tan. \tfrac{1}{2}x \tan. \tfrac{1}{2}y \cos. \omega$$

$$s = \frac{6mn^2}{\tan.^2 \tfrac{1}{2}x \sin. 2\omega\, [\cot. \omega - 3n]}$$

$$\cot. i = \tan. \omega + \frac{2mn}{\tan.^2 \tfrac{1}{2}x \cos.^2 \omega}$$

16. TABLE.

The following table is computed from the above formulae with the date

$$\tan. \Phi = \tfrac{2}{3}, \quad m = \tfrac{5}{4}, \quad q = 0{,}3;$$

$$\therefore \frac{2q+1}{8}\sqrt{\frac{6}{mq}} = 0{,}8$$

$$\sqrt{\frac{1}{6mq}} = \tfrac{2}{3}.$$

ω	n	s	i	$\frac{1}{2}x$	L tan. $\frac{1}{2}x$
0°	0,357	1,669	17° 48′	28° 9′ 19″	9,72851
2	0,352	1,683	17° 51′	28 10 50	9,72897
4	0,347	1,693	17° 57′	28 15 38	9,73043
6	0,343	1,711	18° 2′	28 23 45	9,73288
8	0,339	1,728	18° 8′	28 34 56	9,73625
10	0,335	1,741	18° 17′	28 49 49	9,74072
12	0 332	1,762	18° 25′	29 8 26	9,74626
14	0,328	1,778	18° 38′	29 31 13	9,75300
16	0,3:5	1,788	18° 51′	29 58 30	9,76100
18	0,323	1,822	19° 3′	30 30 52	9,77040
20	0,321	1,846	19° 20′	31 9 10	9,78139
22	0,320	1,926	19° 38′	31 54 30	9,79424
24	0,319	1,917	20° 1′	32 48 30	9,80933
26	0,319	1,963	20° 30′	33 53 24	9,82719
28	0,320	2,018	21° 7′	35 13 36	9,84888
30	0,323	2,098	21° 57′	36 57 0	9,87633
32	0,330	2,202	23° 18′	39 25 40	9,91499
Φ	0,369	2,802	26° 34′	45° 0′ 0″	10,00000

It contains the values of $\frac{1}{2}x$, L tan. $\frac{1}{2}x$, n, s, i for equidistant values of ω. For any other values the corresponding results may be found by an easy interpolation.

For any other value of m the value of n, which corresponds to an equal value of s, may be found with sufficient accuracy for practice from the formula

$$\frac{n'}{n} = \sqrt{\frac{m}{m'}}$$

Thus if $m' = \frac{3}{2}$ and $m = \frac{5}{4}$ as above

$$n' = 0{,}913\, n.$$

The value of Φ adopted is that which is almost universally found for ordinary earth work in dry earth.

17. When the earth slopes to the outer edge of the wall it is recommended to measure the height to c and use this to compute the thickness. Or, what amounts to the same thing, to take n from the table and make the ratio of height to thickness

$$\frac{1}{n} - \tan. \omega$$

This gives a very slight excess of stability.

18. SURCHARGED WALLS.

A wall is said to be surcharged when the backing slopes away from it at the angle of repose to a certain height called the height of surcharge after which it is horizontal. The thickness of the wall is thus somewhat less than it would have to be if the slope were indefinite. According to Rankine we may put

$$n = n_{\Phi} - \frac{n_{\Phi} - n_0}{1 + 2\theta}$$

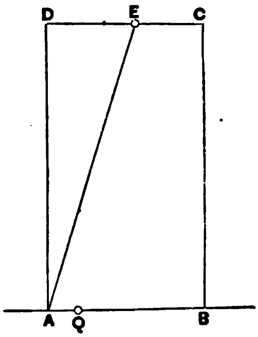
where θ is the height of surcharge, the height of the wall being taken as 1. With the data already used this formula becomes

$$1000n = 369 - \frac{12}{1 + 2\theta}$$

The limitation of height will not decrease the thickness therefore by as much as one thousandth part unless $\theta < 5\frac{1}{2}$.

19. BATTERING-FACED WALLS.

An upright rectangular wall ABCD with center of resistance at Q may be changed



into a battering wall ABCE of equal stability by making DE=3AQ. For the center of gravity of ADE being vertically over Q, this change does not affect the moment of stability. Thus with the same data as before AQ=0,2n; DE=0,6n; and the batter is 1:0,6n.

This change obviously increases the obliquity of the pressure on the base and to guard against sliding we should give the batter a cant backwards.

Any batter less than this may, of of course, be used.

GAS ILLUMINATION.

By Dr. WILLIAM WALLACE, F. R. S. E.

From "Journal of the Society of Arts."

It is impossible to discuss the subject of gas illumination at the present time without referring to the electric light, which many authorities affirm is destined to be the light of the future. If this is so, it might naturally be inferred by those who have only a slight knowledge of the subject, that it is only wasting energy to devote time and study to the improvement of gas lighting, since it must soon be superseded by the more brilliant light obtained from electricity. I have given this matter some attention, and I must say that I have no fear that gas interests will suffer in consequence of the introduction of the electric light for many, many years, if at all. The mere fact that light can be obtained by passing a powerful current of voltaic electricity between carbon points, dates back to a time when gas lighting itself was only in its infancy; and it is now nearly 30 years since the apparatus was so far perfected that the distance between the carbon points was worked automatically; and the improvements recently introduced, if we accept the Jablochkoff candle, and the imperfect arc formed in the Werderman arrangement, have been directed more to the effective production of electricity by mechanical power than to the light itself. Turning over an old periodical a few days since, I came upon a paragraph which I read with some interest, in which it stated that a French *savan* had discovered a plan by which the unsteadiness of the electric light was removed. The date of this announcement is 1853—a quarter of a century since—and even now we are scarcely in a position to say that the unsteadiness of the light has been overcome. The fact is, that we have still a long period of experiment and study before us in regard to lighting by electricity, and although the march of improvement in science is now extremely rapid, I scarcely hope to live long enough to see electricity take the place of gas in the lighting of ordinary dwelling-houses. But even if I am in error in supposing that the enormous difficulties will not immediately be overcome, there is still little, if any, cause for alarm on the part of holders of gas stocks, since, even at the worst, gas is certain to be used side by side with electricity, as long as coal can be got to produce it. The fears entertained recently by shareholders of gas companies, remind me of the beginning of railway engineering, when it was asserted that if railways were allowed to be made there would no longer be any use for horses, and the valuable breeds of the animal in this country would be allowed to die out. We all know that the result was entirely the other way; the railways increased the demand for horses, and they became more valuable and more numerous than ever. Then, again, it was supposed that when gas was used for the lighting of towns the manufacture of candles would cease,

but what is the fact? More candles are made now than ever there were before, and, what is very much to the purpose in connection with my subject to-night, the greatest improvements in the manufacture of candles were made after the gas manufacture was fairly established. Even within my own recollection, candles were burned which required constant snuffing, and so late as 30 years ago artistic designs for snuffers and snuffer-trays were published in art journals. If electricity supplants gas for public lighting, as I believe it may to some extent, it is all the more necessary that we should strive to get more light out of gas, either by improvements in the mode of manufacture, or by better means of burning it, or both, and I am very sanguine that gas lighting during the next 30 years will be developed to an extent of which we can at present form no adequate idea. We have seen some improvements in gas lighting already. What was at one time 12-candle gas, tested by the primitive Argand, became 14-candle gas with the Sugg-Letheby burner, and 16-candle gas with Sugg's London Argand; and all this without sensibly changing the quality of the gas, and, consequently, without conferring any benefit on the public. A real and substantial improvement in gas lighting would be one which would enable the public to get, in ordinary domestic life, something approaching to the illuminating power declared as the result of the official tests, and the object of this paper is to show what has been done up to the present time in this direction. Before passing on to my subject, however, I wish to make just one remark. If the the production of gas is sensibly decreased, the value of the by-products will proportionately rise, the demand for benzole, anthracen, tar oils, pitch, and ammonia, will continue; and, if the quantity produced becomes less, the value of these important articles will undoubtedly increase.

Coal gas is a cheap source of light, the only real competitor in this respect being paraffin oil. The following table gives the comparative values, based on what may be accepted as average prices, although some of them may not be exactly correct at the present time:

Cannel gas, 30 candles, at	4s. 2d.	℔ 1,000 c.ft.	1
Common gas, 16 candles	3s.	"	$1\frac{1}{8}$
Paraffin oil	1s. 6d.	℔ gallon	$1\frac{1}{4}$
Colza oil in moderator lamps	4s 6d.	"	7
Stearine candles	10d.	℔ pound	27
Tallow "	8d.	"	29
Paraffin "	1s. 6d.	"	31
Sperm "	1s. 6d.	"	36
Wax "	2s. 6d.	"	72

In these comparisons, it is but fair to say, the gas is calculated as giving the light obtained when burned in the best known manner, as in the official tests of the gas examiners of the towns where the respective qualities of gas are made.

It will be well to indicate, in a few words, the principle involved in the testing of various gas flames and other sources of light. If a flame of any kind is held at any distance, say a yard, from a screen, in which an opening is made one foot square, and a second screen is placed at the distance of two yards, there will be thrown upon the latter a square figure, which, on examination, will be found to measure exactly 2 feet, and which has, therefore, an area of 4 square feet. If the second screen is moved to 3 yards, the illuminated portion will measure 3 feet square, representing an area of 9 square feet; at 4 yards it will measure 4 feet, giving an area of 16 square feet. We thus see that the space covered by the light increases in proportion to the square of the distance, while the intensity of light decreases in a corresponding degree. To put the matter more clearly to those who have not studied the subject—a flame at a given distance, say a yard, illuminates a given space, say a foot square, but at four times the distance the illuminating effect is diffused over 16 times the area, or 16 square feet, consequently any single square foot at this distance gets only one-sixteenth part of the whole quantity.

I do not propose to enter into any details regarding photometers, all of which are based upon this principle, but I may explain the mode of testing by a simple illustration. I have a space of 100 inches, with a candle at one end, and a gas flame which I wish to test at the other. I have a greased disc moving freely between the two, and by a little practice I can place it in a position in which the two sides are equally illumin-

ated. I now measure the difference between the candle and the disc, and find it to be 20 inches, while that between the gas flame and the disc is 80 inches; the square of 20 is 400, and of 80, 6,400, and the one divided by the other, gives 16 candles as the illuminating power of the gas flame. In practice, the photometer is divided so as to give the illuminating power by direct observation, and many details require to be minutely attended to in order to obtain reliable results.

We are in the habit of talking of certain qualities of gas—16 candles in London, 15 in Birmingham, 14 in Newcastle, 26 in Glasgow, 30 in Edinburgh, but these are not the values of the gas as burned in our houses, warehouses, and shops, but as burned in the manner calculated to give the highest illuminating power. These figures show the possibility of gas illumination, and represent the goal toward which we should strive. I freely admit that it is impracticable, not to say impossible, to obtain in the every-day practice of common life, results as good as those got by means of appliances the most perfect for developing the full photogenic value of the gas, but still a great deal may be done to decrease the reckless waste of light that is constantly going on. I have no hesitation in saying that from 12 to 14 candle power might be obtained in every-day life from what is called 16-candle gas. We stand in a similar position with regard to various forces employed for practical purposes. The engineer calculates the power that should be obtained by the falling of a given weight of water through a given space, but the practical result obtained in a water-wheel always falls far short of the theoretical quantity. In like manner, the force obtained by the combustion of one pound of coal in the boiler of a steam-engine is greatly less than the calculated figure. Still, mechanicians struggle on to obtain better results, and we are constantly improving. Some of the most recent forms of reaction engines show an immense improvement on the water wheels formerly in use, while, in regard to steam power, we have, in the performance of the best descriptions of compound engines, an approach to theory which was formerly deemed impossible of attainment. Such improvements represent so much money saved to the country; and it is equally the case with gas, but with this addition, that a decreased consumption, with the same amount of light, would give increased healthfulness to our dwellings, where the products of the combustion of gas constitute an evil of no small magnitude. It has been said that the man who causes two blades of grass to grow where only one grew before, is a benefactor to his country, and some honor is due also to him who enables us, by improvements in the steam-engine, to get out of one pound of coal the power which formerly required the combustion of two pounds, or, who teaches us how to obtain from one cubic foot of gas the illuminating value for which two feet had previously been expended.

When a porcelain slab is brought over a gas flame a deposit of carbon occurs: the particles exist in the flame, and the contact of the cold slab causes their instant deposition. A similar effect is produced by a current of cold air impinging upon a flame, a portion of which is thus cooled down below the temperature necessary for the combustion of the carbon, and the flame thus exposed to the draught smokes, that is, the finely divided particles of carbon pass into the air unconsumed. In ordinary circumstances, the carbon is consumed in the upper portion of the flame, and if the jet be a good one, and the pressure of gas not too low, no smoke is produced. In the Bunsen burner the gas is mixed with air sufficient to prevent the separation of the carbon, and hence we have a flame which is valueless as a source of light, but convenient for the application of heat. The solid particles of carbon in an ordinary gas flame result from the decomposition of the olefines and other compounds rich in carbon, which are readily decomposed by the action of heat. The same thing occurs if coal gas be passed through a glass tube heated to redness; in this case a deposit of carbon in the interior of the tube occurs at the point where the heat is applied. In gas works a similar effect is produced by the heating of the impure gas in the retorts, in which a deposit of carbon, sometimes three or four inches in thickness, is formed. This carbon was formerly used for the rods or pencils employed in producing the electric light. The presence of the particles of carbon in

a flame renders it opaque, and the degree of opacity varies with the illuminating power. At the bottom of a flat flame, where the oxygen is in excess, the transparency is such that a printed paper may be read through it as if no obstruction intervened, but the upper part almost entirely conceals the printing. The intensity of light depends partly upon the quantity of the carbon particles, and partly upon the heat of the flame by which the carbon is brought up to a greater or less degree of incandescence. Professor Frankland has shown that the light is not entirely due to the separated carbon, and that certain chemical compounds—gases or vapors—from which carbon does not separate by the action of heat, are capable, under some conditions, of giving luminous flames when burned in air. For all practical purposes, however, the original proposition of Davy may be accepted, that the light is radiated from highly heated particles of solid carbon. When air is supplied in excess to a flame, as when the gas escapes through a fine jet at a high pressure, there is little separated carbon, the flame is transparent, or nearly so, and there is very little luminosity. On the other hand, when the flame is large and sluggish, and the air in contact with it is insufficient, the solid carbon is in excess, and a part of it escapes unburnt, giving rise to a smoky flame, in which also the luminosity may be low. What we have to strive after in order to obtain the greatest possible "duty," as mechanical engineers would call it, from gas, is to burn it so as to have the flame as hot as possible, and as near the smoking point as is consistent with the perfect consumption of the carbon in the upper part of the flame. In few words, the whole science of gas lighting is the obtaining a bright flame without smoke. It was at one time time accepted as an axiom that economy in gas lighting could only be obtained by the use of large burners, and that in small jets the contact of air was necessarily so complete that only a feeble light could be obtained. But this is only partially true; more precise and extended experiments have shown that the luminosity depends not so much upon the quantity of gas as upon the conditions under which it is burned. In the case of flat-flame burners, the most essential element is pressure, a high initial velocity giving a low illuminating power, and *vice versa*. I may give a few illustrations from my own experiments—the gas used being of 26 candles illuminating power for five cubic feet per hour. In all the instances I am about to quote Bray's ordinary union jets were used. The gas gave the most unfavorable result when the smallest burner of the series, No. 0, was used under comparatively high pressure—1½ inches—two cubic feet per hour gave an illuminatng effect of 3.21 candles, or calculated to the standard of five cubic feet per hour, eight candles. The best result, on the other hand, was obtained with a No. 8 burner at one inch pressure, when 7.1 cubic feet gave an illuminating effect of 40.63 candles, or for 5 cubic feet, 28.6 candles. Here is a striking contrast, the same gas giving at one time 8-candle power for five feet an hour, at another 28.6, the jets being respectively the smallest and the largest of the series of nine. But let us now take the same quantity of gas under varied conditions of pressure, and we shall see even here very marked differences. Three cubic feet burned at half-inch pressure, and calculated to five feet per hour, gave 25 candles; at one inch pressure, 19 candles; and at 1½ inch, 12½ candles. Here we have the effect simply of pressure, which, in the case of flat-flame burners, is of paramount importance. When common gas is used, the effect of pressure is even more remarkable, the varieties being such that in some cases less than one fourth of the possible amount of light producible is really obtained.

A remarkable effect is obtained with a mixture of cannel gas with about twice its bulk of air. At a low pressure, in an Argand jet with large holes, it gives a fairly luminous flame, while at a high pressure (3 or 4 inches), although the quantity of gas consumed is three times as great, the flame is almost totally non-luminous, and has a greenish tint. The gas used somewhat extensively in the United States, made by saturating air with petroleum spirit, requires to be burned at a pressure not exceeding 0.1 of an inch, which can be obtained only with an Argand with very large holes, or a bat's-wing of peculiar construction, called the American regulating bat's-wing. At ordinary pressures, such as are used for coal gas, there is scarcely any light, and

the flame keeps about a $\frac{1}{4}$ inch or more above the burner.

It is not only on the score of economy that it is desirable to burn gas in such a manner as to afford the greatest possible amount of light. The burning of a moderate-sized jet of gas produces as much carbonic anhydride as the breathing of two grown-up men; and as, in an ordinary apartment, we have usually from three to six of these, the air becomes vitiated with remarkable rapidity. It is, therefore, desirable, in relation to health, to obtain the illumination we require with the least possible expenditure of gas. The sulphur in gas is a very serious drawback to its use. In burning, it is no doubt formed chiefly, if not entirely, into sulphurous anhydride; but it is soon converted into sulphuric acid, which attacks with avidity all the more readily destructible articles in the apartment. So far back as 40 years since, the effects of the sulphuric acid arising from the combustion of gas upon the binding of books and many articles of furniture were noted; and recent experiments have shown that leather, paper, etc., in ill-ventilated apartments, exposed to the emanations from burning gas for a series of years, contain large quantities of sulphuric acid.

There are several qualities of gas in use in this country. The best may be described as Scotch cannel gas, as it is made only in Scotland, where the illuminating power varies from 24 to 30 candles for 5 cubic feet per hour consumed in a union or fish-tail jet: the average may be fairly stated as 26 candles. In London a cannel gas is used in small proportion, the illuminating power of which is about 23 candles; and in Liverpool, Manchester, Carlisle, and probably some other towns, an intermediate gas is manufactured, the illuminating power of which is about 20 candles. The common gas in London, and most other English and Irish towns, has an illuminating power of 14 to 16 candles. In the case of cannel gas, the standard is found by testing the gas by a union jet consuming 5 cubic feet, at a pressure of 0.5 of an inch, while the common gas is tested by Sugg's "London" Argand, consuming 5 cubic feet per hour, at a pressure of about 0.05 of an inch.

The burners at present in use may be divided into the four following classes:—1. Cockspur, or rat-tail. 2. Union, or fish-tail. 3. Bat's-wing. 4. Argand. Of each of these there are a number of modifications.

The cockspur, or rat-tail burner, is the simplest possible form of gas-jet, and it was at one time the only one used for burning gas. It may be made by simply drawing out a piece of glass tube and breaking off the point so as to leave an orifice having a diameter of 1 millimetre or less; but it is usually constructed of cast iron, which is drilled as wide as possible from the bottom, leaving only a thin shell, which is then bored with a fine drill. Two sizes of these were tested, No. 1 having an orifice of about 0.6, and No. 2 of about 0.75 millimetre. These jets are used in Glasgow for lighting common stairs, and the larger sizes were formerly employed for street lamps, but are now discarded in favor of union jets. The following are the results with 26-candle gas:—

No. of Burner.	Pressure in Inches.	Length of Flame in Inches.	Gas per hour.	Illuminating Power in Standard Candles.	Illuminating Power of Five Cubic Feet per hour in Candles.
1	0.5	2	0.45	0.89	9.9
1	1.0	$3\frac{1}{2}$	0.60	1.69	14.1
1	1.5	$4\frac{5}{8}$	0.90	2.40	13.3
2	0.5	$3\frac{7}{8}$	0.80	2.49	15.6
2	1.0	$5\frac{1}{8}$	1.13	3.55	15.7
2	1.5	$7\frac{1}{2}$	1.45	4.53	15.6

These figures show that even with the larger jets no more than 60 per cent. of the real value of the gas can be obtained. I have tried various modified forms of the jet, some having "adamas" tips and contracted at the bottom, or otherwise obstructed so as to diminish the pressure at the point of ignition, but they did not show any marked superiority over those referred to above.

When two rat-tail jets are held at a right angle to one another, the lights coalesce and form a flat sheet of flame. When this discovery was first made, two burners were fitted up in this way, but soon a single burner was contrived which combined the two, and hence was called a "union" jet; it is also known as a fish-tail, from the resemblance of the flame to the tail of a fish. It is a short cylindrical tube with a flat top, in which the two

orifices are drilled at about 90 degrees to one another, and meeting in the center. The union jet is much improved by substituting for the metal top porcelain or stoneware, the principal advantage gained being that the orifices remain clean and constant in size, while those of iron gradually rust up and require to be frequently cleaned in order to give a satisfactory light, and are consequently enlarged. Some fish-tail burners are made entirely of a kind of stoneware or of steatite, but these are troublesome to remove when they get broken. The best form of burner is that with a brass body and porcelain top. Such burners are made by Leoni of London, Bray of Leeds, and other makers, but usually with some means of reducing the pressure. The fish-tail burner is not suited for burning at a high pressure, under which the two flames refuse to spread out into a flat sheet, but form an irregular flame, at the same time emitting a most disagreeable hissing or blowing sound. This effect may also result from other causes, such as a sharp bend in the gas-supply tube, a speck of dust in one of the orifices of the burner, or, in fact, anything that disturbs the even and quiet flow of the gas. One singular example of this is the following:—If a union jet is burning five cubic feet of gas at 0.5 inch pressure, and a portion of gas is led away by means of a tube inserted a few inches below the flame, although diminished in volume, it immediately begins to blow.

In testing flat flames, the custom has invariably been to present the flat side to the disc of the photometer; but, although the results so obtained are satisfactory in comparing one flat flame with another, they cannot fairly be compared with rat-tail or Argand flames, which give an equal light all round. The edge of a bat flame gives considerably less light than the side, but the difference between the two depends very much upon the richness of the gas, or, in other words, the opacity of the flame. The following example may be given:—A union jet, consuming five cubic feet of cannel gas, at 0.5 inch pressure, gave a light of 27 candles when tested in the ordinary manner with the flat side towards the photometer disc; but the edge gave only 23 candles, and when rotated, so as to give the flame in every position, the average result was as nearly as possible 26 candles, showing that the ordinary test gave one candle too much, or nearly 4 per cent. In the case of paraffine flat-flame lamps, the difference between the front of the flame and the average all round varies from 4 to 10 per cent. In the latter case, the flame is intensely opaque, and of a deep yellow color. All the figures given in this paper refer to the flat side of the flame, and this must be borne in mind in comparing flat with round flames.

The following Table gives the results obtained with Bray's union jets, without obstruction to retard the flow of gas and reduce its pressure. Gas by ordinary test, 26 candles:—

No. of Burner.	At 0.5 Inch Pressure.			At 1.0 Inch Pressure.			At 1.5 Inch Pressure.		
	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
0	1.15	1.96	8.52	1.55	2.35	7.60	1.80	2.45	6.80
1	1.45	3.77	13.00	2.15	5.28	12.28	2.85	5.47	9.60
2	1.70	4.85	14.27	2.50	6.74	13.48	3.10	7.62	12.30
3	2.35	8.98	19.10	3.40	11.73	17.25	4.30	13.67	15.90
4	2.85	11.97	21.00	4.05	15.44	19.06	5.00	16.62	16.62
5	3.25	13.84	21.30	4.50	18.78	20.87	5.55	21.90	19.73
6	4.10	19.60	23.90	5.70	25.60	22.46	—	Gas blows.	—
7	4.75	24.76	26.00	—	Gas blows.	—	—	"	—
8	5.00	26.00	26.00	—	"	—	—	"	—

This Table gives instructive information as to the effects of mass or quantity of gas and of pressure. As regards mass, we see that at the same pressure the light afforded by 5 cubic feet of gas per hour varies from $8\frac{1}{2}$ to 26 candles, according to the quantity burned, the lowest result being obtained with about 1 cubic foot per hour, and the highest with 5 cubic feet. This last result—*i.e.*, 26 candles for 5 cubic feet of gas per hour, burned in a union jet at 0.5-inch pressure, is taken as the standard of comparison in all the experiments on cannel gas. The ratio of illuminating power to quantity is nearly the same at higher pressures, and there is no difficulty in deducing the general law that the value in illuminating effect per cubic foot of gas increases with the mass of the flame.

The effects of pressure are not less striking, and might have been more so had the gas been tested at lower pressures than 0.5-inch and higher than 1.5-inch. The results obtained with a jet consuming 5 cubic feet per hour gave 26 candles at the low pressure, and only 16.6 at 1.5-inch, showing a loss of lighting power amounting to about 36 per cent.; 3 feet per hour, calculated to 5 feet, gave at the low pressure 21 candles, at the high pressure 12.3 candles; the burner being a No. 4 in the one case, and a No. 2 in the other. The medium pressure gave results intermediate between these. At the higher pressures some of the larger sized burners became useless, as already explained.

As, in practice, it is found impossible to distribute gas at a pressure of less than 12 or 15-10ths of an inch of water, various contrivances for breaking the force of the gas have been invented. Among union jets of this kind, the simplest, perhaps, is that of Leoni, consisting of a brass and an iron tube, which fit into one another, and between which a thin film of cotton wool is placed. This is a very good burner, but it cannot be depended upon for delivering exact quantities of gas. Bray has constructed a very good burner, similar to those already mentioned, but having a double ply of cotton cloth stretched across a metal ring placed in the tube, in order to relieve the pressure. The same manufacturer has more recently invented another burner, in which the reduction of pressure is attained by passing the gas through an orifice in a porcelain plate cemented into the lower part of the burner. He calls these "Special" burners, and they are of two kinds—one intended for general use, and the other for street lamps, in which the orifices are somewhat smaller, and in which, consequently, the pressure is further reduced. Morley's patent burner is of brass and vase-shaped, with a porcelain top, and at the bottom one or two small orifices in the metal for admitting the gas. Williamson's jet is similar in principle, but more complicated in construction. Da Costa's burner consists of a hollow vase stuffed with iron turnings, into which an ordinary iron union jet is screwed. There are others, but all have the same object in view, and the simpler and cheaper burners, such as Leoni's and Bray's, accomplish it as successfully as those of more complicated construction, and these have, therefore, been selected for a series of comparative trials, all being made with 26-candle gas. Some of the burners referred to are called regulators, but this is a mere name, for it is obvious that they merely obstruct the flow of the gas, the quantity delivered rising as the pressure is increased. In Bray's "Special" burner the two holes forming the "union" jet are placed at an angle of 120°.

In both series of the "Special" burners, in which the pressure is much reduced, the best results are obtained at 1-inch pressure, while, at .5 inch, the flames are sluggish, and, in some cases, show a tendency to smoke. This is not the case, however, when common gas is used.

Mr. Holdsworth, of Bradford, has introduced a simple arrangement which he calls a gas-feeder, which has been adopted rather extensively in the manufacturing towns of Yorkshire. It is simply a little wedge-shaped piece of lead pierced in the centre with a hole, the area of which is less than that of the holes in the burner, and this is fixed in the gas pipe, several inches from the burner. Several sizes are made, to suit varying circumstances of local pressure, as well as different sizes of burners, and if fitted up by an intelligent workman, they accomplish the end in view very successfully.

Many years ago, Mr. Scholl, of London, adopted the system of placing a

BRAY'S "REGULATOR" UNION JETS FOR CANNEL GAS.

No. of Burner	At 0.5 Inch Pressure. Gas per Hour.	At 0.5 Inch Pressure. Illuminating Power.	At 0.5 Inch Pressure. Illuminating Power per Five Cubic Feet.	At 1.0 Inch Pressure. Gas per Hour.	At 1.0 Inch Pressure. Illuminating Power.	At 1.0 Inch Pressure. Illuminating Power per Five Cubic Feet.	At 1.5 Inch Pressure. Gas per Hour.	At 1.5 Inch Pressure. Illuminating Power.	At 1.5 Inch Pressure. Illuminating Power per Five Cubic Feet.
0	1.00	2.72	13.6	1.50	3.13	10.4	2.00	3.21	8.0
1	1.15	3.75	16.3	1.80	4.30	11.9	2.45	4.40	9.0
2	1.50	5.63	18.7	2.30	7.25	15.8	3.00	7.60	12.6
3	1.80	7.97	22.1	2.75	10.11	18 4	3.55	11.37	16.0
4	2.40	11.26	23.4	3.60	15.21	21.1	4.30	15.32	17.8
5	2.60	12.76	24.5	4.35	20.40	23.4	5.10	22.19	21.7
6	3.15	15.95	25.3	4.95	25.42	25.7	5.80	27.51	23.8
7	3.80	20.07	26.4	6.05	32.75	27.1	..	gas blows.	..
8	4.70	24.76	26.3	7.10	40.63	28.6	..	"	..

BRAY'S "SPECIAL" UNION JETS FOR GENERAL USE.

No. of Burner	At 0.5 Inch Pressure. Gas per Hour.	At 0.5 Inch Pressure. Illuminating Power.	At 0.5 Inch Pressure. Illuminating Power per Five Cubic Feet.	At 1.0 Inch Pressure. Gas per Hour.	At 1.0 Inch Pressure. Illuminating Power.	At 1.0 Inch Pressure. Illuminating Power per Five Cubic Feet.	At 1.5 Inch Pressure. Gas per Hour.	At 1.5 Inch Pressure. Illuminating Power.	At 1.5 Inch Pressure. Illuminating Power per Five Cubic Feet.
0	1.44	5.51	19.13	2.16	9.22	21.34	2.59	10.80	20.85
1	1.55	6.11	19.71	2.36	10.33	21.88	2.87	12.00	20.91
2	1.86	7.50	20.16	2.76	12.38	22.43	3 36	14.51	21.59
3	2.10	8.90	21.19	3.10	14.27	23.01	3.75	17.29	23.11
4	2.44	10.94	22.42	3.62	17.69	24.43	4.41	20.83	23.60
5	2.71	13.39	24.70	4.13	21.13	25.58	5.16	26.17	25.36
6	3.12	15.42	24.71	4.76	24.40	25.63	5.71	28.66	25.09
7	3.63	18.43	25.39	5.51	28.65	26.00	6.70	34.33	25.62
8	4.28	22.26	26.00	6.39	34.37	26 89	7.92	40.67	25.67

BRAY'S "SPECIAL" UNION JETS FOR STREET LAMPS.

No. of Burner	At 0.5 Inch Pressure. Gas per Hour.	At 0.5 Inch Pressure. Illuminating Power.	At 0.5 Inch Pressure. Illuminating Power per Five Cubic Feet.	At 1.5 Inch Pressure. Gas per Hour.	At 1.5 Inch Pressure. Illuminating Power.	At 1.5 Inch Pressure. Illuminating Power per Five Cubic Feet.	At 1.0 Inch Pressure. Gas per Hour.	At 1.0 Inch Pressure. Illuminating Power.	At 1.0 Inch Pressure. Illuminating Power per Five Cubic Feet.
0	1.30	4.85	18.65	1.96	8.22	20.97	2.34	9.70	20.73
1	1.46	6.04	20.68	2.21	9.57	21.65	2.63	11.45	21.77
2	1.73	7.28	21.04	2.56	12.00	23.44	3.01	14.86	24.68
3	2.07	9.36	22.61	3.00	14.64	24.40	3.57	17.68	24.69
4	2.24	10.73	23.95	3.33	16.57	24.88	4.05	20.39	25.17
5	2 68	13.15	24.53	4 08	21.17	25.94	4.85	25.67	26.46
6	2.97	14.77	24.86	4.45	23.73	26.66	5.37	28.87	26.88
7	3.44	17.37	25.25	5.31	28.36	26.61	6.43	34.32	26.69
8	3.84	19.21	25.01	5.92	31.22	26.37	7.23	37.32	25.81

small piece of platinum between the two orifices of the union jet, the result being that the initial velocity with which the gas escapes is spent by striking against this plate, and the gas ascends in a somewhat sluggish flame, which, in the case of cannel gas, has a tendency to smoke, and is easily blown about by currents of air. This is the case also with all union jet flames burned at very low pressures. and, practically, a jet of this kind cannot be burned much below 3-10ths or 4.10ths for small sizes, and 5-10ths for large sizes consuming four or five cubic feet per hour. Scholl's "perfecter," as he has called it, has been used extensively in London and other towns for common gas, but it is not suitable for the richer gas used in Scottish towns.

A flame formed by a jet of gas issuing with considerable velocity possesses a certain degree of stiffness, and resists, to some extent, the influences of currents of air. This is particularly necessary in the case of cannel gas, since, whenever the flame is much deflected by air currents, a portion of the carbon arising from the heating of the richer hydrocarbons (*e.g.* olefines, benzole, &c.) passes off unconsumed, and a smoky flame is the result. In practice, it is necessary to sacrifice a certain proportion of the possible illuminating value, in order to give the flame sufficient stiffness to resist currents of air.

Next to the union jet, the "bat's-wing" is that most commonly used for burning gas. It is simply a little tube closed at one end, in which a straight slit is cut, varying in breadth from about 2-10ths to one millimetre. It is made of cast-iron, brass, porcelain, or steatite; the best form being that having a brass body and steatite top. The flame of the bat's-wing is wider and shorter than that of the union jet, and, in order to be equally effective, requires to be burned at lower pressures. It is particularly adapted for large flames burning from 3½ to 5 cubic feet of gas per hour. With rich cannel gas (25 to 30 candles) it gives results at least equal to the union jet, and with gas of 18 to 22 candles, it is decidedly superior.

The following table gives the results of tests of a series of steatite bat's-wing burners manufactured in Germany. Gas, 26 candles:

No. of Burner.	At 0.5 Inch Pressure.			At 1.0 Inch Pressure.			At 1.5 Inch Pressure.		
	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
2	1.10	4.24	19.27	2.35	9.05	19.25	3.15	11.56	18.35
3	1.45	5.68	19.58	2.65	10.02	18.90	3.55	13.20	18.59
4	1.90	8.76	23.05	3.10	12.71	20.50	4.00	15.41	19.26
5	3.40	16.18	23.80	5.20	24 07	23.14	—	Gas blows.	—
6	4.05	19.09	23.57	—	Gas blows.	—	—	"	—

The considerable loss of light experienced when gas is consumed in bat's wing burners at any but comparatively low pressure, has given rise to many efforts to combine with the jet an apparatus to reduce the pressure of the gas before it issues from the narrow slit. Various burners having obstructions have been constructed, of which Brönner's is one of the best known, It consists of a somewhat pear shaped brass body, with a steatite top, similar to those of which the results are given above, and at the bottom a small piece of steatite in which is an oblong slot. There are, for cannel gas, six sizes of bodies, the sizes depending upon the area of the slots, and five sizes of tops; and as these screw into one another, there are 30 possible combinations. In none of these combinations does the pressure of the gas at the point of ignition, exceed 0.5 of an inch with an

initial pressure of 1.5 inch, while in some it is only 0.2, and in some it is so low that the flame smokes and is useless. The rate of combustion is dependent on three conditions—first, the area of the opening at the bottom; secondly, the area of the slit in the burner; and thirdly, the initial pressure of the gas. The range of combinations enables one to select a burner to suit almost any description of gas or any standard of pressure. The accompanying table gives the results of tests at 1 inch and 1.5 inch, with 26 candles. The burners are not adapted for lower pressures than 1 inch.

For common gas (*i.e.*, of 14 to 16 candles) a different series of tops is provided, in which the areas are considerably greater than in those made for cannel gas, and in which the pressure is reduced to from 0.1 to 0.3 of an inch. These burners cannot be used with cannel gas, although with common gas they are exceedingly effective, and are much in use, especially in London:

BRONNER'S BURNERS FOR CANNEL GAS.

At 1.0 Inch Pressure.					At 1.5 Inch Pressure.				
No. of Burner.	No. of Top.	Cubic Feet per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	No. of Burner.	No. of Top.	Cubic Feet per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
2	2	1.20	5.07	24.13	2	2	1.40	5.25	18.75
2	3	1.40	6.64	23.71	2	3	1.95	7.37	18.90
2	4	..	Smokes	..	2	4	2.30	10.33	22.46
2	5	..	"	..	2	5	2.40	11.24	23.42
2	6	..	"	..	2	6	..	Smokes	..
2¼	2	1.40	5.53	19.75	2¼	2	1.90	8.30	21 84
2¼	3	1.70	8.48	24.94	2¼	3	2.30	10.14	22.04
2¼	4	2.03	10.33	25.49	2¼	4	2.70	12.08	22.37
2¼	5	..	Smokes	..	2¼	5	2.85	14.29	25.07
2¼	6	..	"	..	2¼	6	3.00	15.21	25.35
3	2	1.45	6.27	21.62	3	2	2.00	8 48	21.20
3	3	1.90	8.66	22.89	3	3	2.50	11.34	23.63
3	4	2.13	11.24	26.39	3	4	2.80	14.84	26.50
3	5	..	Smokes	..	3	5	3.15	17.04	27.20
3	6	..	"	..	3	6	3.25	18.07	27.80
3¼	2	1.50	5.81	19.36	3¼	2	2.12	8.85	20.87
3¼	3	1.95	8.30	21.28	3¼	3	2.55	12.63	24.76
3¼	4	2.55	12.08	23.68	3¼	4	3.00	14.47	26.12
3¼	5	2.80	14.38	25.68	3¼	5	3.50	18.07	25.81
3¼	6	3.00	15.38	25.97	3¼	6	3.60	19.45	27.01
4	2	1.60	6.36	19.87	4	2	2.30	9.77	21.24
4	3	2.10	10.69	25.45	4	3	2.90	13.83	23.84
4	4	2.65	13.37	25.23	4	4	3.30	17.06	25.85
4	5	3.45	17.61	25.52	4	5	4.10	21.57	26.30
4	6	3.55	18.07	25.45	4	6	4.20	22.40	26.66
5	2	1.77	7.38	20.85	5	2	2.60	9.68	18.81
5	3	2.30	11.90	25.87	5	3	3.30	13.64	20.67
5	4	3.30	15.40	23.33	5	4	4.00	19.31	24.14
5	5	4.10	20.74	25.29	5	5	5.00	25.36	25.36
5	6	4.30	22.68	26.37	5	6	5 30	27.66	26.10

This table shows that it is easy, with properly adjusted bat's-wing burners, to obtain, with a consumption of from 3 to 5 cubic feet per hour, at least the full effect of illumination exhibited in the standard mode of testing already referred to; and that, even with a consumption of 2 cubic feet, a very favorable result may

be obtained. In no case is the loss of light with bat's-wing burners so great as with badly arranged union jets.

Many other descriptions of improved bat'-wings have been constructed, some of which I have tested. The "Clegg" bat's-wing, manufactured by Sugg, has a steatite top, and a conical brass body closed at the bottom, and with a slit cut in it with a fine saw. The respective sizes of the slits above and below determine the consumption of gas and the pressure at the point of ignition. In Silber's bat's-wing made by the Silber Light Company, one burner is placed above another, both being of steatite, the slit of the lower one being much smaller than that of the upper, and connected by a vase of brass. Only the three smallest sizes of these are suitable for rich cannel gas, the larger ones being intended for gas of lower quality. The result obtained with 26-candle gas are given in the following table:

CLEGG AND SILBER BAT'S-WINGS.

Burner.	At 0.5 Inch Pressure.			At 1.5 Inch Pressure.			At 1.0 Inch Pressure.		
	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
Clegg No. 2.	2.00	9.15	22.87	3.40	14.77	21.72	4.45	18.30	20.56
Do. " 3.	2.90	13.00	22.41	4.45	21.11	23.72	5.70	27.04	23.72
Do. " 4.	4.20	20.37	24.25	6.45	31.20	24.19	—	Blows.	—
Do. " 5.	4.80	23.92	24.92	—	Blows.	—	—	"	—
Silber A....	0.95	3.07	16.16	1.50	6.31	21.03	1 90	10.03	26.40
" B....	1.55	7.34	23.68	2.35	12.07	25.68	3.00	15.04	25.07
" C....	2.20	11.24	25.54	3.30	17.27	26.17	4.25	23.12	27.20

Several varieties of regulating bat's-wings have been invented by Sugg, Witthoft, Winsor, and others, the principle of their construction being to check the flow of gas by means of a plug regulated by a screw. At a given pressure in the pipes the burners may be regulated to deliver any desired quantity of gas, and in the experiments on the Winsor and Sugg burners quoted below, they were regulated so as to burn the number of cubic feet per hour corresponding with the numbers marked on the burners. Gas used equal to 26 candles:—

SUGG'S "WINSOR" BAT'S-WING.

No.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
2	2	9.60	24.00
3	3	15.00	25.00
4	4	19.87	24.84
5	5	25.20	25.20
—	—	—	—

SUGG'S REGULATING BAT'S-WING.

No.	Gas per Hour.	Illuminating Power.	Illuminating Power per Five Cubic Feet.
2	2	9.20	23.00
3	3	15.34	25.56
4	4	19.90	24.88
5	5	24.75	24.75
6	6	28.74	23.95

If two bat's-wing flames are brought together, especially if the slits be narrow, the gas of low quality, and the pressure somewhat high, the illuminating power of the united flame is greatly in excess of the sum of the two tested separately. Upon this principle is constructed a double-slit bat's-wing, the slits being about one millimèter apart, which is used in Manchester and other towns in England, and which is an excellent burner for gas not exceeding 20-candle power, but gives a somewhat smoky flame with gas of high quality.

The only other bat's-wing that requires

further to be noticed, is the patent regulating bat's-wing used in the United States of America, where it was introduced in 1871, and which is practically the only flat-flame burner capable of burning advantageously the "air gas," made by saturating air with the vapor of petroleum spirit. It consists of a very much elongated iron bat's-wing, with exceedingly narrow slit, surrounded by a brass tube at the distance of about 2 millimèters. Into the space between the two, gas is admitted by a wide orifice (the amount being regulated by a screw), and this gas ascends entirely without pressure, while the force of the gas issuing from the narrow slit spreads it out into a fine soft flame. This burner gives excellent results with gas of all qualities, but its shape is not adapted to the gas-fittings in use in this country, and it has not been used here except for air gas made for private houses.

Argand burners are exclusively used in the photometric testing of common gas, and they are also employed rather extensively for lighting shops and public buildings, but to a limited extent for private houses. They give a higher photometric effect with common gas than any flat-flame burner known; and even with cannel gas, the best descriptions, especially those of Sugg and Silber, give results which approach very near to those obtained when the gas is tested at a comparatively low pressure by large-sized fish-tail or bat's-wing burners.

The original form of Argand was a brass double cylinder with, above, an iron ring perforated with small holes, and below, a "crutch" or forked tube, by which the gas was introduced at opposite sides. A wide and short glass chimney was used, but this was afterwards modified in a variety of ways with a view to making the current of air impinge more directly upon the flame, and so increase the intensity of combustion. The holes being small, the gas escaped at a comparatively high pressure; and the character of the flame, both as to volume, shape, and luminosity, depended partly upon the initial velocity with which the gas escaped from the burner, and partly upon the shape and dimensions of the funnel. The enlargement of the holes, enabling the gas to escape at a moderate pressure, was proposed by the late Dr. Letheby, who was associated with Mr. Sugg, by whom a great many improvements in Argand burners have been introduced. The Letheby burner raised the apparent quality of London gas from 12 to 14 candles, and a further increase of two candles was obtained by Sugg's "London" Argand, now generally accepted as the standard burner for testing gas made from common coal. In this burner the principle is recognized of permitting the gas to escape practically without pressure, the shape and volume of the flame being determined by the narrow funnel and a "cone" of thin metal, which serves to throw the current of air into close contact with the outside of the flame. The upper portion of the burner is of steatite, and, instead of the ordinary "crutch" below, the gas is introduced by three very narrow tubes. A number of sizes of this burner are made, of which details are given below; but the following are the various dimensions of the standard burner used in photometry: Diameter of steatite top, external, 0.84 inch; internal, 0.47 inch; number of holes, 24; diameter of holes, .04 inch; chimney, $6 \times 1\frac{3}{4}$ inches for gas of 14 candles, and 6×2 for gas of 16 candles. The narrow funnel and the cone restrict the quantity of air to very little more than is required to burn the gas, thus avoiding the diminution of light which results from a too rapid combustion of the gas, and the cooling effect of a large volume of air. The pressure of the gas inside the steatite top is considerably less than 0.1 of an inch, and that required to pass five feet per hour through the complete burner is about 0.2 of an inch.

In the burner introduced by Mr. A. M. Silber, the steatite top with wide holes (about one millimèter, or .04 inch) is also adopted; but the body of the burner is considerably elongated, and the so-called "cone" is long and cylindrical, with a curved top. A very essential feature in the Silber Argand is an air-tube introduced into the center of the jet, which is said to carry a portion of the air to the upper part of the flame, and which certainly has a remarkable effect in steadying it. The chimney is $8 \times 1\frac{3}{4}$ inches, and, in consequence of the form of the "cone," is kept so cool at the bottom

that it may be handled without difficulty while the flame is burning. Chimneys of 10 inches high are also used, but, while the consumption of gas is thereby increased, the illuminating power per cubic foot of gas remains almost quite constant. Mr. Silber has discovered the remarkable fact that a globe or vase placed below his Argand increases the illuminating power considerably, and I have had an opportunity of verifying his statement, both as to common and cannel gas, the increase with the former being about a candle, and with the latter about 1½ candles. The effect of placing a vase below an ordinary union jet was also tried, but no increase of light was obtained, while the flame showed a distinct tendency to "blow." The flame of the Argand should have its illuminating power increased 6 per cent. by passing the gas through a glass vase (or cylindical metal box, which answers the purpose equally well) is a phenomenon which appears to be at present incapable of explanation.

The following table gives the results of photometric tests of various Argand burners, with cannel gas of 26 candles illuminating power. From 3 to 4 cubic feet of gas per hour were burned in each case, and the result calculated to the usual standard of 5 feet per hour.

	Size of Chimney.	Illuminating Power.
German porcelain Argand with cone (40 small holes)........	8 × 1¾	17.80
Leoni 40-hole burner, "adamas" top, with cone......	7 × 1¾	18.18
Sugg-Letheby 15 holes in steatite ring, perforated gallery ..	7 × 2	18.86
American regulating Argand, brass, 40 very large holes....	5 × 2	21.03
Sugg's "London" Argand, 24 holes, with cone & regulator.	7 × 1¾	22.40
Silber 40-hole burner, steatite top, cone, and centre tube...	8 × 1¾	22 54
Silber 32-hole burner, steatite top, cone, and centre tube...	"	23.08
Silber 24-hole burner, steatite top, cone, and centre tube...	"	25.04
Silber 24-hole burner, with glass vase below.................	"	25.61

The following tests were made with various Argands in order to test the effect produced by the cone and center tube of the Silber burner.

	Pressure at Inlet of Burner Inch.	Gas per Hour Cubic Feet.	Illuminating Power.	Illuminating Power for Five Cubic Ft.
Sugg's "London" Argand, 24 holes.	0.19	3.30	15.00	22.73
Sugg's do., without cone...........	—	2.60	11.80	22.70
Sugg's do., older pattern, 36 holes.	0 17	4.00	16.75	20.94
Sugg's do., without cone...........	—	4.00	17.00	21.25
Silber's 24-hole burner, complete....	0.05	4.00	19.20	24.00
Silber's do., without cone, but with air-tube...... ..	—	4.15	19.00	22.89
Silber's do., without air-tube, but with cone...........	—	3.80	17.20	22.63
Silber's do ,without cone or air-tube..	—	3.40	13.10	19 26

These tests show that the cone, by increasing the draught, enables a larger quantity of gas to be burned, an effect which could be obtained equally well by increasing the height of the chimney; and the air-tube of the Silber burner also produces a similar effect, increasing at the same time the heat and the illuminating power of the flame, and its stability. Indeed, the Silber burner without cone and center tube, and especially when the latter is removed, gives so unsteady a flame that it is practically useless, while in its complete condition it gives the steadiest flame of any Argand yet constructed.

A series of experiments were made in order to ascertain the relative dimensions of the inlet and the outlet of various burners. The upper steatite portion of each burner was removed and fitted up in a little bit of apparatus extemporized for the purpose, so that gas could be passed through the holes, while the bottom portions were simply screwed on in the usual manner, and the gas allowed to escape without lighting it. In all the trials the pressure of the gas was maintained steadily at 0.2 of an inch of water. The numbers represent cubic feet of gas per hour:

	Bottom.	Top.	Complete Burner.
Sugg-Letheby 15-hole burner	16.7	28.7	14.6
Sugg 24-hole standard "London" Argand	4.9	28.8	4.5
Sugg 36-hold (older pattern)	6.1	29.1	6.0
Silber 34-hole	17.7	29.5	17.0
Silber 40-hole	19.1	28.8	18.7

These results show that the pressure of the gas is checked much more efficiently at the bottom of the burner by Sugg's arrangement than by that of Silber, and, in fact, the latter has usually attached to it a small regulator, adjustable by a screw, without which, and when regulated only by a stopcock, a disagreeable hissing noise is produced by the passage of the gas through the almost closed stopcock, unless the latter is far removed from the burner.

The "Bec a Bengel," or Bengel Argand burner, used for gas-testing in Paris, has a porcelain top with 30 rather small holes, a brass cone, and at the bottom what is called a "panier," constructed of porcelain, and pierced with numerous small holes for the admission of air. The chimney is $8 \times 1\frac{3}{4}$ inches. With 26-candle gas it burned 2.5 cubic feet, and gave a light of 10.8 candles, or, for 5 feet per hour, 21.6 candles.

Sugg has constructed a series of "London" Argands, burning from 3 to 12 cubic feet per hour of common gas, and from $1\frac{1}{2}$ to $7\frac{1}{2}$ cubic feet of cannel gas per hour. These from A to I resemble in every respect the standard "London" burner already described; K has, in addition, a single or rat-tail jet in the center, and that marked double is formed of two concentric Argands. They gave the following results:—

(*See Table on following column.*)

Burner.	Number of Holes.	Chimney.	Height of Flame.	Gas per Hour.	Illuminating Power.	Illuminating Power for Five Cubic Feet.
A	15	$6 \times 1\frac{1}{8}$	$2\frac{1}{4}$	1.85	7.67	20.73
B	18	"	$2\frac{3}{4}$	2.65	11.90	22.45
C	21	$6 \times 1\frac{3}{4}$	3	2.85	12.63	22.16
D	24	$7 \times 1\frac{3}{4}$	3	3.25	13.74	21.14
E	27	"	3	3.40	14.67	21.57
F	30	"	$3\frac{1}{4}$	3.72	15.97	21.48
G	33	$8 \times 1\frac{3}{4}$	$2\frac{3}{4}$	4.50	19.13	21.25
H	36	9×2	4	5.05	21.17	20.96
I	40	"	4	5.30	22.30	21.04
K	42	"	$4\frac{1}{2}$	6.50	28.40	21.84
Double	54—21	$10 \times 2\frac{1}{2}$	6	7.80	36.40	23.33

It is right to state that all these burners are constructed to burn common rather than cannel gas. A Silber Argand of 24 holes, with chimney 8 by $1\frac{3}{4}$, was tested at the same time for comparison, and gave, for a consumption of 3.75 cubic feet per hour calculated to 5 feet, an illuminating power of 24.02 candles, a somewhat higher result than was obtained with any of Sugg's series, and proving that Silber's Argand is well adapted for burning cannel gas.

Experiments were made in order to ascertain the loss of light resulting from the use of globes of different kinds, and of various shapes. The loss is always considerable, and in many cases excessive, and it results partly from the absorption of light from the material of the globe, and partly from the draught caused by the ascension of the heated air in the confined space. As regards material, a piece of clear window-glass, held in front of a gas-flame, diminishes the light to the extent of about 10 per cent.; but in the case of a clear globe it is, in some cases, less, owing to the reflection from the surface furthest from the photometer. Globes frosted or ground all over, technically known as "moons," absorb about 25 per cent. of the light when well shaped, and opal or "cornelian" globes 40 to 50 per cent., according to the thickness and quality of the glass. The following results were obtained with globes of different sizes ground all over, and show the effect of increased draught in diminishing the light:—

	Per Cent.
6-inch globe caused a loss of	25
$7\frac{1}{2}$ " " "	$27\frac{1}{2}$
10 " " "	33

All these globes had the usual-sized openings below—about $1\frac{3}{4}$ inches in diameter. Experiments were made with clear $7\frac{1}{2}$-inch globes, having openings below, varying from $2\frac{5}{8}$ inches to 1 inch in diameter. The source of light was a

Brönner bat's-wing, No. 5 top, No. 4 bottom, burning, under a pressure of 1 inch, 3.35 cubic feet of gas.

		Candles.	Per Cent.
The naked flame gave a light of		16.8	
With clear globe, opening below	2⅜ in.,	15.4;	loss 8.3
" "	2¼ in.,	15.2;	9.5
" "	2 in.,	13.6;	19.0
" "	1½ in.,	13.0;	22.6
" "	1 in,	12.0,	28 6

With the two larger-sized openings, the flame was perfectly steady, with that of two inches there was a slight flickering caused by the draught; this was more marked with the 1½-inch opening, and was excessive with that of one inch, making the flame practically useless as a source of light. It is evident, therefore, that the openings of the globes should be as wide as possible, and not less than 2¼ inches. The cornelian globes used in Brönner's system of gas lighting have an aperture of 2⅜ inches diameter, and Sugg has introduced globes of similar material which he calls "Albatrine," but with openings of about 4 inches diameter. These globes are constructed of various sizes, to suit certain burners, both bat's-wing and Argand, and the combinations are known by certain names, such as the Westminster, Viennese, Frankfort, Italienne, Parisienne, &c. Some of these arrangements are fitted with regulators, with the intention of maintaining a constant pressure.

One of the difficulties connected with gas illumination is that the pressure in the mains varies considerably in different parts of a town, and at different hours of the day and night. One result is, that a system of lighting, adapted for a part of a town situated in a low level, will show inferior results in a more elevated situation. A rise of 10 feet gives, roughly, a tenth of an inch of increase of pressure, so that it may easily happen that in the same town or city the pressure in one place may be one inch, while in another it may be 2½ inches. Again, the pressure of the gas, as sent out from the gas-works, is altered from time to time, in accordance with the consumption, and as public works, shops, &c., are suddenly lit up or extinguished at certain hours, private consumers are annoyed, in the one case by the falling off in the amount of light, and in the other by a flaring flame and hissing sound, both of which are very irritating. The cure for these evils is to be found in the use of governors or regulators. Every district of a town, the elevation of which is such as to effect appreciably the pressure of the gas, should have a governor, which may either be self-acting, to maintain a constant pressure throughout the day, or to vary sympathetically with the governor at the gas-works. Many of these have been invented, among which may be mentioned those of Cathels, Peebles, and Foulis. The pressure in the mains should not be reduced below 12 or 14-tenths of an inch, but as even that is too high a pressure for the economical burning of gas, each house should have a regulator in order to reduce the pressure constantly to about 7 or 8-tenths. Some of these regulators are dependent on the action of the gas upon a broad leather disc, attached to which is a ball and socket valve, while others have metal or glass bells floating in mercury, and acting upon a valve of the same kind. Both of these work satisfactorly when properly constructed. Among the best dry regulators are those of Sugg of London, and Peebles of Edinburgh, while the best mercurial governor that I have seen is that of Busch of Oldham. In the case of public works and other buildings consisting of several floors, a regulator should be placed on each floor. Street lamp regulators are of great importance, and great attention has been given to the perfecting of them by various ingenious mechanicians. The kind the largest number of which are in use at the present time resembles the dry house regulator already mentioned, the construction being quite similar. These little instruments are made by a great many gas engineers, among whom Sugg and Peebles may be named. The principle involved in the action of the apparatus will be at once understood by a glance at the sectional drawing I have placed upon the wall. It is a regulator, not of volume, but of pressure, and hence the quantity of gas consumed in any street lamp provided with it depends upon the burner. There is an objection to this regulator, and it is a serious one— the leather diaphragm becomes in time hard and stiff, and ceases to act freely, and, unless it be renewed at intervals,

say 12 or 18 months, the instrument is by no means satisfactory. The next street lamp regulator in point of period of introduction is Giroud's rheometer. This beautiful little instrument, which delivers a constant volume of gas, consists of a short cylinder containing glycerine, in which floats a bell of very thin metal, and formed at the top into a cone, the apex of which passes through an orifice in the cover of the cylinder. In the bell itself there is a small hole, through which the supply of gas to the burner must pass. An increase of pressure causes the bell to rise, and the cone to enter the orifice above, thus reducing the area of the aperture through which the gas has to make its way to the burner. The regulation of the rheometer is very perfect, but it ceases to be effective in some eventualities which occasionly occur.

The most recent street-lamp regulator may be called a dry rheometer. It delivers, like the instrument just noticed, a constant volume of gas, but the bell, or substitute for it, instead of floating in a liquid, is simply supported while in action by the pressure of the gas. A regulator of this kind is first indicated in the book published by Giroud, but I am not aware that he ever actually reduced his idea to practice. Victor Bablon, of Paris, patented in 1875, "an apparatus for regulating the flow of lighting gas," in which the float, if it may be so called, is a disc of thin metal connected to a small hollow spindle. It has been introduced somewhat extensively in France, but is almost unknown here. It works with somewhat greater friction than the "needle governor" of Peebles, which is, to my mind, the perfection of gas regulators. In a little cylinder stands a so-called needle, on the point of which rests a flanged cone of exceedingly thin metal. At one side of the cylinder there is a small tube leading away the gas, and by means of a screw working into the side tube, the instrument can be made to deliver any desired number of cubic feet, which it does with surprising accuracy, provided that the pressure of the gas is not less than eight-tenths of an inch. In trials I have made I have not found the variations of volume at different pressures to exceed one per cent. With such a regulator as this, it would be possible to employ Argand burners for street lamps. These burners, when of the best kind, are exceedingly sensitive to quantity of gas. If you have a Sugg or Silber Argand regulated so as to be near the smoking point, and so giving the highest illuminating value that the gas is capable of yielding, the smallest additional supply of gas will cause the flame to smoke. I have here one of Silber's Argands fitted to a needle governor, and you will have an opportunity of seeing the regularity of the flame under different conditions of pressure. I should have mentioned, before the needle governor, the invention of Flürscheim, patented in this country by Borradaile in 1877, but it resembles Bablon's instrument closely, and differs from it chiefly in details.

One other description of regulator remains to be described—that which may be used in connection with the ordinary burners in our apartments. It must necessarily be small, in fact, it should not be much larger than the burner itself. Sugg has, for a number of years, supplied a regulator of this kind, consisting of a leather diaphragm with ball and socket arrangement, but it is a little uncertain in its action, and, so far as my experience goes, not altogether satisfactory; and, besides, it is too large to be useful except as part of a special system of lighting. Peebles has been endeavoring to reduce his needle governor to similar dimensions, and although he does not claim to have yet produced an altogether perfect instrument, in so far as it requires about .8 of an inch of pressure to put it in action, he has great hopes that he will yet be successful.

In this paper I have attempted to indicate the process recently made in the way of developing the photogenic power of coal gas, and the direction in which further improvements may be looked for. I shall be glad if my remarks have the effect of attracting attention to a subject of such interest and importance.

Before concluding, I wish to make one other remark. Last night, on arriving in London, I saw the Thames Embankment illuminated by electricity, and was very much satisfied with the appearance. Then I went across Waterloo Bridge, and saw those lamps in the Waterloo Road; and I must say I was very much struck

with the magnificent light afforded. That certainly was a step in the right direction, although I am of opinion that we are striving too hard to light up our streets. I do not see any necessity for lighting up the streets to make them equal to daylight. What we want is to see where we are going, to avoid being robbed at night, and so on, but I think it is a great mistake that we should strive to make our streets as light as they are during the day. However, there may be economy in these very large burners, and I think there is, though I have not tested them, and I am not able to speak positively. We are very much obliged to Mr. Sugg for lending us these two lamps to-night.

THE TRANSMISSION OF POWER BY ELECTRICITY.

From "The Engineer."

The various reports that have recently been issued relating to the economy of the electric light, all concur in stating that for street purposes gas is more economical than electricity. The concensus of opinion, on the other hand, is rather startling in its unanimity when the lighting relates to large open spaces, or to that which requires penetrating power and brilliancy. This, however, does not surprise any one who understands the theory of the electric light, and the methods of its production. A few weeks since the daily papers, with excess of zeal, produced for the edification of their readers a series of spurious problems, the solution of any one of which was expected to enable electricity to ring the death knell of gas. Among others, the divisibility of the light had great charms, and ever and anon it was discussed as the brilliant discovery of some patentee. The solution of the problem—if, indeed, there is such a problem to solve—still remains amongst the unknown. The multiple are and its capabilities were as well known to electricians ten years ago as they are to-day. Professor Tyndall testifies to this when he says:—"The principles which regulate the division of the current and the development of its light and heat are perfectly well known. There is no room for a 'discovery,' in the scientific sense of the term, but there is ample room for the exercise of that mechanical ingenuity which has given us the sewing machine and so many other useful inventions."

The production of light by electricity has not wholly engrossed philosophers, and of late a good deal has been said concerning the transmission of power to considerable distances by electricity. It may not be uninteresting to explain some of the facts to such of our readers as are not well versed in electrical science. Electricity, as is well known, in many cases acts as if it were a fluid; hence the old hypotheses of one fluid and two fluids. Just as we can lead water by means of pipes from a higher level to a lower level, so we can conduct electricity from one point to another. These points can no more be taken at random with electricity than with water. They must not only be connected by suitable materials, but must be in peculiar electrical conditions. Modern electricians use the term "potential" to indicate an electrical condition, and so state that electricity passes from a point of higher to one of lower potential just as water descends a hill. Whenever and wherever electricity is generated, there is matter existing in these two states, showing, as is commonly said, attractive or repulsive properties, or, more often still, being designated as simply in a positive or a negative condition. We are inclined to the opinion that the phenomena seen are always similarly produced, and that the negative phenomena are due to a differential action; in other words, that electrical repulsion does not exist, but everything is due to attraction. This is heretical, but if we err we do so in good company. One of the fundamental laws of electricity is, that sooner or later the matter thus differently constituted regains its normal condition. We can facilitate or retard this return by the interposition of different materials between the two

points. Thus, if we connect the points by means of a metal wire, the normal state will soon be reached; but, on the other hand, if we use a rod of vulcanite or glass, the return to the normal condition may, under favorable circumstances, be indefinitely retarded. We term metals conductors, because of this property, and such substances as wax, vulcanite, or glass, non-conductors or insulators. The best conductor is silver, but pure copper is almost as good. Then come in the list respectively gold, cadmium, zinc, tin, iron, lead and platinum. Considering these metals, if the conductivity of silver at 0° Cent. or 32° Fah. be represented by 100, that of platinum is nearly represented by 8; increasing the temperature of the metal, however, decreases its conducting power; and at 100° Cent. or 212° Fah. silver and platinum conduct in the proportion of 71 to 6.6. It would not be difficult to explain the cause of the decreased conducting power at the higher temperature; but this is out of our way at present. The fact, however, has to be considered by the electrician when engaged with dynamic electricity, as in the case of the electric light, inasmuch as one of the problems before him is the conduction of a current to a greater or less distance. He requires his current to act at a certain point, and does not wish to lose any more of his power than he can help before he brings it to that point. No substance is a perfect conductor, and no substance is a perfect insulator. Silver presents some resistance to the passage of the current, so does copper and, to a far greater extent, do platinum and iridium. A long wire has a greater resistance than a short wire of the same area, a wire of small section than one of a large. Now the heating effect of a current depends a good deal upon the resistance it encounters. Taking time as constant, the equation $H=C^2R$ gives the heat effect in terms of current and resistance. Another law states that the resistance of a wire varies directly as it length, and inversely as the area of its section, or as the square of its radius. The current, according to Ohm's law, is expressed by the formula $C=\frac{E}{R}$, where E is the electro-motive force of the motor, and R the resistance of the circuit. From such considerations as these it will be seen that calculations can be made as to the size of conductor of any given material required to carry a given current. Dr. Siemens, in his well-known Glasgow address, stated that a 2in. copper wire would convey 1000-horse power from a waterfall to a distance of thirty miles. Mr. T. B. Sprague, one of our more careful and conscientious electricians, in commenting upon this, concludes that, for the above purpose, it would be necessary to provide (1) dynamic converters capable of developing 1000-horse power; (2) 1900 tons of copper rod properly insulated; (3) a dynamo-electric machine capable of generating a current at starting 2000-horse power; and (4) means of cooling that machine, and dissipating rapidly a constant energy equal to 758-horse power. Dr. Siemens admits that, if Mr. Sprague's data are correct, his 2in. wire would have to be increased to 3in., which he maintains would be ample. The cost of such copper wires as these would be enormous, and militate strongly against the general introduction of the electric light. It has been suggested that iron, as being so much cheaper, would obviate this difficulty; but as we have indicated, iron being comparatively a bad conductor, its sectional area must be larger than that of copper, and thus in the long run the cost of conductors would not be greatly diminished by its use. The problem being left in this state in England, Professors Thomson and Houston in America took it up, and their conclusion is rather startling to those who are discussing the problem here. They say: "It is possible, should it be deemed desirable, to convey the total power of Niagara a distance of 500 miles or more by a copper cable not exceeding ½ in. in thickness. This, however, is an extreme case, and the exigencies of practical working would not require such restrictions as to size." We shall now endeavor briefly to indicate the *modus operandi* pursued by these gentlemen, so that readers will be then in possession of the general details of this particular problem.

Suppose two machines connected by a cable of, say, one mile in length, one of these machines being used for the production of a current, the other for the conversion of the current into power.

The two terminals not connected are put to earth. "Let us suppose that the E M F of the current is unity. Since, by the revolution of one machine, a counter electro-motive force is produced in the other, the electro-motive force of the current that flows is manifestly the difference between the two." The resistance of the two machines being made equal to 1, and the connections to .01, the current will be, according to Ohm's law, equal to $1 \div .01 = .99$. If to this system be added an additional machine and a second converter, also another mile of wire, the current will be $2 \div 2.02$, and so on, the addition of 1,000 miles of wire requiring the addition of 1,000 producing and 1,000 converting machines. The expense of this would be very great, and the statement is made merely to show that the problem taken in this light is not unsolvable. In such an extreme case a difficulty would arise in obtaining the requisite insulation, as the electro-motive force would be very great. The authors calculate that "for the consumption of 1,000,000-horse power a cable of about 3 in. in diameter would suffice under the conditions stated." In the results given the authors do not seem to have considered the heating effects upon the wire of small section for short distances, which would, we think, materially affect their calculations; and, again, it must be remembered that to convey the electricity in this way is one thing, but to use the current for lighting purposes is another. The lamps must be arranged either in series or in multiple arc, and while in the latter case the resistance could be kept as low as required, other details would come in so as to require extensive modification of these calculations. The problem then remains—how to convey large currents, obtained from natural sources of power, economically to long distances?

NEW AND SIMPLE METHOD OF RIVER TRAINING.

By A. GEPPERT.

From "Wochenschrift des Oest. Ingenieur-und Architekten Vereines," published by Institution of Civil Engineers.

THIS mode of river training may be briefly described as consisting in the construction of a partition of sheet piling across the land submerged during floods, and so placed as to connect the two points of the river between which the course is to be straightened. On either side of the sheet piling and along the entire length, a wide trench is excavated, the bottom of which corresponds to the level of ordinary water line. The greater portion of the flood water is thus made to flow off through this artificial channel, and in doing so tends to scour a new river bed in it.

The method has recently been applied in the river Lech, at "The Gechtle," and the author gives an account of the works, and of the results obtained. The pile planking extends over a length of 1,867 feet, and intersects a large island which obstructs the direct course of the river. It is 4 feet high from the bottom of the ditch, and is formed of piles 15 feet 6 inches long, driven at intervals of 6 feet 3 inches, and planked on one side. The piles have a diameter of from $8\frac{1}{4}$ to $9\frac{1}{2}$ inches, and are shod in the ordinary way. The deal planks are $2\frac{1}{16}$ inches thick. The trench was made 6 feet 3 inches wide on each side of the line of piling, the soil of the excavation being simply thrown on the banks. The total cost of the work came to 9*s.* 6*d.* per lineal yard.

During the summer of 1877 various floods occurred in the Lech, which, although continuous, did not attain to any height. The last one alone reached the top of the piling and produced notable changes in the bed of the river, without, however, damaging in any way the planking. In a plan the author shows the course of the river both before and after the flood. It gives a fair idea of the effect which the work has had on the course of the river. After the water subsided it was found that the river had taken a new bed all along the first third of the pile planking. Then, encountering higher ground in front, it had deviated to the left and had fallen back into the former bed of its left channel. On

the other side of the planking, the right branch had also slightly altered its course, and had been considerably reduced through silting. These first results were deemed so satisfactory, considering the small height to which the floods had risen, that it was decided to widen out the unaffected parts of the ditch before next year's floods. It is thought probable that the river will then scour out its bed along the entire length. As soon as this result will have been achieved, the author will report again on the subject.

THERMODYNAMICS.

By HENRY T. EDDY, C. E., Ph. D., University of Cincinnati.

Written for VAN NOSTRAND'S MAGAZINE.

II.

15. THE LAW OF VARIATION OF INTERNAL ENERGY.—When a unit of a given substance is caused to pass from one state of volume and pressure to another, the total increment of its internal energy depends alone upon its initial and final states, and in no way depends upon the route by which the passage is effected.

By (4), $di=dh-dw$ . . . . . (25)

$$\therefore\ i_2-i_1=\int_1^2 dh-\int_1^2 dw \quad . \ . \ . \quad (26)$$

In Fig. 2 let the substance pass from state 1 to state 2 by some route x: then, as shown in art. 10, the area between x and the adiabatics e_1 and e_2

$$=\int_1^2 dh=\int^1 de,$$

is the heat imparted; and the area included between the ordinates p_1, p_2, x and the axis of v

$$=\int_1^2 dw=\int_1^2 pdv,$$

is the external work performed. Hence by (26) the increment of internal energy is the difference of these areas, but in this subtraction the common area 123 is canceled, and the remainder

$$i_2-i_1=e_2p_2e_1-p_2e_1p_1,$$

is independent of the route x.

It appears from reasoning exactly like that employed in art. 14 that (26), which may also be written

$$i_2-i_1=\int_1^2 tde-\int_1^2 pdv \quad . \ . \quad (27)$$

is an exact integral, since its value depends alone upon the states 1 and 2.

16. THE LAW OF VARIATION OF INTERNAL WORK.—When a unit of a given substance passes from one state of volume and pressure to another, the total increment of its internal work depends alone upon the initial and final states, and not upon the route.

By (1) $dm=di-ds$ . . . . (28)

$$\therefore\ m_2-m_1=\int_1^2 di-\int_1^2 ds \quad . \ . \ . \quad (29)$$

But di is and exact differential by art. 15; and ds is also an exact differential by art. 6 or art. 8: and, as has been shown in art. 14, the statement that any algebraic expression is an exact differential, and the statement that it depends for its value, upon its initial and final states alone, are synomymous.

17. GENERALITY OF THE SECOND LAW.—The second law has been applied in art. 10 to determine the rate at which heat is imparted to a substance. Rankine, however, seems to apply this law only to determine the rate at which external work will be performed. We wish here to point out its wider scope, and have, with this object in view, so worded the statement of it in art. 4 as to express its generality.

The three ways of expending the heat imparted which are detailed in art. 7 are related to each other in some purely dynamical way, such that the proportion of the whole which is expended in each of these separate effects depends on two

considerations only: viz., the kind of substance to which heat is imparted, and the amount of energy acting. The second law asserts that, for a given kind of substance, these effects depend upon the energy acting and not on the state of the substance. The question here suggested is this: What are we entitled to regard as separate "effects" such as are spoken of in the second law? It seems sufficiently clear, that the effects mentioned in art. 7 can be regarded as separate effects to each of which the law applies, and hence to either of their sums. It may be possible to show that some one of these effects is a composite effect, to each of whose parts the law also applies. For example, the internal work may be written thus:

$$dm=\left(\frac{dm}{dt}\right)_v dt+\left(\frac{dm}{dv}\right)_t dv \quad . \quad . \quad (30)$$

$$dm=\left(\frac{dm}{dt}\right)_p dt+\left(\frac{dm}{dp}\right)_t dp \quad . \quad . \quad (31)$$

$$dm=\left(\frac{dm}{dp}\right)_v dp+\left(\frac{dm}{dv}\right)_p dv \quad . \quad . \quad (32)$$

and it would seem that the second law could be applied to some of the components of the internal energy which are expressed by these equations.

Especially is this the case if we accept the molecular theory of matter, for then certain of these components acquire a mechanical significance.

18. Diagrams of Energy.—The diagrams which we have thus far employed, have graphically represented the state of a unit of any given substance by using its volume and pressure as rectangular co-ordinates; and in such a volume-pressure diagram it appears from elementary considerations, that area represents energy or work. In accordance with this idea is the expression for the increment of external work previously employed, viz., $dw=pdv$ . . . . (10) which at the same time is an elementary area of such a diagram.

Equation (10) integrated around the cycle of the indicator card is the work performed per stroke.

It has also been shown in art. 10, that the energy of the total heat imparted in causing a unit of a substance to pass along the route x from the state 1 to the state 2, is represented by the area included between the curves $e_1 x e_2$, when e_1 and e_2 are adiabatics through 1 and 2 respectively, which extend to infinity.

It is, however, equally possible to represent graphically the state of a substance by using as rectangular co-ordinates any two variables which determine its state. In a valuable paper on this subject, J. Willard Gibbs* treats in detail the entropy-temperature diagram, and the volume-entropy diagram.

In an entropy-temperature diagram, it appears from the equation of art. 11, viz.

$$dh=tde \quad . \quad . \quad . \quad . \quad (18)$$

that here also area represents heat imparted or energy; and upon such a diagram, lines of equal volume and equal pressure (called isometrics and isopiestics) can be drawn from experimental data, just as adiabatics (*i.e.* isentropics) and isothermals are drawn upon the volume-pressure diagram. Indeed, the entropy-temperature diagram is found to be specially suited to assist the mind in grasping the thermodynamic relations flowing from the second law.

19. Carnot's Cycle of Greatest Efficiency.—Let a unit of weight of any given substance pass through a series of states represented on a diagram of energy by a closed figure; then, at the end of this complete cycle, the substance is restored to its initial state as to temperature, internal energy, etc. Hence, the area of the closed cycle, which represents the external work, represents also the heat imparted, minus the heat rejected. Draw an adiabatic through each of the two points 1 and 2 of the cycle whose difference of entropy is greatest. Call the upper part of the cycle from 1 to 2 the route x, and the lower part from 2 to 1 the route y, then x and y together form the closed cycle.

The heat imparted while the substance is carried along an element of the route x lying between a pair of adiabatics which are very near each other, is, by (18), $dh_x = t_x\, de$, and the heat rejected on the route y between the same pair of adiabatics is, $dh_y = t_y\, de$.

$$\therefore \frac{h_y}{h_x}=\frac{\int_1^2 t_y\, de}{\int_1^2 t_x\, de} \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

* Graphical Methods in the Thermodynamics of Fluids. By J. Willard Gibbs, Professor of Mathematical Physics, in Yale College. Trans. Conn. Acad. Vol. II, part 2.

is the ratio of the total heat rejected to the total heat imparted.

The heat rejected is a loss, and in order that this loss may be as small a part of the total heat imparted as possible, it is evident that each element $\int_1^2 t_y\, de$ must be as small as possible, while each element of $\int_1^2 t_x de$ must be as large as possible.

Suppose that the highest available temperature of the source of heat is t_1, and the lowest attainable temperature of the refrigerator which receives the rejected heat is t_2, both being of unlimited capacity; then it is evident that the proportion of lost heat is the least possible when $t_x = t_1$ and $t_y = t_2$; and in this case let $h_x = h_1$ and $h_y = h_2$, $\therefore$ by (33),

$$\frac{h_2}{h_1} = \frac{t_2 \int_1^2 de}{t_1 \int_1^2 de} \quad \therefore \text{ by (20)}, \quad \frac{h_2}{h_1} = \frac{t_2}{t_1} \quad . \quad (34)$$

But in case the loss is least the efficiency is greatest.

$$\therefore \frac{h_1 - h_2}{h_1} = \frac{t_1 - t_2}{t_1} \quad . \quad . \quad . \quad . \quad (35)$$

Equation (35) expresses the greatest efficiency of an engine which uses any working substance between the limiting temperatures t_1 and t_2, provided the working substance returns periodically to its initial state.

The conditions of greatest efficiency are then these: all the heat imparted to the working substance must be received by it along the highest available isothermal; all the heat rejected by it must be lost along the lowest isothermal; the passages between the isothermals must be along adiabatics.

This result is independent of the nature of the working substance, which need not be homogeneous, but may undergo changes of state such as liquefaction, vaporization or dissociation in course of the cycle, provided it returns to its initial state at the end of the cycle. It appears from (35) that the efficiency is increased more by decreasing the temperature t_2 of the refrigerator by 1° than by increasing the temperature t_1 of the source by 1°.

20. Partial Differential Coefficients of Volume, Pressure, Entropy and Temperature.*—Let Fig. 3 be part of a volume pressure diagram in which ad is parallel to the axis of v and ad' is parallel to the axis of p. Let tt and $t't'$ be a pair of isothermals whose difference of temperature δt is small. Also let ee and $e'e'$ be a pair of adiabatics whose difference of entropy δe is small.

The pair of isothermals with the pair of adiabatics together inclose a quadrilateral $abcb'$ which is ultimately a parallelogram.

Complete the figure by drawing the various parallels to the axis of v and p which are seen in Fig. 3.

Then, $ad = \delta v$, is the variation of volume when pressure is constant, corresponding to a variation of temperature δt.

This is expressed in the ordinary notation of calculus thus: $ad = \left(\frac{dv}{dt}\right)_p \delta t$ in which the subscript p is used to signify that p is constant in the differential coefficient.

Now since 1° of temperature is an arbitrary magnitude according to the scale adopted, let us assume it small, and let $\delta t = 1°$, then $ad = \left(\frac{dv}{dt}\right)_p$.

The units of distance and force can also be assumed small, so that the unit of work shall also be small. By these assumptions we shall be able to make $abcb'$ differ as little as we please from being an exact parallelogram. Hence we see that if these adiabatics and isothermals form part of a system drawn in accordance with (21), (22) and (24),

$$\text{then} \quad \delta t = 1, \quad \delta h = 1, \quad \delta e = 1, \quad . \quad (36)$$

and ultimately we have,

$$\left.\begin{aligned} ad &= \left(\frac{dv}{dt}\right)_p, & ad' &= \left(\frac{dp}{de}\right)_v, \\ ah &= -\left(\frac{dp}{de}\right)_t, & ah' &= -\left(\frac{dv}{dt}\right)_e, \\ af &= \left(\frac{dv}{de}\right)_t, & af' &= \left(\frac{dp}{dt}\right)_e, \\ ai &= \left(\frac{dp}{dt}\right)_v, & ai' &= \left(\frac{dv}{de}\right)_p \end{aligned}\right\} \ldots (37)$$

* Theory of Heat. J. Clerk Maxwell, Chap. IX.

Again, the areas of the following parallelograms are equal, because standing on the same bases and included between the same parallels.

$$\left.\begin{array}{r} abcb'=abed=adgh=ad.ah, \\ abcb'=abji=afki=af.ai, \\ abcb'=ab'j'i'=af'k'i'=af'.ai', \\ abcb'=ab'e'd'=ad'g'h'=ad'.ah'. \end{array}\right\} \quad (38)$$

But by art. 13 we may write (36) thus:

$$abcb'=\delta h=1 \quad . \quad . \quad . \quad (39)$$

$$\therefore\ ad.ah=af.ai=af'.ai'=ad'.ah'=1 \ (40)$$

$\therefore$ by (37)

$$\left(\frac{dv}{dt}\right)_p\left(\frac{dp}{de}\right)_t=-1 \quad . \quad . \quad (41)$$

$$\left(\frac{dv}{de}\right)_t\left(\frac{dp}{dt}\right)_v=+1 \quad . \quad . \quad (42)$$

$$\left(\frac{dp}{dt}\right)_e\left(\frac{dv}{de}\right)_p=+1 \quad . \quad . \quad (43)$$

$$\left(\frac{dp}{de}\right)_v\left(\frac{dv}{dt}\right)_e=-1 \quad . \quad . \quad (44)$$

Divide the following equations, member by member, and reduce:

$$(41)\div(42)\ \therefore\ \left(\frac{dv}{dt}\right)_p\left(\frac{dt}{dp}\right)_v\left(\frac{dp}{dv}\right)_t=-1 \quad (45)$$

$$(42)\div(44)\ \therefore\ \left(\frac{dv}{de}\right)_t\left(\frac{de}{dt}\right)_v\left(\frac{dt}{dv}\right)_e=-1 \ . \ . \ (46)$$

$$(44)\div(43)\ \therefore\ \left(\frac{dp}{de}\right)_v\left(\frac{dv}{dp}\right)_e\left(\frac{de}{dv}\right)_p=-1 \ . \ . \ (47)$$

$$(43)\div(41)\ \therefore\ \left(\frac{dp}{dt}\right)_e\left(\frac{dt}{de}\right)_p\left(\frac{de}{dp}\right)_t=-1 \ . \ (48)$$

Also multiply (45) and (47) member by member and reduce, [*i.e.*, divide the product of (41) and (44) by the product of (42) and (43)], then clear of fractions;

$$\therefore\ \left(\frac{dv}{dp}\right)_t\left(\frac{de}{dt}\right)_v=\left(\frac{dv}{dp}\right)_e\left(\frac{de}{dt}\right)_p \quad . \quad . \quad . \quad (49).$$

The four partial differential coefficients of (49) are not found in (41) to (44), and their values in terms of the coefficients in (41) to (44) are expressed by (45) to (48). The equations in this article can be proved equally well by considering Fig. 3 to be an entropy temperature diagram.

21. Differentials of Heat.—Since the state of a unit of any given homogeneous substance is completely determined when any two of the variables $v\ p\ t$ are given, it is evident that the variation of its state with respect to any of its properties, such as its increment of heat, or its increment of internal energy, etc., can be expressed in terms of the increments of two only of these variables.

$$\therefore\quad dh=\left(\frac{dh}{dt}\right)_v dt+\left(\frac{dh}{dv}\right)_t dv \quad . \quad . \quad . \quad . \quad (50)$$

$$dh=\left(\frac{dh}{dt}\right)_p dt+\left(\frac{dh}{dp}\right)_t dp \quad . \quad . \quad . \quad . \quad (51)$$

$$dh=\left(\frac{dh}{dp}\right)_v dp+\left(\frac{dh}{dv}\right)_p dv \quad . \quad . \quad . \quad . \quad (52)$$

For brevity and clearness of conception:—

Let k_v = specific heat with v constant.
" k_p = specific heat with p constant.
" l_v = latent heat of dilatation per unit of increase of v with t constant.
" l_p = latent heat of dilatation per unit of increase of p with t constant.
" h_p = heat received per unit of increase of v with p constant.
" h_v = heat received per unit of increase of p with v constant.

From these definitions we derive the following equations which are further reduced by help of (18) and (41), (42), (43), (44).

$$\therefore\quad k_v=\left(\frac{dh}{dt}\right)_v=t\left(\frac{de}{dt}\right)_v \quad . \quad . \quad . \quad . \quad (53)$$

$$k_p=\left(\frac{dh}{dt}\right)_p=t\left(\frac{de}{dt}\right)_p \quad . \quad . \quad . \quad . \quad (54)$$

$$l_v=\left(\frac{dh}{dv}\right)_t=t\left(\frac{de}{dv}\right)_t=t\left(\frac{dp}{dt}\right)_v \ . \ . \ . \ (55)$$

$$l_p=\left(\frac{dh}{dp}\right)_t=t\left(\frac{de}{dp}\right)_t=-t\left(\frac{dv}{dt}\right)_p \ . \ . \ . \ (56)$$

$$h_v=\left(\frac{dh}{dp}\right)_v=t\left(\frac{de}{dp}\right)_v=-t\left(\frac{dv}{dt}\right)_e \ . \ . \ . \ (57)$$

$$h_p=\left(\frac{dh}{dv}\right)=t\left(\frac{de}{dv}\right)_p=t\left(\frac{dp}{dt}\right)_e \ . \ . \ . \ (58)$$

The subscripts of h and k state which one of the variables is considered constant, but the subscripts of l state which one of them is variable.

Equations (50), (51), (52), may now be written

$$dh=k_v\,dt+l_v\,dv \quad . \quad . \quad . \quad . \quad (59)$$

$$dh=k_p\,dt+l_p\,dp \quad . \quad . \quad . \quad . \quad (60)$$

$$dh=h_v\,dp+h_p\,dv \quad . \quad . \quad . \quad . \quad (61)$$

which may be applied to any change of state of a homogeneous substance equally with (3), (4), (5).

1°. Let the change be isothermal, then $dt=0$.

$$\therefore \text{ by (61), } \left(\frac{dh}{dp}\right)_t = h_v + h_p \left(\frac{dv}{dp}\right)_t \quad (62)$$

or

$$\left(\frac{dh}{dv}\right)_t = h_p + h_v \left(\frac{dp}{dv}\right)_t \quad (63)$$

2° Let the change be at constant volume,

$$\therefore \text{ by (60), } \left(\frac{dh}{dt}\right)_v = k_p + l_p \left(\frac{dp}{dt}\right)_v \quad (64)$$

or

$$\left(\frac{dh}{dp}\right)_v = l_p + k_p \left(\frac{dt}{dp}\right)_v \quad (65)$$

3° Let the change be at constant pressure

$$\therefore \text{ by (59), } \left(\frac{dh}{dt}\right)_p = k_v + l_v \left(\frac{dv}{dt}\right)_p \quad (66)$$

or

$$\left(\frac{dh}{dv}\right)_p = l_v + k_v \left(\frac{dt}{dv}\right)_p \quad (67)$$

Equations (62) to (67) may be written by help of (53) to (58) thus:

$$l_p\, l_v = h_v\, l_v + h_p\, l_p \quad (68)$$

$$l_p\, l_v = t(k_v - k_p) \quad (69)$$

$$l_p\, l_v = h_v\, l_v - t k_p \quad (70)$$

$$l_p\, l_v = h_p\, l_p + t k_v \quad (71)$$

Equations (62) to (67) may also be written thus:

$$\left(\frac{de}{dp}\right)_t - \left(\frac{de}{dp}\right)_v = \left(\frac{de}{dv}\right)_p \left(\frac{dv}{dp}\right)_t \quad (72)$$

$$\left(\frac{de}{dv}\right)_t - \left(\frac{de}{dv}\right)_p = \left(\frac{de}{dp}\right)_v \left(\frac{dp}{dv}\right)_t \quad (73)$$

$$\left(\frac{de}{dt}\right)_v - \left(\frac{de}{dt}\right)_p = \left(\frac{de}{dp}\right)_t \left(\frac{dp}{dt}\right)_v \quad (74)$$

$$\left(\frac{de}{dp}\right)_v - \left(\frac{de}{dp}\right)_t = \left(\frac{de}{dt}\right)_p \left(\frac{dt}{dp}\right)_v \quad (75)$$

$$\left(\frac{de}{dt}\right)_p - \left(\frac{de}{dt}\right)_v = \left(\frac{de}{dv}\right)_t \left(\frac{dv}{dt}\right)_p \quad (76)$$

$$\left(\frac{de}{dv}\right)_p - \left(\frac{de}{dv}\right)_t = \left(\frac{de}{dt}\right)_v \left(\frac{dt}{dv}\right)_p \quad (77)$$

By equations (53) to (58) we also obtain

$$\frac{k_v}{l_v} = \left(\frac{de}{dp}\right)_v = -\left(\frac{dv}{dt}\right)_e \quad (78)$$

$$\frac{k_p}{l_p} = -\left(\frac{de}{dv}\right)_p = -\left(\frac{dp}{dt}\right)_e \quad (79)$$

$$\frac{h_v}{h_p} = -\left(\frac{dv}{dp}\right)_e \quad (80)$$

$$\frac{k_v}{h_v} = \left(\frac{dp}{dt}\right)_v \quad (81)$$

$$\frac{k_p}{h_p} = \left(\frac{dv}{dt}\right)_p \quad (82)$$

$$\frac{l_v}{l_p} = \left(\frac{dp}{dv}\right)_t \quad (83)$$

Equations (78), (79), (80), and (83) are equivalent respectively to (46), (45), (47) and (48).

We may also write (49) thus

$$k_v \left(\frac{dv}{dp}\right)_t = k_p \left(\frac{dv}{dp}\right)_e \quad (84)$$

22. Equations of Condition.—By the help of (59), (60), (61) and (10) we may write (25) in either of the following forms

$$di = k_v\, dt + (l_v - p)dv \quad (85)$$

$$di = k_p\, dt + l_p\, dp - p dv \quad (86)$$

$$di = h_v\, dp + (h_p - p)dv \quad (87)$$

$$\text{But, } dv = \left(\frac{dv}{dt}\right)_p dt + \left(\frac{dv}{dp}\right)_t dp \quad (88)$$

$\therefore$ by (86) $di=$

$$\left\{k_p - p\left(\frac{dv}{dt}\right)_p\right\}dt + \left\{l_p - p\left(\frac{dv}{dp}\right)_t\right\}dp \quad (89)$$

Now (85), (89), and (87) are exact differentials by art. 14; hence the "equation of condition of integrability" applies to each of them; and it may expressed thus:

$$\left(\frac{dk_v}{dv}\right)_t = \frac{d}{dt}\Big\{l_v - p\Big\}_v \quad (90)$$

$$\frac{d}{dp}\left\{k_v - p\left(\frac{dv}{dt}\right)_p\right\}_t = \frac{d}{dt}\left\{l_p - p\left(\frac{dv}{dp}\right)_t\right\}_p \quad (91)$$

$$\left(\frac{dh_v}{dv}\right)_p = \frac{d}{dp}\Big\{h_p - p\Big\}_v \quad (92)$$

Effecting the differentiations expressed in these equations and reducing we have, by help of (41) to (44) and (55) (56):

$$\left(\frac{dl_v}{dt}\right)_v - \left(\frac{dk_v}{dv}\right)_t = \left(\frac{dp}{dt}\right)_v = \frac{l_v}{t} \quad (93)$$

$$\left(\frac{dl_p}{dt}\right)_p - \left(\frac{dk_p}{dp}\right)_t = -\left(\frac{dv}{dt}\right)_p = \frac{l_p}{t} \quad (94)$$

$$\left(\frac{dh_v}{dv}\right)_p - \left(\frac{dh_p}{dp}\right)_v = 1 \quad . \quad . \quad . \quad . \quad . \quad (95)$$

23. THE INTEGRATING FACTOR t^{-1}.—To show algebraically that (59) and (60) have an integrating factor $=t^{-1}$, suppose that the unknown quantity z is an integrating of these equations, then

$$zdh = zk_v\, dt + zl_v\, dv \quad . \quad . \quad . \quad . \quad (96)$$

$$zdh = zk_p\, dt + zl_p\, dp \quad . \quad . \quad . \quad . \quad (97)$$

are exact differentials which fulfill the condition of integrability, which condition may be expressed thus:

$$\frac{d}{dv}\left(zk_v\right)_t = \frac{d}{dt}\left(zl_v\right)_v \quad . \quad . \quad . \quad . \quad (98)$$

$$\frac{d}{dp}\left(zk_p\right)_t = \frac{d}{dt}\left(zl_p\right)_p \quad . \quad . \quad . \quad (99)$$

Effecting the differentiations we have

$$z\left\{\left(\frac{dl_v}{dt}\right)_v - \left(\frac{dk_v}{dv}\right)_t\right\} = k_v\left(\frac{dz}{dv}\right)_t - l_v\left(\frac{dz}{dt}\right)_v (100)$$

$$z\left\{\left(\frac{dl_p}{dt}\right)_p - \left(\frac{dk_p}{dp}\right)_t\right\} = k_p\left(\frac{dz}{dp}\right)_t - l_p\left(\frac{dz}{dt}\right)_p . \ (101)$$

By help of (93) and (94), (55) and (56) we may write (100) and (101) thus:

$$z\left(\frac{dp}{dt}\right)_v = k_v\left(\frac{dz}{dv}\right)_t - t\left(\frac{dp}{dt}\right)_v\left(\frac{dz}{dt}\right)_v \ . \ . \ . \ (102)$$

$$-z\left(\frac{dv}{dt}\right)_p = k_p\left(\frac{dz}{dp}\right)_t + t\left(\frac{dv}{dt}\right)_p\left(\frac{dz}{dt}\right)_p \ . \ . \ . \ (103)$$

It is known from the theory of differential equations that z may have, in general, any one of an infinite number of values. If possible, let one of these values have the form

$$z = f(t) \quad . \quad . \quad . \quad . \quad (104)$$

then, $\left(\frac{dz}{dv}\right)_t = 0$, and $\left(\frac{dz}{dp}\right)_t = 0$. (105)

Hence (102) and (103) reduce to the same form, viz.,

$$\frac{dt}{t} = -\frac{dz}{z} \quad \therefore \quad z = t^{-1} \ . \ . \ . \ . \ (106)$$

This result shows that the supposition made in (104) is possible, and that the unknown form of function in (104) is that expressed in (106).

24. PERFECT GASES.—The experimental law of Gay Lussac for perfect gases is

$$\frac{pv}{t} = \frac{p_0 v_0}{t_0} = c \quad . \quad . \quad . \quad (9)$$

in which c is a constant to be determined for each gas. Hence for perfect gases,

By (55) $l_v = t\left(\frac{dp}{dt}\right)_v$, $\therefore\ l_v = p$, (107)

By (56) $l_p = -t\left(\frac{dv}{dt}\right)_p$, $\therefore\ l_p = v$, (108)

$\therefore$ by (69), $k_p - k_v = \frac{pv}{t} = \frac{p_0 v_0}{t_0} = c$, (109)

Regnault* has shown, experimentally, that k_p, the specific heat at constant pressure, may be regarded as constant for perfect gases, and hence k_v, the specific heat at constant volume, is also constant for such gases.

We can now show that $k_p - k_v$ the additional heat necessary to cause the expansion of the gas is all expended in external work. Since p is constant,

by (10), $\int_1^2 dw = p\int_1^2 dv = p(v_2 - v_1)$ (110)

By (9), $\frac{p(v_2 - v_1)}{t_2 - t_1} = \frac{p_0\, v_0}{t_0}$ . . . (111)

When $t_2 - t_1 = 1°$, we have by (109), (110), (111),

$$k_p - k_v = p\,(v_2 - v_1) \quad . \quad . \quad (112)$$

hence no work is done against internal forces during the expansion, and k_v is the true specific heat. Hence for perfect gases, $dm = 0$.

Known gases are very nearly perfect in states distant from the point of liquefaction, but as they approach that point internal attractions are called into play which assist the condensation and cause the volume to be less than that of a perfect gas at the same pressure and temperature.

25. ISOTHERMALS AND ADIABATICS OF PERFECT GASES.—From (9) it appears that when t is constant,

$$pv = b, \quad . \quad . \quad . \quad (113)$$

in which b is a constant which, by (9), is known when the temperature of the isothermal is given. Hence the isothermals of perfect gases are rectangular hyperbolas, which is the experimental law of Mariotte.

In order to obtain the adiabatics of perfect gases, let us apply the general equations (59), (60), to this case, which by (107), (108), may also be written thus:

* Relation des expériences. Paris, 1862.

$$dh = k_v\,dt + p\,dv \quad . \quad . \quad (114)$$

$$dh = k_p\,dt - v\,dp \quad . \quad . \quad (115)$$

In order to integrate, we must (arts. 14 and 23) divide by t,

$$\therefore \text{ by (109)} \quad de = k_v \frac{dt}{t} + c\frac{dv}{v} \quad . \quad (116)$$

$$de = k_p \frac{dt}{t} - c\frac{dp}{p} \quad . \quad (117)$$

$$\therefore\ e - e_0 = k_v \log_{\cdot e} \frac{t}{t_0} + c \log_{\cdot e} \frac{v}{v_0} \quad (118)$$

$$e - e_0 = k_p \log_{\cdot e} \frac{t}{t_0} - c \log_{\cdot e} \frac{p}{p_0} \quad (119)$$

These results are general and express the total entropy imparted during any change of state of a perfect gas from any given initial state denoted by the subscript 0.

Now let, $k_p \div k_v = n$ . . . . . (120)

$$\therefore \text{ by (109)}, \quad k_v = \frac{c}{n-1} \quad . \quad . \quad . \quad . \quad (121)$$

and $$k_p = \frac{cn}{n-1} \quad . \quad . \quad . \quad . \quad (122)$$

in which $n = 1.41$ nearly, for perfect gases.

We may now write (118), (119) thus

$$e - e_0 = k_v \log_{\cdot e} \frac{t v^{n-1}}{t_0 v_0^{\,n-1}} \quad . \quad . \quad . \quad (123)$$

$$e - e_0 = k_v \log_{\cdot e} \frac{t^n p^{1-n}}{t_0^{\,n} p_0^{\,1-n}} \quad . \quad . \quad . \quad (124)$$

We may also eliminate temperature from (118), (119)

$$\therefore\ (e - e_0)(k_p - k_v)$$
$$= c\left(k_p \log_{\cdot e} \frac{v}{v_0} + k_v \log_{\cdot e} \frac{p}{p_0}\right) \quad . \quad (125)$$

$$\therefore\ e - e_0 = k_v \log_{\cdot e} \frac{p v^n}{p_0 v_0^{\,n}} \quad . \quad . \quad . \quad (126)$$

In order to obtain the equations which express adiabatic change, in (118), (119), let

$$e - e_0 = 0 \quad . \quad . \quad . \quad . \quad (127)$$

$$\therefore\ \log_{\cdot e} \frac{t}{t_0} = (n-1) \log_{\cdot e} \frac{v_0}{v}$$

$$= \frac{n-1}{n} \log_{\cdot e} \frac{p}{p_0} \quad . \quad . \quad . \quad (128)$$

$$\therefore\ \frac{t}{t_0} = \left\{\frac{v_0}{v}\right\}^{n-1} = \left\{\frac{p}{p_0}\right\}^{\frac{n-1}{n}} \quad . \quad . \quad (129)$$

$$\therefore\ \frac{p}{p_0} = -\left\{\frac{v_0}{v}\right\}^n, \text{ or } pv^n = p_0 v_0^{\,n} = a \,(130)$$

in which a has different constant values for the different adiabatics. The adiabatics of perfect gases are hyperbolic in form, but they cross the isothermals somewhat as represented in Figs. 1 and 3.

26. Flow of Perfect Gases.—Let the converging curved lines in Fig. 4 represent the outlines of a stream of gas moving from a place of greater to a place of less pressure. It is of no consequence whether the sides of the stream be conceived to be bounded by solid walls, or other portions of the gas. Draw any surface cutting all the stream lines of the stream at right angles, and also a second surface infinitely near the first and parallel to it.

Let f = area of the surface cutting the stream.

" dz = thickness of lamina between the two surfaces.

" dp = difference of pressure on opposite surfaces of lamina.

" v = volume of one cubic unit of gas in the lamina.

" v = velocity of the stream.

" g = acceleration of gravitation.

Then, $f dz$ = volume of the lamina,

" $-f dp$ = moving force acting on the lamina,

" $\dfrac{1}{gv}$ = mass of one cubic unit of gas in the lamina.

" $\dfrac{fdz}{gv}$ = constant mass of lamina in any position along the stream.

Now from the elementary principles of dynamics the acceleration of any mass is equal to the moving force divided by the mass moved.

$$\therefore\ \frac{d}{dz}\left(\frac{\mathrm{v}^2}{2}\right) = -\frac{gv\,dp}{dz}$$

$$\therefore\ -v\,dp = d\left(\frac{\mathrm{v}^2}{2g}\right) \quad . \quad . \quad . \quad . \quad (131)$$

As is well known, (131) expresses the energy of motion imparted to a unit of weight of the gas.

In ordinary cases, the efflux of a gas

occurs with such rapidity that no heat can be communicated to it during the process, in which case the efflux is adiabatic.

To find the velocity of flow in this case from the state 1 to the state 2, it will be most convenient to express vdp of (131) in terms of t by (129)

$$\therefore\ v=v_0\left(\frac{t_0}{t}\right)^{\frac{1}{n-1}} \quad . \quad . \quad . \quad . \quad (132)$$

$$dp=\frac{np_0}{(n-1)\,t_0}\left(\frac{t}{t_0}\right)^{\frac{1}{n-1}} dt \ . \ . \ . \ . \ (133)$$

$$\therefore\ \frac{\text{v}^2}{2g}=\frac{-cn}{n-1}\int_1^2 dt=\frac{cnt_1}{n-1}\left(1-\frac{t_2}{t_1}\right) . \ (134)$$

From (134) the velocity of efflux v can be expressed by (129) in terms of the initial and final pressures or volumes, as well as in terms the temperatures employed in (134), for by (129)

$$\frac{t_2}{t_1}=\left(\frac{v_2}{v_1}\right)^{1-n}=\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}=r \ . \ . \ . \ . \ (135)$$

$$\therefore\ \frac{\text{v}^2}{2g}=\frac{cnt_1}{n-1}\,(1-r) \quad . \quad . \quad . \quad . \quad (136)$$

in which we can use either value of r given in (135).

The greatest velocity occurs when

$$r=0, \quad \therefore\ \text{v}=\left(\frac{2gcnt_1}{n-1}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (137)$$

is the velocity with which a gas flows into a vacuum.

But to find the conditions for the flow of the maximum quantity per second, it is necessary to make $\text{v} \div v_2$ a maximum; for this is the weight of gas flowing across a unit area per second.

$$\frac{\text{v}}{v_2}=p_1 r^{\frac{1}{n-1}}\left\{\frac{n(1-r)}{c(n-1)t_1}\right\}^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (138)$$

$$\therefore\ r=\frac{2}{n+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

is the condition for a maximum.

$$\therefore \text{ by (136), } \text{v}=\left(\frac{2gcnt_1}{n+1}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (140)$$

is the velocity of maximum flow.

27. Perfect Gas Flowing into a Vacuum.—Suppose that any gas, as the atmosphere, flows at a constant initial temperature t_1 and pressure p_1 into a receiver in which the temperature is $t_2=nt_1$, in which n has the value before given.

It is evident that the work done by the exterior pressure in forcing a unit of gas into the receiver is $p_1v_1=ct_1$, and the energy already existing in a unit of gas as sensible heat is, by art. 24, $k_v\, t_1$ for k_v is the true specific heat; hence, the total energy gained by the receiver with each unit of gas is the sum of these quantities.

$$\therefore \text{ by (121)} \quad (c+k_v)t_1=nk_v\,t_1 \quad . \quad (141)$$

is the total energy gained with each unit of gas. This energy gained, if used only in heating the unit of gas which carries it in, will raise that unit to a temperature,

$$t_2=nt_1 \quad . \quad . \quad . \quad . \quad (142)$$

since k_v will raise it 1°. But when the gas already in the receiver has a temperature $t_2=nt_1$, the energy of the gas which enters will be employed in heating itself alone.

When the receiver is empty at the beginning of the flow, the energy carried into it can be expended only in heating the gas itself. The gas consequently rises to the temperature $t_2=nt_1$, and continues at that constant temperature during the influx, provided no heat is lost or gained by processes other than those mentioned. This remarkable result was first obtained, but in a different manner, by Bauschinger.

28. Flow of Imperfect Gases.—No one of the more perfect gases obeys precisely the law of Gay Lussac, when the absolute zero of temperature is that defined by the third law. But each gas may be taken to obey a law expressed by the equation

$$\frac{pv}{t'}=\frac{p_0v_0}{t_0'}=c' \quad . \quad . \quad . \quad (143)$$

in which t' is temperature measured by a thermometer constructed, as in art. 9, of the particular gas under consideration. In that case t' will differ slightly from t, and the difference, $t-t'$, will be different for different gases. By reason of difficulties met with in experimenting on gases at widely different pressures, Thompson has proposed to measure the amount by which any gas diverges from the law of Gay Lussac by causing it to flow through a tube at one point of which is an obstruction, called a porous plug, which causes a slight difference of press-

ure in the tube on either side of it. If the gas is imperfect, there will be a slight change of temperature after passing the plug, as may be seen from the following discussion:

By (56) and (60), $dh=k_p\,dt-t\left(\frac{dv}{dt}\right)dp$ (144)

Now let the difference of pressure on the two sides of the plug be dp, then

by (131), $dh=-vdp$ . . . (145)

for the only energy supplied, when the gas forms a steady stream, is that producing the motion through the plug. Therefore by (144) and (145)

$$k_p\,dt=\left\{t\left(\frac{dv}{dt}\right)_p-v\right\}dp \quad . \quad (146)$$

which vanishes for a perfect gas, but in an imperfect gas

by (143), $k_p\,dt=c'(t-t')\,\frac{dp}{p}$. (147)

In integrating this equation $(t-t')$ is considered constant for any given gas within the range of the experiment,

$$\therefore\ k_p\,(t_2-t_1)=c'(t-t')\log.\frac{p_2}{p_1}\ .\ (148)$$

In (148) we can without sensible error measure the difference of temperature by the gas thermometer, *i.e.*,

$$t_2-t_1=t_2'-t_1' \quad . \ . \ . \ . \quad (149)$$

Hence $t-t'$ can be found, since all the other quantities in (148) can be determined experimentally.

Experiments by Joule and Thompson gave a slight decrease in temperature after passing the plug for all the permanent gases except hydrogen, which was heated very slightly. Air was cooled slightly, but carbonic acid gas was cooled much more than oxygen, nitrogen or air. The cooling effect is much diminished at high temperatures, and there is doubtless a particular temperature for each gas at which it is perfect.

29. Sound in a Perfect Gas.—For the sake of simplicity, consider a uniform tube of the gas extending in the direction in which the sound is propagated, and having a cross section of one square unit. Suppose that the tube is cut at right angles by two moving planes which remain at a small constant distance from each other, and that both of these cutting planes move along the tube with the velocity with which sound itself moves. Let the first plane cut the tube at a point where the specific pressure and volume of the gas are the same, as in the still gas which is unaffected by the sound wave. Then the lamina between the planes receives through the front plane the same quantity of gas per second as if moving with the same velocity through still gas. Also, since the lamina, in moving with the same velocity as the sound wave, remains stationary in the same part of the wave its density remains stationary, hence it rejects the same quantity of gas through the second plane that it receives through the first.

Let v=velocity of sound which is also the velocity of the gas at first plane.

" $\mathrm{v}+d\mathrm{v}$=velocity of gas at second plane.

" v=volume of a unit of gas at the first plane.

" $v+dv$=volume of a unit of gas at the second plane.

" q=quantity of gas, by weight, entering and leaving the lamina per second.

" dp=difference of pressure at first and second planes.

Then; $\mathrm{v}=qv$ . . . . . (150)

is the volume swept through by either plane face of the lamina.

Also, $d\mathrm{v}=qdv$ . . . . (151)

is the difference of velocities of the gas at the two faces of the lamina.

And, $-dp=\frac{q}{g}d\mathrm{v}$ . . . (152)

is the force acting on the mass $q\div g$ to cause the acceleration $d\mathrm{v}$.

$$\therefore\ q^2=-g\frac{dp}{dv},\ \therefore\ \mathrm{v}^2=-\left(\frac{dp}{d\,v}\right)_e gv^2\ .\ (153)$$

The propagation of sound is so rapid that the gas undergoes an adiabatic change, and this is expressed by making entropy constant as we have done in (153).

By (130), $\left(\frac{dp}{d\mathrm{v}}\right)_e=-\frac{np}{v}$, $\therefore\ \mathrm{v}^2=ngpv$ (154)

$\therefore$ by (109), $\mathrm{v}=\sqrt{cngt}$ . . . (155)

which is the ordinary formula for the velocity of sound.

By (137) the velocity with which a gas enters a vacuum exceeds the velocity of sound in the ratio

$$\sqrt{2} : \sqrt{n-1} = 2\tfrac{1}{5} \text{ nearly.}$$

By (140) the velocity of greatest flow is less than the velocity of sound in the ratio

$$\sqrt{2} : \sqrt{n+1} = \tfrac{9}{10} \text{ nearly.}$$

THE THEORY OF TURBINE WHEELS.

A REVIEW OF PROF. TROWBRIDGE'S RECENT ARTICLE.

By Prof. WM. H. BURR.

Written for Van Nostrand's Magazine.

From the deservedly high position occupied by Professor Trowbridge, his late article on Turbines will be likely to convey permanent impressions of a very erroneous character.

His objection seems to be, that Weisbach, Rankine and others "insist" that, for the best performance of a wheel, the water shall enter without shock; while he holds that an equal efficiency, at least, may be obtained if the relative velocity be not tangential to the buckets at the point of entrance into the wheel. In other words, if resistances be left out of consideration, it is a matter of no importance whether there is or is not shock. Now if the consideration of resistances be omitted, and if "shock," be of no importance one way or the other, then it follows that a formula for curved floats ought to give precisely the same result whether the relative velocity of entrance be tangential in direction or not. In fact the formulæ of Weisbach show this exactly, as do those also of Bresse and Rankine. In order to verify this statement let any one turn to the expressions for the efficiency of a turbine in Art. 260 of Dubois' translation of Weisbach's work on water wheels, Eq. 21, Art. 15 of Mahan's translation of Bresse on "Hydraulic Motors" (John Wiley and Son, 1869), and Eq. 6 Art. 174 of Rankine's "Steam Engine and other Prime Movers," 5th edition; in no one of these expressions is there anything found depending upon the angle which the bucket makes with the circumference of the wheel at the point of entrance into it. Now the matter of shock, or no shock, depends upon the value given to that angle and since the expressions for the efficiency do not in any wise depend upon such a value, they manifestly are *not* based upon the "theorem or axiom" that the "water must enter the wheel without shock."

What those authors *say* is that the "water must enter the wheel without shock" in order to get the best results, simply because, if there is shock, eddies, consequent upon the impact of *water in water*, will be produced giving rise, as is well known, to no small loss of energy, so far as useful effect is concerned. This element is of such uncertain value, however, in a turbine that the authors above mentioned made no attempt to allow for it in their expressions for efficiency, but state explicitly that the wheels which they treat are supposed to run under the most favorable conditions possible.

Again in the first part of his article Professor Trowbridge seems to hold that what he considers the erroneous views of the authors above cited, leads them to make what Weisbach designates as the angle β so large, that no work can be done by *impulse*, and would seem to imply that this is a disadvantage. Now Bresse, in Art. 16 of the work already named, advises that this angle be made a little less than 90°, *which will allow of some work being done by impulse.*

Weisbach, however, in art. 254 of Du Bois' translation, advises as high as 100° to 120°, and so far as I am able to determine from the data before me, Mr. Francis has never made the angle less than 90°, but as high as 119° in his "Centre-Vent" Boott wheel, thus effectually preventing all work by impulse.

Prof. Trowbridge's objections, there-

fore, can not in any sense hold against the investigations of the authors named. The faults on which he bases his objections do not exist.

The influence of the indirect effect of shock in loss of energy by induced eddies, in Weisbach's formula for the efficiency may easily be seen by referring to the Art. 260 already cited. His expression for the efficiency is;

$$\eta=1-\frac{\zeta\left\{1+\left(\frac{r\sin\delta}{r_1\sin\alpha}\right)^2\right\}+4\left(\sin\frac{\delta}{2}\right)}{\zeta\left\{1+\left(\frac{r\sin\delta}{r_1\sin\alpha}\right)^2\right\}+2\cot\alpha\sin\delta}\;\ldots\ldots(a)$$

Now the manner in which that equation is established shows that the loss of energy by the shock considered would cause a term of the form, $\zeta_1 f(\beta)$, to enter the numerator of the second term of the second member; ζ_1 being a co-efficient of resistance for the impact of water in water and $f(\beta)$, a term dependent upon the angle β, the angle at which the buckets cut the circumference of the wheel at the point of entrance into it. It is evident that this term would diminish the efficiency, as it should.

Mr. Francis himself virtually points out the existence of this term for his "Centre-Vent" wheel, on page 69 of Van Nostrand's last edition of his "Lowell Hydraulic Experiments," though in that case the impact is, to a considerable extent, probably, due to another cause which will be indicated further on.

It will now be shown that the formula for the efficiency given by Bresse is identical with Prof. Trowbridge's, and that the same holds for Weisbach's when resistances are omitted, which is assumed by Prof. Trowbridge.

The general expression for the efficiency which Prof. Trowbridge arrives at is

$$E=1-\frac{v_r^2\sin\gamma^2}{v_1^2}$$

Let $v_r \sin\gamma$, the absolute velocity of escape from the wheel, be represented by w; then

$$E=1-\frac{w^2}{v_1^2}\;\ldots\ldots\ldots\ldots(b)$$

The quantity v_1 is the velocity due to the whole head available in any particular case.

On page 89 of the edition of Mahan's Bresse, already noticed, if w be put for v^1 and v_1^2 for $(2gH)$, as must be done to bring both formulæ to the same notation, there results this expression for the efficiency, which is identical with Eq. (b):

$$\mu=1-\frac{w^2}{v_1^2}\;\ldots\ldots\ldots\ldots(c)$$

In Eq. (a), Weisbach's expression, if resistances be omitted and H represent the total fall, there will result;

$$\eta=1-\frac{4v^2\left(\sin\frac{\delta}{2}\right)^2}{v^2\cot\alpha.\sin\delta}$$

But Weisbach shows that,

$$w^2=4v^2\left(\sin\frac{\delta}{2}\right)^2,$$

and

$$2gH=v_1^2=2v^2\cot\alpha.\sin\delta;$$

hence

$$\eta=1-\frac{w^2}{v_1^2},\;\ldots\ldots\ldots\ldots\ldots(d)$$

This again is Professor Trowbridge's formula.

It is clear, therefore, that any objection which holds against the formulæ of Weisbach, Bresse, and consequently Rankine, holds with equal force against the formula of Professor Trowbridge. Also, that if there be omitted from Weisbach's expression that which makes it of real practical value, or, in other words, if we make it about as loose an approximation as we can well have (since the efficiency must be less than unity in any case), there results the formula of Professor Trowbridge. Equations (b), (c) and (d) are, however, all correct under the supposition of no resistances.

The expressions for efficiency and best velocity of rotation arrived at by Weisbach will be tested by the results of some of the best experiments on Turbines that probably were ever made.

Reference is made to the experiments of Mr. Francis in the Tremont Turbine and Centre-Vent Wheel at the Boott Cotton Mills, as given in the third edition of his "Lowell Hydraulic Experiments;" also his experiments on a Swain Turbine, published in the *Journal of the Franklin Institute* for April, 1875; and, finally, to the experiments on a Swain Turbine by Hiram F. Mills, C.E., at Lowell, Mass.,

in June 1869, published in the Franklin Institute Journal and republished in 1872 by the Swain Turbine Company in a business pamphlet.

The experiments chosen will be those for a full gate only, as those are the only ones to which the formulæ directly apply.

Some of the angles used I have been obliged to measure from such plates as I have at hand, and it is barely possible that the values taken may be a little in error, but not not enough to make any sensible difference in the results.

The extreme outer radius of the Fourneyron wheel and the interior radius of the inward and downward flow wheels, have been diminished by a little less than one-half the shortest distance between the buckets for obvious reasons.

The notation used is that of Weisbach, and has no connection with that used before.

Weisbach's expression for efficiency, as already explained, is eq. (a); his formula for best velocity of the interior circumference of the Fourneyron and exterior circumference of the others, is found in Art. 251 of Du Bois' translation before mentioned; it is denoted by v_1. Inasmuch as Weisbach's formulæ are to be tested, his value of $\zeta=0.075=\zeta$, will be used.

TREMONT WHEEL.

For experiment No. 30, 3d edition, Lowell Hydraulic Experiments, pages 32 and 33:—

$$\alpha=28^\circ \qquad r=4.1 \text{ ft.}$$
$$\beta=90^\circ \qquad r_1=3.4 \text{ ft.}$$
$$\delta=11^\circ \qquad h=12.9 \text{ ft.}$$
$$v_1=0.66\sqrt{2gh}=19.2 \text{ ft.}$$

Experiment gives: $v_1=18.05$ ft.

$$\eta=1-0.16=0.84$$

Experiment gives: $\eta=0.794$

CENTRE-VENT BOOTT WHEEL.

For experiment No. 30, pages 66 and 67, 3d edition, Lowell Hydraulic Experiments—

$$\alpha=9^\circ.5 \qquad r=3.95 \text{ ft.}$$
$$\beta=119^\circ.0 \qquad r_1=4.67 \text{ ft.}$$
$$\delta=11^\circ.0 \qquad h=13.378 \text{ ft.}$$
$$v_1=0.716\sqrt{2gh}=21.01 \text{ ft.}$$

Experiment gives: $v_1=19.71$ ft.

$$\eta=1-0.0756=0.924$$

Experiment gives: $\eta=0.797$

This particular experiment was taken because the results are about the mean of those of seven, giving essentially the same efficiency.

SWAIN TURBINE.

Experiment 134 of those made by Mr. Francis at Lowell, in Feb., 1875, and published in the *Journal of the Franklin Institute* for April, 1875.

The value of δ is taken to be a half of the sum of its values at the highest and lowest points of exit from the wheel. It is not at all probable that this is *exactly* right, but it is about as near as can be obtained, and it is evident that not much of an error is involved.

All molecules of water, also, have approximately the same length of path in the wheel, hence the value of r is taken as if the wheel were wholly an inward flow one

$$\alpha=14^\circ \qquad r=22.5 \text{ inches.}$$
$$\beta=95^\circ \qquad r_1=36.0 \text{ inches.}$$
$$\delta=\frac{18+7}{2}=12^\circ.5 \qquad h=12.41 \text{ feet.}$$
$$v_1=0.70\sqrt{2gh}=19.8 \text{ feet.}$$

Experiment gives: $v_1=21.48$ feet

$$\eta=1-0.079=0.921$$

Experiment gives: $\eta=0.834$

SWAIN TURBINE.

Experiment, No. 11 of the series, made by Hiram F. Mills, C.E., at Lowell, Mass., June, 1869.

Observations made on the data of the previous example, apply here also.

$$\alpha=22^\circ \qquad r=12.2 \text{ inches.}$$
$$\beta=104^\circ \qquad r_1=21.0 \text{ inches.}$$
$$\delta=\frac{20+7}{2}=13^\circ.5 \qquad h=14.255 \text{ feet.}$$
$$v_1=0.723\sqrt{2gh}=21.9 \text{ feet.}$$

Experiment gives: $v_1=21.8$ feet.

$$\eta=1-0.113=0.887$$

Experiment gives: $\eta=0.822$.

Weisbach's formula gives the efficiency of the wheel simply as a user of water, implicitly considering, therefore, the wheel friction as useful work. If Prof. Trowbridge's figures, therefore, be taken, there should be added to each of the above experimental efficiencies the frac

tion 0.03. Then taking them in the order of the examples they would stand as follows:

$$\eta=0.824$$
$$\eta=0.827$$
$$\eta=0.864$$
$$\eta=0.852$$

A critical comparison may now be made between the results of calculation and experiment. Mr. Francis' table for the Tremont Turbine shows that, with an essentially full gate, v_1 in the six experiments for which the efficiency was 0.79 or over, varied from 17.3 feet to 18.40 feet, while the formula gave $v_1=19.2$. With this last value of v_1 the Tremont Turbine gave a corrected efficiency of over 0.81.

The corrected experimental efficiency above is 0.824; the formula gives 0.84; this disagreement is certainly small.

The analytical results, then, for the Tremont Turbine surely are to be classed as excellent, and formulæ which can be depended upon to give as good results are worthy of the confidence of an engineer.

The analytical and experimental results for the second example, the Center-Vent Wheel, do not agree as closely, and, indeed, an examination of the wheel shows why they can not. The distance between the wheel and the guides is more than two-thirds the shortest distance between any two consecutive guides, and the angle a is taken so small (9.5° only) that the entering stream is directed nearly tangentially to the wheel. There must result, therefore, some impact of water in water in the annular space between the guides and buckets, consequently, considerable loss of energy by eddies, etc., will follow. The angle β was also so chosen, as Mr. Francis himself shows (page 69), that there is appreciable impact of water in water at the section of entrance into the wheel.

This shows why there ought not to be any impact in entering the wheel, and it is the very thing that Wiesbach, Bresse, and Rankine guard against.

The investigations of those authors show that loss of energy was to have been anticipated from such injudicious choice of the angles a and β. It is to be noticed that much larger values of a are found in the Swain Turbines taken, and that considerably larger efficiencies are obtained.

It has already been shown how the formula for the efficiency should be changed to meet this case, and how it (the efficiency) would be decreased by the change. The analytical value of v_1 is certainly as near the experimental as could be expected under these circumstances. It will be shown further on how a term can be introduced in the value of v_1 so as to make it apply to this case, and that the effect of such a term would be to decrease the velocity v_1. The discrepancies, then, in this case should really strengthen one's confidence in the formula; in other words they are confirmatory. This is so because the formulæ do not apply to all the circumstances of the present case, whereas if the proper terms were introduced in accordance with the general theory on which they are based the resulting changes would be in the proper direction.

It will also be shown that the formulæ of Wiesbach indicate a high pressure at the point of departure from the guides, and the result is confirmed by the experiments. There would be considerable loss under this head but for the admirable arrangements for the prevention of leakage.

The results in the third example are very good. The disagreement in the efficiencies is between 0.05 and 0.06, and in the values of v_1 about the same portion of $\sqrt{2gh}$. Experiment 137 shows that with v_1 equal to 0.70 $\sqrt{2gh}$, nearly, the resulting efficiency was about 0,85, when corrected for wheel friction.

Bearing in mind, also, that exact values for r and δ could not be obtained, it is clear that the operations of the formulæ are most creditable.

The analytical results in the last case agree so nearly with the experimental that they are almost suspicious, nevertheless they are honest calculations and show how thoroughly reliable the formulæ are.

Some other considerations, still more confirmatory, if possible, of Wiesbach's formulæ remain to be noticed.

He shows that if β be greater than $2a$, there will be unbalanced outward pressure at the point of exit from guide curves. In the Centre-Vent Boott Wheel the pressure head at that point may be calcu-

lated from the amount of water discharged and the total area of exit guide orifices, and there is found an unbalancced head of water equal to about 7.7 feet. This is a confirmatory result, because in this case β is several times greater than $2a$.

In the Tremont Turbine there will be found an unbalanced pressure head at the same point of about 6 feet. This was to be anticipated, since $\beta > 3a$.

The formulae indicate, therefore, just what the experiments show, *i. e.*, if small values of a are used, then unusual precautions must be taken to prevent leakage; this certainly was effectually done in the Lowell wheels, including the Swain.

Weisbach and Bresse seem to lay unnecessary stress on the point of taking a and β so that there will be no unbalanced outward pressure at the exit from the guides, for the following reasons:

Equation (a) shows, since the cotangent of a small angle is very large and increases very rapidly as the angle decreases, that a small value of a will give a considerable increase of efficiency over that which will be obtained by a little larger value. It follows from the formulae, therefore, that it is best to take precautions against leakage and make a small, as was done in the Swain Turbines at Lowell.

The general theory of hydraulic resistances, however, show that a minimum limit may be reached below which it is not best to go. For with a very small angle, a, the stream issuing from any guide orifice becomes very thin in comparison with the width of the annular space occupied by the regulating gate, and its direction becomes nearly tangential to the wheel; disturbances caused by impact of water in water result, and there is an induced loss of energy by eddies, &c. Farther if the angle β is not properly chosen the same kind of impact and loss of energy will occur in the wheel.

Figs. 1 and 2 above illustrate this last matter. Fig. 1 belongs to experiment 30 on the Boott Centre-Vent Weeel already treated; ad represents $v_1 = 19.7$ feet; ac, the velocity of exit from the guides, calculated from Mr. Francis' data and found equal to 19.1 feet; and ab the consequent velocity relative to the wheel. The angle bad is found, thus, to be 105°; this should have been the angle β for the actual value of v_1. The value taken, however, was 119° and loss of energy by impact of water in water, induced by shock, resulted as Mr. Francis shows.

Fig. 1.

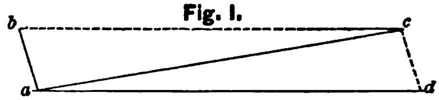


Fig. 2.

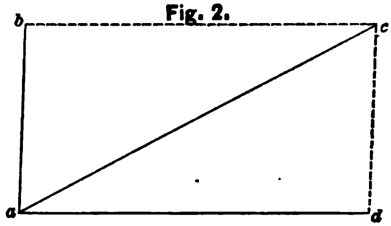


Fig. 2 belongs to the experiment 30 in the Tremont Turbine; $ad = v_1 = 18.05$ feet; ac = exit velocity from guides $= 21.1$ feet; and ab is the velocity relative to the wheel. The angle bad is found to be 87°, while β was taken at 90°. There can be little or no impact in this case, as the analytical results show.

From the manner of establishing Weisbach's value for v_1, the best velocity of rotation, it will be seen by examination that the case of a partially closed gate may be allowed for by placing for ζ the expression $\left\{\zeta + \zeta_1 . f\left(\frac{1}{h}\right)\right\}$ in which $f\left(\frac{1}{h}\right)$ is a function of the reciprocal of the height of the regulating gate, and ζ_1 an empirical quantity dependent upon the impact of water in water.

Since this will increase the denominator of the expression for v_1, it follows that the formula would indicate a decrease of the best velocity for a partially closed gate. An examination of the experimental results is confirmatory. Let the experimental results of Mr. Francis in the seventy-two inch Westerly Swain Turbine already used be referred to. Then taking from two to four of the best results (whose efficiences do not differ essentially from each other and from the highest) for each of several different heights of regulating gates; the following quantities are then deduced:

Experiments 133, 134 and 135:
Height of gate, 13.08 inches.
Mean $v_1 = 0.757\sqrt{2gH}$.

Experiments 117 and 118:
Height of gate, 11.00 inches.
Mean $v_1 = 0.741\sqrt{2gH}$.

Experiments 102, 103 and 104:
Height of gate, 9.00 inches.
Mean $v_1 = 0.705\sqrt{2gH}$.

Experiments 77, 78, 79 and 80:
Height of gate, 7.00 inches.
Mean $v_1 = 0.699\sqrt{2gH}$.

Experiments 50, 51 and 55.
Height of gate, 5.00 inches.
Mean $v_1 = 0.688\sqrt{2gH}$.

Experiments 33, 34 and 35:
Height of gate, 3.00 inches.
Mean $v_1 = 0.659\sqrt{2gH}$.

The variation in these results is as nearly uniform as could be expected under the circumstances, since only from two to four results in each case could be found with efficiencies near enough the maximum. They show, however, just what the formula would indicate, *i.e.*, a decrease in v_1, for a decrease in height of the regulating gate.

The other series of experiments will be found on examination to show the same thing, and it is not necessary to reproduce them here.

The formula for the efficiency would show a decrease in it, also, as has before been indicated.

There is only one thing more to be touched upon, which, indeed, might have been alluded to before. If the four wheels examined had been run with the best velocities indicated by the formula of Weisbach, then the experimental results show that in no case would the efficiency have fallen below the maximum actually obtained, by more than the decimal 0.015. Considering the complicated action of the water in a Turbine this is certainly a most excellent result.

Summing up then, it is seen that for the effect of a partial gate on efficiency and best velocity; for the effect of the relative values of a and β on the unbalanced pressure at point of exit from guides, and impact in the same vicinity, the formulæ indicate just the kind of variation which the experiments show.

The qualitative characteristics of the formulæ are correct; the empirical quantities which enter them only need to be determined by such admirable experiments as those of Mr. Francis, to give the quantitive results their exact values.

The formulae for efficiency and best velocity, as they stand, give most excellent results for full gate and proper design.

In the light of these facts it can hardly be said that mathematical investigation has done little or nothing to aid the engineer in designing the turbine, or that the formulae of Weisbach are inapplicable to modern Turbines.

Before closing it may be well to state that Professor Trowbridge makes a wrong use of the co-efficient of resistance

$$\left\{\left(\frac{1}{0.97}\right)^2 - 1\right\},$$

for he multiplies it by the square of a velocity which does not exist at the point of exit from the guides. His v_1, as he uses it, is the velocity due to the total fall, and that is a very different quantity from either the velocity in the guides or that in the wheel, and the latter are the ones he should use.

Erratum—The Mississippi as a Silt Bearer.—There was an unfortunate transposition of the text of the above article in our last issue.

To be read as was intended, the reader should pass from the paragraph ending on 3d line from foot of 2d column, page 222 to the paragraph beginning on the 12th line 1st column, page 226.

The omitted text should be inserted immediately after the paragraph in the 2d column, page 230, which ends with "foregoing table."

REPORTS OF ENGINEERING SOCIETIES.

Engineers' Club of Philadelphia.—The first issue of the Proceedings has just come to hand. There is much promise of valuable additions to engineering literature by this active organization.

The following is the list of papers in the current number:

I. Oil Lands of Pennsylvania. Charles A. Ashburner.

II. House and Street Drainage of Philadelphia. By Rudolph Hering.

III. Bearing Piles. Rudolph Hering.

IV. The proposed removal of Smith's Island. Lewis M. Haupt.

V. Water-Supply for a Stamp Mill. By W. F. Biddle.

VI Empirical Formula for Strength of Wrought Iron Beams. P. Roberts, Jr.

VII. The Scales of Maps. Lewis M. Haupt.

VIII. The Strength of Wrought Iron in Structures. P. Roberts, Jr.

PROCEEDINGS OF THE BRITISH INSTITUTION OF CIVIL ENGINEERS.—The following Excerpt Minutes of the Proceedings have been, receivod through the kindness of Mr. James Forrest, A.I.C.E.:

Engineering Progress in Foreign Countries. By Messrs. Vernon Harcourt, Clark, Bauerman and Higgs.

Heating and Ventilating Apparatus of the Glasgow University. By W. W. Phipson, M.I.C.E.

Harbor and Dock Works. The Avenmouth Dock. John Bower Mackenzie, M.I.C.E.

The Harbor of Belfast. Thomas Ross Salmond, M.I.C.E.

Whitehaven Harbor and Dock. John Evelyn Williams, M.I.C.E.

Coal Washing Apparatus at Besseges, France. By J. B. Marsant.

LIVERPOOL ENGINEERING SOCIETY.—This Society held its usual fortnightly meeting at the Royal Institution on Wednesday evening last. Mr. M. E. Yeatman, M.A. President occupied the chair. A paper on "The Design and Construction of Sewers" was communicated by Mr. Graham Smith, past President In opening his subject, he remarked, that sewers should not only be constructed in such a manner to carry with dispatch to the outfall the sewage which might find its way into them, but they should likewise be built in a manner such that no portion of the sewage should percolate through them into the surrounding earth. Cholera for instance, might be spread indefinitely by matter discharged from the stomach or bowels of a cholera patient gaining access to any source of water supply. He quoted the General Board of Health returns to show that zymotic diseases largely exist and are much due to improperly constructed sewers and imperfect drainage arrangements. The question of the proper forms of sewers to be adopted under various circumstances was dealt with at some length, and the author advocated the circular section when a large and constant flow could be depended upon, and the egg-shaped sewer when a variable flow had to be accommodated, as the smallness of the invert increases the scouring action of a small quantity of sewage, whilst the increased size of the upper portion provides for any augmentation of flow. He considered that the success or non-success of any system of drainage depended upon the manner in which details were arranged. These questions and such others as the ventilation of sewers and the materials employed in their construction were fully considered.

IRON AND STEEL NOTES.

BRIGHT steel surfaces may be ornamented by painting the patterns intended to be left bright in Brunswick black. If the ornament is to be dead upon a bright ground the patterns must be left untouched, and the ground painted over. Aquafortis—diluted nitric acid—should then be poured upon the exposed parts of the steel, and in a few minutes it will be seen to have eaten sufficiently into the metal. Wash off the aquafortis with water. and Brunswick black may be removed with turpentine. If the steel is made blue by exposure to heat, the blue color can be removed, where it is not required, with white vinegar or other weak acid, the parts to remain blue being protected by Brunswick black. On the parts from which the blue is removed, further variety may be gained by painting additional ornaments in Brunswick black, and exposing the remainder of the ground to the action of the aquafortis. Gilding on steel was formerly performed with a spirit, but now it is best to send the work to an electro-gilder's, first painting over those parts not to be gilt with Brunswick black. The gilding may be performed at home by the following method:—It is known that if sulphuric ether and nitro muriate of gold are mixed together, the ether will, by degrees, separate from the acid nearly the whole of the gold, and retain it for a long time in solution. Take ether thus charged and with a soft brush paint the parts of the design intended to be in gold, and after giving the ether time to evaporate, rub over the parts thus gilt with a burnisher.

RAILWAY NOTES.

IN Europe it is common to indicate on posts the gradients of different sections of a railway at the points of change, for the benefit of the enginemen and others. Formerly on the Continent it was almost universally the custom to express the gradients in the form of a vulgar fraction, as is still done in England. Thus a grade of 1 in 100 would be indicated on the post by $\frac{1}{100}$. A short time ago this was changed on some Prussian lines to the form of a decimal fraction, as 0.01 for the grade above mentioned. The Prussian Ministry of Trade has now given instructions that this new method be abandoned and the old one substituted, the reason given being that the decimal fractions are often misunderstood by the enginemen.

PAPER AND STEEL *v.* CAST IRON RAILWAY WHEELS.—Much interest was excited in the railway world sometime since by the introduction in America of paper instead of cast iron wheels. There is no more important department of a railway than the rolling stock, and no part of the rolling stock of greater consequence than the running gear of trains. The paper wheel body is an American invention, of which Messrs. John Brown & Co., Atlas Works, Sheffield, are the makers for England. The paper wheel carries a steel tire, and it is claimed that the latter tire being truly turned,

and the two wheels on an axle being easily fitted to the same size, the pair will run much more quietly and easily than cast iron wheels of ordinary American make. Practical working, however, is better evidence than any quantity of "claims," and the *Chicago Railway Review* gives some interesting details of the records of the mileage made by the paper wheels as adopted by the Pullman Palace Car Company. The mileage of each trip is computed within one quarter of a mile, and the company, which has used the paper wheel with steel tire about nine years, knows exactly what its wheels are doing. The paper wheel costs £18 and runs 450,000 miles in 2.88 years. For convenience of reckoning call this period three years. At the end of three years the original cost with 7 per cent. compound interest, amounts to not quite £16. During this period nine cast iron wheels have been used, costing £2 16s., allowing a rebate of £1 each for the worn out wheels, and calculating on simple interest at 7 per cent., the cost of the wheels for this service amounts to £18 6s., showing a saving in the case of paper wheels of £2 6s., and with compound interest considerably more. In computing the cost for the second period of three years, a much greater saving would be shown, since a renewal of the tire only, at a cost of £7, is necessary, instead of a fresh cost of £18 for a new paper wheel. The experience of other railways bears out the records of the Pullman Company. The paper steel-tired wheel is used on the Central Vermont, Connecticut River, Cleveland, Columbus, Cincinnati and Indianopolis, Pittsburg, Cincinnati and St. Louis, and the Chicago and Alton line. Engine truck wheels have been found to run 800,000 miles before the tire requires renewal. The subject seems to be one which scarcely receives the attention it deserves in this country. We have as yet no data upon the relative durability of paper wheels, and those with wrought iron or wood bodies as commonly used in England.

RAILWAYS IN CHINA.—When it was first understood that the Woosung and Shanghai Railway had been purchased by the Chinese Government, it was thought railways had obtained a footing in China. The news of its removal almost wholly as so much old iron, and that as the easiest means of transport the carriages were pitched into the sea to convey them to Formosa, the iron work tossed into barges, and the engines, partly in pieces, put into the cases in which they were sent to China in, and then the whole thrown on to the beach Formosa, soon, however, dispelled this idea, and hope of work for railway engineers in this field died away. It seems that the railway was laid while the governorship of Kiang Su was in the hands of a friend to progress, and who made no objections. When, however, the matter of the murder of Mr. Margery had to be settled, it was urged by Chinamen, who did not know much of Western ideas, that as a Chinaman had been killed on the Woosung railway, the work of the English, they had nothing to complain of. The only way of removing this notion was to make the railway Chinese property, and Li-Hung-Chang ordered the Governor of Kiang Su to buy the railway with whatever funds were at his disposal. In the meantime a new governor had been appointed who was not very well pleased with the order to buy the line, and he seems to have made up his mind that though he must buy it, the Minister of Public Works had no power to make him work it. Ting-Futai, the Governor of Formosa, was however, desirous of having a railway in his island, and wrote to the Governor of Kiang Su to that effect. The governor, in reply, said he had a railway he did not want and he accordingly pitchforked it over to Formosa in the way described The Woosung Railway had been at work just long enough to allow Chinamen to see what could be done with it, and that having had it they were inconvenienced without it. The result is that railways have been very much talked about since the removal, and have now many friends. Li Hung-Chang, one of the most advanced statesmen in China, has long advocated them, though he has worked under a disadvantage. Almost all the statesmen of China are Tartars; very few are Chinese, and the latter, when they do rise to positions, rise with great difficulty. The loss of the Woosung Railway has had the effect of stirring up interest in the matter, and with a good reason for believing that China will before very long accept the great civilizer, a well known English engineer has gone to China to negotiate for new lines, when the Celestial makes up his mind to admit them. It is also reported that Li Hung-Chang has contracted with Mr. Arnold Hague, a geologist and mining expert, for the purpose of prospecting for minerals in North of China.

A STRIKING illustration of the superiority of railways to the old coaches and diligence in point of safety has been published by M. Gorilaux, a French statistician. Before the introduction of railways, he estimates that one passenger was killed out of every 300,000 persons traveling by coach or diligence; while in addition there was one injured out of every 30,000 travelers. In the first twenty years of railway traveling in France, that is to say, from 1835 to 1855, the average number of passengers killed in railway accidents was one in every 2,000,000, and one injured in every 300,000. In the next twenty years, from 1855 to 1775. the safety of railway traveling had so much improved, that the average number of killed in accidents on the French lines amounted to only one person in every 6.000,000 passengers, while the injured numbered but one in every 600,000. Hence in the first twenty years, a railway journey was from seven to ten times as safe as a journey by the diligence. In the coach, a passenger ran seven times the risk of being killed, and ten times the risk of being injured that he did on the railway. In the next twenty years, says Herepath, he was twenty times more secure against the risk of a violent death or of injury, than if he used the lumbering old diligence, which, moreover, took five or six times as much of his time to perform a given journey. At the present time, M. Gorilaux estimates that the real danger to the life or limb of passengers from causes beyond their own control is some-

thing far less considerable than those we have just quoted. He calculates that on French railways, there are now actually killed only one person in every 45 000,000 passengers and one injured in every 1,000,000. On these results M. Gorilaux bases a computation by which he arrives at the striking fact in France that a passenger would have to travel at the rate of forty miles an hour, ten hours a day, without intermission for a period of 7,439 years, before he would have to fear an accident of a fatal character.

ENGINEERING STRUCTURES.

The piercing of the St. Gothard tunnel advanced 21.7 meters on the Goeschenen side, and 30.1 meters from the Airolo side, or 52.1 meters during the week ending the 26th ult. During the week ending the 1st inst., the progress was 24.3 and 10.2 meters respectively at the Goschenen and Ariolo sides, or 35.2 meters. The boring now extends to 6421.3 meters on the Goschenen side, and 5906.9 on the Airolo side, leaving 2571.8 meters to be pierced in the center of the mountain.

The Calais-Douvres.—From the last report of the London, Chatham, and Dover Railway Company we learn that the running of the twin Channel steamer Calais-Douvres during the past season appears in most respects to have justified expectatation. She performed the trip from Dover to Calais and back for seventy-eight days with great regularity, making good and quick passages, and affording the public a large increase of comfort, and a very material diminution of those peculiar evils and annoyances heretofore incident to the sea transit. The general appreciation of her merits is shown in the fact that during the season upwards of 55,000 passengers crossed in her, or an average of 715 per day, a number which would on many occasions have involved the necessity of running two of the ordinary mail boats. This may be regarded as in some measure a set-off against the unquestionably heavy cost of work ing a ship of the size and capacity of the Calais-Douvres.

The St. Gothard Tunnel.—M. Colladon recently gave some interesting particulars respecting this work in a paper communicated to the Academie des Sciences by M. Tresca. It seems that besides the excessive hardness of the beds of serpentine and quartz, and insufficiennt hydraulic power on the Airolo side, owing to the lowness of the water last winter, there has been a very heavy infiltration in the south portion, amounting to over 3,000 gallons per minute in the advance gallery. The jets of water had often the force of those from a fire engine pump, due to great heads. Another difficulty has been caused by a mass of decomposed felspar mixed with gypsum found under the plain of Andermatt, under the site of an ancient lake, and about 190 yards from the entrance. This plastic material swells on contact with moist air with irresistible force, capable of crushing the strongest timber, and even arches of granite 3.28 ft. thick. In some of these parts boring by hand advances about 1 metre in three or four days, while through granite, with compressed air and mechanical perforation, a regular advance of 10 ft. to 13 ft. in twenty-four hours has been achieved, and through gneiss as much as 20 ft. The volume of water from the Tremola—Airolo side—having been found insufficient, M. Favre brought water in an aqueduct 3.270 ft. long from the Tessin to work three new turbines, 16.4 ft. diameter, making 50 to 60 revolutions under a head of 262 ft., and four compressors. The old bronze turbines, 3.87 ft. diameter, working under a head 595 ft., which have made some 155,000,000 revolutions per annum, are in good preservation, after four or five years' service, and still work usefully. On each side of the tunnel there are, at present, sixteen air compressors in action, serving both for aeration and for boring operations. They send into the tunnel air under a pressure of eight atmospheres, sufficient to drive eighteen to twenty drills, making 150 to 160 strokes per minute, and effect good ventilation of the part already bored, which is at present 6,800 yards on the north side and 5,900 yards on the south. The transport of materials is effected by horses in the more advanced part of the tunnel, and by compressed air locomotives in the portions near the mouths. The Compagnie du Gothard, which has possession of the line, with the exception of the tunnel, has suspended work for two years. The excess of its actual over its first estimated expenditure is put at nearly 100,000.000f., principally owing to errors described in our corresponding article last year. The works of the tunnel have not been interrupted a single day for six years, and its cost, notwithstanding unforeseen difficulties, will exceed the estimates little, if at all.

An Inland African Sea.—M. Ferdinand de Lesseps has read a paper to the French Academy of Sciences with reference to the interior African sea proposed by Captain Boudaire. M. de Lesseps sailed from Marseilles in November, 1878, and landed in the Bay of Gabes, into which flows the little river Melah, selected by Captain Boudaire as the channel communication between the Mediterranean Sea and the Chotts. Up this small stream the sea penetrates to a distance of about five miles at high tide. The last expression will naturally attract attention as the general belief has always hitherto been that no tide existed in the Mediterranean, with the exception of the Gulf of Venice, where a rise and fall of about 2 ft. occur. Such, however, is not the case, for at Gabes the tide rises to 8 ft. and sometimes 10 ft. The Tunisian Chotts commence at about 14 miles from the sea and extend to a distance of 250 miles. The ground which separates them from the sea would be easily worked. M. de Lesseps states that he noticed on the banks of the Melah some cliffs which appeared to consist of hard stone. He sent one of his companions to procure some specimens, and that gentleman broke off some pieces, which he placed in a bag in his pocket. When they came to be examined they were found to have all been reduced to small fragments, in fact, they consisted of nothing but agglutinized sand. A

portion of the material was handed to M. Daubrée for analysis, and he will make a report to the Academy on the subject. M. de Lesseps states that all the local traditions in Tunis agree as to the former existence there of an inland sea, and his opinion is that its re-establishment would be very easy. However, he said that nothing could be done until Captain Boudaire's mission of levelling and sounding was accomplished, and that would occupy at least six months. In reply to an observation by M. Cosson, M. de Lesseps stated that the formation of an inland sea would not interfere with the existing oases, as they are 15, 20, or even 30 meters above the proposed water level.

ORDNANCE AND NAVAL.

Difficulties are again experienced with the special 6 in. rockets ordered for the Afghan expedition. They have passed an unsatisfactory proof, two out of three having prematurely burst their cases during flight. Weighing nearly 100 lbs., which is four times the weight of the largest ordinary rocket, their cases were made of proportionate strength, wrought iron ⅛ in. in thickness being employed. Based on the practice of the smaller missiles, this was calculated to afford ample resistance to the explosive gases escaping in rear; but the calculation has been proved to be incorrect and the strength of the iron cases insufficient, and they have been ripped open as if they were made of brown paper. The consequence is a dilemma, for the thickness of the cases cannot be much increased without serious detriment to the range and efficiency of the rocket, and it is evident that the cases as at present constructed are quite unserviceable. Fortunately, only 120 were ordered—the odd twenty being for experimental purposes—and they are not nearly completed. The probability is that they will be abandoned, at least for the present, as impracticable, and that the army in India will make the best of the 9-pounder and 24-pounder rockets, which, so far as rockets have any merit, did fairly well in Abyssinia and Ashantee.

The Austrian Guns of the Uchatius Bronze Steel.—A report upon the new Austrian 6 in. gun of bronze steel has been sent to the Royal Artillery Institution at Woolwich by Major J. F. Owen, Royal Artillery, who has for five years past been employed in the Royal Gun Factories, Royal Arsenal, Woolwich. He states that the field guns manufactured of the Uchatius metal in 1875 have given such satisfaction that the Austrians are now making on the same pattern much larger guns with a bore of 6 in. diameter. Although a breech-loader the 6 in. gun fires a charge of nearly 18 lbs. of powder with a shell weighing 85½ lbs., and has given velocities of 1,476 ft. at the muzzle, which is equivalent to a very satisfactory performance against ironclad ships. In accuracy and range the guns have also given good results; and as to endurance, Major Owen states that they have shown themselves as strong as the field guns. Notwithstanding considerable erosion of the bore the accuracy of fire had not diminished, and the powder chamber has remained intact. Altogether the report is favorable to the gun, but Major Owen remarks that although the adoption of the new metal by the Austrians may have been judicious for the time being on economical grounds, there being a number of old bronze guns on hand, and also because the manufacture could be speedily carried out in their arsenal, he does not suppose that in the long run the material will be able to hold its own against steel, in the production and manufacture of which very great progress has lately been made, as exemplified at the Paris Exhibition.

The Thunderer Disaster.—A series of important experiments, suggested by the bursting of the gun on board the Thunderer, were carried out recently with the 38-ton guns on board the Dreadnought at Portsmouth. The experiments were conducted by Captain Herbert, of the gunnery ship Excellent, and among those present were Mr. George Rendel, the inventor of the hydraulic system of loading, Admiral Fanshawe, Rear-Admiral Foley, and Captain Lebrano, from the Italian Embassy. The purpose of the experiments was to ascertain whether the projectiles, after being rammed home by the hydraulic hammer at certain angles of depression, had a tendency to follow the rammers when withdrawn, and, secondly, when arrested by a wad, to determine the amount of force required to overcome the friction of the obstruction. A common shell was first tried without either a wad or a gas-check when it was found to slip back readily. A similar shell was then tried without a wad, but with a gas check affixed, with the same result. In both these instances the gun was depressed 14°, which is rather more than is requisite to load the Thunderer's guns. A common shell was next inserted in the gun, with gas-check and wad. It did not start, and it required the efforts of six men to pull it out, after three trials. A wad was also placed a few feet in the gun, without a shell, when it was found that it required eight men to pull it back. A new wad, suggested by Mr. Rendel, was afterwards tested. It is formed of lamina of different thicknesses, stitched together. A wad of single thickness took the strength of three men to remove; a wad of two thicknesses took four men to pull it out; while a wad of three lamina required five men to overcome it. The gun was afterwards charged with a common shell without wad. At seven degrees of inclination it slipped forward, and when the angle of depression was increased to nine degrees it ran forward 6 feet 3 inches. A dummy projectile was next tried, having a contrivance fitted to its base, the object of which was to enable the shot, should it slip forward, to draw the cartridge with it, and thus prevent the gun, in the circumstances, from going off. In two instances, the dummy did not take hold of the charge on being extracted, but in the third it brought out the cartridge. The last experiment was highly important. The gun was loaded with a Palliser shot, by means of the hand gear, without either wad or gas-check, but from a

horizontal position; The slide was then lifted to the upper step and the gun depressed and struck heavily on the port sill, but without starting the shot. The result of the experiment so far goes to prove that, without a wad, the projectiles will slip at seven degrees and over, but that with a wad the shot runs no chance of starting at the angle used for loading.

The gunnery experiments on board the Dreadnought, sister ship of the Thunderer, were concluded at Portsmouth on Wednesday. Previous to resuming them Captain Herbert and Mr. George Rendel paid a visit to the Inflexible, which is being fitted with the latter's hydraulic gear. But here a great improvement has been introduced, not, however, as regards the principle of the hydraulic system, but in respect to the construction of the ship itself for its application. The upper deck in wake of the turrets has an undulatory appearance, so that the guns may be rotated under the cover of the highest part to load. By these means the 80-ton guns will be loaded from below, at the reduced angle of 8°, instead of 14° on board the Dreadnought, or 11½° on board the Thunderer. The experiments of Wednesday were of a very different character from those of the preceding day, and were undertaken for the purpose of determining the truth or fallacy of certain theories which had been put forward, and more particularly by several correspondents in the *Times*. It required no elaborate experiments to prove that a shot had a natural tendency to slip back when the muzzle of the gun was depressed; and that at an angle of 11° or 14° of inclination it would certainly do so by a wad, or by the foulness of the bore when it frequently holds, especially with a gas-check leaving little windage. But it was alleged that the wads used on board the Thunderer and the Dreadnought were not a sufficient protection against the starting of the shot, and that this starting might be assisted down the incline, first, by the pressure of the condensed air at the rear of the shot after being rammed home, and which had no means of escape; secondly, by the suction or vacuum produced in front of the projectile by the rammer on being withdrawn, and which imparted a tendency of the shot to follow the rammer; and, lastly, by the united force of these two disturbing influences. Arguments of this kind could only have been put forward by persons practially unacquainted with the hydraulic system of loading, and more particularly with the character of the rammer head. Nevertheless, as such opinions had gained currency, it was resolved to test their value by experiment, and the trials on board the Dreadnought, were for the purpose of ascertaining the force of the alleged air-pressure behind the shot and the alleged suction between the wad and the rammer head, to push the projectile forward. For this purpose the ventilating holes in the rammer were plugged up—as it seems there are no apertures in the rammers used on board the Thunderer—and a mercurial pressure gauge was attached to the vent of the gun and a second one to the rammer head. As the shot was forced rapidly down the bore of the gun for half its distance by the first length of the rammer the mercury rose an inch in the tube, indicating a pressure of ¼ lb. per square inch of the shot's area—a perfectly insignificant amount; but before the second half of the telescopic rammer (which works less rapidly than the first) could force the shot home upon its seat even this pressure had vanished, the air escaping through the grooves of the rifling and the interstices of the sponge. The charging was next tried with a view of testing the vacuum. Here, again, the result was found to be perfectly satisfactory and reassuring, the vacuum being quite harmless. But the most unexpected result of the experiment was the discovery that the vacuum which was found to be produced in rear of the projectile, and the tendency of which was to pull it back, was only to an infinitesimal degree less appreciable than the vacuum found in front, and which had a tendency to pull it forward. The experiments thus far have proved eminently satisfactory so far as the system adopted for loading is concerned, and go far to show that the Thunderer's gun exploded through the starting of the shot—which remains to be proved, and which will probably have to be ascertained by experiment—the slipping back can only be attributed to the omission of the wad during the rapid practice. In describing the last and most important experiment of Tuesday it was stated that the round was loaded without either wad or gas check and while the gun was in a horizontal position. This was an error. Both on board the Thunderer and the sister ship, the 38-ton guns are loaded by hand at precisely the same depression as is required by the hydraulic apparatus, the advantage of horizontal loading having had to be sacrificed to the necessity of adding 3 feet to the gun. Both wad and gas-check were used on the occasion, and the object of the experiment was to ascertain if a wad capable of holding the shot and charge securely from any risk of slipping forward could be put up by the hand rammer which has been provided to replace the hydraulic rammer in the contingency of the latter failing. The hydraulic rammer was accordingly removed and the alternative gear employed, but in the same position; and not only did the wad hold so well that the shot could not be started by any inclination or jerking of the gun, but it required the combined efforts of eight men to remove it. The supreme purpose of the experiments has been to ascertain the precise amount of the tendency of the shot to run forward, and to compare it with the amount of hold of the wad provided to prevent the slipping. At 14° the force of the shot to move forward is 1 in 4, or, allowing for friction, the tendency does not exceed 80 lbs. even in a clean gun, while, on the other hand, the wads provided for the ship are found to require over half a ton and upwards to pull them out. At 8° the tendency is 1 in 7, or 100 lbs., but, deducting the same allowance for friction, the result is an equilibrium of force. It may be worth stating that wads of some sort are required for all guns, even on the broadside.

The experiments at the Royal Gun Factories in the Royal Arsenal, Woolwich, to throw light upon the causes of the explosion of the 38-ton gun on board Her Majesty's ironclad

turret ship Thunderer were completed on Tuesday morning, and the tendency of projectiles to slip forward when the muzzles of the guns are depressed has been tested with each description of heavy gun in the service. In every case the shot slipped, sometimes at a depression of 8°, and always before reaching the angle of loading by the hydraulic machinery, which is about 10 or 12°. The bore of the 38 ton guns is 16 feet 6 inches in length, and it is the opinion of the authorities at the Royal Gun Factories that the shell in the Thunderer's gun had slipped to a position about one-fourth of the way, which would correspond externally with the front of the trunnions. Major Owen, Royal Artillery, the representative of the War Office, has arrived at Malta, but has not yet communicated his conclusions to the authorities at home. It is understood that, notwithstanding the frightful nature of the shock caused by the explosion, the turret of the Thunderer still revolves, and that the fellow gun in the turret could in a very short time have been worked and fought had it been necessary to do so. The committee appointed by the Admiralty to inquire into the recent disaster consists chiefly of naval officers now at Malta, Mr. Bramwell, F. R.S., acting as assessor. Captain Noble, late of the Royal Artillery, will attend to represent Sir Willliam Armstrong & Co., the manufacturers of the hydraulic machinery for loading and working the guns in the fore turret on board the Thunderer.

BOOK NOTICES.

TURBINE WHEELS. By PROF. W. P. TROWBRIDGE, of Columbia College. Van Nostrand's Science Series No. 44. New York: D. Van Nostrand. Price 50 cts.

The object of this essay is to show that the theory of Turbines as set forth by Rankine, Weisbach, Bresse, and others, is not a satisfactory elucidation of the principles upon which the modern turbines are based.

The author says: "It may be shown, I think, that if Boyden and Francis had followed strictly the rules of construction laid down in the works alluded to, they would have failed in their efforts to construct Turbines giving any considerable increase of efficiency over the old Fourneyron or Journal wheels of European design or construction."

The discussion of the essay is intended to apply to the later forms of Turbines, but the formulas are applicable to nearly all water wheels, by making simple and proper suppositions in regard to the quantities which enter into them.

MANUAL OF PRACTICAL CHEMISTRY. THE ANALYSIS OF FOODS AND THE DETECTION OF POISONS. By ALEXANDER BLYTH, F.C.S. London: Charles Griffin & Co. For sale by D. Van Nostrand. Price $5.00.

This work does not present, in any part, the ordinary scheme for qualitative analysis. To employ the book to advantage, the reader must be familiar with the methods and materials of a laboratory.

The treatise is in two divisions, land is further subdivided into "parts" as follows:

First Division. Analysis of Foods.

Part I, Ash, Sugar, Starches; II, Wheat, Flour, Bread; III, Milk, Butter, Cheese; IV, Tea, Coffee, Cocoa; V, Alcohol, Alcoholic Liquors; VI, Vinegar; VII, Mustard, Pepper, Almonds, Annatto.

Second Division.

Part I, Introductory; II, Prussic Acid, Phosphorus, Chloroform; III, Alkaloids and Vegetable separated for the most part by Alcohol; IV, Animal Poisons; V, Inorganic Poisons.

In the first division will be found most of the valuable information that is not obtainable from other sources, and in this portion the author has presented a large collection of analysis of the common things of the household.

INSTRUCTIONS FOR TESTING TELEGRAPH LINES AND THE TECHNICAL ARRANGEMENT OF OFFICES; WRITTEN IN BEHALF OF THE GOVERNMENT OF INDIA, UNDER THE ORDERS OF THE DIRECTOR GENERAL OF TELEGRAPHS IN INDIA. By LOUIS SCHWENDLER. Vol. I. London: Trübner & Co. New York: D. Van Nostrand. 1879. Price $4.00.

The first volume of this work, which has recently been issued, is in substance a revised compilation of the contents of a series of circulars of instruction relating to the practical work of the Government telegraph lines in India, which were prepared by Mr. Schwendler at intervals from 1869 to 1876, and printed for the especial use of the staff in that department. The very general demand among members of the profession in other countries for copies of these circulars, finally led Mr. Schwendler to undertake the preparation of a second and revised edition in a more convenient and accessible form. The instructions in the present work, although thus prepared for a special purpose and under the influence of local circumstances, are almost equally well adapted to the use of practical electricians in other countries. In fact, it may be said, that the peculiar difficulties which are met with in the construction, maintenance and administration of telegraphic lines in India, will be found to exist to a much greater extent on the American continent than in any part of Europe, and therefore it is not unlikely that Mr. Schwendler's work will ultimately prove of more real value in this country than anywhere else outside of India.

The present volume is devoted entirely to the subject of testing; the portion relating to the technical arrangement of offices, etc., being reserved for the succeeding volume, which is intended to be published some time in the course of the present year.

The first volume is divided into two parts. The first part treats of the apparatus employed in line testing, *viz.*, the so-called "Wheatstone" bridge, and the differential galvanometer. Of these two methods, Mr. Schwendler very properly gives the most decided peference to the former. The differential galvanometer, he says, for the same bulk and cost, can never combine the same accuracy and sensibility within wide limits as the bridge.

The mathematical theory of the bridge under various conditions is given at great length, and with an imposing array of formula. A theoretical and an actual plan of a testing board is given at the end of this section, which embodies several very valuable features not often found, if at all, in the ordinary arrangements familiar to electricians. For example, the galvanometer coil is wound in two sections and provided with a commutator, which enables the sections to be placed in series or in multiple arc, according as the resistance to be measured is great or small. Another commutator provides for interchanging the position of the battery and galvanometer in their respective diagonals, a great convenience when very large and very small resistances are required to be measured by the same apparatus.

The second part is devoted solely to the examination and solution of the problems which practically arise in the electrical testing of telegraph lines; the determination of their general electrical condition, the localization of faults, etc., etc. Although this chapter is very full, and exceedingly well arranged, there is, perhaps, little or nothing contained in it which is new to the well-informed electrician. An unusual degree of attention is devoted to the methods of deducing the true electrical condition of the line from the apparent condition as shown by the results of the actual tests, a matter which has not by any means received due attention in most of the existing test-books. The directions for checking and eliminating untrustworthy measurements, in making up averages, are also of great value, since this is a matter that frequently leads to error on the part of inexperienced practitioners. So far as we have been able to discover, almost every condition likely to come up in practice has been duly provided for, and this, by the way, is an advantage which has resulted from the manner in which the book has grown up, piecemeal, as it were, in accordance with the requirements of actual service, as manifested from time to time.

Not the least useful part of the work are the various appendices, giving actual examples, taken from the records of the Indian lines, which show the practical application of the methods set forth, in detail. Specimens of all the blank forms, for records, reports, etc., which are there used, are given, and altogether this part of the work will hereafter prove of great service to the officers of other administrations, who are desirous of organizing a system of testing for the purpose of securing similar results.

We think it would have added very much to the practical value of Mr. Schwendler's work, if he had devoted a reasonable amount of space to the theory and practice of line testing with the tangent galvanometer, which, although not to be compared with the differential apparatus in accuracy of its results, is nevertheless a most admirable instrument for ordinary every-day work, in the hands of the average operator. By means of it, it is easy to keep the run of the condition of the lines by going over them rapidly every morning, and in case any fault is discovered, the more accurate differential methods may be resorted to for quantitive measurements, as is now the general practice in this country. The excellent and convenient instruments constructed by Mr. Phelps have proved very serviceable for this work, and have received frequent approval from foreign electricians who have seen them in use here. It is not very probable that so well-informed and experienced an electrician as Mr. Schwendler, is altogether ignorant of the value of the tangent galvanometer when used in this way, and we are, therefore, disposed to find a more probable explanation in the fact that the conditions of the Indian service are unlike ours in one important respect, viz., the much greater average length of circuits and distances between testing stations, and the much smaller number of wires in each office. This renders a more considerable degree of accuracy necessary, and at the same time enables it to be obtained with less inconvenience. Mr. Schwendler's precise habits of mind may also have led him to undervalue the "rough and ready" processes and loose approximations of this mode of measurement. He is a skillful mathematician, as the pages of the work under consideration sufficiently attest; and doubtless has the true mathematician's horror of inexact results. Nevertheless, it is to be hoped he will in a future edition take up this matter, and handle it with his accustomed skill.

The typographical execution of the work is excellent. The type is clear and open, and the paper and press work unexceptionable. The succeeding volume, treating, as it will, upon a branch of telegraphic engineering which has been almost entirely overlooked not alone by the writers of test-books, but even by the contributors of the professional journal, cannot fail to be of great practical value. Its appearance will be awaited with interest, especially in this country, where far greater attention appears to have been paid to this subject than has been the case abroad.—*Journal of The Telegraph.*

Army Sacrifices; or Briefs from Official Pigeon-Holes. By James B. Fry, Colonel Adjutant General's Department, and Brevet Major-General United States Army. New York: D. Van Nostrand. 1879. Price $1.00.

This is a neat volume of sketches which have been "grouped together for the purpose of illustrating the services and experiences of the regular Army of the United States on the Indian frontier." Notwithstanding our innumerable Indian fights, there are very many Americans who have no correct idea of the real life of a soldier in the far West. Here, in the East, officers and soldiers have a comparatively easy time of it; but the ordinary duties in the West are not only without glory, but are attended with dangers and sufferings no less terrible and trying than those of the great wars whose victims receive the unstinted sympathy of their fellow-men, and whose heroes are held up forever to the admiring gaze of millions. The savage foe, the sudden storm, the pitiless wind, the biting frost, the frequent hunger, encountered every year by the larger part of the

Army, call for an aggregate of courage, skill, and fortitude which is perhaps not found elsewhere in the world. The book before us is a recital of some of the more thrilling incidents which have occured from time to time west of the Mississippi River. Most of the sketches are of actual occurrences, and are presented as examples of the dangers and privations to which our soldiers in active service in the Indian country are continually exposed and the gallantry and fortitude they display. So far as these are concerned, the book is a creditable reproduction of official reports, elaborated, perhaps, by confirmatory information of a reliable character. Some of them would have been improved by the elimination of certain introductory remarks at the beginning of each new piece, and the striking out of the too frequent notification that "the first effort of Indians in making an attack is almost invariably to stampede the animals." Nor can we approve of two or three selections, which, to our mind, have no connection whatever with Army life, and which we cannot believe form any part of official records. For instance, the description of the "Penitentes," a religious sect of New-Mexico, while highly interesting in its hideous details, is certainly out of place in a book illustrative of Army sacrifices; as is also the story of "Bill and Dan," two industrious brothers, who, unfortunately for them, no doubt, "met a tragic fate," but one not more tragic than that suffered by hundreds of other frontiersmen.

There are soldiers who fell in the cheerful performance of arduous and dangerous duties whose deeds and death are almost forgotten, save by their families and a few frontiersmen who, in their wanderings, may cross the trails or streams which bear their names. Fortunately, this book contains very many examples of this sort, records of unsurpassed bravery and endurance, of the deeds of men who were not afraid of death, and who did their whole duty as best becomes a soldier. Fiction itself can present no more pathetic incidents or more stirring tales of heroism than some which are so graphically presented to us by General Fry. The story of Lieut. Mellen's sufferings—the summons to duty at a distant post, the lonely ride, the swollen river, the noble horse and gallant rider breasting the turbid waters, the sudden slip and struggle amid the icy waves, the grasp for life, the long dead blank, the painful awakening to a possible death from cold and starvation, the terrible efforts to regain the saddle, the agonizing ride, and the final amputation of both feet—all these were but actual occurrences, growing out of the execution of a simple order in the ordinary routine of an officer's life on the frontier. Take, also, the case of Brevet Lieut-Col. Forsyth, of which we shall have more to say further on, and of Major-Gen. George L. Hartsuff, a hero of the civil war, who died in this city, in 1874. This gallant soldier's fight, when but a Lieutenant, with a band of Seminoles in Florida, in December, 1855, is a wonderful story. The description of Roman Nose, a great war leader, is worthy of quotation. "He was," says Gen. Fry, "one of the finest specimens of the untamed savage. It would be difficult to exaggerate in describing his superb physique. A veritable man of war, the shock of battle and scenes of carnage and cruelty were as the breath of his nostrils; about 30 years of age, standing six feet three inches in height, he towered, giant-like, above his companions. A grand head, with strongly marked features, lighted by a pair of fierce black eyes, a large mouth with thin lips, through which gleamed rows of strong white teeth, a Roman nose, with dilated nostrils like those of a thoroughbred horse, first attracted attention, while a broad chest, with symmetrical limbs on which the muscles under the bronze of his skin stood out like twisted wire, are some of the points of this splendid animal. Clad in buckskin leggings and moccasins elaborately embroidered with beads and feathers, with a single eagle feather in his scalp-lock, and with that rarest of robes, a white buffalo, beautifully tanned and soft as cashmere, thrown over his naked shoulders, he stood forth the war chief of the Cheyennes." The occasion was a council at Fort Ellsworth, late in 1866, at which the Indians protested against the building of railroads, because they were driving away the buffalo, and thus interfering with their hunting grounds.

Roman Nose was afterward killed while leading a large band of savages against Major George A. Forsyth, of the Ninth Cavalry. Forsyth was then, as he is now, on Gen. Sheridan's staff. He had accompanied his chief to the far West, and chafing under the restraint and inactivity of his position, begged a command in the field. It was finally decided that he might lead a force of 50 men, who had been specially hired for the occasion. This little force was soon in the saddle and in active pursuit of the Indians. On the night of the 16th of September, 1867, Major Forsyth's force was surrounded by nearly a thousand savages, and then followed one of the most heroic and effectual struggles in the history of mankind. The Indians rushed in upon the heroic half a hundred time and time again, but as often were they received by the well-directed fire of musketry. At the close of the first day, every horse and mule was dead, Lieut. Beecher, the second in command, was killed, three men, including the doctor, lay dead in the "trenches," two others were mortally, and 17 severely wounded. Before 10 o'clock, Forsyth himself had been shot in the thigh; a few hours afterward a ball entered his left leg below the knee, completely shattering the bone, and before night a third ball grazed the top of his forehead, chipping out a piece of the skull. Men volunteered to go out in search of relief. Day after day the fight was renewed and all the food our heroes had was raw mule meat. But presently, it grew putrid, could no longer be eaten, and the pangs of hunger began. By the seventh day the Indians had entirely disappeared, but the beleaguered force were now to weak to move. Men became delirious, and the wounded ones suffered dreadfully. The shattered bone of brave Forsyth's leg stuck through the skin, and

maggots took possession of the horrible sore. The eighth day wore away, and then, on the morning of the ninth day, friends arrived. The empty honor of a "brevet" was the only recognition Forsyth received for his heroic conduct in this affair, and well may Gen. Fry ask, "Can bravery, gallantry, and devotion to duty flourish under a military system in which such services are neither rewarded nor remembered?"

In his preface, Gen. Fry says it is unavoidable that the recitals in those pages exhibit the worst features of the Indian character, such not being the purpose of the work. "Driven continually behind our rapidly-advancing frontier, plundered and abused by the more powerful and aggressive race, without one particle of redress for any wrong done him by the white man, and knowing no law but that of retaliation and vengeance, it is not strange that the barbarian should indulge in bloody deeds." After discussing this question at some length, the General arrives at certain conclusions, which, briefly summarized, are: First, localize the Indians, subdividing tribes into bands, and securing the title of land to them in common by a deed of trust; secondly, place these locations under martial law, to be administered upon white men and Indians alike; and, thirdly, permit all proper intercourse, especially inter-marriage, between the whites and Indians.—*N. Y. Times.*

MISCELLANEOUS.

Mr. Galloway's theory relating to the part played by floating coal-dust in mines, is receiving much support. M. L. Simonin recently stated certain facts which he considers prove that in the majority of cases it is to the heating of the coal-dust diffused in the galleries of the mines to which explosions are due. Referring to the catastrophe in the Jabin Mine, at St. Etienne, February 4th, 1876, he states, on the authority of the manager, that the mine in question contains very little fire-damp, and that the precautions hitherto taken, with an exclusive reference to that gas, are not sufficient Others must be taken against the extremely fine coal-dust, which at the moment of the explosion of slight amounts of fire-damp, or even of blasting powder, liberates rapidly a part of the coal-gas which it contains) and propagates the explosion, reproducing the cause of the evil with so much the greater energy as the current of air is more violent. Thick crusts of coke—2 or three centimeters—prove this fact, and explain how it is that extensive tracts in which fire-damp has never been observed are burnt like the rest of the workings. Hence, it appears that the precautions to be taken in fiery mines are complex whenever the coal-dust is rich in gas and very finely divided. Explosions may then ensue even in mines where fire-damp is unknown. There is no need to suggest the existence of cavities full of carbonic oxide, or of gaseous hydrocarbons, and suddenly laid open by a blow from a miner's pick. The *Chemical News* suggests that the attempt, recently made in some mines, to screen the coal below ground, is a dangerous mistake, as calculated to increase the quantity of dust diffused in the air of the mine.

Tire Fastenings.—A new and valuable method of fastenings tires, invented by Mr. Kaselowsky, a German engineer, has successfully stood the test of a series of experiments. A dovetailed groove is turned in the inner face of the tire, and a similar one in the outside of the skeleton, so that, when the tire is slipped on, the two come opposite to each other and form a channel of dowel-shaped section going all round the wheel. Into this channel is run some easily fusible metal (by preference pure zinc), which, on cooling, makes a firm connection between the tire and wheel. In carrying out the operation the tire is only slightly heated, a shrinkage of $\frac{1}{1500}$ being found ample, and is then brought over the skeleton, which is laid in a horizontal position, and forced upon it. The zinc is then immediately run in through the holes cast in the skeleton, if of cast metal, or drilled in other cases; thus the zinc is at once prevented from cooling while being run in, and is compressed, and thus rendered much stronger, by the subsequent contraction of the tire. That this mode of fastening, in addition to its simplicity and cheapness, offers full security, both against sideways shifting, and in case of breakage of the tire, has been proved by experiments made in the central workshops at Frankfort.

In a paper read before the French Association, in August, M. Daubree, director of the School of Mines of Paris, says that one of the most remarkable characters of the rocks which have undergone mineralogical transformations, comprised under the name of "metamorphism," is that the rocks thus transformed are often associated, occupying together considerable territory, while in other regions, the same rocks still more extensive, do not present like modifications. These transformations are probably due to the influence of heat produced by mechanical actions, that have left their traces in the bendings and foldings of the strata. M. Daubree, after a series of experiments on the heat produced in rocks by interior movements, draws the following conclusions: (1) The rocks were already in a solid state at the period when they followed the action which contorted them. (2) Many of these rocks during these movements acquired a laminated structure. (3) Certain effects of regional metamorphism may be derived simply from the heat which has been developed in the rocks by mechanical action. (4) Fossils have been destroyed by trituration in the interior movements of such rocks as have become changed in texture or assumed a crystalline state. "Finally," says M. Daubree, "in rock masses where metamorphism has been developed on a great scale, and far from any eruptive rock, the heat which has presided over the transformation of the rocks, and the appearance of new species of minerals, may have been caused by the very mechanical actions which these rocks underwent." M. Daubree thus seeks an explanation of the evidences of heat action in the mechanical action resulting from loss of heat, to which Mallet ascribed volcanic and earthquake phenomena.

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THERMODYNAMICS.*

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III.

30. PERFECT GAS FLOWING FROM ONE VESSEL TO ANOTHER.—Let p_1 v_1 t_1 q_1 be the pressure, specific volume, temperature and total weight respectively of the gas in the first vessel in its initial state, and let p_2 v_2 t_2 q_2 designate the initial state in the second vessel.

Let same gas flow from the first vessel to the second, and let the final state of the first vessel be designated by the subscript 4. and the final state of the second vessel by 3.

$$\therefore\ q_1-q_4=q_3-q_2 \quad . \quad . \quad (156)$$

is the quantity of gas which has flowed from the first vessel to the second,

also, $$q_1v_1=q_4v_4 \quad . \quad . \quad . \quad (157)$$

and, $$q_2v_2=q_3v_3 \quad . \quad . \quad . \quad (158)$$

are the volumes of the first and second vessels respectively.

Also, if no heat is imparted to or withdrawn from the gas, the expansion in the first vessel is adiabatic, but the second vessel has the energy imparted to it which is withdrawn from the first vessel.

$\therefore$ by (129) or (135), and by (157)

$$\frac{p}{p_1}=\left(\frac{v}{v_1}\right)^{-n}=\left(\frac{q}{q_1}\right)^{n}=\left(\frac{t}{t_1}\right)^{\frac{n}{n-1}} \quad . \quad . \quad (159)$$

express the state in the first vessel at any time.

By (10), $$\int_1^4 qdw=\int_1^4 qpdv \quad . \quad . \quad (160)$$

is the work of expansion in the first vessel from the state 1 to the state 4 performed in driving gas into the second vessel. Since the expansion is adiabatic, this work can be expressed by help of (159) in terms the single variable p.

$$\therefore\quad \int_1^4 qdw=-\frac{q_1v_1}{n}\int_1^4 dp=\frac{q_1v_1}{n}(p_1-p_4) \quad . \quad . \quad (161)$$

is the work of expansion gained by the second vessel. It can be expressed in terms of either other variable by help of (159).

Again, $$\int_1^4 sdq=k_v\int_1^4 tdq \quad . \quad . \quad (162)$$

is the energy contained in the gas gained by the second vessel in the form of heat. But by (121), (109) and (159)

$$\int_1^4 sdq=-\frac{q_1v_1}{n(n-1)}\int_1^4 dp=\frac{q_1v_1}{n(n-1)}(p_1-p_4) \quad . \quad . \quad . \quad (163)$$

The total energy transferred from the first to the second vessel is the sum of (161) and (163)

* Figs. 2, 3 and 4 which should have been inserted in the April number of the Magazine will be found at the end of this article.

$$\therefore \int_1^4 q\,dw + \int_1^4 s\,dq = \frac{q_1 v_1}{n-1}(p_1 - p_4) \quad . \quad (164)$$

This can be otherwise readily shown, for

by (141) $\quad nk_v \int_1^4 t\,dq \quad . \quad . \quad . \quad (165)$

is the total energy transferred. But (165) is n times the last member of (162), hence its value is n times the last member of (163), which was to be proved.

By writing the subscript 4 in (159) we find the relations between the quantities p_4, v_4, t_4, q_4, which designate the second state of the first vessel.

To find the second state of the other vessel; the energy of sensible heat in the initial state of the gas in this vessel, is by art. 23,

$$k_v\, q_2 t_2 = \frac{q_2 v_2 p_2}{n-1} \quad . \quad . \quad (166)$$

and the energy imparted by the first vessel is (164): the sum (164) and (166) must be equal to the total energy of sensible heat of the gas in the state 3, which is $k_v\, q_3\, t_3$. Now divide by k_v and we have

$$q_3\, t_3 = q_2 t_2 + q_1\, t_1 \left(1 - \frac{p_4}{p_1}\right) \quad . \quad (167)$$

in which q_3 is given by (156), hence t_3 can be found from (167)

Also, by (9), $\quad \frac{p_2 v_2}{t_2} = \frac{p_3 v_3}{t_3}$

$\therefore$ by (158), $\quad \frac{p_2}{q_2 t_2} = \frac{p_3}{q_3 t_3} \quad . \quad . \quad (168)$

Now divide (167) by $q_2\, t_2$

$\therefore$ by (167), $\frac{p_3}{p_2} = 1 + \frac{q_1 t_1}{q_2 t_2}\left(1 - \frac{p_4}{p_1}\right) \quad . \quad (169)$

from which p_3 can be found. We have now shown how to find each of the variables p_3 v_3 t_3 q_3 in terms of known quantities, when a given quantity $q_1 - q_4$ flows from one vessel to the other.

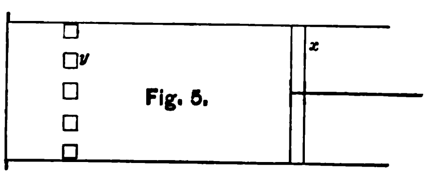


Fig. 5.

31. Hot Air Engines.—In Fig. 5 let x be a solid piston working in a cylinder by which the volume of the air, or other perfect gas, contained in the cylinder and in vessels connected with it may be made to vary. Let x be called the working piston. And let y be a perforated piston also working in a cylinder. On moving y air is transferred through it from one side of it to the other. Let y offer no resistance to the passage of air through it, so that the pressure of the entire mass of air confined by the working piston x is the same on whichever side of y it may be. The piston y serves to divide the whole confined air into two parts, which we shall suppose to be at different temperatures but at the one pressure. Let the whole space at the left of y be called the furnace, and let the air in the furnace be all at the one temperature. Call the space at the right of y the clearance, and let all the air in it be at another temperature. The piston y is the supply piston of the furnace, since by it air is transferred from the clearance to the furnace.

It is seen that by moving x the space occupied by the clearance is altered, but the volume and pressure are varied in both furnace and clearance. By moving y the total volume of furnace and clearance is unchanged, but the space occupied by one is increased at the expense of the other, while the air transferred from one to the other has undergone a change of temperature, and consequently there has been also a change of the specific volumes and pressures in both furnace and clearance.

The simple arrangement just described will enable us to discuss various different forms of air engines, without regard to the special contrivances which may be employed for imparting and rejecting heat, or for causing the working air to pass through either of the possible cycles which will be assumed.

We shall confine our attention principally to the efficiency of the various cycles assumed for the working substance, as the determination of the volume swept through by the working piston and other practical matters of the first importance in actually designing an engine must be omitted in the present brief, theoretical discussion.

One special contrivance, called the regenerator, should, however, be noticed in this connection, on account of the important role it plays in modifying the cycle of changes through which the

air passes. When an air engine is supplied with a regenerator we shall conceive that the piston y is this regenerator, which in that case will be supposed to be constructed in such a manner as to store up all, or a part of the heat of the air passing through it from the furnace to the clearance, and to restore it to the air passing from the clearance to the furnace. The regenerator may be conceived to consist of many sheets of wire netting. If no conduction of heat occurs in the regenerator, the heat will be stored and restored at the same temperature, so that the passage of the air through it may be regarded as being attended by an adiabatic variation of the working substance, when by working substance is meant both the air and the material of the regenerator.

32. Working Air and Cushion Air.—That part of the air in any air engine to which heat is imparted, and from which heat is rejected is called the working air. That part of the air which experiences only adiabatic changes is called the cushion air, and being alternately expanded and compressed, performs as much work during expansion as is expended in compressing it. Hence the external work performed depends in no way upon the cushion air. In practice, part of the cushion air remains in the furnace, but most of it is usually in the clearance. The cushion air however has an important effect on the size of the cylinder. Since it expands at the same time as the working air, the proportion of cushion air to each unit of working air should be as small as possible, in order that the space swept through by the working piston x may be as small as possible.

33. Air Engine with Adiabatic Expansion.—1°. Let both x and y move to the right in such a manner that the initial temperatures t_1 and t_2 of the clearance and the furnace are unchanged during this first operation: hence $p_1=p_2$ is also unchanged. This operation involves heating all the air which passes through y into the furnace from t_1 to t_2 at the instant it passes through y.

The total amount of heat imparted to a unit of air in carrying it from the clearance to the furnace at constant pressure is

$$\text{by (115),} \qquad h_2-h_1=k_p\,(t_2-t_1) \quad . \quad (170)$$

2°. Let x and y move to the right in such a manner that an adiabatic expansion occurs in both the furnace and clearance without transfer of air through y.

Since the pressure, the specific volumes and the temperatures t_1 and t_2 were unchanged during the first operation, we have,

$$p_1=p_2, \text{ and } p_3=p_4 \quad . \quad (171)$$

and also, by (171) and (135),

$$\frac{t_4}{t_1}=\left(\frac{v_4}{v_1}\right)^{1-n}=\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{n}}=\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}=\left(\frac{v_3}{v_2}\right)^{1-n}=\frac{t_3}{t_2} \quad . \quad . \quad . \quad (172)$$

in which the subscript 4 designates the state in the clearance at the end of the second operation, and 3 the state in the furnace.

3°. Let x and y move toward the left in such a manner that the temperatures t_4 and t_3 remain constant during this third operation. This involves cooling all the air from t_3 to t_4 at the instant it passes through y into the clearance.

The total amount of heat rejected in transferring a unit of air from the furnace to the clearance is

$$\text{by (115),} \qquad h_3-h_4=k_p\,(t_3-t_4) \quad . \quad (173)$$

4°. Let x and y move to the left in such a manner that an adiabatic compression occurs without transfer of air through y, and let the compression continue until at the end of this fourth operation the air is restored to its initial state in both furnace and clearance. Since the pressure, the specific volumes and the temperatures t_3 and t_4 were not changed during the third operation, (172) applies to this fourth operation when the subscripts 4 and 1 designate the states of the clearance at its beginning and end respectively, while 3 and 2 designate the corresponding states of the furnace.

The total external work performed must be the heat imparted (170), minus the rejected (173).

The efficiency ε, is the ratio of the work performed to the heat imparted.

$$\therefore\ \varepsilon=\frac{(t_2-t_1)-(t_3-t_4)}{t_2-t_1}$$

$$\therefore \text{ by (172),} \quad \varepsilon=\frac{t_2-t_3}{t_2}=\frac{t_1-t_4}{t_1} \quad . \quad . \quad (174)$$

Hence by art. 18, an air engine working in the cycle, above indicated, is equiva-

lent in efficiency either to a perfect engine between the temperatures t_2 and t_3 of the beginning and end of the expansion of the working air, or to a perfect engine between the beginning and end of the expansion of the cushion air. As t_2 is the temperature of the source and t_4 that of the refrigerator, the efficiency above given would in all cases be less, and practically much less than the efficiency of a perfect engine between these limiting temperatures, which efficiency is by art. 18,

$$\varepsilon = \frac{t_2 - t_4}{t_2} \quad . \quad . \quad . \quad . \quad (175)$$

The only manner in which the value of (174) can be made to approach (175) is by making t_1 approach t_2, and hence t_4 also approach t_3, but this would require a very large working cylinder in proportion to the power developed, a difficulty which has been inherent hitherto in the designs of air engines.

Now let a regenerator be applied to an air engine working in the cycle just described, in such a manner that it stores up during the third operation (during which air is carried from the furnace at a temperature t_3) a sufficient quantity of heat so that during the first operation the regenerator shall restore the air as it enters the furnace to the same temperature t_3.

$\therefore$ by (115), $\quad k_p\,(t_3 - t_1) \quad . \quad . \quad . \quad (176)$

is the heat stored and restored by the regenerator per unit of working air.

Also, $\quad k_p\,(t_2 - t_3) \quad . \quad . \quad . \quad (177)$

is the heat imparted by the furnace,

and $\quad k_p\,[t_3 - t_4 - (t_3 - t_1)] \quad . \quad . \quad (178)$

is the heat rejected per unit of working air.

$$\therefore \ \varepsilon = \frac{(t_2 - t_3) - (t_1 - t_4)}{t_2 - t_3}$$

$$\therefore \text{ by (172)}, \quad \varepsilon = \frac{t_2 - t_1}{t_2} = \frac{t_3 - t_4}{t_3} \quad . \quad (179)$$

It is seen by comparing (179) with (174), that an air engine working in this cycle has its efficiency increased by a regenerator in case t_3 is greater than t_1, but its efficiency is diminished in case t_3 is less than t_1. Since, however, as has been previously stated, t_1 must be almost equal to t_2 in case the efficiency approaches the case of perfect efficiency contemplated in (175), it appears that in general a regenerator would be useless in an engine having a cycle of this kind, for t_3 should be less than t_1.

The cycle of this engine is imperfect because the temperature of the working substance is increased by imparting heat to it, and lowered by losing heat, whereas for a perfect cycle, by art. 18, the heat should be imparted along one isothermal and rejected along another, which is not the case in the cycle which has been discussed.

34. Air Engine with Isothermal Expansion.—Let the cycle of operations which the working air is made to undergo be such that the temperature t_2 of the furnace is constant during all the operations, it being that of the source of heat; and the temperature t_1 of the clearance also remains constant during all the operations, it being that of the refrigerator. The first and third operations will then be the same as in art. 33, but in the second and fourth operations heat is imparted and rejected in such a manner that the working air undergoes isothermal expansion and compression instead of the adiabatic variations supposed in art. 33.

By (115), $\quad k_p\,(t_2 - t_1) \quad . \quad . \quad . \quad (180)$

is the heat imparted during the first operation. The heat imparted to a unit of the working air during the isothermal expansion is

by (119), $\quad t_2(e_1 - e_2) = c t_2 \log_e \frac{p_2}{p_1} \quad . \quad (181)$

By help of (122) and (135) we have

$$t_2(e_1 - e_2) = k_p\, t_2 \log_e r \quad . \quad . \quad (182)$$

which is the heat imparted during the isothermal expansion from the initial pressure p_1 to the final pressure p_2.

$$\therefore \ k_p\,[t_2 - t_1 + t_2 \log_e r] \quad . \quad . \quad (183)$$

is the total heat imparted to a unit of the working air. Similarly

$$\therefore \ k_p\,[t_2 - t_1 + t_1 \log_e r) \quad . \quad . \quad (184)$$

is the total heat rejected per unit of working air during the third and fourth operations.

$$\therefore \ \varepsilon = \frac{t_2 - t_1}{t_2 + \dfrac{t_2 - t_1}{\log_e r}} \quad . \quad . \quad (185)$$

This cycle is more perfect as the ratio of expansion is increased.

By (135) the ratio of expansion $v_2 \div v_1$ is related to the quantity r in the following manner.

$$v_2 \div v_1 = r^{\left(\frac{1}{1-n}\right)} \quad . \quad . \quad . \quad . \quad (186)$$

Let a regenerator be applied to an air engine working in the foregoing cycle. Then the quantity of heat stored during the third operation and restored during the first operation per unit of air is expressed by (180), hence the total heat imparted by the furnace is during the second operation, and is expressed by (182).

Similarly, the total heat rejected is during the fourth operation and is got by changing t_2 into t_1 in (182). Let the temperature of the furnace and clearance be t and t' respectively.

$$\therefore \ \varepsilon = \frac{t-t'}{t} \quad . \quad . \quad . \quad (187)$$

is the efficiency of an air engine with a regenerator working in this cycle. As is seen, this engine is of perfect efficiency, and it is made so by the action of the regenerator, which fact confirms the statement made in art. 28 to the effect that the changes undergone in passing the regenerator are adiabatic, since no engine can have a perfect efficiency, as this one has, by art. 18, except its cycle consists of adiabatics and isothermals alone.

35. Ericsson's Hot Air Engines.—Two kinds of hot air engines have been put in practical operation by Ericsson. The first was constructed with a regenerator and the second without a regenerator.

It is difficult to know with certainty what cycle is described by the working air in these engines.

It has been usually assumed that Ericsson's engine with the regenerator had approximately an isothermal expansion, but that this was the fact appears more than doubtful, for it is known that air is a bad conductor of heat, and hence it would be very difficult, not to say impossible, with the arrangement of furnace which was actually employed to supply to the air during its expansion the heat necessary to prevent a fall of temperature. It seems probable that the expansion was more nearly adiabatic, but that during the outflow the air was again heated to its initial temperature, *i. e.*, to the temperature of the commencement of the expansion. The effect of this would be to cause the efficiency of the engine to have a value near that expressed by (179), instead of the perfect efficiency expressed by (187). This great loss of efficiency might probably be largely overcome by an arrangement of furnace such as has been proposed by Rankine * and employed in the hot air machine of Lemoine. †

A complete discussion, however, of this matter can only be made when we know the rate at which heat is imparted by the furnace at different parts of the stroke. If the working air is maintained at a constant temperature it is evident that heat flows through the walls of the furnace uniformly, for that flow is determined by a constant difference of temperature between the outside and inside of the walls through which the heat passes. An isothermal expansion can therefore be effected only when the parts of the stroke are so arranged as to require a uniform expenditure of heat. It is needless to say that such an adjustment is not that of a crank and piston stroke.

In Ericsson's engine with a regenerator, the compressed air was supplied to the furnace by a separate pump in such a manner that the air entering the regenerator from the pump was at nearly the same temperature as that issuing from the regenerator after giving up its heat to the regenerator. Under such circumstances the occurrence of any conduction or radiation of heat in the regenerator in the brief time during which it was necessary for it to retain the stored up heat, would result in a loss of efficiency.

The reason of this is that conduction or radiation is a process of transferring heat from a higher to a lower temperature without doing the work which by Carnot's principle it might be made to perform during such transference. It seems probable that the heating previously mentioned, which occurred during the outflow, increased the efficiency of the engine, and that a still further heating would be economical as tending to make up for the loss occasioned by

* Steam Engine. Art. 275.

† Novelle Mechanique Industrielle. L. Pochet, Paris, 1874, p. 176.

conduction and radiation in the regenerator.

In Ericsson's air engine without a regenerator, the very complicated mechanical arrangements employed in causing the stroke of the compressing pump do not permit any simple investigation of the rate at which heat is supplied, but evidently the cycle of the working substance lies between that of an adiabatic and an isothermal expansion, and the efficiency which is not large, has a value between that given by (174) and (185).

Ericsson's engines both draw their supply of air from the atmosphere and discharge it again into the atmosphere. The principal advantage of this arrangement is that by it all other provision for rejecting heat is rendered unnecessary, unless possibly it may be best to cool the condensing pump, whereas in case the same working air is used again and again some form of refrigerator must be employed. This usually involves the expenditure of work in pumping a considerable quantity of water.

The principal disadvantage of discharging air into the atmosphere is the small working pressure during the stroke which involves a very large working cylinder in proportion to the power developed.

36. STIRLING'S HOT AIR ENGINE.—In the hot air engine constructed by Stirling the working air is introduced into the furnace with little or no change of volume, hence the pressure and temperature rise during its introduction, and in like manner fall during its discharge from the furnace.

1°. To find the efficiency of an engine in which the working air in Fig. 5 is transferred with increase of temperature from the clearance to the furnace at constant volume, and then expanded along an adiabatic, after which it is transferred with decrease in temperature from the furnace to the clearance, and, lastly, it is compressed along an adiabatic to its initial condition.

Disregard the work stored and restored by the cushion air, which will have no effect upon the final result, and let the subscripts 1, 2, 3, 4, designate the successive states above described, then

$$\text{by (59)}, \qquad h_2-h_1=k_v\,(t_2-t_1) \quad . \quad . \quad (188)$$

is the heat imparted to a unit of weight of air in raising its temperature from t_1 to t_2, and $h_3-h_4=k_v\,(t_3-t_4)$ . . (189) is the heat rejected in cooling a unit of air from t_3 to t_4.

But since the second and fourth operations are adiabatic, and also

$$v_1=v_2, \quad \text{and} \quad v_3=v_4, \quad \text{by (129)}$$

$$\frac{t_2}{t_3}=\left(\frac{p_2}{p_3}\right)^{\frac{n-1}{n}}=\left(\frac{v_3}{v_2}\right)^{1-n}=\left(\frac{v_1}{v_4}\right)^{1-n}=\left(\frac{p_1}{p_4}\right)^{\frac{n-1}{n}}=\frac{t_1}{t_4} \quad . \quad . \quad . \quad (190)$$

$$\therefore\ \varepsilon=\frac{(t_2-t_1)-(t_3-t_4)}{t_2-t_1}$$

$$\therefore \text{ by (190)}, \quad \varepsilon=\frac{t_1-t_4}{t_1}=\frac{t_2-t_3}{t_2} \quad . \quad (191)$$

from which it appears that the efficiency of this cycle is the same as that found in (174), viz: that of a perfect engine working between the temperatures of the beginning and end of the expansion.

If a regenerator be applied to an engine working in this cycle the efficiency will be found, just as in (179), to be

$$\varepsilon=\frac{t_2-t_1}{t_2}=\frac{t_3-t_4}{t_3} \quad . \quad . \quad . \quad (192)$$

By comparing the cycle discussed in art. 33 with this, it appears that this has the advantage of performing the same work with a smaller working cylinder than that.

2°. Let the working air be received into and discharged from the furnace at a pair of constant volumes, as just considered, but let the expansion and the compression be each isothermal instead of adiabatic. Let the temperature of the source be t_2 and that of the regenerator t_1

$$\text{then,} \qquad k_v\,(t_2-t_1)+k_p\,t_2\log_e r \quad . \quad (193)$$

$$\text{and,} \qquad k_v\,(t_2-t_1)+k_p\,t_1\log_e r \quad . \quad (194)$$

are respectively the total heat imparted and the total heat rejected.

$$\therefore\ \varepsilon=\frac{t_2-t_1}{t_2+\dfrac{t_2-t_1}{n\log_e r}} \quad . \quad . \quad (195)$$

is the efficiency of this circle, which is somewhat greater than that of the isothermal expansion whose value is expressed by (185). It likewise has the advantage of a smaller working cylinder.

If a regenerator be applied to this

engine, the heat stored in the third operation is restored during the first operation, and the cycle becomes one of perfect efficiency between the temperatures t_2 and t_1.

The same difficulties are in the way of our knowing the precise cycle of Stirling's engine as of Ericsson's. The working air has an expansion intermediate between an isothermal and an adiabatic. Which engine is theoretically more advantageous in form, Stirling's or Ericsson's, in case each uses the same air continuously is largely a matter of the arrangement of the furnace for heating the air, with the advantage apparently a little in favor of Stirling's engine. As has been before remarked, in such a system, the confined air should by at a high pressure, even at its greatest expansion, in order that the cylinder may be small. Furthermore, Stirling's engine cannot (as Ericsson's can) make the atmosphere its refrigerator. The same considerations apply to the conduction and radiation in the regenerator of Stirling's engine which have been previously adduced in regard to that of Ericsson's engine.

An interesting and valuable discussion of these and other air engines may be found in six articles published in *Engineering*, Vol. XIX (January to July, 1875), in which it is shown that the practical difficulties of construction were more of them overcome in Stirling's engine than in any other; the principal remaining difficulty having been the great heat to which the furnace was exposed which soon destroyed it. Yet if this can be remedied without its being necessary to decrease the temperature of the furnace, the ultimate superiority of air engines over steam engines, will be thereby rendered almost certain, since any such high temperature is not to be thought of in a steam engine by reason of the great pressure which necessarily accompanies it, but in an air engine the pressure at any temperature is a matter entirely within control. Another difficulty inherent in all air engines is the rise of temperature which occurs when the engine is temporarily stopped without stopping the fire.

37. Transmission of Power by Compressed Air.—If air be drawn from the atmosphere at an initial state expressed by $p_1\ v_1\ t_1$ and be driven by a compressing pump into a receiver in which its state is $p_2\ v_2\ t_2$, it is at first compressed from p_1 to p_2 and then driven at pressure p_2 into the receiver. The work performed during both these operations will depend upon the kind of compression adiabatic, isothermal or otherwise, through which the air passes, during the first operation.

The expression for the total work performed in forcing a unit of air into the receiver is

$$w=p_2v_2-p_1v_1-\int_1^2 pdv \quad . \quad . \quad (196)$$

in which the integral of $-pdv$ is the work of compression, during the first operation, $p_2\ v_2$ is the work during the second operation, and $p_1\ v_1$ is the assistance experienced by reason of the pressure of the external atmosphere.

1°. If the compression is adiabatic we can reduce (196) by means of (109), (129) and (135), as in art. 26, to the form

$$w_a=c(t_2-t_1)+\frac{c}{n-1}\int_1^2 dt$$

$$\therefore\ w_a=\frac{cnt_1}{n-1}(r-1) \quad . \quad . \quad (197)$$

in which r has the value given in (186) and can be expressed by (135) in terms of temperatures, specific pressures or volumes.

2°. If the compression is isothermal we have, by (109)

$$pv=p_1v_1=p_2v_2=ct_1 \quad . \quad . \quad . \quad (198)$$

by means of which (196) reduces to

$$w_t=-ct_1\int_1^2\frac{dv}{v}=ct\log_e\frac{v_1}{v_2}=\frac{-ct_1}{n-1}\log_e r \quad . \quad . \quad . \quad (199)$$

in which r has the value given in (186), but not that given in (135). The air after compression is conveyed in pipes to a distant point and there made to drive a piston.

3°. In case the air expands adiabatically, the total work done per unit of air in expanding from the state 3 to the state 4; when found by a process like that employed in obtaining (197) will be

$$w_{a'}=\frac{cnt_3}{n-1}(1-r') \quad . \quad . \quad . \quad (200)$$

in which t_3 is the temperature before expansion, and

$$r'=\left(\frac{v_3}{v_4}\right)^{1-n} \quad . \quad . \quad . \quad . \quad (201)$$

which can also be expressed as in (135), in terms of pressures or temperatures. Such an expansion can be practically effected only by heating the air before expansion, for by reason of the intense cold produced the vapor of water which is always found in air is congealed and so closes the valves, ports, etc.

4°. In case the expansion is isothermal, the total work performed will be

by (199), $$w_i'=ct_3 \log_e \frac{v_3}{v_4}=\frac{ct_3}{n-1}\log_e r' \quad . \quad . \quad (202)$$

in which the expansion is from the state 3 to 4, and r' is to be found from (201). In this case heat must be imparted to the air during the expansion.

5°. When the air is used without expansion and at full pressure, the total work performed per unit of air will be

$$w'=(p_3-p_1)v_3 \quad . \quad . \quad (203)$$

38. Efficiency of Compressed Air.—Either adiabatic or isothermal compression may be employed, and then the air used in combination with adiabatic or isothermal expansion or with no expansion as in (203). The efficiency of these combinations are as follows:

1°. When adiabatic compression is combined with adiabatic expansion,

by (197) and (200) $$\varepsilon=\frac{t_3(1-r')}{t_1(r-1)} \quad . \quad (204)$$

in which t_3 the temperature just before expansion, would usually if no artificial means were employed heat the air, be nearly equal to to t_1 the initial temperature of the air. As before explained, such expansion is impracticable for air containing moisture.

If, however, heat be artificially supplied just before expansion to raise the temperature of the air from the normal temperature t_1 to the same temperature t_3, the amount supplied will be

$$k_p(t_3-t_1)=\frac{cn}{n-1}(t_3-t_1), \quad . \quad (205)$$

hence if this be regarded as part of the expenditure of energy the efficiency becomes

$$\varepsilon=\frac{t_3(1-r')}{t_1(r-1)+t_3-t_1} \quad . \quad . \quad (206)$$

2°. When the compression is adiabatic and the expansion isothermal, heat must be supplied during expansion to an amount expressed, as in (182),

by $$t_3(e_3-e_4)=\frac{cnt_3}{n-1}\log_e r' \quad . \quad (207)$$

which is the same as (202) the work done.

$$\therefore\ \varepsilon=\frac{t_3\log_e r'}{t_1(r-1)+t_3\log_e r'} \quad . \quad . \quad (208)$$

3°. In case the compression is adiabatic and the air is used at full pressure, the efficiency is found by dividing (203) by (197).

4°. When isothermal compression is combined with adiabatic expansion by (199), (200) and (205)

$$\varepsilon=\frac{nt_3(1-r')}{n(t_3-t_1)-t_1\log_e r} \quad . \quad . \quad (209)$$

when the air is artificially raised from t_1 to t_3 before expansion.

5°. When isothermal compression is combined with isothermal expansion by (199), (202) and (207)

$$\varepsilon=\frac{t_3\log r'}{nt_3\log_e r'-t_1\log_e r} \quad . \quad (210)$$

6°. The efficiency of isothermal compression combined with no expansion is found by dividing (203) by (199).

When $t_3=t_2=t_1$, and $p_3=p_2$, by help of (198) we find the efficiency in this case to be

$$\varepsilon=\frac{1-\dfrac{p_1}{p_3}}{\log_e\dfrac{p_3}{p_1}} \quad . \quad . \quad . \quad . \quad (211)$$

The compressors in use permit a rise of temperature more or less great. The compression may be with sufficient correctness for practical purposes be taken to consist of two parts, of which the first part is adiabatic and involves the rise of temperature, and the second part is isothermal. The work performed in this case is found at once from (197) and (198). The circumstances under which compressed air is employed do not usually permit heat to be imparted to it just before or during expansion, hence it is used either at full pressure or with a comparatively small ratio of expansion. This expansion, owing to the conductivity

of the cylinder, etc., is not adiabatic, neither is it isothermal, but between the two. The heat thus derived from external sources contributes to the useful effect and renders that effect greater than expressed in (200). Zahner* has recently made a valuable practical discussion of this subject.

Errata.—In art. 15, write the equation following (26) thus,

$$\int_1^2 dh = \int_1^2 tde$$

In (130) put + for −.

In (144) put $\left(\frac{dv}{dt}\right)_p$ for $\left(\frac{dv}{dt}\right)$.

In (154) put $\left(\frac{dp}{dv}\right)_e$ for $\left(\frac{dp}{dv}\right)_t$.

Figs. 2, 3 and 4, which were accidentally omitted from arts. 15, 20 and 26 respectively, are here inserted.

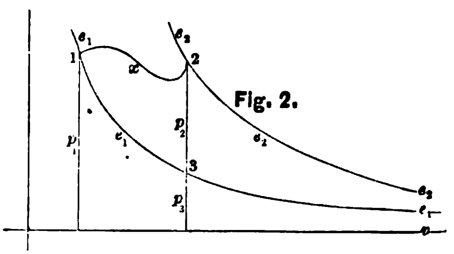


Fig. 2.

* Transmission of Compressed Air. Robert Zahner, M. E. New York: D. Van Nostrand. 1879.

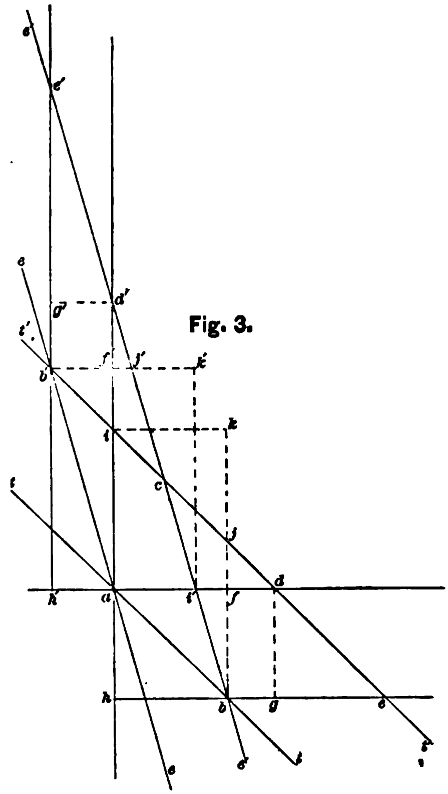


Fig. 3.

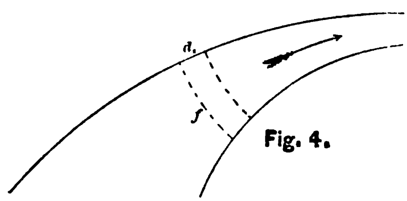


Fig. 4.

SUBMARINE TELEPHONING.

By CHAS. WARD RAYMOND, C.E., Junior of the Society.

Transactions of American Society of Civil Engineers.

A means of communication, direct and at the same time easy and reliable, is and has been a great desideratum by those engaged in submarine work. The method of signaling now employed, by jerking upon the life-line, although, perhaps, sufficient for ordinary work, is, at the best, deficient in rapidity and reliability; and where the work is of such a nature that a word cannot express that which the diver would report to those above, it becomes necessary, in order to facilitate the work, that some other means be adopted.

Several attempts have been made in this direction, but none of them resulted satisfactorily. One may be mentioned which consisted in introducing into the air-supply circuit an air-tight chamber, sufficiently large to admit a person, com-

munication being kept up with the diver through the air-hose, by the assistant in the box or air-tight chamber.

It has been reserved for the telephone to solve the problem, and by the use of this instrument it has been already accomplished by several engineers, with favorable results.

Experiments were begun by the writer about September 1st, 1878,* to test the practicability and utility of the telephone in the submarine work in connection with the construction of the bulkhead wall, now being built by the Department of Docks of this city.

The instruments used at first were two "Phelps' Duplex" telephones, loaned for the experiments by the Gold and Stock Telegraph Company, through the courtesy of Geo. B. Scott, Esq., Supt.

The "Phelps" telephone is peculiarly adapted for this purpose on account of its shape and size. It is oval and flat, 5 inches by 2½ inches, and ⅝ inch in thickness (12.7x6.35x1.59 c. m.).

One telephone was placed in the diver's helmet, and fastened in such a position that, by simply turning his head, the diver could place his mouth or his ear to the instrument. The other telephone was placed on the scow which carried the air-pump and diver's helpers.

The connecting wires were insulated (double-covered paraffined office wire, No. 18) and passed through the air-hose.

Beginning at the helmet telephone, the two wires pass from it through a small opening made in the fan-shaped air-distributor in the back of the helmet, the hole being bushed, to prevent wearing the covering of the wires; passing into the hose, and through the hose until they reach the couplings at which connection is made with the upper telephone. Two couplings, separated by about one foot of hose, are inserted in the hose, at a convenient distance from the air-pump, in this case about 15 feet. To the inside of each coupling is soldered a copper wire, to which the insulated wires from the helmet are fastened. A binding screw is fastened to the outside of each coupling into which the wires from the upper telephone are inserted, thus completing the circuit.

*The idea was conceived in March, 1878, but opportunity to carry out the experiments was not afforded until September.

This was found to work very well so far as communication from the diver to his helper was concerned, but in the contrary direction it was by no means satisfactory, as it was found necessary by the diver, when he wanted to hear, to stop the escape of air from the helmet into the water, which bubbled so as to interfere with his hearing.

This stopping the escape of air is done by the diver stooping over, thereby allowing the air to fill the diver's dress, and, of course, affords only temporary relief.

To remedy this as far as possible, a cloth tube, about 6 inches in length, and of about the same diameter as the escape valve-cap, was fastened by one end over the escape, leaving the other end free. This gave better results, but still not satisfactory.

It was then decided to try an "Edison Carbon Transmitter," which is a form of microphone, and intensifies, or, more properly, increases the volume of the sound. This necessitated the addition of an induction coil and one cell of battery; in this case a "Gravity" battery was used, the arrangement of wires and telephones remaining the same.

The action of the carbon transmitter is quite simple, and on the principle already established, that the secondary or induced current possesses a high degree of intensity, this new arrangement resulted in the perfect success of the experiment. Conversation was carried on with the utmost facility; it was not necessary to give the diver any signal other than a simple "hallo!" It was also found that the diver could talk in the helmet without putting his mouth to the instrument and be heard plainly, and therefore he could continue his work and conversation at the same time.

The battery, induction coil and transmitter were placed on a shelf on the diver's scow, and together occupied no more room than would a Webster's Unabridged Dictionary; the telephone in the helmet occupied but little room, and, of course, was not at all in the way.

The practicability of the telephone for this purpose was thus definitely settled.

These experiments were conducted in depths varying from 0 to 30 feet without any perceptible variation in effect.

It has been reported to the writer that

at a depth of 50 feet the telephone failed. I do not know what the conditions were, but, should any difficulty be met with at that or at greater depths, it can be easily remedied by using a second transmitter, placing it in the helmet on the opposite side from the telephone, and connecting it with its own battery and induction coil—placed on deck—by two extra wires running through the hose to two extra couplings. These four wires may be small in size and occupy no more space in the hose than did the two previously used.

I feel safe in asserting that telephones thus arranged can be used at as great a depth as a diver can work, if not at a greater one.

It may not be out of place here to mention a fact, as curious as it is novel, which was told the writer by Messrs. F. Collingwood and W. H. Paine, Assistant Engineers of the New York and Brooklyn Bridge, that attempts to signal by whistling, with the mouth or fingers, were a decided failure in the caisson when at a depth of about 44 feet. This may possibly be explained on the supposition that, under a pressure of $2\frac{1}{10}$ atmospheres, it might be necessary to change the relative position of the fingers and tongue to produce the sound, which practice in the ordinary condition of the atmosphere enables us to produce. And, also, that in whistling in a dense medium a resistance is met with which is not favorable to the production of the sound, but which, when produced, is conducted with greater rapidity, and consequently is heard more distinctly. From this it may safely be concluded that where the diver can speak the telephone will transmit the sound. As to his being able to hear at that depth there is no doubt.

As regards the utility of the telephone in submarine works, so short a time has elapsed since its introduction into this Department that no conclusions can be drawn other than would naturally present themselves to the intelligent mind.

Much, of course, depends on the nature of the works upon which the diver is employed. In extended general examinations, where the diver has much to report, and where any delay would be inexpedient, its utility would best be appreciated.

Mr. Francis H. Fisher, Assistant Engineer to Mr. Edward S. Philbrick, on the heavy sea wall, in Boston harbor, has kindly furnished me with the results of his employment of the telephone.

He made use, finally, of but one wire, which was wound spirally around the outside of the air-hose, and both hose and wire then wrapped with canvas, to protect the insulated wire from injury. The circuit was completed by attaching one wire to the helmet and the other end to a copper plate suspended in the water from the bottom of the scow. A slight objection to this arrangement may be that communication can only be kept up with the diver so long as the helmet is in the water, and that where the wire passes from the outside through the helmet, the opening must be insulated and air-tight; the wire is apt to become loose and its insulation destroyed. I think the method, herein described, of passing the wire through the hose, much the better way. In this case, two wires form the circuit, are entirely out of the way, are protected by the hose itself, and no other insulation than being covered is necessary. The circuit is thus always complete, whether the helmet is under or above water, as frequently happens in shallow water.

Mr. Fisher testifies to the utility of the telephone in saying: "The necessity of using the telephone arose in providing for a thorough examination of the masonry in the face of the sea wall, below water, in which it was necessary to record the dimensions and positions of the joints around each stone for some courses throughout the wall. The accomplishment of this would have been impracticable, if not impossible, had the diver been obliged to come up to report; but with the telephone it was made in every way successful." Mr. Fisher used a compact form of the Bell Telephone.

From these and other experiments, we believe that submarine operations can be relieved of much expense, both of time and labor, by the use of the telephone. And much submarine work, heretofore considered impracticable, can now be carried on with facility and expedition. In the caisson and in the mine its value would be great, especially so in the latter, where, should one portion of the mine, by an accident, be shut off from the rest, communication by the telephone might be preserved.

THE TURBINE WHEEL DISCUSSION.

A REPLY TO PROF. BURR'S ARTICLE.

By Prof. W. P. TROWBRIDGE, Columbia College.

Written for Van Nostrand's Engineering Magazine.

Professor Burr's criticisms of my article on Turbine Wheels in the last number of your Magazine are prompted, doubtless, by a desire on his part to remove the "impressions of a very erroneous character," which, according to him, my article is "likely to produce."

In his discussion he assumes that I have misunderstood the investigations of Bresse, Rankine and Weisbach, and accordingly supplies his own interpretations of the views held by these authors on the question of impact. At the same time misinterpreting my article on the same subject, in such a way that, like Buttercup in the play, he "mixes us all up."

He intimates that I regard shock as of no importance, while the authors referred to, although "they said" it didn't mean it; or, to speak more plainly, although they "say" that impulse must be avoided, yet in their final results it is quite ignored.

Although Professor Burr's statements on this matter are somewhat confused, yet when carefully scrutinized they amount to this, when taken in connection with the matters of fact to which they relate.

That when the *conditions* of a mechanical problem are laid down and expressed mathematically, the result of combining the mathematical expressions may be and is in this case to eliminate the *conditions* on which they were founded from the final result. A remarkable proposition, new to mathematics, new to mechanics, in fact, new to science.

That Professor Burr does actually maintain such a proposition in his paper (unconsciously of course), and considers it the lever by which he overturns my article, I propose to demonstrate.

Professor Burr admits that the authors, above mentioned, lay down a certain mechanical axiom as a condition essential to the best construction and working of a Turbine wheel; but he says that in their final expressions for efficiency there is nothing whatever depending on such axiom or mechanical condition. This question is one of facts, not of opinions, and I shall show that these authors do lay down such an axiom, as stated in my paper; but as to the last assertion that that condition is finally eliminated the facts are against Professor Burr.

Indeed it might be at once taken for granted that such eminent mathematicians, as those above mentioned, would not lay such stress upon a particular mechanical condition as to insert it as a mathematical condition, and then proceed to eliminate it altogether. I propose to show that they were not guilty of any such folly.

I will first make quotations from Prof. Burr's paper for reference, and then endeavor to point out the fallacies of his deductions.

He states that my "objection seems to be that Weisbach, Rankine and others 'insist' that for the best performance of a wheel the water shall enter without shock:" while I hold "that an equal efficiency, at least, may be attained if the relative velocity be not tangent to the buckets at the point of entrance to the wheel." I do not "object" to the authors named insisting on the principle that the water must enter the wheel without "shock." On the contrary, in discussing the *reaction* wheel, I show that this is as it should be an *essential feature* of the *Fourneyron reaction wheel.* What I do endeavor to demonstrate is that this leading axiom, which all of the authors named impose in advance upon their discussions *limits these discussions* to the *reaction* wheel.

In regard to the last part of the paragraph, "while he (I) holds that an equal efficiency, at least, may be obtained, if the relative velocity be *not tangential* to the buckets at the point of entrance to the wheel. In other words, if the resistances be left out of consideration, it is a matter of no importance whether there is or is not *shock.*" Professor Burr must have

read my article hastily or he would not have received this "erroneous impression."

He will find, I think, that so far from regarding shock as a matter of "no importance," it is one of the chief causes of efficiency and work in Impulse-and-Reaction Wheels. Shock, impact and impulse, being in the sense in which we are using these words synonymous. That these words are synonymous in the discussions of Bresse, Weisbach and Rankine I will presently show.

But I will follow Professor Burr a little further, first, on the same subject.

He says: "Now if the consideration of resistances be omitted, and if shock is of no consequence, then it follows that a formula for curved floats ought to give precisely the same result whether the relative velocity of entrance be tangential in direction or not." "In fact the formulae of Weisbach show this, as do also those of Bresse and Rankine." "In order to verify this statement let any one turn to the expressions for the efficiency of a turbine in art. 260 of Du Bois's translation of Weisbach's work on water wheels, eq. 21 of art. 15 of Mahan's translation of Bresse on Hydraulic Motors, and equation 6, art. 174 of Rankine's "Steam Engine and other Prime Movers."

"*In no one of these expressions is there anything found depending on the angle which the bucket makes with the circumference of the wheel at the point of entrance.*" *Now the matter of shock or no shock depends upon that angle, and since the expressions for the efficiency do not in any wise depend upon such a value they manifestly are* NOT *based upon the theorem or axiom that the "water must enter the wheel without shock.*"

I have made this long quotation in order to allow the authors mentioned to speak for themselves afterwards as to their meaning, and especially as to the statement that "the matter of shock or no shock depends upon the angle which the bucket makes with the circumference to the wheel at the point of entrance".

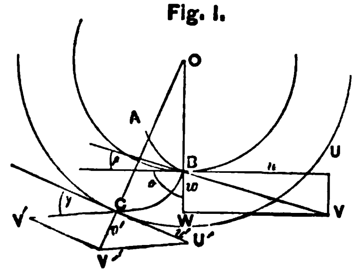
Rankine says: "*In order that the water may work to the best advantage it must enter the wheel without shock and leave it without velocity; for which purpose the velocity of whirl on first entering the wheel must be equal to that of the first circumference of the wheel,**" &c. Here Rankine says nothing about the angle of the blades, but he does lay down the axiom that the water must in no case have an impulsive effect upon the wheel. Again, page 196: "*The greatest efficiency without friction is altered, as has been stated, when $v=ar$.* (Heading of art. 175, Rankine.) In this quotation v=the tangential component of the velocity of the water and $\overline{ar}$ the tangential velocity of the wheel where the water impinges upon it.

No reference is here made to the angle formed by the blades with this circumference, but the condition above quoted is made one which must be imposed for the maximum efficiency.

Let us now refer to Bresse.

The following figure is copied from Mahan's translation of Bresse:

Fig. I.



O is the center of motion of a Fourneyron wheel. AB a guide blade. BC a bucket, BU$=u$ the velocity of the inner circumference of the wheel at B, and BV.$=v$ the velocity of the entering water. BW $=w$ is the relative velocity of entrance to the wheel, CW$'=w'$ is the velocity of exit v' the absolute velocity of exit or the lost velocity.

In establishing the equations from which, with other equations, *his expressions for the efficiency is derived*, Bresse makes the following statement, (see Mahan's translation page 82). "*There still remains to express two conditions for obtaining the best effective delivery. It is necessary that at the point* B (see sketch) *that w* (*i.e., the relative velocity*) at the point of entrance be directed *tangentially to the floats*," and this "axiom" or primary condition of the problem is introduced mathematically by the equation

* Rankine "Steam Engine and other Prime Movers," page 192.

$$\frac{n}{v} = \frac{\sin(\theta+\beta)}{\sin\beta.}$$

(see sketch for the angles θ and β).

This equation of condition is simply that of the sines of the angles being proportional to the opposite sides.

Bresse thus obtains the first of his two "remaining conditions," which is precisely the same as that of Rankine above quoted. It will be seen from the sketch which I have given, as well as from the above quotations, that the question of shock or no shock does not depend upon the angle of the blades alone, but upon the condition mentioned by Rankine, that the wheel velocity and the component of the water velocity in the same direction shall be equal.

For Weisbach's view of this matter we may refer to DuBois' translation page 399. In laying down the theory of the *Reaction Turbine* one of the first conditions named is as follows:

"*In order that the water may enter the wheel without impact this velocity* v (*i.e.*, the velocity with which the water leaves the guide blades) *must be decomposed into two others, of which the first must coincide with the inner velocity* u (*the velocity of the inner circumference of the wheel*) *and the other* w *having the direction of the* entering *water in the wheel*," we have therefore for the last,

$$w^2 = v^2 + u^2 - 2uv\cos\beta.$$

(The notation in Weisbach has been changed to correspond with that of Bresse).

Professor Burr states that the question of shock or no shock depends upon the angle made by the blades with the circumference at the point of entrance of the water, and that *angle* being *eliminated*, the condition itself of shock or no shock disappears from the final result.

The absurdity of the assumption will appear in a stronger light, when the expressions for efficiency are analyzed to which he refers your readers.

That of Bresse is

$$\varepsilon = 1 - \frac{V'^2}{2gH}.$$

This expression it will be seen contains the expression for the lost velocity V', which depends directly upon the *relative velocity of entrance* w, *and the latter depends directly upon the angle*, which Professor Burr says, does not appear in the expression for the efficiency. It is true that it does not appear in the sense that its symbol can be seen, but nevertheless it is there involved, and will not "out" at any one's bidding, and it is mathematically absurd to say that its influence has been eliminated.

Professor Burr cannot dispute these facts if he will read more attentively what Bresse and Weisbach have to say on the subject.

Weisbach expressly states that "*the exit velocity* upon which the *lost velocity directly depends, depends on the entrance velocity*, and Bresse states expressly that the angle β to which Professor Burr alludes should not be *too acute because it would increase the entrance velocity.*

Professor Burr is thus absolutely in the wrong when he states, that the question of "shock or no shock" depends only upon *an angle*, which is eliminated from the expression for efficiency. The question depends upon other elements of the problem, and especially upon the relative velocity of the wheel and water, and indeed if the angle referred to and the direction of the entering water to be fixed in advance, the question depends upon the equality between the wheel velocity and the component velocity of the entering water.

By reference to facts I have shown that he is quite as much in the wrong in asserting that the expression for the efficiency is independent of the angle to which he so often refers.

In regard to the expressions for the efficiency of turbine wheels, Professor Burr lays much stress upon the identity of my formula with those of Bresse and Rankine (when frictional resistances are left out of consideration). He cannot be ignorant of the fact that these formulas have nothing to do with the theory of turbine wheels as far as *form* is concerned, any more than they are the formulas for efficiency of the steam engine or any other motor employing energy. The universal formula for efficiency of all motors is $\varepsilon = 1 - \frac{W'}{W}$; W' representing the energy lost, and W being the whole disposable energy.

In the case of heat engines this takes the form

$$E=1-\frac{T_1}{T_2}$$

In the case of a fly-wheel doing work

$$E=1-\frac{V_2^2}{V_1^2}$$

In the case of water acting by its weight

$$E=1-\frac{h_2}{h_1}$$

In the case of a propeller

$$E=1-\frac{S}{S'}, \text{ \&c., \&c.}$$

The efficiency depending, with a given quantity of disposable energy, upon imposing such *conditions* upon the motor that the energy lost, represented by the numerator in the second member, shall be a minimum.

In my paper on Turbines, I endeavored to show that Bresse, Weisbach and Rankine have imposed such conditions upon the motor which they discuss as to make it a *reaction wheel*, and that these conditions do not apply to the best modern wheels, introduced by Boyden & Francis, because the latter are *impulse*-and-reaction wheels, not restricted by such conditions.

I gave for the first time, as far as I am aware, the theory of such wheels, and stated that if Boyden & Francis had followed the rules laid down by the authors above quoted, they would not have improved the old Fourneyron wheel, and would not have produced the inward flow wheels, which are now becoming unusual.

In regard to the rules laid down by Bresse and Weisbach, applying as they do to purely reaction wheels, they advise that the entrance angle of the water should be nearly 45°.

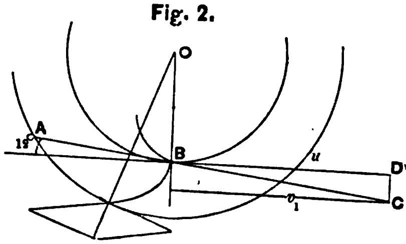
Bresse says, page 92 (Mahan's Translation), the sum $2\beta+\theta$ should be nearly 180°, and θ should be nearly 90°. This makes β as the best angle 45°.

Weisbach, in applying his "arrangement," referred to by Professor Burr as "an admirable arrangement," says that the entrance angle should be about 55° for the best efficiency, but, because the guide blades would then be too divergent, he thinks it better to make the entrance angle smaller.

Now, in some of the best modern wheels this angle β, or a of Weisbach, is made as small as 12°.

Let us see what would be the effect, with this small angle, of enforcing the condition that the "velocity" of the wheel must be equal to the component of the velocity of the water in the same direction. The following sketch will illustrate the construction:

Fig. 2.



Let v_1 represent the velocity of the water entering at an angle of 12°, n the velocity of the wheel, and also the component of the velocity of the water in the same direction. There is no effect from impact or impulse, and the water enters the wheel with the small relative velocity DC, and this is all the available energy with reference to this wheel. In order that it may leave the wheel with a relative velocity equal to the second circumference, its velocity must be increased by the centrifugal effect of the convex surface of the blade, and the wheel would actually have to *pump out* the water. With this small angle the whole energy of the water is made effective if it be allowed to act by impulse, or, in other words, if the velocity of the wheel at its inner circumference be less, in proper proportion, than the component velocity of the entering water. And this is the law of the Boyden & Francis wheels; a direct violation of the axioms or mathematical conditions imposed by the authors who have so often been referred to.

In regard to the results which Professor Burr obtains from using Weisbach's formula for the efficacy for Francis' experiments, he must use certain coefficients, and if he will publish those coefficients and state by what experiments they were derived, as especially applicable to Turbine wheels, he will furnish information which I for one would be glad to obtain. As far as Weisbach's own ex-

amples furnish data, I consider the coefficients mere guess work; there is one thing especially apparent in that author's valuable work on the dynamics of fluids; viz., that coefficients of resistance under varying conditions are exceedingly variable, and when such coefficients are applied to unknown conditions, or new conditions, they must be taken with great allowance, in fact are merely approximate guesses.

Now it was shown by Francis in his experiments that a *variation of velocity of forty per cent. produced a variation of efficiency of two per cent. only.*

The extreme accuracy of Professor Burr's deductions is, therefore, suspicious. Not that Professor Burr would purposely cook the results: I would emphatically disclaim making such an imputation, but that other results could be obtained with as good a show of general accuracy in the use of coefficients I have no doubt.

I suppose Professor Burr is aware of the fact that Weisbach rather contemptuously doubted the correctness of Francis' experiments, in obtaining some of his large efficiencies, and that author surely ought to be a good judge of the applications of his own formulas.

Professor Burr lays much stress on the "admirable arrangement" of Weisbach to prevent "leakage," as Weisbach expresses it, between the wheel and the case.

This "admirable arrangement" consists, for wheels whose buckets are radial, simply in making the guide blades at such an angle that the water enters the wheel at an angle of 45° with the circumference. (See Du Bois' translation of Weisbach, page 405.)

Would Professor Burr recommend all Turbine wheel manufacturers to adopt this "admirable arrangement" in their constructions? I think not; I hardly think his advice would be adopted, at all events, if he should.

Finally, I must correct Professor Burr in another important point. He states that I regard it as a matter of indifference whether there be shock or not. He has become somewhat confused as to what the authors to whom he clings regard as shock, and I have shown conclusively their own interpretation of it, viz. *impulse.* In this sense my paper shows that I regard shock as of very great importance in wheels as they are now almost universally made and run.

What I did say and distinctly conveyed in language that no one should misinterpret was, that undue importance had been attributed to shock or impulse as *to its effects in producing the disturbances which it was the object of the old authors to avoid* by their restrictions. That in limiting their wheels to a particular class, and the best wheels even to an exceptional case of a class, they had, by their rules, shut out the kind of practice now in vogue. That the disturbances which they feared were not so great as had been supposed.

The practical confirmation of my ideas in this respect may be found in the performances of the best wheels now in use, in which this dreaded but very useful shock exists with all its attendant *disadvantages* as well as advantages.

If Professor Burr should still want further instructive evidence of the efficacy of impulse, and of the distinction which Rankine and Weisbach make between impulse and reaction, I would refer him to those pages in the works of these authors which treat of the *impulse* and *reaction* of water upon *vanes.*

In my paper I extended their theories to *Impulse* and *Reaction Turbines,* while they confined their discussions of turbines to *Reaction.* Turbines. I was prompted to do this because I felt that their discussions were not *general enough* to include the best practice of the present day.

I will close this reply, by quoting the well deserved tribute to Boyden and Francis, of an eminent hydraulic engineer, a Frenchman by birth, given in a recent public lecture. He says:

"The Turbine on the Screen is an exact copy of one built by Fourneyron himself forty years ago. * * * Were I able to give you on the same screen the same class of turbines as constructed by Boyden and Francis, you could not avoid admiration for the thoroughness of their researches in finding, applying, and perfecting curves, and for the mechanical appliances whereby the water is divided and prepared, from which curves and appliances were derived those astounding results of the total efficiency of the water power."

THE METRIC SYSTEM.

REPORT OF THE STANDING COMMITTEE ON THE METRIC SYSTEM OF WEIGHTS AND MEASURES, AND ACCOMPANYING LETTERS, PRESENTED AT THE ANNUAL MEETING OF THE BOSTON SOCIETY OF CIVIL ENGINEERS, MARCH 19, 1879.

To the Boston Society of Civil Engineers:

The standing committee on the Metric System of Weights and Measures request your careful consideration of two subjects:

The first is the familiar theme of petitioning Congress. Petitions were sent to the last (the 45th) Congress by the following organizations (and possibly others) besides our own society:

New Jersey State Medical Society; New Haven Engineering Society; Cincinnati Society of Natural History; Medical and Chirurgical Faculty of Maryland; Society of Arts of the Massachusetts Institute of Technology; Boston Society of Medical Sciences; Boston Society for Medical Observation.

The Ohio Association of College Officers also voted to memorialize Congress.

The tenor of their memorials was not precisely the same, but they generally prayed for the exclusive use of the metric system in various branches of the Government service after some future date. The New Haven Engineering Society specified the Post-Office, the Custom-House, the Mint, and the Engineer Corps. A week ago, in New York, the United States Board of Trade voted that the Government be recommended to put the metric system into practice in the Post-Office and Custom House.

The first session of the 45th Congress opened 15th October, 1877. Our last written report to this Society was presented 17th October, 1877; and the Society that evening voted to memorialize Congress for the passage of a resolution inquiring of the heads of the Executive Departments what objections there are to the adoption of the metric system in the Government business and in private transactions, and how long a time should be allowed for such adoption, the same resolve, in short, which had been proposed in the preceding Congress by Hon. Milton Sayler, of Ohio, but had not been acted upon. Mr. Sayler presented our memorial 3d November, 1877, and had it printed in full in the Congressional Record. It named seventeen organizations from whom we had received assurances of co-operation; among them were the Engineers' Club of St. Louis, and the New England Association of Gas Engineers. The resolution as prayed for was promptly passed by the House of Representatives 6th November, 1877, and replies thereto were made about a year ago by twenty-four Government officers of various departments and bureaus. Some presented elaborate reports; some brief and specific answers to the inquiry; a few stated that having nothing to do with weights and measures they were not in a position to judge. As a single illustration of the variety of opinion that has been brought out it may be mentioned that the Surgeon-General of the Army thinks the adoption of the metric system would be fraught with grave danger to the sick soldiers; while, on the other hand, the sick sailors are now being dosed according to it, the medical purveyor's department of the Marine Hospital Service having actually introduced it last year; and it is believed that the health of the Navy has not suffered in consequence.

The House Committee on Coinage, Weights and Measures having digested these various reports, has printed them and many other documents in a considerable volume, along with its own report presented 7th January, 1879. A copy is to-day added to the library of the Society. In a speech of Hon. Levi Maish, of Pennsylvania, printed in the Congressional Record for 8th March, 1879, occur the following sentences: "I introduced a bill in Congress making the use of the metric system obligatory in the custom-houses and post-offices of the country. The committee, however, thought this would be too great a stride to make at once toward the adoption of the new system."

Nevertheless, the report just now mentioned earnestly recommends the early passage of Mr. Maish's said bill.

We have repeatedly expressed the opinion that it rests with Congress to take action upon the metric system. A deliberate recommendation has now been made by a standing committee of eleven members headed by the venerable Alexander H. Stephens, after a full presentation of the facts; and it was voted that 10,000 copies should be printed; we think their recommendation should be acted upon by this Society. Moreover, as it is very evident that the complete introduction of the metric system can only be accomplished by general concurrence, it is manifestly reasonable to begin by asking the largest user of weights and measures, namely, the United States Government, to join in the movement.

We submit the following draft of a new memorial:

To the Hon. the Senate and the House of Representatives of the United States in Congress assembled:

The Boston Society of Civil Engineers respectfully prays that in accordance with the earnest recommendation of the Committee on Coinage, Weights and Measures of the House of Representatives, as expressed in Report No. 53, 45th Congress, 3d Session (p. 34), it may be enacted as follows:

Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled,

Sec. 1. That on and after July first, eighteen hundred and eighty-one, for all postal purposes, fifteen grams shall be substituted for half an ounce, and so on in progression.

Sec. 2. That on or before July first, eighteen hundred and eighty-one, the Postmaster-General shall furnish all post-offices with postal balances denominated in grams of the metric system, at an expense not exceeding fifty thousand dollars, which sum is hereby appropriated, or so much thereof as may be necessary for this purpose.

Sec. 3. That on and after July first, eighteen hundred and eighty-one, the metric system of weights and measures, as legalized in section thirty-five hundred and sixty-nine of the *Revised Statutes*, shall be obligatory in the assessment of duties on imported commodities in the custom-houses of the United States.

Section 3,569 of the *Revised Statutes*, is the law of July, 1866, establishing the equivalents of the metric units in terms of the customary weights and measures.

The form of law proposed is the same as Mr. Maish's bill, with the exception that we have altered by one year the date for it to go into effect, because of the delay of a year in the time of its expected passage.

The 46th Congress opened yesterday, and we respectfully urge upon this society that at the earliest reasonable moment it be

Voted, that the memorial submitted 19th March, 1879, by the Committee on the Metric System, be adopted by the Society, signed by the President and Secretary, and transmitted to both Houses of Congress.

The desired legislation will be made when it is demanded by our citizens; it is hoped that the demand will be prompt and general. The undecided and conservative people who are anxious to proceed very cautiously, and take extremely slow and gradual steps, ought to join in this appeal. Nearly all can agree upon this first step. We don't mean, however, to disguise our own opinion, as stated heretofore, that a simultaneous movement is required for the most economical conduct of the reform; that the establishment of the metric system in the Post Office and the Custom House should be rapidly followed, say in five years, by its exclusive use in all other branches of the public service; and that the people of the United States (and particularly those persons who petition Congress for the change) ought to adopt the metric system in their private affairs directly after beginning to use it in their dealings with the Government. The more quickly the transformation can be completed, the less protracted will the annoyance be; provided, of course, that it be not attempted to accomplish it within shorter limits than are reasonably fixed by the requirements of any one ordinary business. In this connection it may be interesting to quote the opinion of Gen. Humphreys, Chief of Engineers. He says, 3d December, 1877:

"So far as the proposed change would affect the works carried on under charge

of the officers of the Corps of Engineers, it need only be said that while any change in the ordinary and accepted standards must be an inconvenience, yet there is no other reason why the change should not be made, provided sufficient time is given for preparation. It is thought that the French metric standards should not be adopted to the exclusion of the present standards, in this office within a less interval of time than five years after the passage of the act. This limit is fixed as the minimum, in order to allow for the proper careful manufacture, comparison, and distribution of standards, and their duplication in various forms for ordinary use, for the necessary changes in tables and formulæ, and more especially to allow a sufficient interval of time during which a practical familiarity with the new standards may be acquired, particularly by those with whom the business of the Engineer Department is transacted and who are not in the public service, as well as by those not in the public service who use the maps, charts, etc., of the department."

The practical steps toward the actual use of the meter in our own profession will be the topic of the remainder of this report. The little matter of plotting a metric scale upon those plans that have any linear scale has formerly been spoken of as the beginning. The disposition of engineers toward that preliminary measure is illustrated by a letter (a copy accompanies this report) sent last October to the Library Committee of the American Society of Civil Engineers, and by the fact that they purpose, hereafter, to put a metric scale upon the illustrations of papers published in their Transactions, where no objection is made by the author. The Engineers' Club of Philadelphia adopted, April 6, 1878, the report of its Committee on Metric System, which requested "that, in all papers read before the Club, the Metric System be used conjointly with the English, and that, on all maps, sections, and drawings, a metric scale be placed, for comparison with the ordinary mile, foot, or inch scale." They also recommended the introduction of the metric system into machine shops, public schools, and professional literature. The report was printed and widely circulated; copies were courteously sent us, and were submitted to the Society last spring, as may be remembered. The Civil Engineers' Club of the Northwest, devoted a meeting to the metric system, November 6, 1878. The arguments in its favor were very ably presented; and a motion was carried instructing the Secretary to procure for the use of the Club, stationery printed with metric devices. It was stated in the discussion, that on the U. S. Lake Survey the metric scale is being attached to charts. A full report may be found in *Engineering News*, for November 14 and 21, 1878. A final report by the committee appointed three years ago, and decisive action by the Club, have yet to be made.

The American Society of Civil Engineers, by letter ballot, canvassed February 6, 1878, "Resolved, that the further consideration of the metric system of weights and measures be indefinitely postponed," 102 to 57.

We think ourselves that the further consideration of the advisability of adopting the metric system may be superfluous, regarding its ultimate use all over the world as a foregone conclusion; but we believe it is worth while to study the best method of introducing it. We look forward to a time, not far off, when there shall be a general movement for the actual use of the metric measures and the abandonment of the old ones. The first effective substitution in the work of our own profession may be expected in surveys of land. Our second subject, therefore, which we wish to examine at considerable length, is the metric system in the measurement of land; for, in fact, its use there has already begun.

The meter has always been the standard of the U. S. Coast Survey, to take the most conspicuous instance; though, strange to say, its published charts bear linear scales of miles, but not of meters, and their ratios to natural size have sometimes been $\frac{1}{15000}$, $\frac{1}{30000}$ and others that can not be expressed precisely as decimal fractions. During the past two years a re-survey of Boston Upper Harbor has been made in meters, under the direction of the Massachusetts Harbor Commissioners; their last report gives a list of geographical positions of land marks and conspicuous objects with which local surveys can readily be connected, distances being

given in meters only; it contains also a skeleton map of the triangulation bearing a linear scale of meters alone. Passing from large surveys to small ones, Mr. Charles W. Howland, of Rockland, Mass., has been making surveys in meters and corresponding plots for deeds for his patrons in Rockland and neighboring towns, during the past five years or more; and he finds that it works perfectly well.

The reason for beginning the use of the metric system upon this class of work rather than upon any other that falls within the province of the engineer is that it is the least connected with other people's measurements. There is a considerable amount of land surveying of this class; surveys merely for the measurement of area, or for making descriptions and plans for deeds of record, or for dividing a parcel of land by lines fulfilling given geometrical conditions, are nearly independent of other measurements, and the surveyor can use what unit he likes. Whenever he shall become convinced that the speedy adoption of the metric system is assured, he will be able to use the metric measures upon work of this class more readily than elsewhere. Measurements of land, moreover, unlike those of the warehouse and the workshop, are liable to be referred to for many years. The plans that we are making to-day may be preserved in our Registries of Deeds, and be appealed to a century hence. According to general opinion the metric system will then be in use everywhere, and if the coming surveyor shall not find it on our plans, he will suffer a similar inconvenience to that which we who use feet now endure from the chains, rods and links of a former generation.

The introduction of the meter will require but very little effort. Metric chains, tapes, poles and rules cost no more than those of the old-fashioned style. The lion which most of us see in the way is probably the incongruity between the meter and the foot. What we ought to aim at in making the change will be to use the meter only and not to refer to the foot at all; the incongruity will then be a help, not a hindrance. Nevertheless, the interconversion of old and new measurements will occasionly be required at the best; and to people who are on the lookout for obstacles, that prospect is rather formidable; but when we look upon it as an inevitable task which we must contrive to perform in the easiest way we can, we shall see that it is the same sort of thing that we are already (to our sorrow) well acquainted with in the transfer of miles, chains, rods, and links into feet, and in the change of inches and sixteenths into feet and decimals. The change into meters is doubtless of somewhat greater magnitude; but it is not so much greater as might at first sight be supposed; and by an adroit selection of examples, may even be made to appear less. We sometimes see a long string of decimals (.025399772) written to express the value of an inch in meters; to express the value of an inch in *feet* takes just as many decimals, .083333333+, and is not mathematically exact at that. Suppose that we come down to reasonable figures for practical use; for instance, a surveyor who uses decimals of a foot is called upon to give lines or grades, for masons and carpenters, who are building from architect's plan figured in inches; the surveyor is exact enough if he uses three places of figures, and calls 5 inches 0.417 of a foot, and 10 inches 0.833 of a foot; that is, he neglects one part in 1,250, or one part in 2,500; if, on the other hand, he had used a metric tape and three places of decimals, and had called 5 inches 127 millimeters, and 10 inches 254 millimeters, he would have made an error of less than one part in 100,000. To reduce rods to feet you must multiply by 16.5, miles to feet by 5,280, still three significant figures. Take square measure; to reduce acres to square feet you must multiply by 43,560, four significant figures; to reduce square rods to square feet by 272.25, five significant figures; and even to reduce square feet to square inches, you must multiply by 144, three significant figures. On the other hand to reduce square feet to square centimeters, you must multiply by 929; square feet to square meters by .0929; square feet to hektars, by .00000929; which is three significant figures. The computation is made a simple matter by the aid of tables, some of which are very conveniently arranged. A new table by Mr. Emonts, of the American Society of Civil Engineers, will probably be printed soon. For approxi-

mate reckoning which will be sufficient in many cases, a graphical method can be applied. (See accompanying diagram.)

As another illustration of the readiness with which the change can be made, take the matter of draughting on plans. Suppose a surveyor makes a drawing of a building lot on a scale of 10 feet to an inch, that is, $\frac{1}{120}$ or $8\frac{1}{3}$ per thousand of full size; that plan is sent to an architect's office to guide him in making his designs; suppose he wishes to read distances upon it in feet and inches, the nearest scale he can find to it on the common architect's triangular draughting scale is $\frac{3}{32}$ of an inch to the foot, which is $\frac{1}{128}$ of full size; if he should use that scale, therefore, $\frac{1}{16}$ part or $6\frac{1}{4}$ per cent. would have to be deducted from each reading. Suppose, on the contrary, that he should wish to scale distances in meters from the plan; the nearest standard decimal scale would be .008, or 8 millimeters to a meter; and if this were used, $\frac{1}{25}$ part or 4 per cent. would have to be deducted from each reading would have to be deducted from each reading to get the measurement in meters. As another instance, suppose you have to add some work to an old plan on a scale of 100 rods to an inch. To plot feet on it you require a scale of 1,650 feet to an inch, which is not ordinarily to be found engraved ready for use; it is $\frac{1}{19800}$ of nature, that is, one per cent. larger than $\frac{1}{20000}$, and, as paper plans are liable to shrink as much as that (and plans printed on damp paper considerably more than that), a metric scale of .00005, or 5 centimeters to a kilometer, can be applied to plot metric measurements as conveniently as anything. Similarly scales of 20 rods to an inch, 5 inches to the mile, and $2\frac{1}{2}$ inches to the mile vary about one per cent. from 00025, .00008, and .00004.

It is not pretended that these are average cases; they are confessedly selected for a purpose; but to any mind familiar with this matter, examples enough of a contrary tendency will undoubtedly occur to secure a fair judgment. The present discussion is simply intended to give a realizing sense of the fact that the labor of translating quantities into the metric system need not alarm engineers, who can at one time use miles and acres on extensive land surveys, feet and decimals on railroads and building lots, cubic yards on earthwork and masonry, and inches and sixteenths on wooden and iron construction.

By the adoption of the metric system, several material advantages are to be gained in the details of the surveyor's work, which are often overlooked in our general view of its more important and extended benefits.

Apply the metric system to draughting.

1. We shall use the same scales that architects will.

2. We shall use the same draughting scale for all plans from the largest detail drawings to the smallest maps.

3. A plain metric rule, such as every man will carry in his pocket, will afford a moderately convenient scale for reading dimensions on all the commonest sizes of drawings.

4. If we write the scale as a decimal fraction there will be no ambiguity; large scales will be expressed by large numbers; small scales by small numbers.

Apply the metric system to leveling.

1. We shall have a unit of just the right magnitude for nice work. The hundredth of a foot is not considered fine enough for the most accurate leveling; so we undertake to use the thousandth: but that is too fine a division to be read twice alike. The millimeter, which is very near to a mean proportional between the two, is just what is wanted.

2. We shall have a target rod without a vernier; for millimeters can be read from a plain scale.

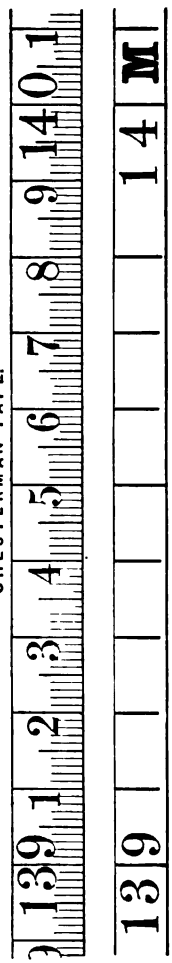
3. We shall have a speaking rod which can be read at a greater distance than at present, or more easily read at the same distance; for the centimeter is more than 3 hundredths of a foot. Intermediate millimeters can be estimated very closely if required.

Apply the metric system to the measurement of length. As to whether it will be more convenient than the old measures, probably opinions differ according to the class of work especially contemplated by the person making the comparison. For suburban house lots where it might be thought finical to give the hundredth of a foot, and negligent to stop with the nearest tenth, the centimeter may be found better suited. For the measurement of the most valuable estates in the business portion of cities,

A SLIDE-ROD SCALE
Submitted by Charles H. Swan, C.E. Providence R.I.
☞ READING 2ᵗᵗ.563

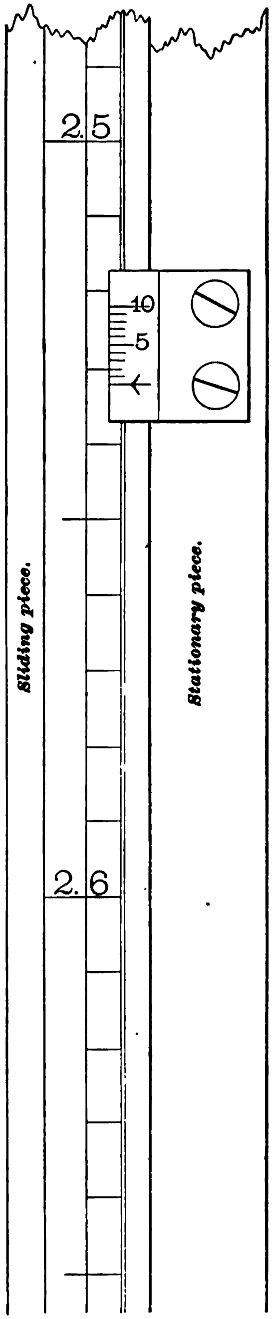
A SPEAKING-ROD PATTERN.
Submitted by F. Brooks.
☞ SHADED PORTION RED

FULL SIZE.



CHESTERMAN TAPE.

PAINE TAPE.



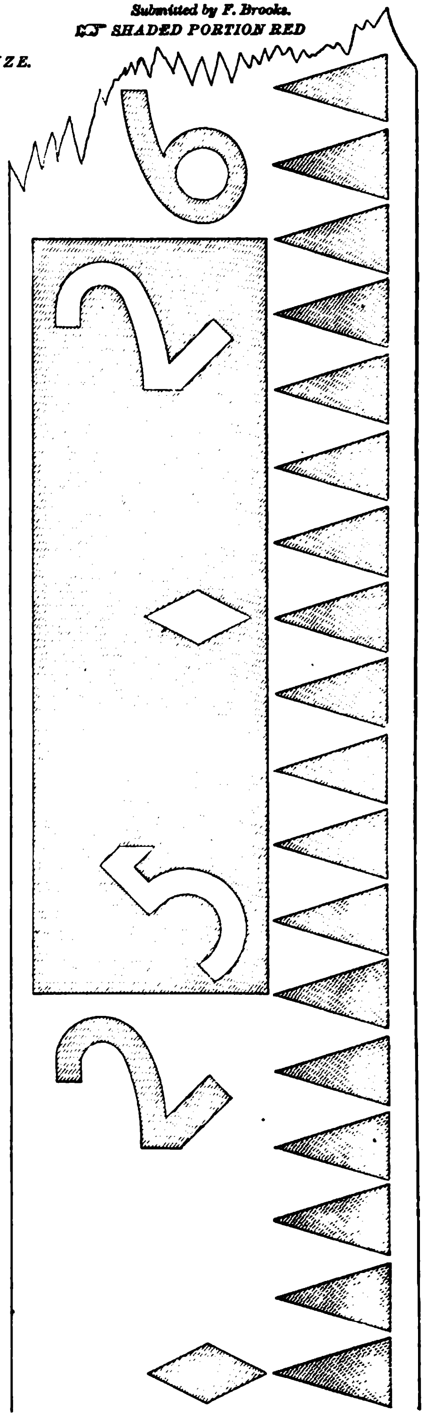
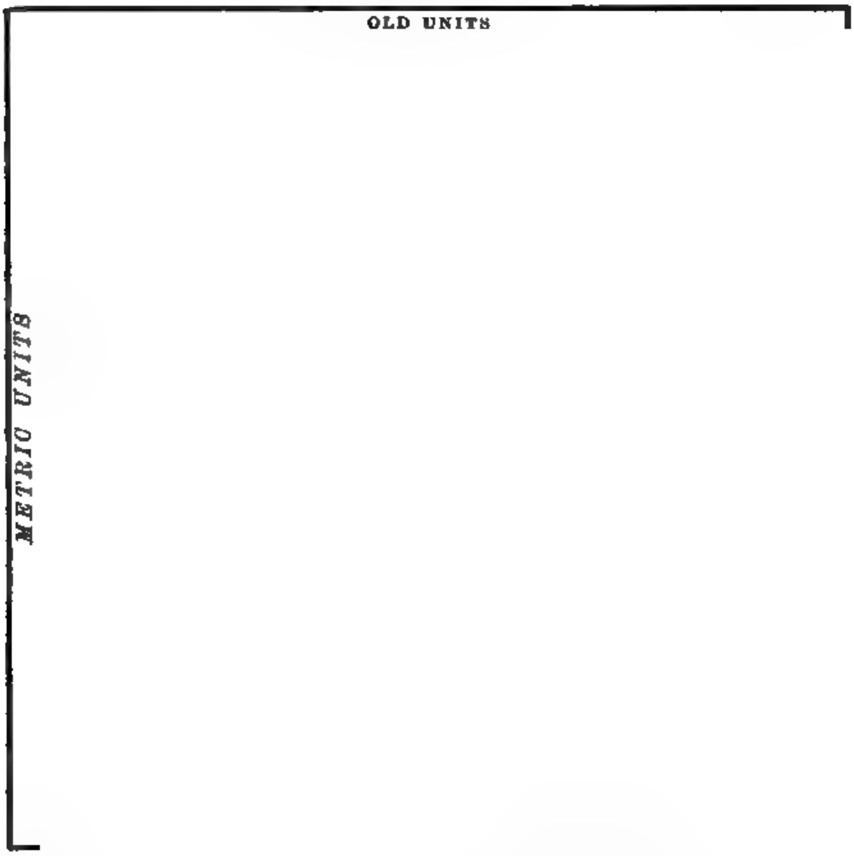


DIAGRAM OF EQUIVALENTS
(Adapted from a German periodical)



there will be some gain in nicety by the use of the millimeter instead of the hundredth of the foot, the thousandth being as unattainable here as in leveling.

We have been speaking of beginning new works of pure land-surveying. There is a great deal of surveying, however, which is more closely bound up with old measures. Where a large tract of land like our Back Bay territory has already been planned and partially laid out, it will plainly be easier to continue the use of the same unit of measurement first employed than to adopt a new one in going on to complete the work or set off additional lots. When surveys of land are made with a view to the construction of buildings or works thereupon, it would obviously be most convenient to apply the same standard of length on the survey that is to be used in the construction; in such surveys the engineer must change very nearly in unison with the manufacturers who will supply his materials in standard sizes and the mechanics who will execute his plans. Upon purely constructive works independent of surveying, the engineer must evidently act precisely simultaneously with artisans and tradespeople, in undertaking to introduce thoroughly and exclusively the use of the metric system.

Some eminent and able men have maintained that in land surveying there is need of especial delay, and that the lapse of time required for making the proposed change must there be greater than in other classes of measurement. To controvert such views we may refer to a letter which we recently addressed to the Congressional committee; a copy accompanies this report.

It is hoped that this subject will

receive the thoughtful attention of the society.

Respectfully submitted,

[SIGNED] FREDK. BROOKS,
" L. FREDK. RICE,
" CLEMENS HERSCHEL,
Committee.

19th March, 1879.

[COPY.]

BOSTON 11TH FEB., 1879.

To the Committee on Coinage, Weights and Measures:

The Boston Society of Civil Engineers having appointed us, the undersigned, a standing committee on the Metric System of Weights and Measures, we desire to present the following discussion of the arguments on the metric system in *land surveys* reported by officers of the Executive Departments and printed last year in Ex. Doc. 71, 45th Cong., 2d Sess.

The Commissioner of the General Land Office says (Part 2, p. 2);—

"The Gunter chain, so long used in this branch of the public service, is of the convenient length of 66 feet."

Now that equals 20.117 meters; and the usual metric chain has the convenient length of 65.62 feet, or 20 meters. The difference is 4½ inches.

He says in explanation of the convenience of the Gunter chain as applied to the sub-division of the public lands (Part 2, p. 2):—

"The legal township of the United States land surveys is approximately a rectangular tract, with sides of six statute miles. This body of land is divided into 36 sections, with sides of 80 chains, each regular section embracing, as nearly as may be, a square mile or 640 acres."

But the law tolerates a variation of 1 chain in 80, and many sections actually vary from a square mile by more than half that amount. Less than half is sufficient to reduce the section to a square of 1600 meters on a side, as frequently remarked by western surveyors, and exhibited by the diagram of the Commissioner of Education (Part 2, p. 8). He shows the quarter sections and the usual smaller divisions; but the metric dimension is adapted to still further quartering than the old mile measurement. It appears therefore, that the same division now established could be continued; that what is now "approximately" 80 Gunter chains, also is, and might be called, approximately 80 metric chains, or 1600 meters; that what is now "as nearly as may be" 640 acres is as nearly as need be ($16 \times 16 =$)256 hektars; that a 40 acre lot might as well be considered 16 hektars; and that the substitution of the metric system would thus require but a very simple computation from the old denominations to the new, and would *not* necessitate the "retracement of many standard and meridian lines now marked in the field," and would *not* destroy those associations which the land system now has with "the social and political life of the people."

The Commissioner of the General Land Office states (Part 2. p. 2);—

"The aggregate quantity of unsurveyed public lands in eleven partially surveyed States and Territories, and in the wholly unsurveyed Territory of Alaska, is about equal to that already surveyed."

After referring to the same fact, Maj. Powell, of the Rocky Mountain Survey, well observes (Part 2, p. 18);—

"If the metric system is finally to prevail in this country, it is desirable that these lands should be measured and conveyed in units of the new system."

The Commissioner of Patents relates the interesting fact that at some places in the Mississippi Valley the ancient French arpent still continues in use, and remarks (Part 2 p. 5);—

"It is because all real estate transactions are matters of permanent record, and permanent records are only changed with great difficulty. To change them involves translation, tedious and accurate computation, the discarding of original records, and opens the door to mistakes and fraud; and the possibilities of these are without end."

We see no occasion for changing or discarding the original records, but we do think that any new measurements ought to be made and recorded in terms of some unit recognized by the laws of the United States. In New England surveys were formerly made in chains and links, but it is now the common practice to use feet and decimals. Our old records remain, and at those rare intervals when we have to refer to them we can translate dimensions from one de-

nomination into the other with tolerable confidence in our accuracy, and with no special temptation to fraud. Changing arpents into feet would be the same sort of thing, though perhaps a little more tedious; and changing feet into meters we find presents no insuperable difficulty. As Maj. Powell very justly says with reference to the metric system (Part 2, p. 18);—

"In land measures its introduction will but slightly inconvenience the people at large, for the measurement of land is practically relegated to skilled persons, as engineers and surveyors; and the conveyancing of lands, to persons skilled in that branch of business."

If a change of unit were really impracticable, it would be true of the City of St. Louis, and not of its suburbs alone; but in the city the antiquated arpent has given place to another anomalous measure, the "front foot." Real estate is now sold in St. Louis, by the foot of street line, although the lots vary in depth. The whole territory of France also has found it possible to change from the arpent to the hektar.

Prof. Hilgard, Assistant in Charge of the U. S. Coast Survey Office, says (Part 1, p. 24);—

. . "all the most valuable real estate, such as lots and streets in cities, has been laid off in this country in even feet, generally even tens of feet, as 50, 60, 80, 100, 150, &c. What adequate motive is there to change these expressions into terms which are necessarily fractional, and in which those foreign nations whose convenience it is proposed to meet have no conceivable interest? What useful purpose is subserved by designating a building-lot 24 by 120 feet in the form of 7.315 by 36.576 meters?"

One weak point about this is that it does not adhere closely enough to the facts. Many of those city lots in exact feet are the lovely creations of the fancy. To draw an illustration from home, a large part of South Boston is laid out in rectangular blocks, several of which have a nominal length from street to street of 500 feet; the plans at the City Surveyor's office show the following series of real distances: 501.23, 501.37, 501.19, 500.76, 500.60, 500. 501.10, 501.23. The general fact is that cities are laid out and division walls built when land is cheap, and when minutely accurate measurement would cost more than it would be worth. In after years when the property becomes valuable and is carefully surveyed, it is found that the actual dimensions are often fractional. In such cases their metric equivalents will be no more irregular.

Whether foreign nations be interested or not, what we seek is our own convenience. In real estate business, as elsewhere, the useful purposes to be accomplished by the introduction of the metric system are so obvious that a very brief mention of them will be a sufficient answer to Prof. Hilgard's questions and to some other remarks of similar tenor, as that "there is nothing to compensate for the hardship and the danger that would ensue from such a change.

One great benefit will consist in uniformity with other measures. If land is surveyed at all it is probably with a view to its being subsequently used in some way more or less connected with such measurements as are adopted for the common purposes of life. If it is to be walled in or fenced around by masons and carpenters who use the metric system, if roads are to be made across it by contractors who use the metric system, trees planted at regular distances by farmers who use the metric system, houses and barns erected by architects and builders who use the metric system, or even graves dug in it by sextons who use the metric system, there will be an element of convenience in having the survey of the land in metric units to begin with.

A second gain will be that we shall effectually get rid of some awkward relations like 7.92 inches in a link, 16.5 feet in a rod, 5,280 feet in a mile, and 43,560 square feet in an acre, and substitute a completely decimal system with a logical nomenclature.

Another advantage relates to our dealings with distant places. If to-day a land-owner in St. Louis desires to borrow money upon his estate consisting of 25 "front feet," a capitalist outside the city is put to more trouble to inform himself about it than he will be when the description shall be made in a form everywhere familiar. If a western railroad company wishes to place its land

grant bonds in the German market, the business could be facilitated by describing the property in kilometers and hektars as used in Germany. If American instrument makers try to sell their goods in foreign countries, as American manufacturers in other departments are so generally doing, it will help them to have the same style of implements for land-measuring used both at home and abroad. Our engineers, by familiarizing themselves with metric dimensions upon their own surveys, will understand better when they read that a foreign work has drained a territory of so many hektars, or that a street in a foreign city is so many meters wide.

It is for just such reasons, presumably, that, as stated by Prof. Hilgard himself (Part I., p. 19),

"In the operation of the Coast Survey the meter is used, and has been employed from the first as the unit of measure."

We think this fact redounds greatly to the credit of the far-seeing officers who originated the practice.

It is for such reasons, probably, that the meter has been frequently used in the map-work of the U. S. Geological and Geographical Survey of the Territories, as mentioned by Prof. Hayden (Part 2, p. 15).

It is for such reasons, also, that upon several recent local surveys in our own Commonwealth the meter has been made the unit. The re-survey of Boston Upper Harbor, under the Harbor Commissioners, has been made in meters. The Boston and Providence Railroad Co. has set up a row of stone posts, two kilometers apart, along its line. Numerous small surveys for deeds of record, etc., in the towns of Abington, Rockland, and Hanover, Plymouth County, Mass., have been made and plotted by the metric system during the past five years.

If to accomplish the change in land units is going to require such an unconscionably long period as has been supposed, we can only urge that no time be lost in getting started.

Very respectfully,

[SIGNED] FREDK. BROOKS,
" L. FREDK. RICE,
" CLEMENS HERSCHEL,
Committee.

[COPY.]

To the Committee on Library of the American Society of Civil Engineers.

GENTLEMEN—May we ask your co-operation in an effort to bring about (as far as possible) unanimity of action in a little matter of professional practice? We observe in your Proceedings a standing request to members, "in papers hereafter presented to the Society to write, in parenthesis, weights or dimensions by the metric system in connection with those of the system in general use;" and we think that the spirit of this recommendation easily includes the addition of a metric scale to such plans as require any linear scale. Another announcement says with regard to illustrations of papers presented for publication, "Always put a lineal scale upon each drawing"; and this direction appears to be followed in a large proportion of cases. We would like to inquire whether in reproducing such illustrations in the Transactions an appropriate metric scale could not be added by the Secretary if neglected by the author. We append a list of several documents, plans, &c., having more or less of a public character, which bear duplicate scales, saying nothing of the private practice in engineers' offices. The metric scale is constructed graphically from the scale already on, the whole operation requiring only a few minutes time.—Respectfully,

[Signed] FREDK. BROOKS,
" L. FREDK. RICE,
" CLEMENS HERSCHEL,

Standing Committee on the Metric System of the Boston Society of Civil Engineers.

[Signed] CHAS. A. ASHBURNER,

Chairman Committee on the Metric System, Engineers' Club of Philadelphia.

[Signed] W. A. NORTON,

President of New Haven Engineering Society.

[Signed] C. W. KELLY,

Secretary of New Haven Engineering Society.

[Signed] HENRY FLAD,

President Engineers' Club, St. Louis, Mo.

[Signed] Samuel S. Greeley,
Chairman of the Metric Committee of the Civil Engineers' Club of the N. W.

List (referred to above).

Annual Report of City Engineer, Boston, 20th Jan., 1877; Annual Report of City Engineer, Boston, 21 Jan. 1878; Report of Newton Water Commissioners, Nov. 1, 1877; Fred M. Hersey's competitive plan for a Park, Boston, May, 1878; Plan, profile, &c., Quebec W. W. in *Engineering News*, Chicago, 16th May, '78; Figs. 629–49 on pp. 656–7, 675 on p. 686, and 1016 on Plate XVII, Drinker's Tunneling, N. Y., 1878; Plan of South Boston Flats improvement, Mass. Harbor Commissioners, 28 Feb. 1878; Providence W. W. Sections and Details of Hope Reservoir, Providence, R. I. 1876 (exhibited at Centennial Exposition); 11 maps in Gray's Atlas of the U. S., Philadelphia, 1877; Boston W. W. Systems of Supply and Drainage Areas, City Engineer's Office, May, '78; Plan 1, showing the Mouth of the Rhone, accompanying Appendix S 12 to Report of the Chief of Engineers for 1875, Part 1. (Of this last probably the original scale was metric, and the British scale was added subsequently).

A NEW PROCESS IN METALLURGY.*

By JOHN HOLLOWAY.

From "Nature."

Long before human art acquired the knowledge of metal-making, prehistoric man had learned to make fire of the dry stems and branches of trees; in the charred fragments of half-burnt wood we recognize a form of carbon, the first simple elementary body produced by man from the complex natural bodies with which he was surrounded. In the knowledge of the use of fire, then, was the first dawn of art, particularly of that art which deals with the reduction of simple bodies from compound minerals. To convert metallic compounds into metallic elements is the domain of the metallurgist, and the means by which this is effected constitute the basis of metallurgic art. Carbon was thus a necessity to metallurgy—with the knowledge of fire the world emerged from the stone age. From those early times down to the present day, no fusion has been effected without using carbon, which in the form of wood, coal, or charcoal, has been the substance invariably used by the metallurgist for the production of heat, and to enable him to decompose and to smelt metal-bearing materials.

The new process, however, we are about to describe, has for its object the smelting of metalliferous substances without the employment of carbonaceous fuel. The sulphides of iron, lead, and zinc are known to be combustible substances of almost universal occurrence, and when burnt under favorable conditions give rise to a great evolution of heat. We have calculated the relative temperatures thus produced, from which it appears that the temperature at which iron pyrites (bisulphide of iron) burns in air under the conditions most favorable to the development of a high temperature is over 2,000° C., protosulphide of iron burning at about 2,225° C. Zinc sulphide, or blende, gives a temperature of 1,992° C., and galena 1,863° C.; while calculations made in a similar manner with coal, assuming it to be completely burnt, show the temperature attainable to be 2,787° C. These mineral sulphides, which are therefore natural and almost inexhaustible sources of heat and energy, can under certain circumstances be burnt more economically than their heat-giving equivalent of coal.

The best means, however, of utilizing this heat-producing property of metallic sulphides is not so apparent as would appear at first sight. Only iron pyrites is sufficiently combustible at a low temperature to burn in the open air, the mass

* A paper with full details of the process was read at the Society of Arts on February 12, 1879.

being raised to the temperature at which the oxidation takes place solely by the union of the sulphur and iron with aerial oxygen. In Spain this is carried on in vast heaps of hundreds of thousands of tons, and the operation extends over many months. The oxide of iron that remains is typical of those mineral substances which, once burned in the primeval operation of nature, gave up their stores of heat and force, and became, as it were, inert bodies.

Going back now to the combustion of carbon, it is well known that it burns at widely varying temperatures, as for example, in our bodies, in a common coal fire, or in a powerful furnace. A great deal of attention and thought have been spent upon the subject of the economy of carbonaceous fuel, and great advances have been made in this direction, yet the expenditure of coal or coke necessary, say, to melt a given quality of metal, still far exceeds the theoretical limit. The main causes of this discrepancy may be accounted for as follows:—(1) That only a fractional part of the oxygen of the air passed into the furnace acts upon the material to be burnt. (2) That the oxygen is not brought in contact with the combustible matter with sufficient rapidity to attain the necessary temperature for the operation. (3) That gases pass off hot and unburnt; these are now, however, frequently utilized.

There is one metallurgical operation in which the first two sources of loss are perfectly avoided—namely, by blowing air through molten crude iron, as in the Bessemer operation, where, by the burning of small quantities of carbon and silicon contained in the crude iron a very high temperature is attained, which is not the case in the process of puddling, where the oxidation is spread over a considerable period of time, although the same constituents are frequently burnt in similar proportions. But even in the Bessemer process the carbon is only half burned, and a large amount of heat escapes with the carbonic oxide and nitrogen. When, however, we blow thin streams of air through molten sulphide of iron lying upon a tuyère hearth, a high temperature is produced by the perfect combustion which ensues in the midst of the sulphides, and no unburnt gases excepting sulphur vapor escape from the surface of the molten mass. Hot nitrogen and sulphurous acid being the only gaseous products of the operation (excepting the small quantities of hydrogen from the aqueous vapor of the air), these may be caused to act upon iron pyrites and other mineral matter. When pyrites is thus heated, an atom of sulphur held in feeble combination is in great part expelled, and thus is obtained protosulphide of iron, with which the operation commences, and which can exist in the molten state. Sulphide of zinc thrown into this bath of molten sulphide is converted into oxide: the sulphides of copper, nickel, and silver do not burn at all so long as sulphide of iron is present, and, accordingly, if oxides, silicates, or carbonates of these latter metals are introduced into the molten sulphide of iron, the iron present will take away the oxygen with which the metals are combined and concentrate them into a regulus of sulphides. But the question then arises, How, after fractional decomposition by oxidation, we can separate the sulphides from the oxides? This is accomplished by the addition of siliceous matter introduced into the furnace with the charge of sulphides, so that in the manner explained are obtained from crude materials five principal classes of products, viz.:—(1) sulphur; (2) sublimates of volatile sulphides and oxides; (3) a slag of silicates of certain more oxidizable metals, principally iron; (4) regulus containing the nickel, copper, and silver; (5) sulphurous acid and nitrogen. Under certain circumstances a sixth class of products may be obtained consisting of the metals copper and lead. Thus, when the sulphides of iron and copper present in the bath are treated continuously with the blast of air without the addition of combustible sulphides, a point at length arrives when the whole of the iron present is oxidized, and the regulus in the bath consists of subsulphide of copper. If now a limited supply of air is introduced, the copper is reduced to the metallic state, with the evolution of sulphurous acid. Further experience in the matter may lead to the adoption of this continuation of the process. Again, sulphide of lead present in the bath may be caused to yield metallic lead by partial oxidation. The sulphurous acid can be made into sulphuric acid

in chambers or condensed to the liquid state. Thus we have in this new process a metallurgical operation, the necessary heat for the decomposition and fusion being entirely obtained by the combustion of the iron and sulphur contained in the materials operated on.

Some large experiments have been made in order to prove the more important points here enunciated. They are all to be found described in the paper upon the subject in the *Journal* of the Society of Arts, dated February 14 and 21, 1879. A brief record of some of the phenomena witnessed at the February experiments at Penistone may not be uninteresting. At seven in the morning on February 12 last a small party of gentlemen arrived at Messrs. Cammell & Co.'s Penistone Steel Works, in order to see the operation from its very commencement. Two Bessemer converters were ready for the experiments; one of these was charged at 10 A.M. with some molten protosulphide of iron (made by fusing some pyrites in a cupola), and a blast of air was driven through the tuyères. Lumps of sandstone were continuously thrown in together with cupreous pyrites. A flame of the burning vapor of sulphur expelled from the pyrites passed from the converter mouth to the chimney shaft; it was from 6 to 10 feet long, blue at the edges and greenish in the body of the flame. About noon this experiment broke down through an accident, after which the product was taken out. An experiment was then commenced by setting fire to some sulphide of iron by means of about 2 cwt. of coal thrown into the vessel to start the combustion; pyrites and sandstone were then thrown in, in lumps, which rapidly melted, this being continued until midnight (over eight hours). The molten mass in the vessel remaining perfectly liquid was, from time to time, partially poured out to make room for the addition of further similar materials. During the whole of the eight hours not an ounce of coal was used, the converter being "fed with stones," and "vomiting forth fire and brimstone," as a gentleman present graphically expressed it. In this latter experiment about eighteen tons of raw pyrites were thus treated, and over four tons of sulphur distilled and afterwards burnt. More than half a million cubic feet of sulphurous acid and nitrogen left the mouth of the converter at a high temperature, taking away with them a considerable fraction of the heat produced by the oxidation. This was very unfavorable to the success of the experiment, as will be readily understood when this great loss of heat is taken into account. With a suitable plant the heated gases would be utilized to drive off sulphur from pyrites, so as to produce the molten protosulphide required to continue the operation. Heat is not only obtained by the oxidation of the metallic sulphides, but also by the oxidation of iron protoxide to peroxide when the contents of the vessel are over-blown. In an experiment made in July last the oxidation was thus purposely continued. "As soon as the subsulphide of copper began to burn a splendid emerald green flame suddenly appeared, lasting about a minute, and all the lines except those of copper and sodium left the spectrum. During the last few minutes of the blow the mouth of the converter was dull and without flame."

Some of the products of these experiments were shown at the Society of Arts; they consisted of crystalline masses of ferrous silicate and blocks of fifty per cent. copper regulus. No sulphur was collected, it being impossible to do so with Bessemer plant, which, in actual operations, will not be used for the process. These experiments, however, enabled those present to witness, in the course of a few hours, the principal effects produced. "A remarkable spectrum was obtained from the burning sulphur vapor; viewed through a small direct vision spectroscope, many absorption bands were seen occurring at apparently regular intervals from the red to the violet. The lines of sodium, lithium and thallium were recognizable, but the majority of the lines are of (as yet) unknown origin, though they are the most important, since the changes furnish indications of the progress of the chemical changes taking place in the vessel. The lithium was, probably, derived from the sand introduced with the pyrites."

The process is peculiarly suitable—

(1) For the treatment of metalliferous substances which cannot be advantageously utilized by other processes. For the extraction of sulphur by distillation,

and simultaneously for the concentration and separation of copper, silver and nickel from such materials in the form of a metallic regulus; while lead, zinc, arsenic, &c., accrue in the sublimates.

(2) For the treatment of cupreous pyrites, large quantities of which exist in many parts of the world where fuel is scarce, and where the present mode of treatment by the cementation (wet) process involves not only the loss of vast quantities of sulphur, which is burnt to sulphurous acid, but causes the destruction of all vegetation within its influence. For example—about one million tons of pyrites, too poor in copper to pay for shipment to the United Kingdom, are annually treated in Spain by the cementation process. Such ores thus treated, containing $1\frac{1}{2}$ per cent. of copper, leave only a small profit, whereas it is calculated that similar ores by this new process will yield a profit more than five times as great.

(3) For the treatment of copper and nickel ores, so as to produce a concentrated regulus without employing carbonaceous fuel.

It is therefore obvious that this process will effect a great revolution in the treatment of metallic sulphides, such as iron, cupreous and nickeliferous pyrites, also copper and nickel ores and the refuse gangue of mining operations, which can thus be smelted without the employment of carbonaceous fuel, the necessary heat being obtained by the oxidation of the metallic sulphides.

THE DESIGN AND CONSTRUCTION OF SEWERS.*

By GRAHAM SMITH, C. E.

From "Iron."

The expeditious removal of fœcal matter and refuse of all kinds from the habitations of the members of any community largely conduces to their individual comfort, and the health of the whole community is much dependent on the efficacy of the measures adopted to effect this end. The water-carriage system may be said to be now universally in operation in this country, and the success or non-success of this system mainly depends upon the design and construction of the sewers. These should not only be constructed in a manner such that they will carry to the outfall with despatch the sewage which may find its way into them, but they should likewise be built in a manner that no portion of such sewage should percolate through them into the surrounding earth. All matter discharged from the stomach and bowels of a cholera patient is infective, and if a taint of the infective material gains access to any well or other source of water supply, it may spread the disease indefinitely. Notwithstanding this fact, the records of the General Board of Health demonstrate that at the present time there are many branch sewers and drains existing in London which allow of the liquid portions of the sewage passing through the joints in the brickwork. These contaminate the surrounding earth, whilst the solid matters remain behind to choke the sewers and pollute the atmosphere with the gases which they generate whilst undergoing the process of decomposition. It is painfully apparent that the early sewers were, as a rule, very imperfectly constructed, and even now many sewage works are improperly carried out, simply because Local Boards are prone to practice a false economy in their public works department, and retain the moneys which ought to be paid as fees to competent men for supervising their work. An examination of Appendix No. 1 to the Annual Medical Report to the Local Government Board will convince the most sceptical that disease is caused in our country to a very large extent through defective sewers and defective drainage arrangements. Forty-two places were inspected, and, with the exception of one being a question of hospital

* Paper read at a meeting of the Liverpool Engineering Society, February 25th, 1879.

accommodation, and another to certain manufacturing processes causing a nuisance, the grounds for inquiry were the presence of zymotic diseases which were attributed by the medical officers in almost every instance to such causes as insufficient and polluted water-supply; imperfect means for the disposal of excrement and refuse; defective sewerage and drainage; and improperly constructed sewers. Sufficient has now been said to show the importance of the subject under consideration, and the effect it has on the well-being and welfare of the inhabitants of our villages, towns and cities.

The laws on which sewers should be designed, laid out and constructed, although by no means of a rule-of-thumb description, are yet sufficiently well known and understood to enable a competent man to treat effectively undertakings of this nature. When dealing with a growing town or village the conditions are complicated, and drainage arrangements may require alteration from time to time; but with thickly inhabited towns no excuse is admissible for a mistake in the form or capacity of sewers. No works or working plans should be undertaken until a thorough study has been made of all the local circumstances which are likely to affect the scheme when completed. The amount of surface drainage due to rainfall and storms, and the quantity of sewage which will require to be carried to the outfall by each vein of the system must be ascertained as nearly as possible. The position of the outfall, so as to allow of a sufficient inclination being given to the sewers of the whole system, is a matter of first importance, and deserving of the most careful consideration. The form of sewer which should be employed in any particular instance depends in a great measure on the quantity of sewage which is likely to flow through it. If a large and unvarying quantity can be depended upon, the circular section is undoubtedly that which should be employed, or even when the minimum flow is such that the sewer runs half full, the circular form has much to recommend it. Circular sewers are more easily and economically constructed than egg-shaped sewers, and are better calculated to resist the external earth-pressures. Nevertheless, where the quantity of sewage varies considerably, the egg shape should be adopted, as the smallness of the invert increases the scouring action of a small quantity of sewage, whilst the increased size of the upper portion provides for any augmentation in flow. This is so, notwithstanding that the circular sewer when running half full exposes a smaller wetted perimeter than the egg-shaped sewer with the same sectional area of stream. In all calculations with rivers and streams, it is usual to suppose that the velocity of flow diminishes with any increase of the wetted perimeter. This is not the case, however, when dealing with narrow channels, in which the increase of depth raises the velocity to a greater extent than it is diminished by the increase of friction. This was demonstrated by the experiments carried out by Mr. John Phillips with the Westminster sewers upwards of thirty years back. These experiments led him, about the year 1847, to design the egg-shaped sewer, with the major diameter one and a half times the minor diameter. A semicircular crown, struck with a radius of half the minor diameter in length. An invert struck with a radius one quarter the minor diameter, and sides with radii of one and a half times the minor diameter. These proportions are found at the present time to give satisfactory results, and have been very generally adopted wherever the egg-shaped sewer has been employed. More recently, however, he has recommended for branch sewers, in which the flow is at times small, a modification of this form in which the radius of the invert is reduced to one-eighth of the minor diameter. He likewise demonstrated by his experiments that, with a view of rendering a sewer self-cleansing, that those of moderately large dimensions should be given a rate of inclination sufficient to cause a velocity of flow of $2\frac{1}{2}$ feet per second. In Latham's "Sanitary Engineering" will be found a table calculated on this basis, which will enable an engineer at once to determine the inclination to be given to any sewer, whatever its form or dimensions, in order to produce this velocity. In small sewers a velocity of flow of not less than $3\frac{1}{2}$ feet per second should be maintained. If the sections and inclinations are properly arranged so as to maintain these veloci-

ties under all circumstances the sewers will require little or no scavenging, and, in fact, may be considered as entirely self-cleansing. In every system where the quantity of sewage is small greater inclinations are necessary than where the quantity is large, and so the lower portions of any system usually require a less inclination than the higher portions, the sewage accumulating as the outfall is approached.

It must be borne in mind, however, that although much is due to sectional form and inclination, not a little of the success of any system of sewers depends upon the manner in which the details are arranged. Especial care must be bestowed upon the form and construction of the junctions of sewers. These must always be so designed that branch sewers may discharge into main sewers, and main sewers into larger main sewers, with a velocity of flow approximating to that in the larger sewer and in the same direction as the flow already existing therein. If this point is not attended to eddies are caused which lead to deposits being formed. The level of the bottom of the smaller sewer should never be made on a level with that of the larger sewer, as the latter is constructed to carry, as a rule, a greater depth of sewage. It stands to reason, therefore, that if the bottoms of the sewers are made on the same level there will frequently be dead water in the smaller sewer, which will lead to a deposit being formed at its mouth. Looking at the matter from a purely theoretical point of view, it will be seen that the sewers should be placed relatively to each other so that the surface of the sewage in each, at the point where the sewers join, may be at the same level. It is scarcely necessary to point out that the fluctuation in the flows in sewers are so frequent that this perfection of arrangement is rarely if ever attainable in practice. It is, nevertheless, an ideal state of things which it should be the aim of all to approach as nearly as possible in actual work. The question of ventilation must also be dealt with, for the proper and efficient ventilation of sewers is one of the most important matters to be considered in any system of drainage. The temperature of the air in sewers is generally higher than that of the external atmosphere. Mr. Haywood found by his investigations, carried out during the year 1858, that the temperatures in the London sewers during the winter were 11.61° Fah. higher, and in the summer 3.12° Fah. lower than the temperature of the external air. The average temperature of the latter for the whole year was 50.24° Fah., and for the former 55.35° Fah. This fact of the temperature in the sewers being greater than that of the external atmosphere has an important bearing on the question. The higher the temperature the more quickly will fermentation and putrefaction take place, and the more freely will the gases generated in the sewers ascend to the upper portions of the system, and find their way into dwellings if proper means are not taken for their disposal and dispersion elsewhere. In a work entitled "Sanitary Drainage of Houses and Towns," by Colonel Waring, an American author, it is stated that the excrement of a typhoid patient continually agitated in contact with fresh air and a fair admixture of water passes through a series of complete chemical changes with no injurious product, but if allowed to remain stagnant, if not properly exposed to the air, or if it gain access to the human circulation before a certain oxidation, it will, like a ferment, reproduce itself and give rise to the conditions under which it was at first produced." . . . "Unventilated and badly constructed sewers are sure agents for the propagation of the disease when once it has taken root." The latter fact is uncontrovertible. Many means have been suggested for ventilating sewers by shafts, fires, and other of the ordinary expedients for producing air-currents. These have all more or less failed, their effect having been found to be entirely local. It is now the generally accepted opinion that the only means of ventilating sewers is by allowing them to ventilate themselves. All manholes should therefore be provided with iron gratings or with ventilating pipes, so that they may not only serve the purpose for which their name denotes they are intended, but at the same time allow of the free entrance and exit of the air to the sewers. By this means the poisonous nature of the gases is to a great extent counteracted, as when largely diluted with air they are considered to be comparatively harmless.

Manholes should be placed at the junctions and angles of all main sewers, and for purposes of ventilation the distance apart of either manholes or ventilating shafts should not exceed 300 feet. No rules can be laid down for the spacing of manholes, but the more frequently they occur the better will be the ventilation and the greater the control over the system. If all gulleys and catchpits are efficiently trapped, and the junctions of all branches made in such a manner that they are never water-locked by the sewage in the main sewer, these manholes and ventilating shafts may be made to act as down-air shafts, and pipes three or four inches in diameter carried from each private drain up the side of the house, which such drain serves, to a few feet above the top of the house, will form efficient up-air shafts. The drains should be trapped just outside the walls of the dwelling. These pipes will then not only serve the purpose of up-air shafts from the main system of sewers, but they will likewise relieve the trap from all pressure and prevent the possibility of any leakage of sewer-gas into the house. This, of course, supposes that no other drains pass under the houses unless bedded in concrete, and trapped at each end immediately outside the walls of the building. Time will not permit the subject of manholes, traps and gulleys, being gone into more fully, the author will therefore devote his remaining remarks to the materials employed in the construction of sewers.

Lime mortars were often necessarily employed before the introduction of Portland cement, and even at the present time chalk and other rich limes are not unknown in sewer construction. These are not capable of resisting the chemical action of sewage, and therefore, although the more hydraulic limes may be capable of affording a sufficiently strong cementing material, they should never be employed, excepting on the crowns of sewers, which are estimated, under normal circumstances, to run about half bore, and even under these conditions, their use in sewer construction is attended with some trouble and risk. For the purpose of backing up a sewer, or filling up a trench in land liable to a settlement, good hydraulic lime concrete is all that can be desired. The water used in mixing the mortar or cement should be free from all organic matter, as should also the sand and gravel which are to enter into the construction in any form whatever. In districts where gravel, or other suitable materials for the making of concrete are to be procured, no better material than Portland cement concrete is to be found with which to construct an efficient and economical sewer. The proportions adopted by engineers in mixing Portland cement concrete for sewer work vary, but 5 to 1 and 7 to 1 may be taken as the limits employed for the main portions of the work. In some cases the whole interior of a sewer is rendered with a thin layer of pure cement or cement mortar, consisting of equal parts of cement and sand. A somewhat more easily-constructed sewer than that in which concrete only is employed is the sewer with an internal ring of brickwork, surrounded entirely with concrete. It is slightly more expensive, but it permits of the invert being formed with hard bricks glazed on one side, such as the Staffordshire blue bricks. The inverts of sewers should always be formed either with these hard bricks or with glazed earthenware blocks, as the friction and erosive action of the detritus carried in the sewage is very great. For sewers of moderate size the earthenware blocks have many advantages, as, when laid on a bed of concrete the bottom of the sewer can be constructed with facility and accuracy. The invert block in one piece is preferable to that made in three pieces, as it is difficult to find men who will make the joint of the latter properly, and when one block only is used it is generally made hollow, and so a continuous subsoil drain is formed under the sewer to carry off water from the trenches during the progress of the work. In large sewers the inverts are preferably made with bricks of the hard and smooth description already pointed out. Softer bricks, such as the "London stock," are, however, more suitable for the construction of the other portions of the work, owing to their offering a better surface for the adhesion of the mortar. A few experiments, carried out by the author with a view to ascertaining to what extent the nature of the brick affects the strength of brickwork may not be here

out of place. In the first experiment twelve bricks similar, but slightly harder than London stocks, were crossed in pairs and united with Halkin lime mortar, mixed in the proportion 1 of lime to 2 of sand. At the end of 168 days the six pairs of bricks were drawn asunder with an average of 553 lbs. for each pair. With hard firebricks tested in every way in a similar manner, the average of six experiments was 474 lbs. for each pair. These experiments tend to show that soft porous bricks are preferable to hard vitrified bricks where tensile strains have to be met in brickwork. For sewers of small diameter glazed earthenware pipes of circular section are preferable to any other system of construction. These pipes should be laid on concrete, and where the roots of trees are numerous they should be entirely surrounded with this material. Puddled clay was at one time much employed for the making of the joints of such pipes, but it has now been superseded by Portland cement, as it was not found to answer satisfactorily. The roots of trees find their way into the sewers through the small fissures which are always liable to be formed in puddled clay, simply by the dryness of the surrounding earth. In conclusion, the author deems it well to remark that the time at his disposal for the preparation of this paper having been limited, he has not been in a position to prepare diagrams, and therefore has been unable to deal in detail with the design and construction of sewers.

THE PUBLIC WORKS OF THE UNITED STATES.*

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THE NAVIGABLE WAYS OF THE UNITED STATES.

I. GENERAL OUTLINE OF EXISTING ROUTES.

Every one knows that the basin of the Mississippi, lying between the Alleghany and the Rocky Mountains, embraces about half of the United States; that the Eastern slope of the Alleghanies presents but a narrow space along the Atlantic in which colonization was nearly confined in the last century, and that on the opposite side, west of the Rocky Mountains, the continent forms an undulating plateau which has a width of nearly 2000 kilometers and more than 2000 meters average height.

Until 1847, when gold was discovered in California, a few pioneers only had ventured beyond the Mississippi and Missouri, into the plain which rises by gentle slope for 800 kilometers in distance to the foot of the Rocky Mountains. To-day one may say the center of gravity of the American continent is found on the Mississippi between St. Louis and Chicago.

It is here the great grain-producing states are grouped. The fundamental problem connected with the establishment of routes of communication, stated in a word, consisted in uniting Chicago with the Atlantic coast from which all the unconsumed products are exported. With this problem the three great navigable routes of the United States are connected.

1. The line from Chicago to Montreal by way of the Lakes and the St. Lawrence.

2. The line from the Eastern extremity of Lake Erie to New York, via. the Erie Canal and the Hudson River.

3. Lastly, the Mississippi from whose mouth vessels sail by sea for the large ports of the Atlantic, Baltimore, Philadelphia, New York, Boston.

I will now enlarge on some of the details of these lines to which will be added some remarks on the most interesting affluent of the Mississippi, the Ohio.

* In an introduction to this article, which is omitted, the author speaks of the collection of drawings and models sent from the United States to the Paris Exposition of 1878, and of the imperfect idea that they conveyed of our great public works. He was asked to describe them more fully and this paper is the result.—D.F.

THE CANADIAN LINE.

Every one knows that the waters of Lake Erie empty into Lake Ontario by way of the Niagara river. A long while ago, these lakes were united by a navigable canal—the Welland Canal. It was constructed for boats of 500 tons. It is about to be enlarged to give passage to vessels of 1000 tons which navigate the lakes.

Six canals, varying in length from 1,200 meters to 18 kilometers were constructed a long time ago around the St. Lawrence Rapids above Montreal. These canals serve for all ascending boats and in descending for all boats loaded with cereals. Rafts and steamboats descend by the river itself. As with the Welland Canal, there is a project for their enlargement. These canals were all built by the Canadian Government. To this end it has expended the sum of about 100 millions of francs. It is estimated that 90 per cent. of the merchan dize transported by this route comes from the United States.

Below Montreal attention is being paid to the improvement of the river, to render it navigable at all seasons for English packets.

THE ERIE CANAL.

The Erie Canal, completed in 1826, has given to New York for more than a quarter of a century, a marked superiority over all other ports of the Atlantic. But this state of things has been modified within the last dozen years, especially by the railways. These new routes (it is said) carried in 1876, 83 per cent. of Western freights, while the Erie Canal carried only 17 per cent.

The canal was at first constructed with a depth of 1^m 22 for boats carrying 76 tons. It was soon enlarged by doubling the locks, increasing the draft to 2^m 13 and the tonnage of the boats to 240 tons.

This enlargement reduced the price of transportation 50 per cent., in conformity with the predictions of the engineers.

THE MISSISSIPPI.

Before the establishment of railways, New Orleans (thanks to the Mississippi) was the New York of the South.

This river is navigable as far as St. Paul, nearly 4000 kilometers.

But from St. Louis to St. Paul navigation is precarious on account of shoals and ice. The depth is reduced sometimes to 1 meter. In 1870 the improvement of all the rapids which are found in this portion of the river was accomplished, viz: at Keokuk and at Rock Island.

At Keokuk a lateral canal was constructed. At Rock Island a channel 60 meters wide 1^m 22 below low water was excavated in the rocky bed of the river.

From St. Louis to New Orleans, over a distance of nearly 2000 kilometers, navigation is but rarely impeded by low water or ice. The average minimum depth of water has varied during the nine years from 1868 to 1876 as indicated in the following table:

	Days.
Less than 1^m22 during	5
More than 1^m22 and less than 1^m83	52
More than 1^m83 and less than 2^m44	103
More than 2^m44 and less than 3^m05	69
More than 3^m05	136
	365

Vessels coming from the sea rarely pass New Orleans, but whatever their draft, they can always ascend as far as Vicksburg and sometimes to Memphis, (1,200 kilometers from New Orleans). From New Orleans to the apex of the delta, for a length of 195 kilometers, the depth of the water is about 30 meters.

All the different passes into which the river divides itself have a bar at their extremities in which the natural depth is less than 5 meters.

As long as commercial navigation was limited to vessels of 400 to 500 tons, drawing from 3^m to 4^m25 of water, the bars at the mouth of the Mississippi presented no serious obstacle. But for about the last quarter of a century, maritime commerce has employed, commonly, vessels of 1000 to 5000 tons, with a draft of from 16 to 23 feet (from 5 to 7 meters). Experience has proved their economy beyond a doubt, notably, for long voyages. Since that time New Orleans, (like Nantes) has scarcely been more than a port for coasters.

For several years preceding 1875, the Federal Government has expended in the neighborhood of 1 million francs in annual dredging, in the South West Pass. A channel from 15 to 20 feet deep (4^m57 to 6^m10) and from 15 to 60 meters wide was thus obtained. But the result was in-

sufficient and doubly precarious, storms sometimes filling up to-day the channel of yesterday, in spite of appropriations.

THE OHIO RIVER.

The Ohio drains in the upper parts of its course carboniferous regions whose area is larger than those of Great Britain, France and Belgium united.

Pittsburg is the center of the coal commerce of the West and South West. In 1876 above 2,500,000 tons of coal were shipped on the Ohio from this point.

From Pittsburg to Cairo, where the Ohio joins the Mississippi, is about 1550 kilometers. Though much less troubled with ice than the Upper Mississippi and the Missouri, the Ohio is subject to greater freshets than any of the other affluents of the great river. The water rises 19 meters at Cincinnati. The coal barges require a depth of $1^{m}83$. Above Cincinnati the depth is sometimes as little as 2 to 3 feet. Below, (that is to say for a distance of 829 kilometers) steamers large or small can generally navigate during eleven months in the year.

The canal from Louisville to Portland, (constructed to avoid the rapids) has been wholly in the hands of the Government since 1874. It cost 16½ millions of francs. By reducing the toll from 2 fr. 50 to 0 fr. 50 per ton a considerable service was rendered to the navigation of the Ohio.

The freight on coal from Pittsburgh to Louisville or to New Orleans is from one to two millimes (0.02 to 0.04 cent.) per ton kilometer. There is not a railway in the world that could carry at so low a price and, consequently, the Ohio is the only route which can supply coal to the centers of populations established on the banks of the Mississippi below St. Louis; to the steamboats which ply its waters and to those which take the sea at its mouth.

PROJECTED CONNECTIONS.

Among the number of connections that are projected between certain navigable ways may be mentioned the following:

1. Canal of large section from Lake Champlain to the St. Lawrence (Caughnawaga).
2. From Troy to Oswego (lake Ontario) by way of lake Oneida.
3. From the James River to the Kanawha (or from Richmond to the Ohio).
4. From the Tennessee to the Chattahoochee (by Macon).
5. From Rock Island to Hennepin.

II. THE EXECUTION OF THE WORKS.

Rivers can be improved with a view to navigation, by dredging, by lowering their bed, by widening, by ridding them of rocks. The end sought is thus attained directly. It can also be attained indirectly by narrowing the bed by longitudinal dikes which concentrate the flow.

Lastly, an improvement much more radical can be obtained by controlling by dams the course of the river, diminishing the velocity and at the same time increasing the depth, in a word, transforming the river into a series of lifts like those of a canal.

A. DAMS.

Americans have no movable dams although they have some fixed ones, and also some canalized rivers. But this question is the order of the day with them—they come to study what we are doing in France, desirous of ascertaining how our various systems can be applied, notably to the Ohio—and to water courses much larger than ours, where the floods are much higher and more sudden and where very few men can be had for manœuvers.

Under these more complex and difficult conditions, American engineers are now studying the problem of movable dams. No doubt but that their spirit of invention will be given full play, and that some day at some new international exposition they will return us with interest the lessons that France is happy enough to furnish them to-day.

Leaving the subject of dams, there only remains for us to examine in America dredging machines and examples of channels narrowed by dikes.

B. DREDGING MACHINES.

Spoon Dredge—Although chains of cup dredges are found in the United States, the apparatus most commonly employed in rivers and canals is a dipper dredge directed by an arm.

It has been sought from time to time in Europe to solve the problem of the managament of this arm, which is 10 or

12 meters long. It is held by and turns on a roller, whose axle carries a wheel joined by a chain to the two extremities of the arm. One man governs it easily. This machine, for which perhaps Americans have too exclusive a preference would be useful in France for accurate dredging.

Shell Dredge.—The shell dredge is an apparatus of which the original idea belongs to Europe, but of which Americans alone have been able to make a practical, accurate, strong and useful machine. The difficult point was to produce from above, by means of a chain, a downward pressure, causing two shells or jaws, mounted on a common horizontal hinge, to penetrate the soil and to join after imprisoning between them the detached materials.

Rakes are employed in the Upper Mississippi to increase, at a small cost, the depth of the channel from 1 meter to $1^{m}30$ on certain sandy shoals. They have been found so successful that the ordinary boats used in commerce sometimes ask for these rakes to attach behind.

The General MacAlester dredge-boat is represented at the palace of the Champ du Mars, as are also the preceding machines.*

Removal of Sub Marine Rocks.—A new method for removing large masses of sub-marine rocks was applied at San Francisco in 1870, and since then has been used at New York as follows:

First a coffer dam is established on the summit of the rock from the interior of which, after being emptied, a well is sunk to the level of the projected excavation.

Galleries are opened radiating from this well, joined by concentric galleries or else by galleries cutting each other at right angles with proper piers, and a roof of sufficient thickness to exclude the water. Thus the whole mass is excavated. So well was this done at San Francisco that at the last, wooden posts were substituted for the pillars which had been left. Finally, casks of powder are distributed in the interior, the water is let in, and then, at high tide, a simultaneous and general explosion takes place by means of electricity. In this system a grade is fixed upon a little lower than the level required for the channel, in order to dispense with picking up the fragments of debris from the excavation, which is sometimes expensive.

Among the processes of sub-marine blasting may be always found that which consists of sinking holes, in which cartridges are placed. For this purpose steam drills mounted on flying scaffolds are used.

In a model (not on exhibition) which was shown me several weeks ago by Mr. Julius H. Stredinger, (one of the assistant engineers to Gen. Newton, at Hell Gate, near New York) the scaffold was not fixed, but was held in place by four sliding legs which carried it, and which is made to rise or fall by means of racks. When it is wished to move the apparatus, a special ponton is advanced and inserted between the legs, and these being raised the pontons support the scaffold which can then be carried somewhere else.

This method, already employed in spoon and shell dredges, avoids the use of long guys which are often so troublesome to other boats or vessels.

C. DIKES FOR NARROWING THE CHANNEL; THEIR USE FOR IMPROVING MOUTHS OF RIVERS.

1ST. TRIBUTARIES OF THE GREAT NORTHERN LAKES.

Americans have improved several mouths of rivers by means of dikes. Combined with dredging, this system has succeeded in opening ports at the mouths of streams which flow into the grand lakes of the North. The bars which obstruct these streams are principally produced, as at the embouchure of the rivers of the Baltic, by the action of the waves on the movable material of the shore.

It can be seen, consequently, that by means of parallel jetties prolonged a sufficient distance from the shore, it has been possible, either to prevent in the future the changes of the shore, or to arrest for a longer or shorter period the movable matter which is carried along the coast, as has been done since 1824 at the mouth of the Oder.

* This machine perfected by Major Howell, was lost in the sea last winter, while *en route* to the mouth of the Sabine River.

Below will be found a table of depths obtained on the bars.

	Originally.	In 1875
	Meters	Meters.
At Chicago........	0.90	4.57
At Milwaukee.....	2.13	5.18
At Racine.........	0.60	4.27
At Michigan City..	0.30	3.66
At Erie...........	0.90	4.57
At Buffalo........	0.20	4.57

2D. THE MISSISSIPPI.

A much more important undertaking has been in process of execution since 1875, viz., the improvement of the embouchure of the Mississippi.

It was debated at first whether it was best to construct a lateral canal, that is to say, a canal connecting the deep portion of the river with the sea, or to undertake the destruction of the bar

The Government engineers, in 1874, submitted a plan for a canal as the only certain solution of the problem. This canal, 11 kilometers long, was to be connected with the river near Fort St. Philips—60 kilometers from the mouth —and from this point was to take an easterly direction in a straight line and empty to the South of Breton Island. The other plan has also found strong supporters, who have protested against the idea of subjecting such an important commercial undertaking to the disadvantages of a lock, and dwelt on the benefits of a navigable pass of sufficient capacity and practicable at all times.

In a word everything possible has been said and written on the subject exactly as was done in France, when, in reference to the embouchure of the Rhine, the same question was submitted to a commission instituted by ministerial decree of Dec. 14, 1843, and whose inquiries were started on a similar plan by Engineer Turrell.

In this situation the President of the United States also appointed a commission who began by visiting all similar works in Europe, and who selected as was done in France in 1852 the plan of improving the bed of the river.

Unlike the work done for the Rhone which was considered as an experiment and for which the small sum of 1,500,000 francs was expended, the dikes designed by the American Commission were estimated to cost a much larger amount of money.

But the partizans of the system, at whose head was Mr. James B. Eads (of St. Louis, Mo.,) offered to execute the work at their own risk. This offer was accepted by Congress, March 3, 1875 and in June following Mr. Eads began work.

Let us examine the question from its technical point of view: What were the arguments entered into on both sides; what works have been executed, and what have been the results up to the present time?

Description.—The head of the passes is 20 kilometers from the sea. The river here divides into three branches which are continually extending, while the bars at the mouths, caused by the deposits of mud, advance into the sea.

The river transports annually in the neighborhood of 190 millions of cubic meters, or more than ten times that carried by the great Rhone.

The South Pass, which was chosen for improvement, has hardly 700 feet (213 meters) of average width. The width of the bar, measured on the axis of the channel between the two curves of $6^{m}.70$ depth, was 3,600 meters in June, 1875, for about 800 meters the depth of the water was scarcely more than $2^{m}50$, consequently his South Pass was seldom or never used by Commerce.

I will add that from 1838 to 1874, during a period of 35 years, the bar had advanced about 30 meters on an average per year (while the annual advance of the bar in the South West Pass had been 90 meters).

Contract.—Mr. Eads was authorized to execute such jetties as he might think proper to produce a wide and deep channel across the bar at the mouth of the South Pass.

The width to be secured between the jetties was not to be less than 700 feet.

The contractor is engaged to secure the following results:

1°. In 30 months from March 3, 1875, a depth of 20 ft. ($6^{m}10$) at least, this depth to be reckoned from the lowest level of high water in the sea where the stream is at low water.

2° A farther deepening of at least 2 feet ($0^{m}61$) in the course of each of the three succeeding years. The whole

depth thus becoming 26 ft. (7m93). If either of these conditions are not fulfilled the contract becomes null and void.

On the other hand, Mr. Eads guarantees to obtain 30 feet of water (9m15), in consideration of which the Government is pledged to pay him, first, the sum of $5,250,000 (26,250,000f.) for the jetties and other works which become the property of the Government; second, an annual sum of $100,000—for the maintenance of the works as soon as the depth of 20 ft. is obtained—up to the final delivery of the works.

Payments of $500,000 on account, will be made when the minimum depth reaches successively 20, 22, and 24 ft. on minimum widths respectively of 200, 250, 300 and 350 ft. The Government will retain as a guarantee the last million dollars for 10 years and $500,000 for 10 years more.

In addition to which the delay of the guarantee will cease to run, and the annual subsidy of 100,000 francs will cease to be paid, during periods when the channel shall be less than 30 feet deep or 350 feet (106m75) wide.

Lastly, the government will have authority to take possession of the jetties at any time, by paying the contractor the sum retained as guaranteed and releasing him from all responsibility.

The Act of Congress of March 3, 1875, explicitly provides that Mr. Eads shall have full liberty as far as relates to the line of the jetties, their construction and method of execution; but stipulates that they shall be substantial and durable and capable of indefinite maintenance at a reasonable annual expenditure.

In any event the Government will not be responsible for losses that Mr. Eads may sustain in the course of the work.

Principle of the System.—Before mentioning what tnis bold contractor has done and obtained up to this time, it is well to indicate the basis of his confidence in the success of the plans.

The South Pass of the Mississippi presents the characteristic features of rivers flowing with tideless seas. After remaining constant for 15 kilometers the transverse section changes.

The banks created by the river itself widen as they approach the sea in which they disappear, at the same time the latter is raised by a slope the back bone of which, constituting the bar, is found some kilometers distant. By prolonging the banks artificially by two jetties it is evident that the bar will be attacked. It will re-form a little farther out as was the case with the Rhone, but a temporary deepening will result. Here is the point made by Mr. Eads. If, said he, jetties sufficiently long are built rapidly in a few years, so that the deposits will take place in the deep portions of the sea (where the gradual foundation on which the bar will elevate itself will take place) the re-formation of the obstacle can be postponed for a century. It is sufficient to take suitable advance on the natural elongation of the pass; for an elongation of 30 meters a year 1 kilometer of dike will give more than 30 years of advance.

EXECUTION.

By the last of the year 1876, the South Pass terminated in an artificial channel 300 meters wide, comprised between two parallel dikes constructed at the same time over their whole length up to a point beyond the crest of the bar where there was originally 35 feet of water (10m. 67).

These dikes are similar to those of Holland, gravel and stone being lacking at the mouth of the Mississippi; one of these was 3650 meters long, the other 2500.

The scour has been further aided by numerous transverse dikes and by dredging.

If one imagines the channel divided into six portions of 600 meters each the deepening obtained from June 1875 to November, 1876 has been:

m.50 in the 2d portion.
1m.33 in the 3d portion.
3m.35 in the 4th portion.
3m.47 in the 5th portion.
3m.39 in the 6th portion.

On November 14, 1876, the two 20 ft. (6m.10) curves on each side of the bar were joined (reckoned from the average sea level at high tide and at low water of the river).

The minimum depth of 6m.10 having extended over a width of at least 60 meters, Mr. Eads received his first payment, on account, of 2,500,000 francs, in the Month of January, 1877. December 15, 1877 the depth having reached 6m.70

over a width of from 60 to 80 meters Mr. Eads received his second payment of 2,500,000 francs.

Present Situation.—Since then six months have passed. What is the present condition of things? Is Mr. Eads still confident of success? I have before me the three following documents:

1. A petition signed the 7th of May 1878, from Mr. Eads to the Secretary of War, with a view of obtaining several modifications in the conditions of the contract.

2. A report on this subject from a special commission consisting of two Government engineers.

3. Lastly, a letter from the Secretary of War transmitting the document to the Senate approving the conclusion of the Commission.

Mr. Eads states that the expenditure made to accomplish the results have considerably exceeded his anticipations, and that the interest is exhausting him. He asks that the time during which the successive depths should be maintained, in order to secure a new payment on account, should be shortened.

Commerce has the greatest possible interest in as speedy a termination of the works as possible. Equal profit to the public treasury would ensue, relieved as it would be from interest paid the contractor of 5 per cent. in the grants which have been complied with in principle but whose payment is postponed.

Besides this, Mr. Eads asks for a provisional reduction of the minimum widths of the channel for the successive depths; remarking that the Commission of engineers of 1874 who prepared the contract abstained from fixing any width, and that this stipulation added afterwards by a legislative committee, assumed relations between the depth and width of the channel which could not be realized in practice. Thus to obtain finally a width of 350 feet (107 meters) a greater depth of channel than 30 feet (9^m. 15) would have to be secured, and that these dimensions exceeded the real needs of navigation. Mr. Eads concludes by asking:

1. That the progress of the deepening be verified foot by foot.

2. That the corresponding widths be modified, following a scale which he indicates. That instead of the increase from 250 to 350 feet when the depth is increased from 24 to 30 feet, the width, on the contrary, shall decrease from 150 feet to 100.

3. Lastly, that the payments be divided with 7 subsidies, diminishing concurrently from $750,000 to $250,000, the whole not to exceed the limits of the total grant.

The reserve guarantee would be maintained at one million dollars.

Lastly, Mr. Eads urges, with incontestible eloquence, the right which he has to the concessions that he solicits.

In the report of Messrs. J. G. Barnard and H. G. Wright, let us consider only some of the technical reviews of the work executed.

To attain the character of solidity and of durability which the law requires, the jetties should be enlarged and riveted with stone. This treatment is particularly necessary at the extremities of the jetties where the dikes subside from the settling of the mattresses and the subsoil, while the sea breaks over them.

The Commission of 1874 estimated that $\frac{1}{8}$ of the jetties would be made of stone and $\frac{7}{8}$ fascines. They had anticipated that more than 150,000 cubic meter of stone would be necessary. Instead of this, only enough stone has been used to aid the sinking of the mattresses, with about 15,000 cubic meters added; and Mr. Eads announces that half as much is delivered. In this case, he is far from the estimated section. However this may be, the Commission is of the opinion that the works are being properly executed, that their success is probable, and that now the government and the country are both interested, not only in the continuance of the work, but in its rapid execution. The Commission argues with the petitioner that 26 feet depth (7^m. 93) is sufficient, and that if 30 feet was talked of it was to allow a margin.

The statement is added that on the bars of the Mississippi the whole depth of the water is available. By way of information the report adds that 85 per cent of the entire shipping of the world does not draw more than 23 feet (7 meters), that 26 feet (7^m.93) draft is the maximum required for the regular traffic on the bar at the entrance of New York. Lastly, that the largest vessel that ever crossed this bar drew only 28 feet (8^m.54).

The report of the Commission is evidently favorable as a whole to Mr. Eads' petition. It advises, however, that Congress alone can authorize the modifications asked for, as far as relates to the widths and to the payments of the subsidies.

This work so full of interest is unrepresented at the Exposition. It is nevertheless remarkable from more than one point of view.

This is one of the boldest enterprises that has ever been risked by private energy.

The results already obtained surpass from a professional point of view, and in material importance, all that has previously been done for the jetting of rivers.

If the results are obtained and completed without the jetty system proving too expensive, a veritable commercial revolution will have been accomplished not only for New Orleans but for the whole basin of the Mississippi.

§ 2.

RAILWAYS—LONG SPAN BRIDGES.

The most important works of art necessary for railways or roads are those which enable them to cross streams.

In America these are almost the only works which merit attention, for extensive cuttings are rare, large embankments rarer still, subterranean works are few and of short length, if we except the Hoosac tunnel, 7 kilometers long, constructed at the expense of the State of Massachusetts to shorten the distance from Boston to Albany and Buffalo. (This tunnel was opened for travel in 1876).

In the greater part of the United States, good building stone is lacking, thus bridges are constructed only in wood and iron. Wooden bridges, the only ones known up to about 1840, are not built now, except as a temporary measure, or when sufficient capital at first is lacking. This is the reason why almost all of the interesting bridges are of iron.

Another notable fact in the United States is that the rivers if they are few in number are, on the other hand, very large.

Their general direction is North or South, like the Alleghanies, the Rocky Mountains, or all the great physical features of the American continent. Thus they constitute barriers which averted the western expansion of colonization, first at the Ohio, then at the Mississippi and, lastly, at the Missouri. The necessity of crossing these streams arose.

Another difficulty of the problem came from the nature of the beds of these rivers which are composed in the majority of cases of moving sand, 10, 20 or 30 meters deep. The difficulties attending the foundations resulted in spacing the points of supports much farther apart than is generally the case in Europe.

Iron bridges of large span form a truly original feature in the public works of the United States. The construction of some of these bridges has cost one million dollars, others two millions, and the St. Louis bridge cost much more. Eminent engineers occupy themselves wholly with iron bridges; large companies have made an almost exclusive specialty of their construction. The Keystone and the Phoenixville of Philadelphia; the American Bridge Co. of Chicago; the Delaware Co. of New York, and the one directed by Mr. Pope in Detroit; the Watson Co. at Paterson, N. J., besides others.

TABLE.

1ST. FIXED SPANS OF TRUSS BRIDGES.

	Spans. Meters.
In 1862 there were in the United States (we believe) but 2 long span bridges, these were designed by Mr. Albert Fink; the spaces were	61.00
Then the Monongahela bridge was built	79.30
And another on the Ohio at Steubenville	97.60
In 1869 2 others on the Ohio at Bellaire	106.75
In 1869 2 others on the Ohio " Louisville	122.00
In 1871 over the Missouri at St. Charles	91 50
In 1871 over the Ohio at Cincinnati	156.00
In 1871 over the Hudson at Poughkeepsie one was commenced which will have 5 spans of	160.12

2D. DRAW BRIDGES—WITH TWO SYMMETRICAL TRUSSES.

		Meters.
Length of the floor	at Chicago	68.62
Length of the floor	at Cleveland	99.12
Over the Mississippi	at Dubuque... at Kansas City	109.80
Over the Mississippi	at Keokuk	118.03
From 1873 to 1875 over the Missouri at Atchison		111.32

[A similar bridge is about to be erected at the port of Marseilles. The plan was

approved April 1, 1878, by the Minister of Public Works. The total length of the floor, including a bearing of $0^m.90$ on each of the two abutments, will be $73^m.80$].

3D. SUSPENSION BRIDGES.

		Meters.
1855	Niagara Bridge (lower) 2 stories	250.20
1860	Pittsburgh Bridge	105.00
—	Wheeling Bridge over the Ohio	308.05
1867	Cincinnati Bridge over the Ohio	322.00
1869	Upper Niagara Bridge	386.44
1877	Minneapolis near St. Anthony, Min.	205.90
1877	Point bridge over the Monongahela. at Pittsburg	244.00
1877	East River bridge under construction	493.00

4th. ARCH BRIDGE AT ST. LOUIS.

A central arch of	158.60
Two others of	157.07

SUPERSTRUCTURE.

A. *Pin Trusses.*—All the large bridges consisting of straight trusses continue to be built on a system much better known to-day in Europe than it was in 1870. The characteristic features may be stated as follows:

A chord of channel beams of 3 or 4 meters length, often united by cast iron boxes, and another chord of eyebars united by pins are connected by posts on ties variously combined, always jointed by pins on the lower chord and sometimes on the other in such a way that each member is never subjected to but a single stress, tension or compression, of which the maximum determines the section necessary to be given to each piece.

This method should conduce to a reduction of weight and a consequent economy. But this is not the only advantage. Others may be found in the small opportunity offered for rust, the metal following the lines of strains; in the opportunity for inspecting and repainting from time to time; in the facility of transportation—each piece isolated having but short length; in the small area offered to the wind by the large lattices, which also do not retain the snow upon the floor of the bridge.

In fact, the persistent use of the system in the United States proves conclusively that it preserves all its merits in the eyes of Americans.

Riveted lattice bridges are only employed in a limited measure. They seem to constitute a specialty for a firm established at Rochester (State of New York) —the Leighton Bridge & Iron Works.

The comparitive merits of the two systems, which differ essentially in the methods of uniting the members, is difficult to determine on account of the lack of comparative data on the weights of metal employed and the cost of labor, as well in the workshop as in erection.

There is another consideration which prevents our employing pin bridges in France.

While the iron and steel employed in American bridges is always of a superior quality, the manufacturers in Europe where riveted bridges are made, are far from regarding a rigorous choice of iron employed in their bridges as equally necessary. Then again these shops have not the special tools which allow Americans to manufacture, so surely and economically, pieces of which the types are limited. These special tools our workshops could procure, but they do not do so for the reason that they would not have use for them.

B. *Suspension Bridges*, invented in America near the end of the last century, have been perfected during the last 30 years.

By an excellent combination of suspended cables and longitudinal trusses and ties uniting the floor to the supports of the bridge; by the inclination given in plan to the cables and the addition of exterior guys; lastly, by improvements introduced in the manufacture of and methods of attaching the wire cables, Americans have succeeded in building bridges, more expensive without doubt, but perfectly substantial, and which have solved problems unapproachable by all other systems.

Since 1870, a large suspension bridge was built at Pittsburg, and the construction of the East River Bridge in New York has been continued.

Point Bridge.—The first, constructed in two years (1873-1875) carries a double track and two side-walks. The floor has a width of $10^m.37$ between centers of rails. The length of the principal span (center to center of piers) is, as I have already indicated, 244 meters. The height left under the bridge at low water is $24^m.40$. The suspension chains are not formed of wires as in the Roebling bridges. They are composed of bars

$6^m.25$ long side by side, and united at their extremities to the next bars by a pin or hinge of $0^m.15$ diameter.

The method employed to give the bridge great stiffness consists mainly in the fact that the chains are connected with the floor by posts not susceptible of flexure, while each half chain acts as the lower chord of an inclined truss, of which the upper chord descends in a straight line from the summit of the tower to the center of the floor. Posts and diagonal bracing all jointed unite the two chords. The upper chord, formed of channel bars and plates, gives a rectangular section of 0^m 56 width and 0^m 33 height. The distance between the two chords is 6^m 71.

The East River Bridge.—The two piers and the abutments of this bridge were completed in 1873, but the continuation of the work has been subjected to long delays on account of financial difficulties. The cities of New York and Brooklyn have now charge of the enterprize instead of the original company.

The superstructure is in course of erection. A section of one of the principal cables can be seen at the Exposition. It is formed of 6000 cast steel wires galvanized. Its diameter is about 0^m45. Its resistance to rupture is estimated at 10 tons.*

II. FOUNDATIONS.

Foundations by compressed air, invented in France in 1841 by M. Triger and applied ten years later to the reconstruction of the Rochester Bridge, were introduced in America in 1855. They have been notably applied there, on a scale unknown in Europe, to the two great bridges at St. Louis and New York.

Among other differences one remarks,

First. The immense area of the caissons (reaching nearly 16 arcs in the New York Bridge).

Secondly. The substitution of massive timber work for iron in the caisson of this same New York bridge, where the thickness of the platform reached 7 meters.

Third. The employment of a machine called a sand pump in the St. Louis bridge.

Fourth. The great working depth ($33^m.70$) under water which was reached in this same bridge.

Fifth. Lastly, and above all, the location of the air chambers at the bottoms of the wells.

Attacked on a grand scale from the end of 1869, the foundations of one of the piers of the St. Louis bridge were far advanced when I visited there in September, 1870. I was very much struck with the air locks which, instead of being placed above the level of the water and removed whenever it became necessary to lengthen the wells of access to the working chamber, were established as a fixture in the chamber itself. The descent was thus made in a central well in the ordinary atmosphere by means of a large cage 3 meters in diameter. In the well *a circular staircase was built*, which later gave place to an elevator.

The descent was thus made to a point 6 feet below the ceiling of the caisson, then by a door on a level with the air chambers, which was two meters in diameter. When the equilibrium of pressure was once established the outside door opened itself, and nothing more was to be done but to jump to the earth from a height of about $0^m.80$. In air thus strongly compressed it became at least necessary to reduce the time of labor to less than an hour. What an advantage not to be obliged to subtract the time necessary to ascend and descend a height equivalent to 10 stories of a Parisian house. What relief to the laborers, generally overcome with fatigue and reeking with prespiration at the end of their task. What convenience for the transmission of orders, for the introduction of tools, in fact for all kinds of communications. Besides the space necessary for the compressed air was much reduced, and it was no longer necessary to construct the portion of the wall situated above the chamber of heavy plate iron.

It was sufficient that it should be protected from any water which might filter through the masonry by an external skin of iron, or much more economically (as was done by Mr. Eads in 1870) by an interior lining of pine wood staves. The publicity given in 1873 to the description of the St. Louis bridge had the effect to bring forward the fact, that in the construction of the Cologne bridge over the Rhine an air chamber had been placed

* *Note by Translator.* Perhaps 10,000 tons is meant.

permanently in the working chamber as at St. Louis. This idea was elaborated jointly in 1869 by M. Masson, builder, and M. Sadi Carnot, *ingénieur des ponts et chaussees.*

It was carried out in the Spring of 1870 under an effective head of water of about 8 meters. This work is represented to-day with the rest in the French section of the Exposition, in the pavilion of the Minister of Public Works.

The reputation of M. Masson will lose nothing by the simultaneous execution of the Cologne and St. Louis bridges. While M. Sadi Carnot should be doubly happy at the importance which the invention to which he contributed has taken in America; an invention carried out perhaps at the right moment, by an intelligent breeze from the banks of the Rhone to those of the Mississippi.

CONCLUSION.

Of the facts before mentioned, the following concluding summary may be made:

Americans have proved that it is possible to construct readily bridges of 100 meters and more of span. In countries where the need is not felt in a similar degree, one should at least conclude from these examples that it is best to look twice before constructing bridges which may impede navigation and prove obstacles to the flow of freshets.

As regards river bridges we can (at least in the present state of the question) retain riveted lattice bridges and leave to Americans those pin bridges of large lattice which they have, nevertheless, made so useful. But all nations can learn from America useful lessons on the art of compressed air foundations.

In that country where the construction of railways is nearly completed, government and public opinion return, after 40 years of interval, to navigable routes. They are occupied to-day not only in creating great lines where they did not exist before, but also with improving and completing existing lines. When traffic is abundant a good navigable line will transport freight much cheaper than a good railway. They are the natural counterweights to the omnipotence of railways and the most efficacious regulator of bulky and heavy freights. To that end, the Federal Government is studying with much care improvements relating to the system of interior navigation. It is itself executing works whose expense can be estimated with exactness and whose results are certain. But it leaves to local interest and private industry the responsibility of adventures similar to those which are being accomplished at the mouth of the Mississippi.

While the government abstains here, Mr. Eads and his associates are going ahead and each is in his place. We can but approve the reserve of the government, but how can we help admiring Mr. Eads, who is wrestling with the bar of the Mississippi, foot by foot, and giving blows which recall those of the gallant warriors of Charlemagne. Let us hope for him, that the extremities of his jetties will neither be carried off by tempests nor engulfed by moving sand, that his mattresses will be imbedded in mud before being devoured by the *teredo navalis.* Lastly, that the bar—temporaily put to flight, will not reappear before the time appointed by its undaunted enemy. Whatever the definite result of this gigantic hydraulic river experiment may be, it has a right, it seems to me, to the sympathy and attention of the engineers assembled at the Exposition, who have a common interest in that progress of which their art will always be capable.

THE following gentlemen have been appointed as Commissioners for the purpose of inquiring and reporting whether, with respect to the influence of fluctuations of atmospheric pressure upon the issue of fire-damp from coal, to the adoption and efficient application of trustworthy indicators of the presence of fire-damp, and generally to systematic observation of the air in mines, to improved methods of ventilation and illumination, to the employment of explosive agents in the getting of minerals, and to other particulars relating to mines and mining operations, the resources of science furnish any practicable expedients that are not now in use and are calculated to prevent the occurrence of accidents or limit their disastrous consequences:—Mr. Warington W. Smyth, F.R.S., Sir George Elliot, M.P., Mr. F. A. Abel, C.B., Mr. Thomas Burt, M. P., Mr. Robert Bellamy Clifton, F. R. S., Professor Tyndall. F. R. S., Mr. Lindsay Wood, and Mr. William Thomas Lewis.

TESTING IRON BY ELECTRO-MAGNETISM.

By A. HERRING, Mining Engineer.

Written for Van Nostrand's Magazine.

[The theory of this process which is believed to be new is briefly this: When a bar of iron or steel is passed through a helix which is traversed by a current of electricity, the bar becomes a magnet of greater or less intensity depending upon the current, the coil of wire, and the mass of metal. If the metal be homogeneous the polarity of the bar or plate may be inferred from known laws of electro-magnetism. If, however, there be a want of continuity of fibre at some point, new poles are established there which a properly-mounted needle will readily detect. But such a condition of the metal constitutes a *flaw;* hence the usefulness of the method.

The details of the testing process and of the apparatus employed are described by the inventor as follows.—Ed.]

Any inequality in density, the slightest crack, flaw, or any molecular change tending to disturb the homogeneity of the iron, is a false magnetic pole the intensity of which is in direct proportion to the extent of the defect. By my process, I locate and measure the intensity of the false poles of any piece of iron, and estimate the extent of the imperfec tions causing such false poles.

A magnetometer sufficiently sensitive for this purpose is a nicely balanced dipping needle. These needles I have made of different forms and sizes to suit them to the work for which they are intended. In testing iron, it is not necessary that it should be magnetized to saturation, but the inducing current should be constant and of uniform intensity. To magnetize the iron to be tested, a coil or helix is used constructed of insulated copper wire, of such size and length as will magnetize it with the least amount of battery power, and the helix of such dimensions as to admit of the iron being passed through it.

To test steel wire 0.20 of an inch in diameter the helix should be composed of about six hundred feet of No. 20 copper wire, insulated and wound in superposed layers upon a spool of rubber or other non-conductor, which should be about two and a half inches in length, with an aperture, one-fourth of an inch in diameter, through it. The helix should be made stationary upon a frame or table. A wooden block about 3 inches in width, with a hole one-fourth of an inch in diameter through it, is then placed in front of the helix in such a position as to allow the wire to pass through both helix and block, and upon this block the dipping needle is to rest, about one-eighth of an inch above the aperture through which the wire is to be passed. The needle to be used is one inch in length, with its axis in the center, and to the negative pole is attached an aluminum pointer, to move over the arc of a circle 4 inches in diameter—the arc divided into degrees—the needle so mounted that when the positive pole is down the pointer shall be at zero on the arc, which is numbered right and left from zero to 90 degrees. This, with one cell of a Watson's battery, and two steel test wires, which I will describe below, completes the apparatus necessary for testing the wire. These test wires should be of the best homogenous steel, about two feet long, of the same size as the wire to be tested, the breaking strain and density of each, known from actual test. No. 1 test wire should be thoroughly annealed, and No. 2 evenly tempered to that degree of hardness at which it will bear the greatest strain.

With a coil of wire to be tested wound on a reel or drum in such a position as to be passed through the helix and block and wound on a second drum provided for the purpose, the battery is charged and carefully tested by means of a galvanometer, and its strength noted, we are ready to begin the test. The battery is connected with the helix so as to bring the positive pole of the helix next to the block on which the dipping needle (with its pointer and arc) is placed. No. 1 test wire is then inserted in the helix and block and drawn through until its center is within the helix. The magnetometer or dipping needle is then placed on the

block immediately over the wire, the block is then moved to and fro along the wire until the pointer rests at zero on the arc, the block and magnetometer are then so fastened that the distance between them and the helix shall remain the same throughout the operation of testing. Test wire No. 1 is now removed and No. 2 put into the helix and block. This will cause the positive pole of the needle to rise in the direction of the helix. When it has ceased to vibrate note the position of the pointer on the arc. This indicates the maximum strength and density, and zero the minimum of the wire. Remove the test wire, and pass the end of the wire to be tested through the helix and block, fasten it to the reel or drum on which it is to be wound, and while an assistant winds it from one drum to the other, note the changes of the pointer, marking the defective places—the defect being in proportion to the variation of the pointer between the maximum and minimum as found by the test wires. Should the pointer pass zero, it indicates a flaw in the wire at the point then passing under the needle. This action of the needle indicates an actual break in the fibers of the metal and is to be estimated as in proportion to the deflection of the needle. Practice soon teaches the proper estimate to place on such defects. By this process we estimate the comparative magnetic intensity of the wire and the tensile strength of every foot of it, without impairing it in the least.

The testing of boiler plates is done in a similar manner, or upon the same principle. The plate is placed in a flat helix, connected with sufficient battery power to magnetize it, and passed under a number of magnetometers or needles arranged with their axis all in a line at the proper distance in front of the helix. These needles must be all of the same size, shape, and weight, and must be magnetized to the same intensity, and, for convenience in operating, may be mounted on a strip of wood or brass of sufficient length to extend across the plate to be tested. They should be placed about five inches apart, and free to rise and fall in the direction of the helix, and each supplied with a graduated arc by which to note the variations of the needles.

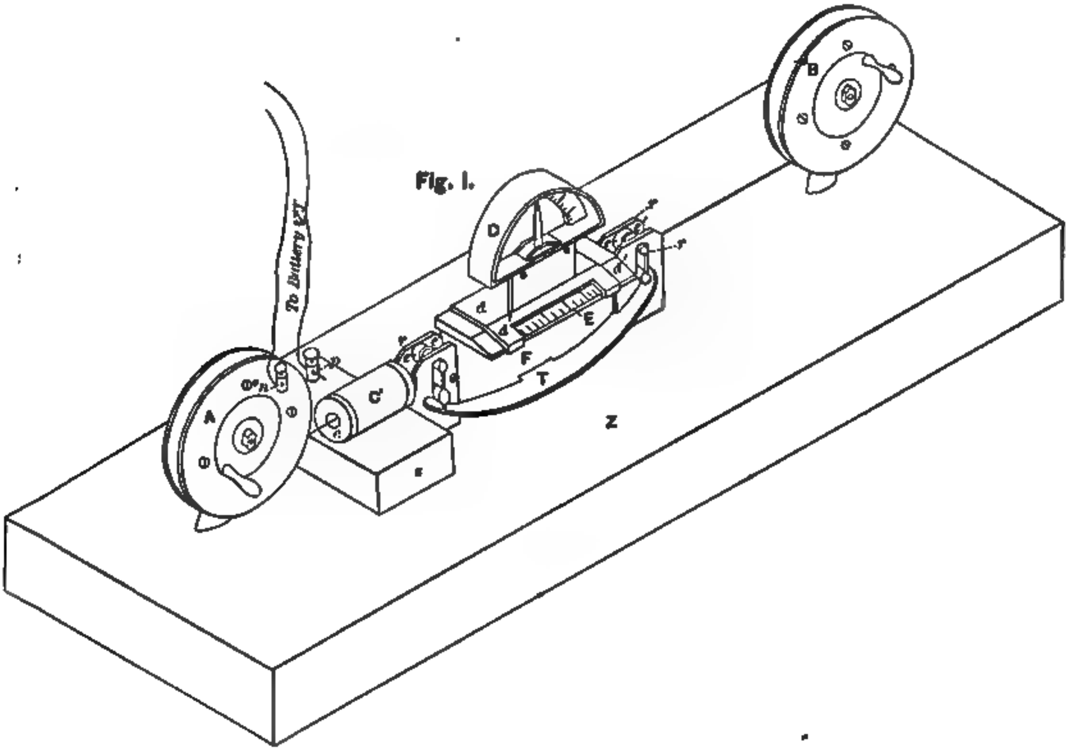
So, in testing iron colums, large shafting, heavy machinery and ordnance, as well as small articles such as pieces of fire-arms, sabres, etc., the apparatus is only modified in shape and intensity to adapt it to the size, weight, and shape of the piece to be tested, embodying the same principles and same laws in all tests. Though the form, size, and magnetic intensity may be changed according to the particular work to be done, we accomplish the same result in all, namely, to point out with certainty the exact point or spot where any flaw or imperfection exists, and also indicate the amount or extent of such defect.

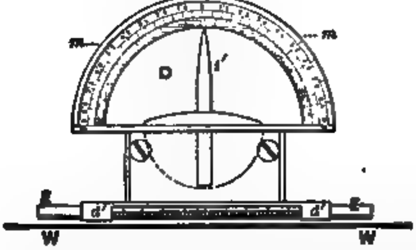
In Figs. 1, 2, and 3 is represented an apparatus adapted for testing the density and tensile strength of steel wire, in which a foundation, as at Z, constitutes a support for the main portion of the apparatus, which is represented in connection with a cell, of what is known as "Watson's battery."

The essential parts of the apparatus consists of drums or reels A and B, a helix, C, a magnetometer, D, mounted and movable upon a scale, E, a frame, F, and a battery.

The helix C rests upon a projecting portion, *z*, of the foundation Z, and is in the form of a spool, the body portion *c* of which is made of hard rubber with an aperture through it from end to end. Upon this spool insulated copper wire *c'* is wound in layers, the ends of which wire are connected to the binding-screws *n* and *p*.

The magnetometer D consists of a magnetized needle or bar of steel, *i*, the negative end of which has been lengthened by the addition of a piece of aluminum, *i'*, in order that the operator during the act of "testing" may read on an enlarged semicircle, *m*, as shown clearly in Fig. 3. The magnetometer D is seated upon a thin metal plate, *d*. This plate has its rear portion or edge bent over and under the rear edge of the scale E, while front portions, as at *d'*, are in like manner fitted to or clasped upon the front edge of said scale, thus retaining the magnetometer in position upon the scale, and also allowing it to be moved longitudinally thereon. A portion of the plate *d* is cut away between the lapped parts *d'*, so that the scale E is exposed to view; and as the magnetometer is moved upon the scale





toward or from the helix the inner edge of the parts *d′* serve to register with the divisions marked off upon the scale, and thus indicate the exact distance from the helix which the magnetometer is to occupy at the commencement of a "test," and which position throughout the test it must maintain. In other words, the magnetometer is so mounted upon the scale E as to allow it to be moved toward or from the helix C, and admit of its being fixed at any desired distance from the helix to suit the different sizes of wire to be tested.

The frame F is constructed at each end with a set of slotted posts, *r r*, and between these posts are two upper rollers, *e e*, having a groove, *e′*, central of their length to receive and guide the

wire *w* while being tested, and also to fellow ungrooved lower rollers, *aa*. The axles of each of these rollers play loosely (up and down) in the vertical slot of the posts *r*. The axles of the lower rollers extend some distance outside of said posts, so as to engage, as indicated in Fig. I, with a spring, T, attached to the frame F, as shown, so that the force exerted upward by the spring will press the wire *w* firmly against the rollers *e e* in their grooves *e'*, thus keeping the wire at a fixed distance from, but near to, the needle *i* while being wound from the reel A to the reel B.

Operation: To practically test steel wire which is from four to five millimeters in thickness, the apparatus should have the following proportions; Drum A should be of such dimensions as to receive the coils as they come from the manufacturers. Drum B should be about two meters in diameter. The helix C should be composed of about two hectometers of No. 14 insulated copper wire, wound in superposed layers upon a spool, the center of which should be made of a non-conducting substance (such as rubber, paper, or hard wood,) with a hole six millimeters in diameter through the length of it. The scale E should be two decimeters in length, and divided into millimeters. The guide-rollers *e e* and *a a* in frame F should be about three decimeters apart. The spring T should be sufficiently stiff to keep the wire *w* firmly to its place in the grooves *e'* of the rollers *e*. Any form of galvanic battery may be used; but the Watson battery, owing to its constancy, seems best adapted to the purpose. Three cells of this battery should be coupled together and connected to the helix C by the binding-screws *p n*. The arc of the magnetometer should be half of a ten-centimeter circle. The arc should be divided, as indicated in Fig. 3, into degrees, or one hundred and eighty equal divisions, with zero at the top, and numbering right and left to ninety. The needle of the magnetometer should be formed of two pieces: first, a steel bar or needle, four centimeters in length, with a pivot in the center; second, an aluminum point, riveted to the negative end of the steel needle, making the length of the needle seven centimeters. In this manner we bring the axis of the needle near the iron to be tested, which insures the greatest deflection from any change in the magnetic intensity of the iron, and gives an extended arc, *m*, on which we are the better able to read the changes of the needle.

To adjust the apparatus for use, the battery C being in operation, a piece of soft-iron wire, perfectly made, five millimeters in thickness and one meter in length, and of known tensile strength, is placed in the helix C and drawn through until the center of the length of the wire rests within the helix. The magnetometer D is then moved along the scale E until the negative end *i'* of the needle points to zero. This will take place when the magnetometer has been moved toward or from the helix, as the case may be, until the positive end of the needle is directly over the point of greatest magnetic intensity developed in the wire. The soft-iron wire is then removed, and the magnetometer remains throughout the test in its position on the scale E which it now occupies.

In place of the soft-iron wire a piece of steel wire of like dimensions and known strength, and perfectly made, is now in like manner placed in the helix. If the steel is of greater tensile strength than the iron, the pointer *i'* of the magnetometer will fall to the right of zero. Take a note of the point on the scale of the arc *m* at which the pointer comes to rest; remove the steel wire; fix the coil of wire to be tested on the reel A; pass one end of the wire through the helix C and between the rollers *a a* and *e e* in the groove *e'* of the rollers *e*, and then attach the end of the wire to the reel B. Now, turn the reel B, and while the wire is wound from the reel A to the reel B note the position of the pointer *i'* upon the scale of the magnetometer, and calculate the strength of the wire from the ratio found between the two test-pieces. For instance, if the breaking-strain of the soft-iron wire which fixed the needle at zero is two thousand pounds, and the steel wire which fixed the needle, say at forty-five degrees, breaks at four thousand eight hundred pounds, the increase in pounds per degree is sixty-two pounds, and the ratio found, which is thirty, will serve to calculate the strength of any portion of the wire being tested, unless the pointer *i'* passes zero to the left,

which indicates an actual flaw in the wire, and the greater the deflection of the pointer to the left of zero the greater the flaw.

In testing large shafting, bars of metal, heavy ordnance, pieces of machinery, and small articles, such as pieces of fire-arms, sabers, bayonets, &c., the apparatus will of course be adapted in intensity to the size, weight, and shape of the piece under examination.

Fig. 4.

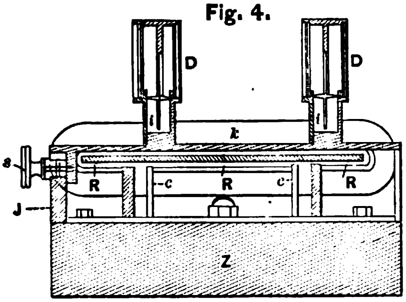


Fig. 4 represents the helix as constructed and arranged for testing plates of iron and steel. By this construction and arrangement we are able to test rapidly and give the density and tensile strength of every square inch of the largest-sized boiler or armor plates. The helix C is flattened or elongated, so as to admit the metal plate R to be passed through it. This helix is composed of superposed coils of insulated copper wire the ends of which connect with binding-screws *n* and *p*. The magnetometers D D are mounted on a movable bar, *k*, that they may be the better moved to and fro in front of the helix, and fixed at any desired distance from it by means of the set-screw *s* to the scale J. In the figure, R indicates a plate of iron or steel position in the act of being tested, the plate resting upon trunnions *c c*, over which it is drawn by a drum.

To test a plate the helix and battery should be suited in size and power to the thickness of the iron or steel to be tested. The movable bar *k* should reach from side to side of the plate, and have fixed to it a number of magnetometers about two decimeters apart. These magnetometers should be of the same construction as that described for testing wire. Connect the poles of the battery with the helix C by means of the binding-screws *n* and *p*, so as to magnetize the plate with the negative pole to the right of the helix; pass the end of the plate through the helix and fasten with clamp, and while an assistant draws it steadily through the helix note the changes of the needle *i* in the several magnetometers.

Having previously ascertained the magnetic intensity of a cubic centimeter of iron and steel of known density and tensile strength, we are enabled to calculate the strength of any part of the plate by a comparison of its magnetic intensity.

Mr. Herring's process is protected by letters patent under date March 11th, 1879.

At a recent meeting of the French Academy of Sciences M. de Lesseps gave an account of the Suez Canal currents furnished by M. Lemasson, the company's engineer, the result of observations taken since 1871 at Port Said, Suez and other stations. Notwithstanding Lake Timsah and the Bitter Lakes, which form two great regulators, the north and south branches of the canal are not unaffected by each other. From May to October the winds cause a rise of level at Port Said and a fall at Suez. This difference of level, which reaches about 15.5 inches, leads to a current from the Mediterranean to the Red Sea—a current which, though interrupted by the tides, drives a considerable volume of water from north to south. In winter, on the contrary, the high winds give the Red Sea a level higher by about 12 inches than the Mediterranean, causing a current from south to north. About 14,126 million cubic feet or about 400,000,000 tons of water yearly pass from one sea to the other. This, coupled with the tides, tends to neutralize the effects of evaporation from the surface of the lakes and to dissolve the basis of salt in the Bitter Lakes. That basis was 32.8 feet in thickness, but is gradually dissolving, especially in the course of vessels. The velocity of these local currents varies between Port Said and Lake Timsah from 0.5 feet and 1.3 feet per second; while in the broader part, between Suez and the Bitter Lakes, it is from 2 feet to 3.6 feet per second.

THE PRISMOIDAL FORMULA.

By J. WOODBRIDGE DAVIS, C. E.

Written for VAN NOSTRAND'S MAGAZINE.

PREFACE.

It would be difficult to estimate the value of the prismoidal formula, or to enumerate the branches of business and professional work in which it is constantly employed. Its advantage over most other rules for mensuration, lies in the fact that its extreme simplicity is coupled with a wonderful generality, even as an exact rule, while it approximates very closely the contents of a host of shapes. It is, therefore, a rule which can be understood and applied by every intelligent person who is not acquainted with the more general, but less simple, rules of higher mathematics, at the same time that it is a rule applicable to a great many of the most common shapes.

But, apart from its immense importance as a practical rule, the prismoidal formula has always been of great interest to mathematicians, and, it may be said, a mystery, on account of its wide range of application. Perhaps everyone, after studying its derivation from the rectangular prismoid, has been startled when first he saw, or heard it announced, that it applies to the sphere and the ellipsoids. Every author of a treatise on mensuration takes delight in demonstrating its applicability to the three round bodies, and in deriving from it numerous special rules. The limits of its applicability have been gradually widened by successive discoveries—often exciting great surprise—of its applicability to shapes before mensurated by more difficult rules, or merely approximated for want of convenient formulæ. There has also been much speculation as to the number and variety of prismoidal shapes, and as to the nature of their connection.

The object of this article is to establish a single formula that shall indicate exactly the limits of the field of application of the prismoidal rule, and shall express the relation which exists between all prismoidal spaces; also, to show the value of this formula as a criterion in determining whether or not the rule apply to given spaces; and, finally, to convey an idea of the vast extent governed by this rule, and to describe, in illustration thereof, several extraordinary shapes more curious than available.

It must be confessed that the writer was almost discouraged from publishing this matter by reading, after his own work had been accomplished, two excellent articles on the same subject, of which mention is made in proper place. There are, however, so many points of distinction as to have induced the writer to persevere. Moreover, the articles referred to were published in mathematical journals, one extinct, and the other, presumably, not widely circulating among engineers.

This paper is condensed from a thesis, bearing name of *Prismoidal Formula*, and deposited in the Engineering Department of School of Mines, Columbia College. In accordance with present design, several of the conclusions, which are merely interesting, perhaps astonishing, are simply stated. For demonstration, if any require it, the reader is referred to that work. Also, three rather extensive notes have been entirely omitted. These are: *Broken Lines represented by Equations of Infinite Degree; Indeterminate Coefficients*, in which the ordinary proof contained in the algebras is criticised; and *Limits to the Number of Roots of Transcendental Equations*. Two chapters, containing discussions on *A New Center of Gravity Formula* and *A New Moment of Inertia Formula*, have likewise been excluded; but, as these latter are the most practical portions of that work, the writer hopes to be able soon to present them in acceptable shape.

PRISMOIDAL FORMULA.

Space, whether illimitable or contained by a certain boundary, may be generated by a right plane, always remaining parallel to two non-parallel straight lines and moving in a direction perpendicular to them, the motion being constantly progressive, and the boundary of the plane at every position being its intersection at that position with the boundary of the space. Thus, the generatrix

passes once through every point of the space, and is, accordingly, said to describe it.

The boundary of the space may be however intricate, and possess any number of convolutions or entirely separate parts.

Letting y represent the area of the generatrix at any position, [when the space is of two dimensions, y is the length of generating line], and letting x denote the distance of generatrix from first limit, we have, when y everywhere between those limits varies according to the same function of x,

$$y=f(x), \tag{1}$$

and for the amount of space, or volume,

$$v=\int_0^l f(x)dx. \tag{2}$$

(1) may be considered the equation of the space, since it indicates the magnitude of the generatrix at every position. It is independent of the shape of boundary. Therefore, the space, whose equation is (1), may be contained by any of an infinite variety of boundaries; and its volume for each is the same.

[In the single case where the space is a plane, and one longitudinal boundary a straight line, (1) is the equation of the other longitudinal boundary as well as of the space].

If, however, between the limits y vary as $f_1(x)$ to a distance m_1, then, abruptly, begin to vary as $f_2(x)$ to the distance m_2 from origin, and so on change the mode of its variation till the second limit be reached, then the equation of the space becomes

$$y=f_1(x)_0^{m_1}, f_2(x)_{m_1}^{m_2}, \ldots f_n(x)_{m_{n-1}}^{l}; \tag{3}$$

and

$$v=\int_0^{m_1} f_1(x)dx+\int_{m_1}^{m_2} f_2(x)dx\ldots\ldots + \int_{m_{n-1}}^{l} f_n(x)dx. \tag{4}$$

The manner of variation of y may be such that it can be represented between given limits by a succession of known functions, only when their number is infinite.

But, however a generatrix may vary in a given space between limits, it must, at a distance x_1, have some magnitude, which let y_1 express.

At other distances, x_2, x_3, ...x_{n+1}, let the magnitudes of y be represented by y_2, y_3, ...y_{n+1}. In the equation of the nth degree,

$$y=A+Bx+Cx^2\ldots\ldots+Kx^n, \tag{5}$$

the $n+1$ coefficients may have such values as to satisfy $n+1$ conditions concerning x and y. Let these be the conditions that when $x=x_1$ y shall be y_1 etc. Then (5) is the equation of a space, which coincides in magnitude with the given space at the distances x_1, x_2, etc. from the origin. If every position of the generatrix be considered, (5) is an equation of infinite degree; and it is the equation of space which coincides in magnitude with the given space at every distance from the origin. Therefore, the generatrix of every space, between any limits, varies according to a single function of its path; and that function is represented by an algebraic expression of infinite degree, in terms of the path, with integral, positive exponents and constant coefficients. If we denote this function by F (x), the equation of every space is

$$y=F(x).^* \tag{6}$$

The volume of a space, to which the prismoidal formula applies, is

$$v=\frac{l}{6}\left\{A+4M+B\right\}, \tag{7}$$

where l is the distance between limits,

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* In note 1 of the thesis mentioned in preface are given examples of broken lines represented by infinite equations. Among them are the perimeter of square, which, as a continuous line, referred to axes parallel to its sides and intersecting at its center, is represented by

$$x^{\infty}+y^{\infty}=r^{\infty},$$

when ∞ is even; and the axes themselves, within square, as one continuous line, represented by the equation

$$x^{-\infty}+y^{-\infty}=r^{-\infty},$$

when the degree is even.

$$x^{\infty}+y^{\infty}+z^{\infty}=r^{\infty}$$

is the equation of the surface of a cube, and

$$x^{-\infty}+y^{-\infty}+z^{-\infty}=r^{-\infty}$$

is the equation of co-ordinate planes within cube, when ∞ is even.

The following also is quoted from this note.

The fact that any irregular shape can be represented by an equation, allows us to use, in treatises on mechanics, the expression dv to denote the content of an infinitesimal section of that shape; also, to use the other differential and integral symbols pertaining to discussions of center of gravity, moment of inertia, etc. Demonstrations, made with aid of these symbols, and the processes they represent, are generally more concise than those wherein the expressions of mere summation of independent particles are employed. When the general principles are applied to known forms, the symbols of the calculus must be used, because they indicate how the summation can be effected. Hence, for uniformity also, it is advantageous to use latter symbols even in general cases.

and A, B, M, are the magnitudes of the generatrix at first and last limits and at the position midway between.

The extent of application of the prismoidal formula may be determined by considering it in connection with equation (6), since this represents all spaces. The F (x) is the sum of any number of algebraic expressions, each of the form Kx^n, n being positive and integral. The prismoidal formula applies to the space represented by one of these, when

$$\frac{l}{6}[K(0)^n+4K(\tfrac{1}{2}l)^n+Kl^n]-\int_o^l Kx^n dx=o$$

$$\text{or,}\quad \frac{Kl}{6}\left\{o^n+\frac{4l^n}{2^n}+l^n\right\}-\frac{Kl^{n+1}}{n+1}=0. \quad (8)$$

This equation is true when the coefficient K is zero, or when l is zero, in either case when the volume is zero. The former is the case of a line or number of lines, of any complexity between any limits. The latter is the case of a generatrix of any magnitude maintaining a single position. The equation is also true when n has any of the values 0, 1, 2, 3.

For all values of n, except zero, the first term in the brackets is zero, and the equation may have the more convenient form

$$Kl^{n+1}\left\{\frac{2^{1-n}}{3}+\tfrac{1}{6}-\frac{1}{n+1}\right\}=0. \quad (9)$$

When n exceeds 5, the first member of this equation is always numerically greater than zero, and, consequently, the equation is false. Also, when n is either 4 or 5, the equation is false. Hence, 0, 1, 2, 3, are the only positive integral values of n, such that the prismoidal formula applies exactly to the space represented by the equation $y=Kx^n$.

Applying now the prismoidal formula to the space $\quad y=F(x),$

and subtracting from the volume thus found, the true volume found by integrating each term, the difference, equated with zero, we find to be

$$\left.\begin{array}{l} K_1 l^{m+1}\left\{\dfrac{2^{1-m}}{3}+\tfrac{1}{6}-\dfrac{1}{m+1}\right\} \\ K_2 l^{n+1}\left\{\dfrac{2^{1-n}}{3}+\tfrac{1}{6}-\dfrac{1}{n+1}\right\} \\ K_3 l^{p+1}\left\{\dfrac{2^{1-p}}{3}+\tfrac{1}{6}-\dfrac{1}{p+1}\right\} \\ \qquad +\text{etc.} \end{array}\right\}=0. \quad (10)$$

If the F (x) consist of terms affected by no other exponents than 0, 1, 2, 3, [when an exponent is zero the wanting term in enclosed factor must be supplied] all the enclosed factors are zero. Hence, the prismoidal formula applies between any limits to the space whose equation is

$$y=a+bx+cx^2+dx^3. \quad (11)$$

If the F (x) include terms having greater exponents than three, then, in eq. (10), the terms corresponding to the exponents 0, 1, 2, 3, are zero; and, if q denote the number of terms remaining, the truth of equation (10) depends upon the values of $2q+1$ independent quantities, viz: the coefficients and exponents of the terms and the distance between limits. Each, consequently, must be a certain function of all the rest, or one of a definite number of particular functions, in order that this equation may be true.

Thus, from equation (10) we see that for given values of the coefficients and exponents, that is, for a given space, there are as many values of l such that the equation shall be true, as the number indicating the degree of F (x) and one more. As many of these are zero as the number indicating the degree of the lowest term above the fourth term in the F(x), and one more. The other values may be positive, negative and imaginary. For instance, the prismoidal formula applies to the space

$$y=a+bx+cx^2+dx^3+ex^4+fx^5+gx^6 \quad (12)$$

$$\text{when}\quad \tfrac{1}{120}el^5+\tfrac{1}{48}fl^6+\tfrac{23}{672}gl^7=0. \quad (13)$$

Solving this equation in favor of l, we find that besides five zero values

$$l=-\frac{7f}{23g}\pm\frac{21}{46g}\sqrt{\frac{35f^2-92eg}{315}}.$$

If the equation of the space be of infinite degree, then eq. (10) is infinite, and there are an infinite number of lengths of the space subject to the prismoidal formula. If these lengths be none of them negative or imaginary, and if they all be consecutive values of l, then this equation of infinite degree represents a space subject to the p. f. [prismoidal formula] between any limits. This is the only possible case of a prismoidal space of higher degree than the third. But, since for every value of l the value of the functions of l expressed in first member of eq. (10), is zero, this is

the equation of a line coincident with the axis of X. Whatever be the coefficients of this equation, they are equivalent to those of the following equation:

$$y=(y)+\left(\frac{dy}{dx}\right)x+\left(\frac{d^2y}{dx^2}\right)\frac{x^2}{2}$$
$$+\left(\frac{d^3y}{dx^3}\right)\frac{x^3}{6}+\text{etc.,} \quad (14)$$

wherein y represents the first member of eq. (10), and $(y), \left(\frac{dy}{dx}\right)$, etc., represent the values of $y, \frac{dy}{dx}$, etc., when $x=o$. The last member of eq. (14) is identical with the first of eq. (10), the coefficients having merely a different form. Since the line represented by either equation is coincident with the axis of X, it is tangent thereto at the origin. Consequently, as many successive coefficients must be zero as the number of succeeding points at which the line still coincides with the axis of X. Because every succeeding point is common to both lines, every coefficient of eqs. (14) and (10) is zero. Now, the factor in brackets, of every coefficient of eq. (10), corresponding to a term of fourth or higher degree in F (x), is not zero. Therefore the K of this term must be zero. On the contrary, the factor in brackets, of each coefficient of a term in eq. (10), corresponding to a term in F(x) of third or less degree, is zero. Therefore, the K of any of these terms may have any value. This reduces the equation of every space, $y=\text{F}(x)$, to the same case expressed by eq. (11).*

Between given limits there may be an infinite number of spaces, whose generatrices vary according to algebraic functions of any degree except the fourth, yet such that the p. f. shall apply to each space between those limits. For a given length, l, measured from origin, the p. f. applies to a space, whose generatrix varies as an algebraic function of sixth degree, eq. (12), when, as derived from eq. (13),

$$e=-120\left(\frac{fl}{48}+\frac{23gl^2}{672}\right)=-\tfrac{5}{2}fl-\frac{115}{28}gl^2.$$

* It will be noticed that the result arrived at in this paragraph might have been directly quoted from the chapters on *Indeterminate Coefficients* in treatises on algebra. But, as the demonstrations in these seemed to the writer imperfect, (for reasons discussed in note 2 of thesis mentioned in preface) he has preferred to use the proof shown in text.

The conditions, $l=o$, $k=o$, since they are practically useless, being now neglected, and, also, all those spaces to which the p. f. does not apply between every pair of limits, there remain, as spaces to which the p. f. universally applies, of all spaces, those only which are represented by the equation

$$y=a+bx+cx^2+dx^3. \quad (11)$$

Eq. (11) is independent of shape of boundary. It may be used, as a criterion, to ascertain whether the p. f. apply or not to spaces of given boundary. It only needs that the general expression for magnitude of generatrix in terms of path be determined. If this be an algebraic expression of degree not higher than third, the formula applies. Otherwise, the formula does not apply. This may also be used as a method of demonstrating that the p. f. does apply. Its application to either case saves us the labor of actually applying the p. f., and, then, some other rule known to be correct, and, finally, comparing results. Thus, eq. (11) serves to generalize and abbreviate greatly the demonstrations of rules for mensurating common solids. To the practical calculator it is both a guide and a guard.

Another use, perhaps not very practical, that can be made of eq. (11), is described as follows. By means of eq. (11) may be obtained expressions for boundaries, which shall satisfy any number of given, non-conflicting conditions, yet shall contain spaces, all subject to the prismoidal rule. In this manner, by imposing unusual conditions, prismoidal spaces of most extraordinary shapes, may be discovered.

Portions of prismoidal space may be negative. Such portions are generated by negative generatrices. For instance, the quadrilateral, represented in Fig. 1,

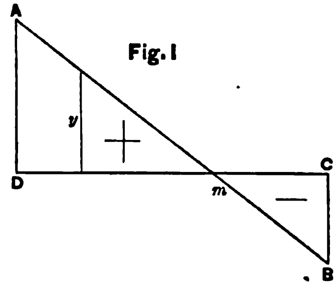


Fig. 1

is generated by the moving line, y. This becomes smaller, and, consequently, the infinitesimal differentials of the area become smaller, as it moves to the right. At the point m it is zero; and here its motion adds nothing to the space. Proceeding, it diminishes at the same rate, becoming, therefore, negative, and producing negative space. The p. f. applies between any limits, only when the space B*C*m is negative. When this is positive, the shape is, in fact, not a quadrilateral, but a hexagon, whose sides are Am, mC, CB, Bm, mD, DA.

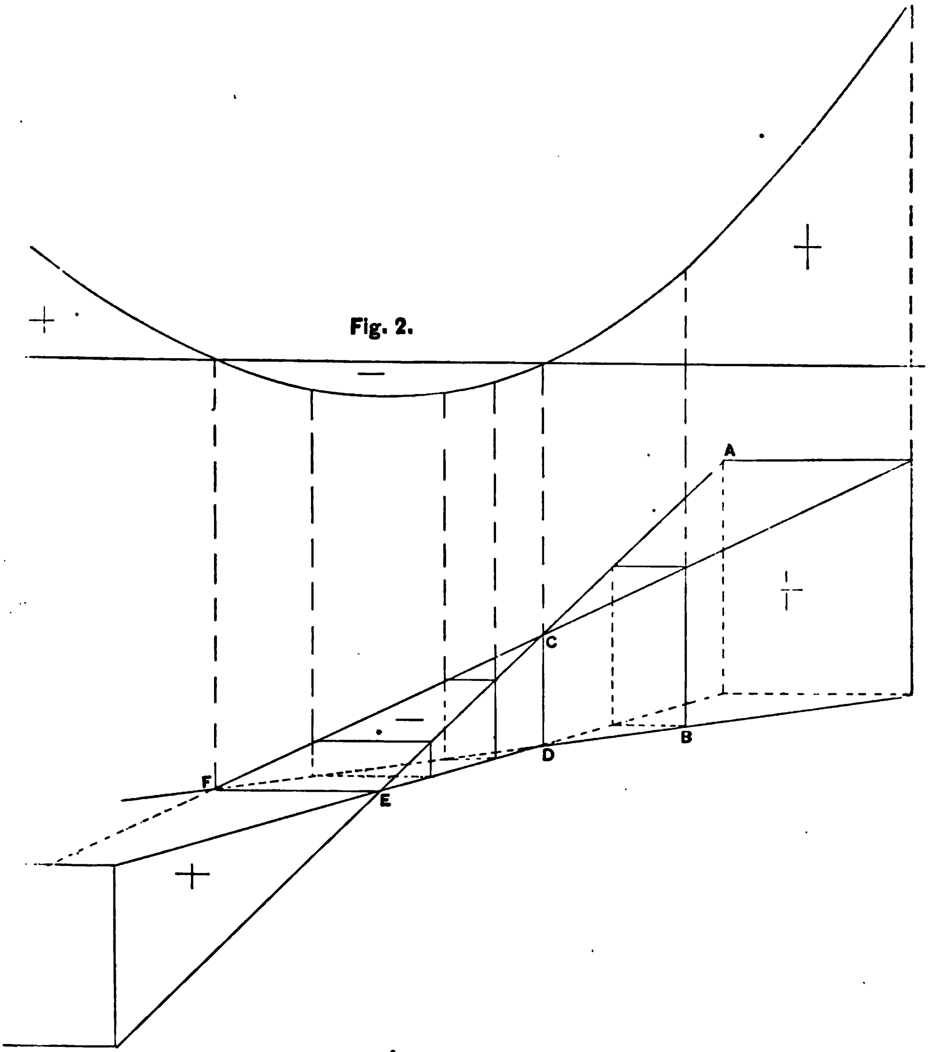


Fig. 2 represents the complete prismoid. This may be defined as *The space generated, between any limits, by the motion of a varying rectangle, whose sides maintain a constant direction, while one vertex proceeds along a straight line.*

Both the product, and the quotient resulting from the division of one by the other, of the rectangle's two dimensions, must be functions of the path, and, therefore, variable.

The rectangular pyramid can be defined in same words, except that *the product must be variable and the quotient constant.*

In the prism *both are constant.*

To complete the four cases, we must consider the space generated by a rec-

tangle whose dimensions vary as $\frac{a}{x^n+b}$, x^n+b. In this shape *the product is constant and the quotient variable.*

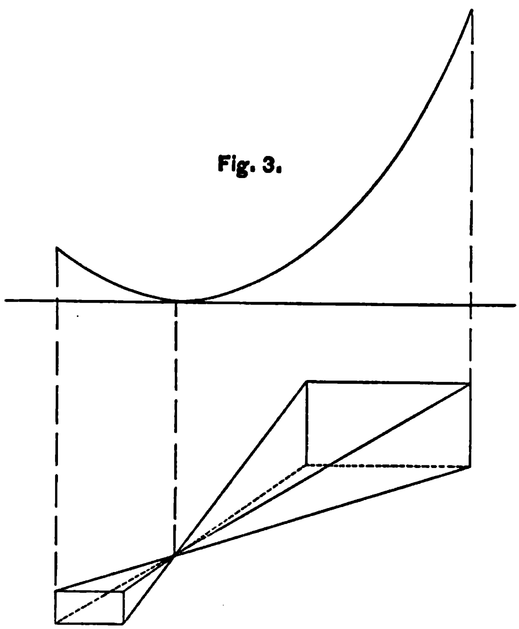
It follows, from this definition of the prismoid, that one pair of faces must intersect in advance of the remaining pair. At this position the generatrix is zero. The immediate space following is negative, as the product of a positive by a negative dimension. The generatrix becomes zero again when the other faces cross, and, proceeding, becomes permanently positive, as the product of two negative dimensions. The generatrix passes through the same series of changes through which the ordinate of the parabola sketched above passes. The generatrix of the lateral surface passes through the same changes as does the ordinate of Fig. 1.

AB is the conventional prismoid. Both varieties of wedges are shown, as also the pyramid CDEF, which, as a prismoid with no end areas, has excited much attention.

The peculiarity of the middle branch of the prismoid is, that the amount of space between any two of its cross sections, as calculated by the trapezoidal rule, is less than the amount by the prismoidal rule, if the space be considered positive. The important fact that there are such sections of a prismoid seems to be unknown to some writers on the applications of the p. f., while others appear to regard it as a singular exception. A conjugate peculiarity of the middle branch is that between any two cross sections thereof, one dimension increases while the other decreases. Considered analytically, *not practically*, it is an universal rule that *between any limits, the amount of space in a prismoid, as calculated by the trapezoidal rule is excessive.*

Fig. 3 is a pyramid. Both pairs of

Fig. 3.



faces cross at the same section. Here the area of generatrix is zero; but immediately on each side it is positive, as the product of two positive or of two negative dimensions. The generatrix of lateral surface passes through same changes as does the ordinate of Fig. 1.

Consider now the triangular warped-faced shape, Fig. 4. It is generated by the motion of a varying triangle, which remains parallel to one plane, while its vertices are constrained to follow any three straight lines. The three faces are hyperbolic-paraboloids.

Referred to the plane through one directrix, as CC′, and the plane of generatrix at any position, as III, the area of generatrix, at a distance x from III, as at

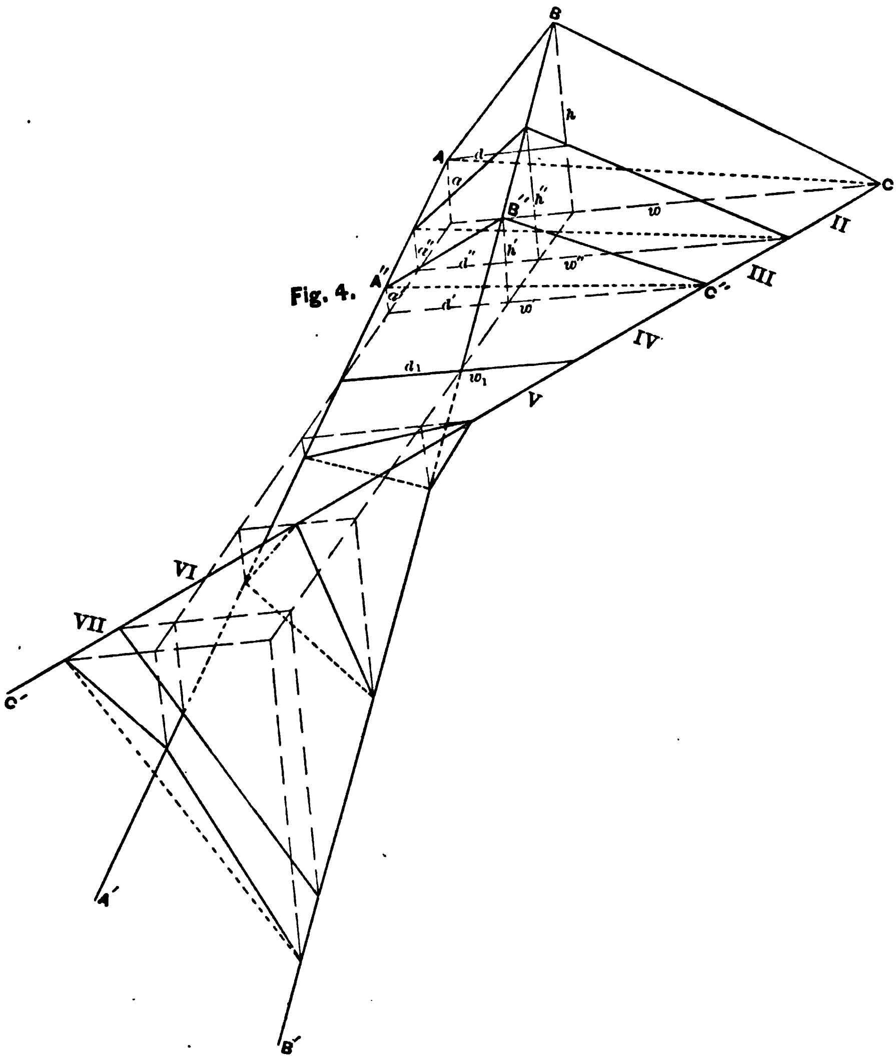


Fig. 4.

II, the measurements of diagram used, is, according to rule B, page 293, of April No. of this Magazine,

$$\tfrac{1}{2}[h''w''+a''(d''-w'')],$$

where, in terms of measurements of two given positions of generatrix,

$$h''=h'+(h-h')\frac{x}{l},\ a''=a'+(a-a')\frac{x}{l},$$

$$w''=w'+(w-w')\frac{x}{l},\ d''=d'+(d-d')\ \frac{x}{l}.$$

Without actually making the substitutions, it is seen that, were they made, the general expression for area of generatrix would appear as a function of x to the zero, first and second powers only. Hence, the p. f. applies between any limits.

The form of the equation of this space,

$$y=a+bx+cx^2,$$

shows that it may contain a negative branch; but, if so, that two consecutive infinite branches are positive. When the generatrix is about to change its sign, it must become zero, or must become a line. [If it become a point, the figure is

a pyramid.] Hence, all the directrices intersect their opposite surfaces, respect ively, in one element common to the three surfaces. The latter immediately enclose negative space. [*i.e.*, space on the opposite side of surface. See Fig. 4, page 293, April No. of this Magazine.] But the directrices must again cross their opposite surfaces in one element, which divides the negative from second positive branch.

This leads to the following theorem, which it would perhaps be difficult to prove by other methods.

If three hyperbolic-paraboloids have a common plane director, and a common element at each of three points of which two of the surfaces are tangent to each other; then it is certain that the surfaces have one other element in common, at which they are in like manner tangent to each other; and it is also certain that no two of them can have a third common element, nor any other point of tangency.

If one directrix be tangent to opposite surface, the surfaces are tangent to each other throughout the element, and, consequently, each other directrix is tangent to its opposite face. Hence, the surfaces do not intersect, there is no negative space, and the two elements mentioned in the theorem are consecutive.

Similar theorems, when the directrices are more than three in number, and when they are curves, can be constructed.

This shape is useful as well as interesting. If CC′ be the center stakes of consecutive cross sections of a cutting, AA′ the slope stakes on one side, then AC, A′C′, represent top lines of regular cross sections, and B,B′ are the "breaks," which render the solid irregular. Every irregular solid is compounded of a regular solid and a number of these triangular shapes, positive and negative, which form the ridges and hollows, as seen along BB′. A study of this shape reveals the fact that the error of applying the trapezoidal rule to each of these warped-faced portions may be excessive when compared with that of analogous plane figures; and that it is equal in importance to that produced on the whole regular solid by use of the same rule. Some writers have thoughtlessly neglected these errors, concluding without investigation that they must be trifling, and that the only error worth correcting is that of the regular part of solid. Others have made the errors of irregular part arbitrary functions of the error of regular part, instead of independent functions of their own measurements.

Another peculiarity of this peculiar shape, though important, is not well known. The portions of Fig. 4, limited by the planes I, IV, and IV, V represented the case of a ridge or hollow which fades out at one section. The volume between IV and I is

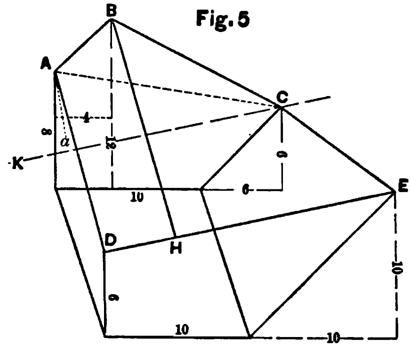
$$[a(2d+d_1)+(h-a)(2w+w_1)]\frac{D}{12}.$$

Here it is seen that to give d_1 any increment, m, increases the volume by $\frac{1}{12}amD$; while to give d the same increment, increases the volume by $\frac{2}{12}amD$. Therefore *the effect of shifting the fading end is half as great as the effect of shifting the other end*, B, *as far in same direction.* Earthwork measurers are very careful to fix the extremity B, but they seldom, if ever, notice the other end. The effect of d_1 on the volume is greater as the opposite face ACC′A′, is more warped. If this be plane, a is zero, and d_1 has no effect.

To illustrate the serious effect the neglect of d_1 has in practice, consider this example of the right side of a railroad solid, which was given by Prof. Gillespie to illustrate his plan of fixing the fading ends. He proposed to establish each of them at a point whose distance from center line should bear the same ratio to the measured distance d_1 of the other end, as the side width w_1 bears to w. The measurements are shown in diagram, Fig. 5. The value of a in this example is found to be 5.1 feet. Substituting this in the formula $\frac{1}{12}amD$, and making m 1 foot, D being 100 feet, we find, for the increment to the volume, 42.5 cubic feet, corresponding to an increment of 1 foot to d_1, or 43.4 cubic feet corresponding to 1 foot measured horizontally. The total effect occasioned by moving the point H from the end D of the line DE to the other end E, is, in cubic yards, $43.4\times20\div27=32.1$. This is equal to half the content of the whole solid ABCDEH, when at its least value.

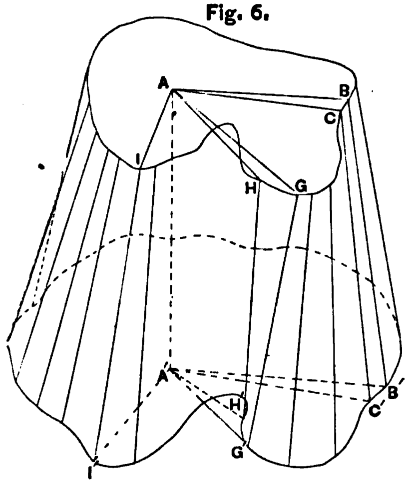
Another use that can be made of this shape [Fig. 4] is to aid in the following demonstration. Consider the space

Fig. 5



shown in Fig. 6, included, terminally, by two parallel planes, and, laterally, by a surface generated by the motion of a right line, always intersecting the planes, and returning finally to its initial position. Assume an axis anywhere, within or without the space, and connect its extremities A,A′, the points of its intersections with the planes, by straight lines with the extremities B,B′,C,C′, of two consecutive elements of lateral surface, as shown in diagram. Denote the area of upper base by E, of lower base by E′, of mid-section by M, and by V the

Fig. 6.



volume of the entire space. Designate the corresponding areas and volume of the space ABCA′B′C′ by dE, dE', dM, dV, and by h, the distance between the parallel planes. Then, since, according to previous demonstration, the p. f. applies to ABCA′B′C′,

$$dV=\frac{h}{6}[dE+4dM+dE']. \qquad (15)$$

The revolution of the warped surface, ABB′A′, generates E,M,E′,V, simultaneously, and completes the generation of all in the same time. Therefore, if an independent line be considered to be generated by the uniform motion of a point in the same time, and x be employed to denote its length at any time, the magnitude of upper base at such time, however irregular the generation be, may be expressed as $F(x)$, according to the demonstration of the introductory part of this paper. Similarly, M,E′,V, at any state are $F'(x), F''(x), F'''(x)$. Now, since the variables of eq. (15) are all functions of a single variable, x, if the integration of all the terms be performed between the limits $x=o$, $x=l$, or, when the independent line is the circumference of a circle, whose radius is unity, between the limits $x=o$, $x=2\pi$, there is obtained

$$V=\frac{h}{6}[E+4M+E']; \qquad (16)$$

that is, the p. f. applies to the space.

If the integrals be confined by closer limits, it results that the p. f. applies to such a space as ABB′I′IAA′.

When the generating warped surface reaches the position AGG′A′, the next dE' is negative, while the corresponding dE is positive, because HH′ crosses the opposite face of the shape AGHH′G′A′A. Further on, both become negative, and remain so until one edge of differential solid again crosses its opposite face.

The limits of the integrals may be extended, such corresponding to the case of the revolution of AB through more than 360°. The unreal shape thus produced, may have parts of double, triple or greater intensities; also parts may be negative or null. The p. f. still applies.

The class of shapes which are bounded laterally by ruled surfaces, are very important for two reasons: *First*, nearly all ordinary prismoidal shapes are included. *Second*, when it is necessary to determine the volume of a space covered by a given unknown surface, it is possible to approximate it, readily and as nearly as is desirable, by means of ruled surfaces; whereas it is impossible by aid of the eye to approximate it by means of surfaces all of whose elements are given curves, because the eye cannot recognize the nature of a flat curve, but can always recognize a straight line. The plane, a

surface composed in every direction of straight lines, is not pliable enough to enable us to use it with accuracy for any but small extents of surface.

It is evident that this class of shapes may have very grotesque forms. For instance, at one place a logarithmic curve may extend from base to base, at another, the sinusoid, at another, a line represented by an algebraic equation of the hundredth degree, and at still another place may be a broken line; yet the generatrix can be moved so as to pass through every point of all.

The following somewhat similar rules are interesting, though they do not include many practical shapes.

The prismoidal formula applies to every shape bounded, terminally, by two parallel planes, and, laterally, by the surface generated by the motion of the ordinary parabola, returning, finally, to its initial position, but, meanwhile, undergoing any series of changes in amount and direction of curvature, position of principal vertex, and direction of containing plane, the principal axis, however, always remaining parallel to any line in either base.

The prismoidal formula applies to any shape contained by two parallel bases, and a lateral surface generated by the motion of a cubic parabola whose plane is always parallel to a given plane, but whose curvature and point of inflexion may pass through any series of changes in amount, direction and position.

It is often necessary to calculate the contents of ungulæ and their frusta. Sometimes the p. f. is applied to these. Let us examine this case in a general manner. Consider the shape whose surface is formed by revolving the line

$$y = a + bx + cx^2 + dx^3 + \text{etc.}$$

about the axis of X. Consider, also, a secant surface composed of straight line, parallel elements, perpendicular to plane XY. The equation of this surface is

$$y' = a' + b'x + c'x^2 + d'x^3 + \text{etc.}$$

At a distance x from origin, the magnitude of generatrix of either segment of the space, is a semicircle whose radius is y, plus or minus a segment of the same circle, included between diameter and chord distant from center by y'. Its area is, consequently,

$$\tfrac{1}{2}\pi y^2 \pm y'\sqrt{y^2 - y'^2} \pm y^2 \sin^{-1}\frac{y'}{y}.$$

In order that p. f. shall apply, this must be equivalent to an algebraic expression of third or less degree. The transcendental factor of the last term must, therefore, be constant. Hence, the proportion,

$$a' : a :: b' : b :: c' : c :: \text{etc.},$$

and $y' : y :: \text{A} : 1 :: a' : a :: \text{etc.},$

in which A is any constant quantity, must subsist. The second term now becomes $\text{A}y^2\sqrt{1 - \text{A}^2}$. From this it is seen that A must be less than unity, and that all coefficients beyond b, b', must be zero.

The interpretation of this is: The shape must be a cylinder, a cone or a frustum of a cone. The secant surface must be a plane, parallel to elements of cylinder, or passing through vertex of cone, or through vertex of the cone whose frustum is considered. On p. 34 of Trautwine's *Civil Engineer's Pocket-Book*, that author describes, and illustrates by diagram, a segment of a cylinder made by a secant plane not parallel to elements, but which is merely required to pass through both bases; and he denominates it as a space to which the p. f. applies exactly. The foregoing demonstration shows that this is not so. It is well to point this out, because an error becomes very important, when contained in a book otherwise so admirable as to claim the full confidence of engineers.

The magnitude of the generatrix of any space, is, at any position, a function of the corresponding ordinates of all the longitudinal elements of its boundary. Therefore, any number of these elements may be arbitrarily fixed, while enough remain free to have such values accorded their ordinates as shall render the value of the function, that is, the area of generatrix, whatever we please in terms of the path; and, as these remaining elements are infinite in number, there are an infinite number of solutions to this problem. This enables us to impose uncommon conditions upon prismoidal space, and, by selecting one of the solutions, to obtain the equation of an extraordinary prismoidal shape.

Thus, let it be required to determine the equation of the boundary of a prismoidal space, such that the line of double curvature,

$$\begin{cases} z=a+bx+cx^2-dx^3+ex^3 \\ y=g+hx^2-x^4+kx^7 \end{cases}$$

shall be contained in that boundary.

Let every cross section, parallel to the plane YZ, be an ellipse with its principal axes parallel to the axes of Y and Z. At a distance x, the equation of the cross section is

$$A^2(z-n)^2+B^2(y-m)^2=A^2B^2.$$

Of the four conditions which may be imposed upon this cross section, we exhaust one in making its perimeter pass through the given line, by assigning to z and y their values in the equations of that line. For the other conditions let

$$m=g+hx^2-x^4+kx^7,$$
$$n=-dx^3+ex^3,$$

and $\quad A=a\ constant,$

whence, $\quad B=a+bx+cx^2,$

in order that the area, πAB, shall be

$$\pi A(a+bx+cx^2).$$

The last expression shows that p. f. applies to the space in question. Substituting in the general equation of the ellipse the values of A,B, m,n, in terms of x, and making x general, we obtain the equation of the shape,

$$\left.\begin{array}{r} A^2(z+dx^3-ex^3)^2 \\ +(a+bx+cx^2)^2[(y-g-hx^2 \\ +x^4-Kx^7)^2-A^2] \end{array}\right\}=o.$$

Let it be required to find the equation of a surface, symmetrical about the axis of Z, which shall contain the lines

$$x=\log.\ (a+z),$$
$$y=b+c\ \sin.\ z,$$

and shall be the boundary of a prismoidal space.

Let the equation of any cross section, parallel to XY, be

$$y^2=[g\sqrt{h^2-x^2}-k(h^2-x^2)]^2.$$

Assigning to x and y appropriate values, we find

$$h=\log.\ (a+z),$$
$$gh-kh^2-b+c\ \sin.\ z.$$

$$\int_o^h ydx=\tfrac{1}{2}g\left\{x\sqrt{h^2-x^2}+h^2\sin.^{-1}\frac{x}{h}\right\}_o^h$$

$$-k\left\{h^2x-\tfrac{1}{3}x^3\right\}_o^h$$

$$=\tfrac{1}{4}\pi gh^2-\tfrac{2}{3}kh^3=$$

$$\tfrac{1}{4}\pi\log\ (a+z)[b+c\ \sin z+k\ \log^2(a+z)] \\ -\tfrac{2}{3}k\ \log^3\ (a+z)$$

$$=m+nz+pz^2+qz^3.$$

The last member expresses the condition that the p. f. shall apply. m,n,p,q, are arbitrary. Assign values to these; solve equation with respect to k; knowing k and h determine g; then, substitute for g, h, k, their values in terms of z, in general equation of cross section, and make z general. The result is the equation of the required surface.

Consider the space bounded by the surfaces,

$$y=o,\ y=w,\ z=o,$$

$$z=ax^my^n+bx^py^q+cx^ry^s+dx^ty^u+\text{etc.} \quad (17)$$

The exponents m,n,p, etc., of equation of top surface, having any given values, it is required to find the relation which must exist among the values of coefficients of this equation, in order that the p. f. shall apply in the direction of X.

Eq. (17) may be placed in this form.

$$z=\left\{\begin{array}{l} a_1+b_1x+c_1x^2+d_1x^3+\varepsilon_1x^4 \\ \qquad +g_1x^5+\text{etc.} \\ +(a_2+b_2x+c_2x^2+d_2x^3+\varepsilon_2x^4 \\ \qquad +g_2x^5+\text{etc.})y \\ +(a_3+b_3x+c_3x^2+d_3x^3+\varepsilon_3x^4 \\ \qquad +g_3x^5+\text{etc.})y^2 \\ \qquad +\text{etc.} \end{array}\right. \quad (18)$$

This last, for any value of x, is the equation of the corresponding transverse element of the surface. Therefore, the result of the multiplication of this general value by dy, and the integration between $y=o$, $y=w$, is the area of the generatrix at the distance x from origin. In order that the p. f. may apply between any limits, this area must equal

$$m+nx+px^2+qx^3, \quad (19)$$

where m, n, p, q, are arbitrary. Then

$$\left.\begin{array}{l} (a_1+b_1x+c_1x^2+d_1x^3+\varepsilon_1x^4+ \\ \qquad g_1x^5+\text{etc.})w \\ +\tfrac{1}{2}(a_2+b_2x+c_2x^2+d_2x^3+\varepsilon_2x^4 \\ \qquad +g_2x^5+\text{etc.})w^2 \\ +\tfrac{1}{3}(a_3+b_3x+c_3x^2+d_3x^3+\varepsilon_3x^4 \\ \qquad +g_3x^5+\text{etc.})w^3 \\ \qquad +\text{etc.} \\ \quad -m-nx-px^2-qx^3 \end{array}\right\}=o, \quad (20)$$

is the criterion.

If we make the sum of all coefficients of the same power of x, equal to zero, eq. (20) is true. That is, eq. (20) is true, and the p. f. applies to shape covered by (17) or (18) when in latter one coefficient of a term containing x to each power, is such a function of w and all other coefficients of terms containing x to same power, as is implicitly expressed by the equation

$$g_1w+\tfrac{1}{2}g_2w^2+\tfrac{1}{3}g_3w^3+\text{etc.}=o. \qquad (21)$$

Since m, n, p, q, are arbitrary, the coefficients of terms containing x to powers less than fourth, need not, if we please, be altered.

We may also, by alteration of the coefficient and exponent of x in any one chosen term of eq. (17), make this the equation of a surface covering prismoidal space. Thus, to one coefficient, as c_1, ε_2 or d_3, may be given such a value in terms of the variable x and all other constants in eq. (20), as to make that equation true.

Suppose eq. (17) contains two or more terms with x^4, and none with x to higher powers, whatever be the powers of y. If, now, in eq. (20), the coefficients, m, n, p, q, have all the sets of values possible, that is, if the expression of third degree, (19), have in succession all the values possible, in each instance one coefficient of x^4, or some other coefficient, may have such a value, according to eq. (21), as to make eq. (20) true. Therefore, there may be as many surfaces of the fourth degree, with respect to x, such that the p. f. applies to the space covered by it, as there are surfaces, altogether, susceptible of representation by a cubic equation. For every additional term in eq. (17) containing x^4, the number of such surfaces becomes infinitely greater. In general, the number of surfaces of any degree, covering prismoidal space, is, at least, equal to the whole number of surfaces of degree next lower, and it may be infinitely greater.

For a particular set of values of m, n, p, q, eq. (20) reduces to the more convenient form

$$\left.\begin{array}{c}(\varepsilon_1x^4+g_1x^5+\text{etc.},)w\\ +\tfrac{1}{2}(\varepsilon_2x^4+g_2x^5+\text{etc.},)w^2\\ +\tfrac{1}{3}(\varepsilon_3x^4+g_3x^5+\text{etc.},)w^3\\ +\text{etc.}\end{array}\right\}=o, \quad (22)$$

which still represents an infinite number of conditions.

Any surface, which varies with x and y but not with z, as

$$y=\alpha+\beta x+\gamma x^2+\varDelta x^3+\text{etc.}, \quad (23)$$

may be substituted for $y=w$; and the variable value of y substituted for the constant value w, in expression (20), or (22), whereafter it will be necessary only to make one constant, as a_1, ε_2, etc., or α, β, etc., a certain function of the remaining quantities, that there may result the equations of the two surfaces, conditioned to enclose a prismoidal space.

It follows from the above that through any number of points, arbitrarily fixed, except that more than one be not in the same perpendicular from base, a surface may be passed, while, also, the width of shape may vary according to any given function of x, so that between any limits the p. f. shall apply to the space enclosed.

Such a surface can be passed through the points.

$$\begin{array}{l}x=1,\ y=4,\ z=4,\\ x=2,\ y=4,\ z=2,\\ x=3,\ y=4,\ z=3,\\ x=5,\ y=4,\ z=2,\\ x=3,\ y=1,\ z=5,\\ x=6,\ y=2,\ z=1,\end{array}$$

and the curve

$$y=x+\tfrac{1}{2},\ z=2\ \text{Nap. log. }(1+y)$$

Thus, through first four points and the curve, at a point in same vertical plane with these, pass a line of fourth degree. Through the last two points, and the two points in same vertical plane of the two curves, pass a cubic parabola. With the three curves as directrices, and the plane ZY as a plane director, generate a surface by motion of a cubic parabola. The area of generatrix, at any distance, x, is now represented by an expression containing one arbitrary quantity. Assign such a value to this as shall make the expression equivalent to a cubic equation.

Between given limits in directions of X and Y, a surface of any degree may be found, such that to the space covered by which the p. f. shall apply in both directions. This can be shown most briefly by an example. Consider the space

$$z=ax^5y+bx^4y^2+cx^2y^4+dx^2y^4$$
$$+exy^2+fx+g.$$

Integrate with respect to y, between

limits o and w, all terms containing x to higher powers than the third, and set result equal to zero, using as criterion (22).

$$o=\frac{1}{2}ax^4w^2+\frac{1}{4}bx^4w^4. \quad \therefore b=-\frac{2ax}{w^2}.$$

Substitute in original equation; then proceeding in like manner for y, l taking the place of w, we obtain

$$o=\frac{1}{3}cy^4l^3+\frac{1}{4}dy^4l^4. \quad \therefore d=-\frac{4c}{3l}.$$

The required equation is

$$z=ax^5y-\frac{2ax^6y^2}{w^2}+cx^2y^4-\frac{4cx^2y^4}{3l}+exy^2+fx+g.$$

In exactly same manner space may be treated, when its boundaries are represented by equations containing polar co-ordinates, r,θ,z. Now the area of generatrix is $\frac{1}{2}\int_o^{2\pi} r^2\,d\theta$, instead of $\int_o^w zdy$. Examples of prismoidal shapes are the following two:

$$r=ax^2\theta^2-\frac{7ax^2\theta^4}{40\pi^2}+c,$$

$$r=\sqrt{ax^4\theta^2-\frac{4}{3}\pi ax^4\theta+bx^2\theta+cx^2\theta^5+dx+g}$$

The equation of every shape, or of every surface, is

$$o=\mathrm{A}r^m\theta^nz^p+\mathrm{B}r^q\theta^sz^t+\text{etc. ad inf.}$$

It may be shown that this space can be made subject to the p. f. by alteration of any one term of its equation.

No equation of finite degree, representing a bounding surface, can define the limits of applicability of the p. f., because surfaces of higher degrees enclose prismoidal spaces. It requires two equations of infinite degree, one the general equation of the surface, and the other the equation of condition, to define the limits. The number of equations of finite degree necessary is infinite.

When the bounding surface of prismoidal space is of high degree, there may be a correspondingly great number of negative parts in the complete space; but the generatrix, being considered as a whole, can change its sign no oftener than three times, since it varies as the ordinate of a cubic parabola.

The formula

$$v=\frac{l}{8}\left\{\mathrm{A}+3\mathrm{B}+3\mathrm{B}'+\mathrm{A}'\right\}, \qquad (24)$$

where the capital letters in parenthesis represent areas of cross sections, dividing length of space into equi-parts, has no greater extent of exact application than the ordinary p. f., though it depends upon four cross sections. So, the formula for rectangle,

$$v=al, \qquad (25)$$

where a is area of mid section, has equal extent of application with the trapezoidal formula, where $a=\frac{1}{2}[\mathrm{A}+\mathrm{A}']$. The same is noticeable in formulæ of wider application.

If we multiply the generatrix of space, $\mathrm{F}(x)$, by dx, and integrate between $-\frac{1}{2}l$, $+\frac{1}{2}l$, we obtain

$$v=l[a+\tfrac{1}{12}cl^2+\tfrac{1}{80}el^4+\text{etc.}]. \qquad (26)$$

This does not depend upon odd powers of x in F (x). For rectangle, (26) reduces to (25); and, since b does not appear, this is true for any trapezoid.

When F (x) is of second degree, another pair of cross sections must be introduced. Then,

$$\mathrm{A}=a-\tfrac{1}{2}bl+\tfrac{1}{4}cl^2,$$
$$\mathrm{A}'=a+\tfrac{1}{2}bl+\tfrac{1}{4}cl^2,$$
$$\mathrm{A}+\mathrm{A}'=2a+\tfrac{1}{2}cl^2,$$
$$\tfrac{1}{12}cl^2=\tfrac{1}{6}[\mathrm{A}-2a+\mathrm{A}'].$$
$$\therefore\ v=\frac{l}{6}[\mathrm{A}+4a+\mathrm{A}'];$$

and this is applicable to spaces of third degree.

If F (x) be of fourth degree, then,

$$\mathrm{A}+\mathrm{A}'=2a+\tfrac{1}{2}cl^2+\tfrac{1}{8}el^4,$$
$$\mathrm{B}+\mathrm{B}'=2a+\tfrac{1}{8}cl^2+\tfrac{1}{128}el^4.$$

Find from these $\frac{1}{12}cl^2$ and $\frac{1}{80}el^4$. Then, (26) is

$$v=\frac{l}{90}[7\mathrm{A}+32\mathrm{B}+12a+32\mathrm{B}'+7\mathrm{A}']. \qquad (27)$$

This is exact when F (x) is of fifth degree.

An interesting but lengthy method of forming these rules is found in Israel Lyon's *Treatise of Fluxions*, London, 1758, pp. 173–8. A very elegant method by Chauncey Wright, is published in the *Mathematical Monthly*, Mass., Oct. and Nov., 1858. Another method is shown in *Boole's Calculus of Finite Differences*.

Numerous writers have furnished theorems of more or less generality, defining whole classes of shapes subject to

the p. f. Want of space compels us to mention the names of a few only, and to omit their rules altogether. These chiefly relate to the shape shown in Fig. 6, and to the shapes formed by revolution of conic sections.

Thomas Simpson in 1750 decomposed the ordinary prismoid into two wedges, and derived the formula

$$v=\tfrac{1}{6}h[\mathrm{WL}+wl+(\mathrm{W}+w)(\mathrm{L}+l)].$$

The ordinary rule follows directly from this. The same mathematician devised the popular rule for mechanical quadratum, which bears his name. He could not have failed to notice the similarity between them.

Charles Hutton in his *Mensuration*, 1770, constructs the formula in its common shape. He demonstrates that it is applicable to all solids generated by the revolution of a conic section, and to their frusta.

Professor Gillespie of Union College demonstrated, in 1857, before the *American Association for the Advancement of Sciences*, that the formula is applicable to the space covered by the hyperbolic-paraboloid. This was a most important extension of its uses.

John Warner made similar demonstrations in his treatise on earthwork calculations, 1861.

Charles Baillairgé, a Canadian, proposed in a treatise on mensuration, 1866, to use the p. f. as an universal rule, for practical purposes, in quadrature and cubature. He mentions no new shapes, to which the formula applies exactly; but simply uses a succession of prismoids to approximate any given space, [Simpson's method], and indicates what shapes need a close assemblage of measured cross-sections, and those whose contents are exactly, or very nearly, represented by the p. f. in one application. His work is designed for artisans unskilled in mathematics, and is, undoubtedly useful in that respect.

Professor Rankine, the Rev. Mr. Billion of Montreal, and Professor R. Steckel of Alsace, France, have given excellent general rules. Those of last two are published in Mr. Baillairge's *Stereometrical Tableau*.

None of the foregoing writers produced widely general rules.

Chauncey Wright, in 1858, in Oct. number of the *Mathematical Monthly*, Cambridge, Mass., obtained, by a brief and elegant demonstration, the cubic equation, (11), as expressing the law of variation in magnitude of the generatrix of prismoidal spaces. This was the solution of the problem which had, probably, engaged the attention of many mathematicians before.

Prof. E. W. Hyde, of the University of Cincinnati, in 1876, published in the July number of the *Analyst*, Des Moines, Iowa, an article entitled, *Limits of the Prismoidal Formula*. He attempted to determine the limits of the formula's applition by means of eqs. representing shapes, or the bounding surfaces. If in eq. (20) of present paper, m, n, p, q, be assigned such values as reduce that equation to the form of eq. (22), and then, to satisfy this, each remaining coefficient as g_1, e_2, etc., be made zero, the resulting class of shapes is that which he discusses. Evidently, the longitudinal elements can only be lines of third degree or less. Using the co-ordinates, x, θ, ρ, he shows that the p. f. applies to the shape,

$$\rho=\sqrt{x^3f(\theta)+x^2f_1(\theta)+xf_2(\theta)+f_3(\theta)},$$

where θ may have any exponent, but x must not be of higher degree than third. This paper is very interesting, and the formulæ include many, though not all, practical shapes.

The present writer was led to investigate the subject, by a desire to compare the range of a new and general center of gravity formula with that of the prismoidal formula. From special trials upon spheroids, paraboloids and other common shapes, they seemed to occupy the same field. The result and the formula itself shall be published shortly. Other than this, the most important result of this paper, eq (11), has been anticipated by Chauncey Wright. The writer has endeavored to make this result, if possible, more general, by commencing with the conception of every space, and by discussing equation (10) thoroughly.

The writer is especially indebted to Prof. E. W. Hyde for information and references.

A VISIT TO A MINING DISTRICT IN CHINA.

By G. J. MORRISON, M. Inst. C.E.

From "The Builder."

In China,—that great land of undeveloped wealth,—there are villages and whole districts where all the men one sees are covered, I might almost say clothed, with coal dust, for they wear little else in summer, and where, as there are no railways, one meets long trains of coal-laden mules, asses or camels, according to the nature of the country.

About ten or twelve miles to the west of Pekin, runs a river called the Hun Ho. This is a shallow river, fordable in many places in dry weather, but after heavy rains it is subject to great floods, which make its passage difficult and dangerous even by a ferry-boat. There is, however, one bridge across it which is always available.

Pekin is situated on an immense plain, but immediately to the west of the Hun Ho the country becomes hilly. Mount Conolly, as it is called by foreigners, which rises to a height of 5,000 feet or 6,000 feet above the sea, is the highest peak in the immediate neighborhood. The whole of these hills abound in coal, and the district is generally termed by foreigners the Chai Taeng district, from the name of a village about forty miles from Pekin, and near which some of the best coal-mines are situated. Being in the neighborhood in August, 1877, I had an opportunity of forming one of a party to visit these hills. The party consisted of Mr. Nicolson, of the Diplomatic service; Mr. Brenan, of the Consular service (who is thoroughly acquainted with the colloquial language) and myself.

Having started early one Monday morning, we crossed the Hun Ho by a ferry at a village called Ma Yu, and at once began to ascend the mountain-road leading to the mines. By ten o'clock we reached the entrance to the Ta Yao, or great mine, situated about eighteen miles west of Pekin, and 1,050 feet above the sea.

The mines here are not worked as most mines are in England, by shafts sunk vertically, but by "adits" or entrances from the side of the hill, as is sometimes done at home. The people look for some place where the seam of coal crops out at the surface. They then begin making a tunnel about four feet wide, and four feet six inches high, carefully lining the side and roof with timber, so as to prevent any of the earth falling in, and they work on in this way until they have arrived at good solid coal, with firm rock above and below, the portion near the surface being generally more or less loose and broken up. When they are fairly in the solid coal, they begin working out coal to the right and the left, as well as in front. If they were to work out a large area without taking any precautions, the roof would fall in; they therefore place temporary props of timber where they are working, and, in addition, they carry forward their main road by carefully continuing the timber work of the tunnel, and also on each side, outside the timber, building a wall of the rubbish which is excavated with the coal, so that, even if the roof of any portion of the working falls, they are sure of having a road through which they can reach the solid coal further in. The coal when originally deposited, was no doubt level, but since that date there have been great upheavals of the strata, and much of the coal now lies at a very steep angle, sometimes as much as sixty degrees to the horizontal. The road or tunnel, therefore, following as it does the seam of coal, is sometimes flat and sometimes steep, and as various local causes make it convenient, during the progress of the work, to carry the road to one side or the other, it happens that the main road in an old mine is as crooked as a corkscrew. There is yet one other peculiarity. Although the tunnel is begun so large that a man can enter by stooping, it often happens that, when the seam is thin and the roof and floor composed of hard rock, the miners grudge the expense of cutting it, and the road, therefore, gets lower and lower until the final limit is reached of a road so low that a man can only just crawl through it. Now, having

come to see the mines, I was determined to enter into one and see the workings for myself. We, therefore, addressed ourselves to some miners whom we found at the mouth of the Ta Yao. At first they thought it was all a joke, and it was a considerable time before they could be persuaded that I was in earnest. Then they said it was impossible; that one foreigner had tried some years previously and had failed; that I would ruin my clothes, and made various other objections, but finally we prevailed. And so, divesting myself of most of my clothing, I took a lamp and prepared to follow my guide to the depths of the mine. The lamp was a curiosity in its way. In principle, it was the same as the ordinary open lamp used in the Scottish mines; but in general appearance it resembled a teapot without a handle. It was carried by a string passed round the head; and, being globular, it rolled about a great deal, and as it must have weighed over 1¼ lbs., it was a serious addition to the troubles of the journey.

On entering, I found that all along the floors there was laid a ladder about two feet wide, having good large rungs eight inches or nine inches apart. The ladder was roughly made, but was tolerably level on the upper surface. My guide, who at first took a fatherly interest in me, at once signed to me to walk on the rungs of the ladder, and not to let my foot slip between, as, in the event of there being a hole, I might break my leg. Very soon the road got so steep that I had to sit on the ladder and let myself down gently, catching the rungs with my heels; then it became so steep that I had to turn my face to the ladder and descend in the ordinary way. For about 600 yards the road was of this varied description, but rather steep on the whole. I estimated the total fall in this distance to be about 500 feet. This part of the journey was tolerably easy. The roof was seldom less than three feet or three feet six inches high, so that the only difficulty consisted in passing the coal drawers, to whom I shall refer presently. After the first 600 yards, however, the affair changed. The road twisted a good deal, sometimes was uphill, sometimes down, and sometimes level, and the roof was very low. The total length of this portion was 1,000 or 1,200 yards; and the total fall must have been between 100 feet and 200 feet. Along almost the whole of this it was necessary to go on hands and knees or hands and feet; but this was not the worst. In several places the roof was so low that I could not go on hands or knees. I am, unfortunately, a trifle larger than an ordinary Chinese miner, and not being acquainted with the place, I probably required one or two inches more space than I should otherwise have done, for it must be remembered that the roof is not smooth, and if one thinks he can crawl under a projecting stone, and is mistaken, he will be painfully reminded of his error by the loss of a large piece of skin from his back, if he is not more seriously hurt. There was but one way for it, and that was to lie on my face, stretch out my arms, and, resting my weight on my hands and elbows, drag myself forward. There were two or three places where I had to do this for a length of twenty yards at a time, and I was very glad when I arrived at the working face, a distance of about a mile from the entrance, and still more glad when I again reached the fresh air, after having spent about three hours below ground. The coal which is here being worked, is a description of anthracite, but not so hard and clean as some found on the other side of the hill. The seam in this mine is only one foot nine inches to two feet thick.

It is extremely difficult to give a fair description of the method of working. Being ignorant of the language, I could ask no questions, and as all old workings are full of rubbish, it is impossible to tell with certainty what was done last, or what is about to be done next. The main road running from the surface was crossed at certain intervals by roads running at right angles. The miner ascends from the road to the working face, and cuts away some coal, which he rolls down hill to the road, whence it is taken away by the drawers. While this is going on he is obliged to put temporary props to support the roof, and after a certain time these become so numerous as to interfere materially with the coal reaching the road, and for this reason the cross roads are placed at the distance which is found by experience to be most convenient. While the miner is working between the

cross roads, the main road is carried lower, and a new road is opened, and another miner is set to work there. It must not, of course, be understood that all this goes on with regularity. There are faults and other difficulties to encounter. Sometimes the coal is too thin to pay, and in an old mine the workings are most irregular; but, judging from what I saw, this is the arrangement aimed at. At the working faces only the coal is cut away, neither floor nor roof being touched; the space in which the miner works is therefore only one foot nine inches high. It is hard to wriggle about in such a place, and the difficulty of working is very great. It is, therefore, not surprising that the output of the mine does not much exceed two piculs for each miner per day. A picul ought to contain 100 catties of $1\frac{1}{3}$ lbs. each; but the wretched practice exists in China of having piculs of 120, 140, or even 200 catties, just as at home we have stones of different weights. I imagine the output in this case was about 3 cwt. per miner.

The tools used by the miner are chisels about twelve inches long, and one-half inch thick, and a small hammer. With these he cuts holes in the coal, and afterwards breaks off lumps of considerable size by means of wedges. The pick is very little used in these mines, although it is used in the bituminous coal mines a few miles distant. The coal is drawn to the surface in baskets fixed to sledges. Each basket contains about 100 lbs. A man or a boy draws the basket by a leather strap, which passes along his back over one shoulder, and back again between his legs to the basket. He progresses on hands and feet, and draws the load after him, the sledge passing smoothly enough over the ladder, whether the ground is levelled or inclined. When the ladder is steep the process looks dangerous, as, if the man were to slip, or anything to give way, the man behind would almost certainly be killed. The maximum work of a man is four journeys per day. In returning to the surface where the road is steep, the drawer sits on his basket and slides down the ladder, using his heels to guide himself, and to regulate the speed.

The mines here are never troubled with water. When any appears, as it does occasionally, the miners stop work for a few days, and it runs away through the fissures in the strata. The ventilation is effected through openings to the surface at various places, which I understood were either old entrances, or places where the workings had come to the surface. It was stated distinctly that in this mine no special air-courses were cut. On the day of my visit the temperature of the mine was much lower than that of the external air, and natural ventilation would be sufficient to keep the mine fresh; but they told us they sometimes used fans below ground—"something like winnowing-machines," about four feet or five feet diameter.

They said that with thirty miners and thirty drawers the output of the mine was about sixty piculs, which I take to be about four tons and a half. The wages of the men are three tias (about 1s. 1d.) per day, and of boys two tias (about 9d.). This coal sells at the mine at three tias and a half a picul. This would appear to be a losing game, but the profit is made on the difference between the miners' picul and the selling picul. The mines are worked for the benefit of the owners of the land, and we were told that they paid nothing to Government, except a yearly present of small amount to the local magistrate.

After hearing this, I fancy most people will be surprised to find that the Chinese know so much and effect so little. The system of working is not unscientific, and only differs in detail from our long wall system at home. Although we place our fans for the ventilation of collieries at the surface, still, in the construction of headings, &c., they are often placed below. The Chinese would probably be unable to cope with large quantities of water; but it must not be forgotton that within a century our own Cornish miners failed in this, and only succeeded through the assistance of outsiders, of whom Watt was the chief. But with all this knowlledge, here is a mine which has been worked for 100 years, and where the workings extend to the distance of a mile, with a daily output of four or five tons.

The great want below as above ground is that of roads. I am tolerably safe in saying that there is no drawing-road in any coal-mine in Great Britain like the one I have described. There are, doubtless,

air-courses as small, but then men only pass through them occasionally. The worst roads are to be found in the Cornish mines, where, unfortunately, it is a common thing for the miners to take more than an hour to get from the surface to their working places, and the owners comfort themselves by thinking that in old workings nothing better can be expected, and by trying to believe that it is the miners' time and strength that are sacrificed, and not theirs.

Dreadful, however, as is the life of these poor Chinese miners, it must not be supposed that they appear to be discontented. Their wages are more than sufficient to provide them with plenty of good food and with houses to live in, and as they wear no clothes below ground, and very little in summer above ground, their tailors' bills cannot be serious matters. They are always civil and ready to chat and joke with anyone, and here, as over the greater part of China, a foreigner has nothing to fear from the common people, unless, either intentionally or accidentally, he offends them.

I believe I am correct in stating that women are never employed in Chinese mines. In this respect the Chinese are far in advance of the Japanese, and, for that matter, of the Sicilians and some other western nations, in whose mines much of the drawing is done by girls. I have seen dozens of naked Japanese men working in the same mine with women, whose clothing was just visible to the naked eye.

Immediately to the west of the mine just described there is a low range of hills, and after passing these the scenery is very fine. The numerous coal-mines occasionally mar the landscape; but some of the cliffs are magnificent. The roads are execrable, taking one up and down hills at almost impossible gradients. Mules and asses are the only beasts of burden that can use them, and wheeled traffic is impossible. Twenty miles a day, as the crow flies, is hard work.

The following day we reached Chai Tang, about forty miles west of Pekin, and we visited some of the bituminous coal-mines in the neighborhood. The seams which we saw are six feet or seven feet thick, and there are some much thicker. The mines appear to be worked on the pillar and stall system; that is to say, large pillars of coal are left to support the roof, and these are worked out when any portion of the mine is about to be abandoned. Here they construct artificial air-courses, and also employ "brattices," or partitions of timber or similar material, to divide the passages in two, so that the air may travel in along one division, and out along the other. When asked about fire-damp, they replied that they were troubled with it occasionally, and in some neighboring mines there had been explosions killing people; but they added, "these only occur when the air does not circulate."

It will thus be seen that they have not a great deal to learn in the theory of mining. The want of roads is not so much felt in these mines as in the thin anthracite seams. These seams are high, and the mines have not been worked so long. One draw can bring out about a ton a day, and one miner can get as much. The tools used are picks with one head, that is to say, in the form of an ˥, and not of a T. We were told that the price at the pit's mouth was 6s. a ton, but I believe it is often sold as low as 4s. The cost of carriage to Pekin is, however, 30s., so the coal is dear enough there.

About thirty-five miles west of Pekin we came upon a Roman Catholic village of 800 or 1,000 inhabitants, called Sang Yu. Not far from this village we came upon a mill for grinding wood and other materials to make joss-sticks. These sticks are about twice the length of an ordinary pencil, and rather thinner, and of a light brown color. They are placed in front of the images in the temples, and when lighted, smoulder slowly, giving off a slight and not unpleasant odor. The mill was driven by a horizontal water-wheel,—a description of turbine. The wheel was eight feet or ten feet in diameter, and the spokes were flat and very wide. The water issued from a spout, and impinging on the spokes, drove the wheel round. The whole available fall of the water was not utilized, but as the axle of the wheel formed the vertical axle of the mill, there was absolutely no gearing whatever, and nothing that required skilled labor to erect, or was likely to get out of repair; and as there was abundance of water-

power to be had, the arrangement was very creditable to the designer.

The next day we visited the Imperial tile manufactory at Lien li ku, about fifteen miles west of Pékin. Here all the yellow tiles and bricks required for Imperial buildings are made, as also large numbers of green, blue, and other colored tiles, for various ornamental purposes. The material used is a hard blue shale, nearly as hard as slate. This is allowed to lie in heaps for some time. It is then ground to powder by granite rollers, on a stone floor thirty or forty feet in diameter. The powder is then stored in heaps, and taken to the works as required. For ordinary work the powder is mixed with a proper proportion of water, and moulded into large bricks, which are laid out to dry for some hours, after which they are dealt with by the modellers. When bricks are to have a moulding on them, say, for coping a wall, the plan of the operation is as follows: Two pieces of wood, each cut to the shape of the moulding, are placed upright on a slab. The clay brick is placed between them, and two men run the mouldings roughly along with chisels. They then apply straight-edges to test the accuracy of their work, and finally rub the edges with moulds somewhat in the same way as plasterers make mouldings at home. The brick is then passed to a third man, who cuts any necessary holes in it, and to a fourth, who trims it off and repairs any defect. The more ornamental tiles and bricks, representing fabulous animals, &c., are first roughly moulded, and afterwards finished off with tools exactly similar to those used for modelling in clay in Europe. Some of this work has some pretension to artistic merit. All the bricks and tiles are baked in ovens, and then, after having the glaze put on, are baked a second time. All the work done at this manufactory appears to be first-rate, and the number of people employed when they are busy is about 500.

Much of the work we saw, particularly the moulding of coping bricks, could no doubt be executed easily by machinery; but seeing that labor is so cheap, I doubt if any advantage could be gained by its introduction. As an example of the paradoxes to be seen in China, it is worthy of remark that these bricks, which deserve to rank with the highest class of artificial building materials in the world, can only reach the capital, distance fifteen miles, by being carried on camels or mules.

The Chinese manners and customs, and modes of thought, differ so entirely from our own, that it is extremely difficult to express any general opinion regarding them. The most contrary views are stated by people who agree perfectly on other subjects, and even the same person is sure to bestow encomiums upon them at one time, and to speak of them as little better than savages at another, in a way that is most perplexing. For instance, the money of the bulk of the inhabitants consists of copper coins, varying in value in different districts, but generally worth about $\frac{1}{25}$ of a penny each. These coins, however, differ so much in size and quality that it takes more time to select and count 1,000 of them than to perform the same operation for 1,000 sovereigns. Is it wonderful, then, that one set of people are fond to talk of the admirable system of the Chinese, which enables the poorest person to get the advantage of a turn of the market in buying a single egg, while others can only look upon it as a system for wasting the time of hundreds of thousands of shrofts (or money-counters) who might be better employed, and for extorting from the poorer classes payments under the name of exchange to an amount which is absolutely appalling. Again, every particle of night-soil is collected and employed as manure on the fields. While one man can talk of nothing but the almost unbearable nuisance caused by the collection and distribution of the unsavory stuff, another can speak only of the science and industry displayed by the people in utilizing the valuable materials which civilized nations allow to run to waste. These differences may, to some extent, be explained by saying that one set of people are inclined to look at the ends aimed at, while another set look more at the means employed; but I fear it will be long before the views of writers on China cease to present contradictions which appear inexplicable.

On one point most people are agreed, and thrt is, that of all the defects of China, want of means of internal com-

munication is one of the greatest. They have a few bridges, some useful (like the bridge at Foochou, over the river Min, which is a work of considerable magnitude), and some ornamental (like the marble bridge near the Palace in Pekin, which is unquestionably a most elegant structure); but while a few people may talk of such works as evidences of enterprise and public spirit, most people can only consider them to be monuments of disgrace to a nation which knows what such things are, and which will not take the trouble to repair the old ones and build new ones!

Of roads, I may say, although I have been forced to employ the word occasionally, that they are non-existent. It is the fashion among authors at present, when they cannot find English words to express their meaning, and sometimes, I suspect, when they are not very clear as to the meaning they wish to express, to employ words from some other language. Any one describing the tracks through China, may be excused if he makes use of the Chinese name "Loo," for it is a fact, and one for which I am thankful, that it has no equivalent in English.

ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION.

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IV.

§ 5.

FREE VORTEX MOTION.

We have already seen under what circumstances it is impossible for rotational motion to exist in a fluid mass. If the fluid in its initial condition has irrotational motion—or, if it be at rest, and motion is induced by a system of conservative forces, then the motion will always be irrotational; *i. e.*, if the quantity

$$udx+vdy+wdz$$

is at any time an exact differential it will always be one. The conditions for this quantity being an exact differential are

$$\frac{dw}{dy}-\frac{dv}{dz}=o,\ \frac{du}{dz}-\frac{dw}{dx}=o,\quad \&c.$$

Suppose that these quantities are not equal to zero, but that we have

$$\xi=\frac{1}{2}\left(\frac{dw}{dy}-\frac{dv}{dz}\right),$$

$$\eta=\frac{1}{2}\left(\frac{du}{dz}-\frac{dw}{dx}\right),$$

$$\zeta=\frac{1}{2}\left(\frac{dv}{dx}-\frac{du}{dy}\right);$$

The quantities ξ, η and ζ, as is well known, denote the components of angular velocity around the axes of x, y and z, respectively, of a particle whose velocities parallel to these axes are u, v, w. That these quantities should have certain definite values different from zero is, of course, the condition that vortex, or rotational motion exist in the liquid. These values of ξ, η and ζ, pre-suppose that we know the values of u, v and w. A problem that now immediately presents itself for solution is to find the values of u, v and w, supposing ξ, η and ζ to be given.

Assume three functions U, V and W such that

$$u=\frac{dW}{dy}-\frac{dV}{dz},$$

$$v=\frac{dU}{dz}-\frac{dW}{dx},$$

$$w=\frac{dV}{dx}-\frac{dU}{dy},$$

The quantities u, v and w must satisfy the equation

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=o.$$

It is found without difficulty that this equation will only be satisfied by the above values of u, v and w if the following conditions hold,

$$\frac{d^2U}{dx^2}+\frac{d^2U}{dy^2}+\frac{d^2U}{dz^2}=-2\xi,$$
$$\frac{d^2V}{dx^2}+\frac{d^2V}{dy^2}+\frac{d^2V}{dz^2}=-2\eta,$$
$$\frac{d^2W}{dx^2}+\frac{d^2W}{dy^2}+\frac{d^2W}{dz^2}=-2\zeta,$$
$$\frac{dU}{dx}+\frac{dV}{dy}+\frac{dW}{dz}=o.$$

The integrals of the first three of these equations are well known to be given by

$$U=\frac{1}{2\pi}\int\int\int\frac{\xi' dx'dy'dz'}{\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2}},$$
$$V=\frac{1}{2\pi}\int\int\int\frac{\eta' dx'dy'dz'}{\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2}},$$
$$W=\frac{1}{2\pi}\int\int\int\frac{\zeta' dx'dy'dz'}{\sqrt{(x'-x)^2+(y'-y)^2+(z-z)^2}};$$

where x', y', z' are the co-ordinates of any other point in the vortex element, and ξ', η', ζ' are the angular velocities at this point; the denominator,

$$r=\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2}$$

denotes the distance between this point and the assumed point to which the U,V, W refer. Before going further it will be convenient to give two of Helmholtz's definitions. The line passing through any point and coinciding at all times in direction with the instantaneous axis of rotation of that point is called a *vortex line.* If we consider a number of vortex lines passing through every point in the perimeter of an infinitely small surface, they will cut from the rest of the fluid a filament which is called a *vortex filament*, or a vortex filament is an infinitely small filament of the fluid whose bounding surface is made up of vortex lines.

Now, in our equations giving U, V, W, the points x, y, z and $x' y', z'$ are supposed to lie on the same vortex filament; we can represent an element of this filament by $d\tau$, then our equations become,

$$U=\frac{1}{2\pi}\int\frac{\xi' d\tau}{r},$$
$$V=\frac{1}{2\pi}\int\frac{\eta' d\tau}{r},$$
$$W=\frac{1}{2\pi}\int\frac{\zeta' d\tau}{r}.$$

where the integrations are of course extended over all the space which is supposed to be filled with vortex filaments. Now to examine the condition

$$\frac{dU}{dx}+\frac{dV}{dy}+\frac{dW}{dz}=o.$$

By differentiation we have

$$\frac{dU}{dx}=-\frac{1}{2\pi}\int\int\int\frac{(x-x')\,\xi' dx'dy'dz'}{r^3}$$
$$\frac{dV}{dy}=-\frac{1}{2\pi}\int\int\int\frac{(y-y')\,\eta' dx'dy'dz'}{r^3}$$
$$\frac{dW}{dz}=-\frac{1}{2\pi}\int\int\int\frac{(z-z')\,\zeta' dx'dy'dz'}{r^3}$$

Integrating by parts we have,

$$\frac{dU}{dx}=-\frac{1}{2\pi}\int\int\frac{\xi' dy'dz'}{r}+\frac{1}{2\pi}\int\int\int\frac{1}{r}\frac{d\xi'}{dx'}dx'dy'dz'$$
$$\frac{dV}{dy}=-\frac{1}{2\pi}\int\int\frac{\eta' dx'dz'}{r}+\frac{1}{2\pi}\int\int\int\frac{1}{r}\frac{d\eta'}{dy'}dx'dy'dz'$$
$$\frac{dW}{dz}=-\frac{1}{2\pi}\int\int\frac{\zeta' dx'dy'}{r}+\frac{1}{2\pi}\int\int\int\frac{1}{r}\frac{d\xi'}{dz'}dx'dy'dz'$$

Now since

$$\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}=o$$

throughout the entire mass of the fluid we have

$$\frac{dU}{dx}+\frac{dV}{dy}+\frac{dW}{dz}=-\frac{1}{2\pi}\left\{\int\int\frac{\xi' dy'dz'}{r}+\int\int\frac{\eta' dx'dz'}{r}+\int\int\frac{\zeta' dx'dy'}{r}\right\}$$

This can readily be changed into a surface integral. If $d\sigma$ denote an element of the surface of the vortex filament and $\cos\alpha$, $\cos\beta$, $\cos\gamma$, denote the direction cosines of the outward normal to this surface, we have

$$dx'dy'=d\sigma\cos\gamma,\ dx'dz'=d\sigma\cos\beta,$$
$$dy'dz'=d\sigma\cos\alpha;$$

therefore our integral becomes

$$\frac{1}{2\pi}\int\frac{1}{\sigma}\left[\xi'\cos\alpha+\eta'\cos\beta+\zeta'\cos\gamma\right]d\sigma$$

taken over the entire surface. Now from the equation

$$\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}=o$$

we have also

$$\int\int\int\left(\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}\right)dx\,dy\,dz=o$$

by integration this becomes

$$\iint\xi\,dy\,dz+\iint\eta\,dx\,dz+\iint\zeta\,dx\,dy=o$$

or as a surface integral,

$$\int(\xi\cos\alpha+\eta\cos\beta+\zeta\cos\gamma)d\sigma=o$$

consequently

$$\frac{dU}{dx}+\frac{dV}{dy}+\frac{dW}{dz}=o.$$

Substituting now the obtained values of U,V,W, in the equations $u=\frac{dW}{dy}-\frac{dV}{dz}$ &c... and we obtain for u, v, w, the following values:

$$u=-\frac{1}{2\pi}\int\int\int\frac{1}{r^3}[\zeta'(y-y')-\eta'(x-x')]dx'dy'dz'$$

$$v=-\frac{1}{2\pi}\int\int\int\frac{1}{r^3}[\xi'(z-z')-\zeta'(x-x')]dx'dy'dz'$$

$$w=-\frac{1}{2\pi}\int\int\int\frac{1}{r^3}[\eta'(x-x')-\xi'(y-y')]dx'dy'dz'$$

or, as these may be expressed,

$$u=\frac{1}{2\pi}\int\left(\zeta\frac{d\frac{1}{r}}{dy}-\eta\frac{d\frac{1}{r}}{dz}\right)d\tau$$

$$v=\frac{1}{2\pi}\int\left(\xi\frac{d\frac{1}{r}}{dz}-\zeta\frac{d\frac{1}{r}}{dx}\right)d\tau$$

$$w=\frac{1}{2\pi}\int\left(\eta\frac{d\frac{1}{r}}{dx}-\xi\frac{d\frac{1}{r}}{dy}\right)d\tau$$

Representing each of these differential expressions by u', v', w' respectively we see that u', v', w' are the increments of u, v, w, which correspond to the element $dx\,dy\,dz$ of the vortex filament. Writing for convenience of reference the equations

$$u'=\frac{d}{2\pi}\left(\zeta\frac{d\frac{1}{r}}{dy}-\eta\frac{d\frac{1}{r}}{dz}\right)$$

$$v'=\frac{d\tau}{2\pi}\left(\xi\frac{d\frac{1}{r}}{dz}-\zeta\frac{d\frac{1}{r}}{dx}\right)$$

$$w'=\frac{d\tau}{2\pi}\left(\eta\frac{d\frac{1}{r}}{dx}-\xi\frac{d\frac{1}{r}}{dy}\right)$$

we see that they give rise to the equation

$$\xi u'+\eta v'+\zeta w'=o.$$

This shows that, considering u', v', w' as the components of a certain new velocity, the direction of the resultant

$$\sqrt{u'^2+v'^2+w'^2}$$

of these components is at right angles angles to the direction of the axis of rotation of the element $d\tau$. Again, we have

$$u'\frac{dr}{dx}+v'\frac{dr}{dy}+w'\frac{dr}{dz}=o$$

and the direction of this resultant is also at right angles to the line r joining the element $d\tau$ to any other. Thus we see that each rotating element of the fluid mass implies in every other element a velocity whose direction is at right angles at the same time to the axis of rotation of the first and to the line joining the two elements—*i.e.*, at right angles to the plane containing the second element and the axis of rotation of the first. It is easily shown that

$$\sqrt{u'^2+v'^2+w'^2}=\frac{d\tau}{2\pi}\sqrt{\xi^2+\eta^2+\zeta^2}\,\frac{\sin\vartheta}{r^2}$$

when ϑ denotes the angle between r and the axis of rotation. From this equation we see that the magnitude of this induced velocity is directly proportional to the volume of the first element, its angular velocity and the sine of the angle between the line joining the two elements and the axis of rotation; and also inversely proportional to the square of the distance between the elements. Denote the angular velocities at the time $t=o$ by ξ_o, η_o, ζ_o then the last equations of chapter I become

$$\xi=\xi_o\frac{dx}{da}+\eta_o\frac{dx}{db}+\zeta_o\frac{dx}{dc}$$

$$\eta=\xi_o\frac{dy}{da}+\eta_o\frac{dy}{db}+\zeta_o\frac{dy}{dc}$$

$$\zeta=\xi_o\frac{dz}{da}+\eta_o\frac{dz}{db}+\zeta_o\frac{dz}{dc}$$

a, b, c being the co-ordinates of an arbitrary particle we have for the co-ordinates of another situated indefinitely near this $a+da$, $b+db$, $c+dc$, and at the time t the co-ordinates will be x, y, z, $x+dx$, $y+dy$, $z+dz$; now suppose that at $t=o$ we have

$$\frac{da}{\xi_o}=\frac{db}{\eta_o}=\frac{dc}{\zeta_o}$$

that is da, db, cd proportional to the initial angular velocities—and suppose further that the direction of this indefinitely small line coincides with that of the axis of rotation. Call the common value of these ratios ε, ε being an indefinitely small quantity, independent of the time; then we have

$$da = \xi_0 \varepsilon,\ db = \eta_0 \varepsilon,\ dc = \zeta^0 \varepsilon,$$

Substituting these values in the above equations, and we have

$$da = \xi\varepsilon,\ dy = \eta\varepsilon,\ dz = \zeta z,$$

or

$$\frac{dx}{\xi} = \frac{dy}{\eta} = \frac{dz}{\zeta}$$

consequently the direction of the line joining the indefinitely near elements will at all times coincide with the direction of the axis of rotation. This combined with our definition of a vortex line shows us that every particle of fluid that lies on a vortex line at any instant will always remain there. If we call ω the resultant angular velocity we have

$$\omega = \sqrt{\xi^2 + \eta^2 + \zeta} = \frac{1}{\varepsilon}\sqrt{dx^2 + dy^2 + dz^2}$$

or the angular velocity so varies as to remain always proportional to the distance between the two particles. We have all along supposed the density of the fluid equal to unity. Remembering now our definition of a vortex filament, we see that any vortex filament must remain composed of the same fluid particles. Calling now k the cross section of any filament and l an indefinitely small length of the filament, that is

$$l = \sqrt{dx^2 + dy^2 + dz^2}$$

we have kl=const.; but l is proportional to ω, therefore ωk=const. or the product of the angular velocity of an infinitely small portion of any filament into its cross section is a constant. Call now k_1, k_2 the cross sections of a filament at points whose angular velocities are given by ω_1, ω_2; and let $d\tau$ denote an element of the filament;

$$\int d\tau \left(\frac{d\xi}{dx} + \frac{d\eta}{dy} + \frac{d\zeta}{dz}\right)$$

$$= -\int d\sigma [\xi \cos(nx) + \eta \cos(ny)$$

$$+ \zeta \cos(nz)] = -\int d\sigma \omega \cos(\omega n).$$

Now from the values of ξ, η, ζ, we see that the factor of $d\tau$ in the left hand integral is $=o$;

$$\therefore \int d\sigma\omega \cos(\omega n) = o$$

But at the ends of the portion of the vortex filament that we are considering, we have $\cos(\omega n) = \pm 1$, and for all other points $\cos(\omega n) = o$, and consequently our integral is equivalent to

$$\omega_1 k_1 - \omega_2 k_2 = o$$

or the product of the angular velocity and the cross section is a constant throughout the vortex filament. As each rotating element of the fluids implies rotation in every other element, we have that all the particles of fluid must be in motion, and consequently from the definition of vortex lines we see that these lines and consequently the vortex filaments cannot terminate within the fluid, but must either terminate in its surface or must return into themselves; the former of these cases is illustrated by the vortices formed in running water, and the latter by smoke rings.

If in the expression

$$\int\int\int d\tau \frac{d(w\mathrm{V} - v\mathrm{W})}{dx} + \frac{d(u\mathrm{W} - w\mathrm{U})}{dy} + \frac{d(v\mathrm{U} - u\mathrm{V})}{dz}$$

we substitute for u, v and w their values in terms of the derivatives of U,V,W, we find that this is equal to zero, and from it we obtain the striking equation

$$\int d\tau u \left(\frac{d\mathrm{W}}{dy} - \frac{d\mathrm{V}}{dz}\right) + v\left(\frac{d\mathrm{U}}{dz} - \frac{d\mathrm{W}}{dx}\right) + w\left(\frac{d\mathrm{V}}{dx} - \frac{d\mathrm{U}}{dy}\right)$$

$$= \int d\tau \mathrm{U}\left(\frac{dw}{dy} - \frac{dv}{dz}\right) + \mathrm{V}\left(\frac{du}{dz} - \frac{dw}{dx}\right) + \mathrm{W}\left(\frac{dv}{dx} - \frac{du}{dy}\right)$$

or

$$\int d\tau u^2 + v^2 + w^2 = 2\int\int\int d\tau [\mathrm{U}\xi + \mathrm{V}\eta + \mathrm{W}\zeta]$$

But we may write for U, V, W the values

$$\mathrm{U} = \frac{1}{2\pi}\int \frac{\xi' d\tau'}{r}$$

$$\mathrm{V} = \frac{1}{2\pi}\int \frac{\eta' d\tau'}{r}$$

$$W=\frac{1}{2\pi}\int\frac{\zeta' d\tau'}{r}$$

$$\therefore d\tau u^2+v^2+w^2=\frac{1}{\pi}\int\frac{d\tau'}{r}[\xi'\xi'+\eta\eta'+\zeta\zeta']$$

But the expression for the energy of the fluid is

$$T=\tfrac{1}{2}\int d\tau(u^2+v^2+w^2)$$

and by virtue of the above we have

$$T=\frac{1}{2\pi}\int\int\frac{d\tau d\tau'}{r}[\xi\xi'+\eta\eta'+\zeta\zeta']$$

We can now take up the simplest case of vortex motion viz., that in which the motion is parallel to one plane. If we assume this plane as xy and further make the motion independent of z we have the angular velocities around x, and y, and the velocity in direction of z equal to zero, or

$$\xi=\eta=w=o$$

We have thus

$$u=\frac{dW}{dy},\ v=-\frac{dW}{dx},\ \zeta=\frac{dv}{dx}-\frac{du}{dy}$$

from which

$$-\zeta=\frac{d^2W}{dx^2}+\frac{d^2W}{dy^2}$$

which gives

$$W=-\frac{1}{\pi}\int\zeta\log\rho\, d\sigma$$

when $d\sigma$ is an element of the plane xy. Of course as ζ is independent of z we might have obtained this by integration from our general value of W before given. Here ρ represents the distance of the element $d\sigma$ in the plane xy from any other point in that plane. Each vortex filament implies in any other particle of the fluid a velocity whose components are,

$$-\frac{1}{\pi}\cdot\frac{\zeta d\sigma}{\rho}\cdot\frac{d\rho}{dy}\text{ and }\frac{1}{\pi}\cdot\frac{\zeta d\sigma}{\rho}\cdot\frac{d\rho}{dx}$$

and whose magnitude is

$$\frac{1}{\pi}\cdot\frac{\zeta d\sigma}{\rho}\cdot$$

The direction of this velocity is given by the cosines

$$-\frac{d\rho}{dy},\quad\frac{d\rho}{dx}$$

and of the line ρ by

$$\frac{d\rho}{dx},\quad\frac{d\rho}{dy}$$

or the direction of the velocity is at right angles to ρ. Assume two quantities x_0, y_0, which define by the equations

$$x_0\int\zeta d\sigma=\int x\zeta d\sigma,$$
$$y_0\int\zeta d\sigma=\int y\zeta d\sigma.$$

Now if we regard ζ as the density of a mass distributed over the element $d\sigma$ of the plane xy we see that x_0, y_0 will represent the co-ordinates of the center of gravity of this mass. Now l, the length of an indefinitely small portion of our vortex filament cannot alter, consequently by virtue of the equation kl=constant, k or $d\sigma$ cannot vary with the time and by virtue of $k\omega$=constant, ω or ζ cannot vary with respect to the time; consequently, we have, by differentiating with reference to t

$$\frac{dx_0}{dt}\int\zeta d\sigma=\int\frac{dx}{dt}\zeta d\sigma$$

$$\frac{dy_0}{dt}\int\zeta d\sigma=\int\frac{dy}{dt}\zeta d\sigma$$

Substituting

$$\frac{dx}{dt}=u=-\frac{1}{\pi}\int\frac{\zeta' d\sigma'}{\rho},\ \frac{y-y'}{\rho'}$$

$$\frac{dy}{dt}=v=\frac{1}{\pi}\int\frac{\zeta' d\sigma'}{\rho},\ \frac{x-x'}{\rho}$$

we have

$$\frac{dx_0}{dt}\int\zeta d\sigma=-\frac{1}{\pi}\int\int\rho\rho' d\sigma d\sigma'\frac{y-y'}{\rho^2}$$

$$\frac{dy^0}{dt}\int\zeta d\sigma=\frac{1}{\pi}\int\int\rho\rho' d\sigma d\sigma'\frac{x-x'}{\rho^2}$$

The double integrals are $=o$ therefore

$$\frac{dx_0}{dt}=o\quad\frac{dy_0}{dt}=o$$

or the center of gravity of the filament does not change with the time. In the case of only one vortex filament let us write

$$\int\zeta d\sigma=m$$

for particles as at finite distance from the filament we have

$$u=\frac{dW}{dy}\ v=-\frac{dW}{dx},\ W=-\frac{1}{\pi}m\log\rho$$

for particles indefinitely near the filament we see that W, u, v, are infinite and depend upon the cross section of the fila-

ment and the angular velocity ρ. We also know that at the center of gravity of the filament u and $v=o$. Each particle of fluid that is at a finite distance from the filament we see has a uniform velocity of $\frac{m}{\pi\rho}$ and moves in a circle whose center is the center of gravity of the vortex filament. Suppose we now assume a number of filaments whose cross section is indefinitely small. Write in general

$$\int \zeta_i \, d\sigma_i = m_i$$

and let $x_i\ y_i$ denote the co-ordinates of the centers of gravity of the filaments at the time t and ρ_i their distances from the point (xy). Then for all points at finite distances from the filaments we have as before

$$u=\frac{dW}{dy},\ v=-\frac{dW}{dx},\ W=-\frac{1}{\pi}\Sigma m_i \log \rho_i$$

Now each filament inducing a certain amount of motion in every other particle of the fluid induces motion in the centers of gravity of every other filament, therefore the filaments change their places in the fluid. But here, as before, that portion of u, v which each vortex filament gives to its own center of gravity is $=o$. Suppose that the point from which the ρ_s are measured is one of the centers of gravity, for example $x\ y$. This will materially simplify the investigation by confining us exclusively to the influence of the system of vortices upon its different members and as this point x, y, is an arbitrary point no generality is lost. We have thus

$$u_1=\frac{dW_1}{dy_1}\quad v_1=-\frac{dW_1}{dx_1}$$

$$W_1-=\frac{1}{\pi}$$

$$(m_2 \log \rho_{12}+m_3 \log \rho_{13}+\ldots m_i \log \rho_{1i})$$

or briefly

$$W_1=-\frac{1}{\pi}\sum_{i=2}^{i} m_i \log \rho_{1i}.$$

We can now assume a function Q such that

$$Q=-\frac{1}{\pi}\Sigma m_i m_j \log \rho_{ij}$$

and then we have

$$m_1\frac{dx_1}{dt}=\frac{dQ}{dy_1},\ m_2\frac{dx_2}{dt}=\frac{dQ}{dy_2},\ \ldots\ldots$$

$$m_1\frac{dy_1}{dt}=-\frac{dQ}{dx_1},\ m_2\frac{dy_2}{dt}=-\frac{dQ}{dy_2},\ \ldots\ldots$$

A complete system of integrals cannot be in general obtained, but by observing one peculiarity of Q we can obtain two integrals—

$$\rho_{ij}=\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}$$

If we increase x_i, x_j, or y_i, y_j, by the same quantity, ρ_{ij} will be unaltered; this gives us then

$$\Sigma\frac{dQ}{dx_i}=o,\ \Sigma\frac{dQ}{dy_i}=o$$

or

$$\Sigma m_i\frac{dy_i}{dt}=o,\ \Sigma m_i\frac{dx_i}{dt}=o$$

from which

$$\Sigma m_i x_i = \text{const.}$$

$$\Sigma m_i y_i = \text{const.}$$

or the center of gravity of the system of vortex filaments is unaltered. Again since

$$m_i\left(\frac{dx_1}{dt}dy_i-\frac{dy_i}{dt}dx_i\right)=o$$

we have

$$dQ=o,\ \therefore\ Q=\text{const.}$$

the equation of the lines of flow of the fluid. Introduce now polar co-ordinates,

$$x_1=\rho_1\cos\vartheta_1\quad x_2=\rho_2\cos\vartheta_2+\ldots.$$

$$y_1=\rho_1\sin\vartheta_1\quad y_2=\rho_2\sin\vartheta_2+\ldots.$$

we have by these substitutions

$$m_1\rho_1\frac{d\rho_1}{dt}=\frac{dQ}{d\vartheta_1},\ m_2\rho_2\frac{d\rho_2}{dt}=\frac{dQ}{d\vartheta_2}\ldots\ldots$$

$$m_1\rho_1\frac{d\vartheta_1}{dt}=-\frac{dQ}{d\rho_1},\ m_2\rho_2\frac{d\vartheta_2}{dt}=-\frac{dQ}{d\rho_2}\ldots$$

If now we increase all the ϑ's by the same quantity, Q will evidently remain unaltered and we have the equation,

$$\Sigma\frac{dQ}{d\vartheta_i}=o$$

The first row now gives by addition

$$\Sigma m_i\frac{d\rho_i}{dt}=o$$

or

$$\Sigma m_i \rho_i^2=\text{const.}$$

Now let us suppose d to remain unchanged but ρ to become $n\rho$, then $\log\rho$ becomes $\log\rho+\log n$, and $\log\rho_{ij}$ becomes $\log\rho_{ij}\,1+\log n$, and in consequence Q will be increased by

$$-\frac{1}{\pi}[m_1m_2 \log n + m_1m_3 \log n + m_2m_3 \log n + \ldots . m_j\, n_j \log n +]$$

or

$$-\frac{1}{\pi} \log n\, \Sigma\, m_i\, m_j$$

and consequently we have

$$\Sigma \frac{d\,Q}{d \log \rho_i} = -\frac{1}{\pi} \Sigma\, m_i\, m_j$$

or

$$\Sigma \rho_i \frac{dQ}{d\rho_i} = -\frac{1}{\pi} \Sigma\, m_i\, m_j$$

But we have also

$$\frac{d\,Q}{d\rho_i} = -m_i\, \rho_i \frac{d\vartheta_i}{dt}$$

Substituting this value gives

$$\Sigma\, m_i\, \rho_i^2 d\vartheta_i = \frac{dt}{\pi} \Sigma m_i\, m_j$$

Assume now the case of only these vortex filaments existing in the fluid.

The equations $\Sigma m_i x_i =$ const.; and $\Sigma m_i y_i =$ const. become

$$m_1\rho_1 \cos\vartheta_1 + m_2\rho_2 \cos\vartheta_2 + m_3\rho_3 \cos\vartheta_3 = C_1$$
$$m_1\rho_1 \sin\vartheta_1 + m_2\rho_2 \sin\vartheta_2 + m_3\rho_3 \sin\vartheta_3 = C_2$$

Multiply the first equation by cos ϑ, the second by sin ϑ and add,

$$m_1\rho_1 + m_2\rho_2 \cos(\vartheta_2 - \vartheta_1) + m_3\rho_3 \cos(\vartheta_3 - {}_1\vartheta) = C_1 \cos\vartheta_1 + C_2 \sin\vartheta_1$$

Again multiply the first by sin ϑ, and the second by cos ϑ, and add

$$m_2\rho_2 \sin(\vartheta_2 - \vartheta_1) + m_3\rho_3 \sin(\vartheta_3 - \vartheta_1) = C_2 \cos\rho_2 - C_1 \sin\vartheta_1$$

Again,

$$Q = -\frac{1}{\pi}[m_1m_2 \log\rho_{12} + m_1m_0 \log\rho_{13} + m_2m_0 \log\rho_{23} + \ldots]= \text{const.}$$

and

$$m_1\rho_1^2 + m_2\rho_2^2 + m_3\rho_3^2 = \text{const.}$$

Through these four equations we may express any four of the quantities ρ_1, ρ_2, ρ_3, $\vartheta_2 - \vartheta_1$, $\vartheta_3 - \vartheta_1$ interim of the fifth; for example ρ_1; then the equations

$$m_1\rho_1 d\vartheta_1 = -\frac{dQ}{d\rho_1} dt$$

and

$$\sum_1^3 m_i \rho_i^2 d\vartheta_i = \frac{dt}{\pi} \sum_1^3 m_i m_j$$

will enable us to express ϑ_1 and t as functions of ρ_1 and afford a complete solution of the problem.

Assume now only two vortices, and take the origin of co-ordinates at their common center of gravity. This point does not move, and we have

$$\frac{d\vartheta_1}{dt} = \frac{d\vartheta_2}{dt}$$

Q in this case becomes

$$-\frac{1}{\pi} m_1 m_2 \log(\rho_1 + \rho_2) = \text{const.}$$

and also

$$\Sigma m_i \rho_i^2 = m_i \rho_i^2 + m_i \rho_i^2 = \text{const.}$$

from these two equations we obtain

$$\rho_1 = \text{const.}, \quad \rho_2 = \text{const.}$$

Again the equation

$$\Sigma m_i \rho_2^2 d\vartheta_1 = \frac{dt}{\pi} \Sigma m_i m_j$$

becomes

$$m_1\rho_1^2 d\vartheta_1 + m_2\rho_2^2 d\vartheta_2 = \frac{dt}{\pi} \cdot m_1 m_2$$

giving

$$\frac{d\vartheta_1}{dt} = \frac{d\vartheta_2}{dt} = \frac{1}{\pi} \cdot \frac{m_1 m_2}{m_1\rho_1^2 + m_2\rho_2^2}$$

If the direction of rotation of both vortex filaments is the same $m_1 m_2$ [which depend on ζ_1 and ζ_2] have the same sign. But suppose $m_1 = -m_2$, then

$$\frac{d\vartheta_1}{dt} = \frac{1}{\pi} \frac{m_1}{\rho_2^2 - \rho_1^2}$$

But we have now for the center of gravity

$$x = \frac{m_1 x_1 - m_1 x_2}{m_1 - m_1} = \infty$$

$$y = \frac{m_1 y_1 - m_1 y_2}{m_1 - m_1} = \infty$$

or the center of gravity of the two filaments lies at infinity. Their velocities = their angular velocity x by the distance from the center of gravity differ from each other by an infinitely small quantity and can be expressed by

$$\frac{\rho_1 + \rho_2}{2} \frac{d\theta_1}{dt}$$

but by our preceding equation giving the value of $\frac{d\theta_1}{dt}$ this is

$$= \frac{1}{2\pi} \frac{m_1}{\rho_2 - \rho_1}$$

the direction of the motion is, of course, perpendicular to the line giving the cen-

ters of gravity of the two filaments. The particles of fluid lying between the filaments move forwards in the same direction as do the filaments, the one-half way between them moving four times as fast. Let us suppose that the vortex filaments at the beginning of the motion lie on the axis of x at equal distances from the origin, then the particle above referred to will lie at the origin. Also write $\frac{\rho_2-\rho_1}{z}=a$ the absolute distance of each filament from the origin. We have then for the co-ordinates of the filaments at the time t (a, y') and, by virtue of what has been said the co-ordinates of the particle originally at the origin will be $(0,4y')$. The equations of the lines joining these two points are

$$\begin{vmatrix} x, & y, & 1 \\ o, & 4y', & 1 \\ \pm a, & y', & 1 \end{vmatrix} = o$$

The intersections of these lines with $y=o$ are given by

$$x=\pm\frac{4a}{3} \text{ or } x=\pm\tfrac{2}{3}(\rho_2-\rho_1)$$

that is, the lines joining the particle half way between the filaments with the centers of gravity of the same pass through fixed points on the line joining the original positions of the centers of gravity of the filaments. The points lie outside of the original positions of these centers of gravity and at an absolute distance from

$$\text{them}=\tfrac{1}{3}(\rho_2-\rho_1)=\tfrac{2}{3},\ \frac{\rho_2-\rho_1}{z}.$$

The particles of fluid that lie in the plane bisects at right angles the line joining the two vortex filaments will remain in this plane. If this plane be considered as a fixed boundary, we have by considering one of the filaments the case of a filament moving parallel to a fixed plane which limits the extent of the fluid.

Let us now assume that the vortex filaments are so arranged as to form the continuous surface of an elliptic cylinder of finite cross section, and further assume ζ as constant for every point of this cross section. As the same particles of fluid constantly remain in any vortex filament, the bounding ellipse of the cross section of the fluid will always be composed of the same fluid particles. The equation of this line will be a function of x, y and t, and may be written for the moment as

$$f(x,Y,t)=o.$$

Then by virtue of the above we have generally

$$\frac{df}{dt}+\frac{df}{dx}\frac{dx}{dt}+\frac{df}{dy}\frac{dy}{dt}=o$$

or

$$\frac{df}{dt}+u\frac{df}{dx}+v\frac{df}{dy}=o$$

But,

$$u=\frac{d\mathrm{W}}{dy},\ v=-\frac{d\mathrm{W}}{dx}.$$

Therefore this equation becomes

$$\frac{df}{dt}+\frac{d\mathrm{W}}{dy}\frac{df}{dx}-\frac{d\mathrm{W}}{dx}\frac{df}{dy}=o.$$

Now the general equation of our ellipse is

$$f=\alpha x^2+2\beta xy+\gamma y^2-1$$

when α, β, γ are functions of t. Assume another system of co-ordinates coinciding with the axes of the ellipse and also passing through its center; then

$$x'=x\cos\theta+y\sin\theta$$
$$y'=-x\sin\theta+y\cos\theta.$$

Call a and b the semi-axes of the ellipse then we have for its equation,

$$\frac{x'^2}{a^2}+\frac{y'^2}{b^2}=1.$$

Substituting for x', y', their values as given in terms of x and y, and this becomes,

$$x^2\frac{b^2\cos^2\vartheta+a^2\sin^2\vartheta}{a^2b^2}+2xy\frac{[(b^2-a^2)\cos\vartheta\sin\vartheta]}{a^2b^2}$$
$$+y^2\frac{b^2\sin^2\vartheta+a^2\cos^2\vartheta}{a^2b^2}=1$$

Comparing this with

$$\alpha x^2+2\beta xy+\gamma y^2=1$$

and we obtain,

$$a^2b^2\alpha=b^2\cos^2\vartheta+a^2\sin^2\vartheta$$
$$a^2b^2\beta=(b^2-a^2)\cos\vartheta\sin\vartheta$$
$$a^2b^2\gamma=b^2\sin^2\vartheta+a^2\cos^2\vartheta$$

In these a and b are constant, but α, β, γ, ϑ are functions of t. Now W satisfies the equation

$$\frac{d^2\mathrm{W}}{dx^2}+\frac{d^2\mathrm{W}}{dy^2}=-\zeta$$

for all points in the interior of the ellipse, and its value is obtained by integration of the equation

$$\frac{\mathcal{E}}{2}ab\int_o^\infty \frac{1-\frac{x^2}{a^2+\lambda}-\frac{y^2}{b^2+\lambda}}{\sqrt{(a^2+\lambda)\,(b^2+\lambda)}}\,.\,d\lambda$$

The integral of this is

$$\frac{\mathcal{E}}{2}ab\left(2\log\frac{1}{a+b}+1\right)-\frac{\mathcal{E}}{a+b}\,(bx'^2+ay'^2)$$

or, since we only use the derivatives of W, we may write it,

$$W=C-\frac{\mathcal{E}}{a+b}\,(bx'^2+ay'^2)$$

for all interior points, and for points at the boundary. If now we write for brevity,

$$A=b\cos^2\vartheta+a\sin^2\vartheta$$
$$B=(b-a)\cos\vartheta\sin\vartheta$$
$$\Gamma=b\sin^2\vartheta+a\cos^2\vartheta$$

we have

$$W=C-\frac{\delta}{a+b}(Ax^2+2Bxy+\Gamma y^2).$$

Let us examine again the condition that we obtained for the bounding line of the cross section of the cylinder, viz:

$$\frac{df}{dt}+\frac{df}{dx}\frac{dW}{dy}-\frac{df}{dy}\frac{dW}{dx}=o$$

we have

$$\frac{df}{dt}=\frac{da}{dt}x^2+2\frac{d\beta}{dt}xy+\frac{d\vartheta}{dt}y^2$$
$$\frac{df}{dx}\frac{dW}{dy}=-(ax+\beta)(\vartheta Bx+\Gamma y)\frac{4\delta}{a+b}$$
$$-\frac{df}{dy},\frac{dW}{dx}=(\beta x+\vartheta y)(Ax+By)\frac{4\delta}{a+b}.$$

Equating to zero the co-efficients of x^2 xy, and y^2 separately we have

$$(a+b)\frac{da}{dt}=4\mathcal{E}(aB-\beta A),$$
$$(a+b)\frac{d\beta}{dt}=2\mathcal{E}(a\Gamma-\gamma A),$$
$$(a+b)\frac{d\gamma}{dt}=4\mathcal{E}(\beta\Gamma-\gamma B).$$

If ϑ cannot be determined as such a function of t as to satisfy these equations then will our equation of condition

$$\frac{df}{dt}+\frac{df}{dx}\frac{dW}{dy}-\frac{df}{dy}\frac{dW}{dx}=o$$

hold for all points in the cross section.

Forming the derivatives of a, β, γ with respect to t and transforming them by reference to the values of A, B, Γ we find that the function of t sought is given by the equation

$$\frac{d\vartheta}{dt}=2\,\mathcal{E}\frac{ab}{(a+b)^2}.$$

We have thus the value of the angular velocity with which the cylinder rotates around its axis. The rotation of the cylinder also induces relative motions among the component vortex filaments. These are obtained by regarding x' and y' as functions of t. We have by differentiation

$$\frac{dx'}{dt}=y'\frac{d\vartheta}{dt}\,\frac{dy'}{dt}=-x'\frac{d\vartheta}{dt}$$

and the other components of the velocities in the directions of x' and y' are

$$\frac{dW}{dy'},-\frac{dW}{dx'}$$

Therefore we have by combining these

$$\frac{dx'}{dt}=\frac{dW}{dy'}+y'\frac{d\vartheta}{dt}$$
$$\frac{dy'}{dt}=-\frac{dW}{dx'}-x'\frac{d\vartheta}{dt}$$

Now,

$$\frac{dW}{dy'}=-\frac{2a\mathcal{E}}{a+b}y',\text{and},\frac{dW}{dx'}=-\frac{2\mathcal{E}bx'}{a+b}$$

and

$$\frac{d\vartheta}{dt}\mathcal{E}=2\frac{ab}{(a+b)^2}.$$

Therefore

$$\frac{dx'}{dt}=-\frac{2\mathcal{E}a^2}{(a+b)^2}y',\ \frac{dy'}{dt}=\frac{2\mathcal{E}b^2}{(a+b)^2}x'.$$

Differentiate each of these for t and with

$$\theta=2\mathcal{E}\frac{ab}{(a+b)^2}$$

and we have after integration of the resulting well-known powers

$$x'=a\,l\cos(\theta t+i)$$
$$y'=b\,l\cos(\theta t+i)$$

when l and i are the constants of integration and determine the particle of fluid to which x' y' have reference. $\rho>1$ because then for the cases when $\cos(\theta t+i)=1$ we should have $x'>a$ which cannot be, $\therefore$ l is a proper fraction. θ of course denotes the angular velocity of the cylinder, or

$$\frac{d\vartheta}{dt}=\theta$$

from which $d=\theta t$.

Solving the equations

$$x'=x\cos\vartheta+y\sin\vartheta$$
$$y'=-x\sin\vartheta+y\cos\vartheta$$

for x and y gives

$$x=x'\cos\vartheta-y'\sin\vartheta$$
$$y=x'\sin\vartheta+y'\cos\vartheta$$

Substituting for x', y' and ϑ their values these become

$$x=al\cos(\theta t+i)\cos\theta t \\ -bl\sin(\theta t+i)\sin\theta t$$

$$y=al\cos(\theta t+i)\sin\theta t \\ +bl\sin(\theta t+i)\cos\theta t$$

by expanding the quantities cos sin $(\theta t+i)$ and collecting the terms these equations may be written

$$x=\frac{a+b}{2}l\cos(2\theta t+i)+\frac{a-b}{2}l\cos i$$

$$y=\frac{a+b}{2}l\sin(2\theta t+i)-\frac{a-b}{2}l\sin i$$

by differentiation with respect to t we obtain

$$\frac{dx}{dt}=-(a+b)\theta l\sin(2\theta t+i)$$

$$\frac{dy}{dt}=(a+b)\theta l\cos(2\theta t+i)$$

from which

$$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=(a+b)\theta l=2\zeta l.\frac{ab}{a+b}$$

and also,

$$\left(x-\frac{a-b}{2}l\cos i\right)^2+\left(y+\frac{a-b}{2}l\sin i\right)^2=\left(\frac{a+b}{2}\right)^2l^2$$

From these equations we see that each vortex filament moves in a circle with uniform angular velocity, the time of rotation being evidently $\frac{\pi}{\theta}$; and that the position of the center and the radius of the circle varies for different filaments. Suppose one of the semi-axes a or b to be infinitely greater than the other; this gives $\theta=o$, and consequently $\vartheta=o$, or the straight line which has become the limiting case of the ellipse does not rotate. If $a=b$ our ellipse becomes a circle, and we have $\theta=\frac{\zeta}{2}$; in this case there is no change of the relative positions of the vortex filaments, but they all rotate around the common central axis with angular velocity ζ.

THE TREATMENT OF IRON TO PREVENT CORROSION.*

By Professor BARFF, M. E.

From "Journal of the Society of Arts."

It is now more than two years ago since I had the honor of introducing to your notice in this room a process for the prevention of corrosion in iron; and it was through the Society of Arts that it was first made known to the public. My paper then met with a very flattering reception, both from this Society and from the public press. An article appeared in the *Times*, which, written with great ability, gave to it a claim to public consideration, and with one or two trifling exceptions, it was well spoken of in all directions. The exceptions to the general desire manifested to receive well a process which, if practicable, was highly desirable, as being likely to increase largely the use of iron, and to enable it to replace, for certain uses, other metals which were injurious to health, were not founded on reasons of sufficient force to cause me to notice and answer them; and some of them, which could only result from the inner consciousness of those who made them, without an atom of proof, have been fully answered by the work which has been subsequently carried out on a larger scale at my laboratory at Kensington. I was blamed by some for bringing out my invention too soon; but I think that you will see that, by following the course I did, I was enabled to

* A Lecture before the Society of Arts.

give specimens to those who have very thoroughly tested for a long time the goodness of the process; so that I, to-night, stand before you armed with a large amount of experience, and with testimony from others on whose word and judgment reliance can be placed. In answer to some remarks made by the Chairman on the night when I read my first paper, Admiral Selwyn stated that the black oxide of iron had stood the action of sea water for ages, and he therefore advised us to look to nature for a proof of its enduring properties, on the shores of New Zealand, where quantities of it had existed unchanged since the creation of the world. No one, who has a right knowledge of its properties, could doubt its power to resist atmospheric influences, and even the action of sea water; but the doubt that did exist in the minds of many was whether it could be produced artificially on iron, so as to keep its place, and enable the iron beneath it to resist their action as well; or, rather, I should say, whether its adherence to the iron was so complete and perfect as to protect it from them. Pieces of iron will be shown you which were exhibited on that occasion—which have been exposed ever since, and you will be able to judge for yourselves whether the protection afforded by the black oxide is complete and perfect, or not. I feel that I ought, before proceeding further, to notice briefly one or two of the exceptions to which I have alluded in the reception which my first paper met with. One was that the process was not original; that I had no right to claim as my discovery what had been known to chemists for years. I will quote from my paper, and then you will see whether I deserve this charge or not. "In every school where chemistry is taught, in the most elementary lecture on hydrogen, the pupils are told that, if they pass steam over red-hot iron filings, contained in an iron tube, they will be able to collect and burn the hydrogen gas at the opposite end of the tube to where the steam enters. For a long time it was thought that the particles of black oxide formed by this decomposition of the steam were pulverulent, and could not be made to cohere into a solid mass." It is manifest that I could not claim as my invention what I stated was already known. Another exception was that the process might be of use for small articles—pots, pans, &c.,—but that it could not be applied to large articles, and, even if it could, it would so materially weaken the iron that dependence could not be placed on its strength; in fact, if I remember rightly, a solemn warning was given to persons not to trust to it. Now, that the process is only applicable to pots and pans, &c., the articles before you will disprove. At the beginning of my experiments—I did not wish to incur a great outlay, and, therefore, the chambers, or muffles used were not large. A year and a half ago, I had a chamber built of fire brick, and that has been in use ever since. In it articles six feet long have been treated; and if the chamber were 12 feet long, or 20, articles of such lengths could be treated as well as those which you see before you. As to the action on the strength of the iron, bars treated have been tested for breaking and tensile strain, and the result is that the strength of the iron is not affected, and the persons who tested them assert that they would not hesitate to use the process because of any injurious effect which it has on the strength of iron. I need not do more this evening than briefly remind you that my process consists in oxidizing the surface of iron by means of superheated steam. In my former paper a description of the rusting of iron will be found; it is enough for my present purpose to state that the black oxide of iron is unaltered by any of the ordinary influences which produce red rust, and which therefore cause the destruction of iron.

The points which I have to bring before your notice this evening are those which two years' experience has enabled me to discover in the method of working the process; and these are very important, because formerly there was a want of certainty in performing it which gave very unequal results. During the last eighteen months I have been able to give constant attention to it, which I was not able, from my engagements, to do before, and now I can assert that it can be conducted with ease and with perfect certainty. In the specification of my patent, the method of performing the necessary operations is given, but considerable practical experience is required, which it

is impossible to describe in writing. In the earlier experiments performed at my laboratory at Kilburn, it was often found that the coating of black oxide scaled off wrought-iron articles. This is never the case now.

This scaling resulted from an insufficient and irregular supply of steam to the muffle during the operation, whereby air was not excluded, but was often forced in from the want of a sufficient pressure of water on the superheating pipes. Air must be completely excluded from the oxidizing chamber, because, if the oxidation of the iron depend, during any part of the process, on the oxygen in the air, such oxide formed will not adhere to the iron properly. This I have proved by submitting iron to oxidation by dry air, and in every case wrought-iron has, when so treated, lost its coating, which has flaked off in scales; and in the case of cast-iron, the oxide on exposure comes off in a very short time, and therefore does not provide perfect protection to the iron. If, however, the air forced into the chamber be moist, the same result occurs with wrought-iron, but with cast-iron the coating formed does adhere for a time, and the length of its adherence is proportionate to the quantity of moisture present in the air. If the air be forced into the ordinary chamber from a vessel in which it is in contact with water, and if the temperature of the room in which this vessel is be high, as in such a case it must be, the quantity of moisture converted into steam, when at the temperature of the iron to be oxidized, will be great, in fact, enough to oxidize the iron, for very little steam is required to oxidize a great weight of iron; but then the oxygen of the air will take part in the action, and wherever the iron is oxidized by the oxygen of the air its adherence will not be complete, and though by being mingled with the other oxide it may have a certain amount of stability, yet in a short time it will come off. I exhibit two specimens in illustration of this, one of cast the other of wrought iron, both of which have been exposed in the open for some time; the piece of cast iron did not rust for some time after it was exposed, but the wrought iron flaked and rusted at once. It appears, therefore, to be absolutely necessary to secure a good result, that air must be completely excluded from the oxidizing chamber.

For a long time I experienced considerable trouble from the appearance of small spots of rust on articles otherwise well coated, which were immersed in water. The spots of rust appeared to increase in size, but on examination it was found after washing off the rust, which could be easily removed, that it originated from small openings in the coating of black oxide. It required a magnifying glass to see these openings; the rust did not spread by more of the iron surface rusting, but because the rust formed in these minute cracks was carried out by the water in which the articles were, and was therefore diffused about. Such rusting has no effect on the strength of the iron, and after a few cleanings it ceases altogether. However, I felt that it was very necessary to prevent it, and that led me to seek carefully for its cause. When iron is heated it expands, when cooled it contracts. If iron be heated in an oxidizing chamber it expands, its pores, so to speak, open. If a jet of superheated steam be admitted at a temperature lower than that of the iron in the chamber, the iron will contract, and then will decompose the steam—of course it must be at a sufficiently high temperature to do so. Now, the iron will gradually get hotter, and will expand again, and the first thin coating of black oxide will be in part cracked, and as the oxide goes on forming it will in part cover and fill up these cracks, but I think—in fact I am sure—that it does not do so perfectly, and hence some of them remain, the iron at the bottom of them being coated with but a very thin film of oxide. Reasoning in this way, I came to the conclusion that no contraction must be allowed to take place in the iron after the oxidizing action had commenced, and to secure this, the ordinary chamber is always kept at a much lower temperature than the superheater; and now it is never allowed to rise above five or six hundred deg. Fahr. before the superheated steam is admitted, and the steam is never allowed to pass in at a temperature less than one thousand degrees. This for a long time has been our invariable plan of work, and in no case whatever have we experienced any failure as long as the apparatus was sound. Many have tried

experiments, independently of me, but in all cases I have heard complaints that they have not succeeded, and I feel sure that the want of success has been due to one of the causes I have mentioned, or to another which I described in my former paper, viz., the presence of moist steam.

When asked by my lamented friend, your late able Secretary, to read another paper on my iron process, I willingly consented to do so, because it would give me the best method of making public and explaining the difficulties I had met with and overcome, but which were still troubling others who take a real interest in the process. It has always been my opinion that the best way of forming the black oxide on iron is to conduct the process by means of superheated steam alone, because the steam, being the source of heat to the iron, raises its temperature to that at which it can decompose steam, so that oxidation commences immediately the iron is hot enough. When the iron is heated in the chamber, before the steam is allowed to act upon it, there is always danger of air getting into the chamber and forming a film of oxide before the steam gets to work, and this is a thing to be avoided. I have only been able to experiment with superheated steam alone on a small scale, and the large chamber has flues up its sides which would conduct off the heat if it were attempted to raise its temperature by superheated steam alone. I may be here misunderstood. The flues at the sides of the chamber would cause cold air to circulate round it, and the heat from the superheated steam would thus be conveyed away. The experiment I did perform was with an iron muffle, similar to that which was used in the early experiments. This was surrounded with fire-clay, to act as a non-conductor of heat. Steam at 1,500 deg. Fahr., was injected into it for a short time, and then the articles to be treated were put inside it, and the steam was again let in. In a short time the muffle and its contents became red hot, and, after a few hours, were found to be well coated with black oxide. I could not work this process on a larger scale, for I have already, through the assistance of a friend, expended a large sum of money in experiments, which have resulted in my being able to state that my process is now commercially perfect, and is waiting the enterprise of gentlemen in the iron trade to take it up and use it. When I last had the pleasure of addressing you I spoke with diffidence, and I could not give definite answers to those who questioned me; now I can speak with confidence, and though I cannot perhaps reply to all questions to your entire satisfaction, yet I can assert that there is no reason why this process should not be largely adopted. Adopted it will be, for it is a success, and has been proved to be so by the testimony which I have the pleasure of submitting to you. But, before submitting these testimonials, allow me to say a few words about the properties which this artificially-formed black oxide possesses, as to which I could not testify in my first paper. It gives great hardness to the surface of iron, when the coating is sufficiently thick; by this I mean when it is even less than one-sixteenth of an inch. An ordinary flat rasp will not remove it without great labor; it resists emery powder, as I stated in my first paper; but now I have proved that it will for a long time resist a rasp, and will remove pieces of steel from it. This has been witnessed by many, among these by M. St. Yves, Engineer in Chief of the Ponts et Chausées, Paris, who was sent over to report on the process, and Professor Frankland. Substances which adhere to iron, zinc and enamel will not adhere to it. Saucepans in which arrowroot and other sticky substances are cooked can be cleaned with the greatest ease, after they have been oxidized, a simple wipe removing all dirt. I exhibit a saucepan which has been in use at my house for two years, and a wrought iron stew-pan which has done about six months' service. I have a whole set of stew-pans at home, and my cook prefers them much to any others. Dr. Mills, of Glasgow, testifies to the same property. You see before you a urinal which was in constant use at my laboratory for months, and was then sent to be used here two months ago. There are no deposits on it. I had water evaporated in an oxidized pan for six weeks—common tap water; the water never boiled, but was slowly evaporated. The deposit found was removed with a duster; it did not stick to the iron. This

is a matter of great importance to boilers, and for pipes through which water is to be conveyed.

Now, articles coated can be submitted to a high temperature, even a red heat, without the coating being injured or disturbed.

I have written to most of the gentlemen to whom specimens were sent two years ago, after my first paper, and most of them have kindly replied. Their letters I have published, even those which show that at present my process does not meet their requirements. Where it has not answered, I have added in a note my own remarks for what they are worth. At present I fear that iron wire cannot be treated successfully—the wire can be treated and will not rust, but it cannot be bent to a sharp curve without the coating coming off. I show a specimen to prove that the wire, when not bent, does not rust, and that articles made of wire can be made non-rustable provided they be not stretched beyond a certain point. Riveted iron plates can be most successfully treated; the process tightens the rivets and assists the caulking; the plates before you show this. I have not solved the question of riveting plates after treatment, but I am sanguine that I shall be able to do so. Weights are treated for the Government, and submitted by Mr. Chaney to tests, and the process is now recommended by that Department for the standard weights throughout the country. I also exhibit two specimens, one of oxidized, and the other of common iron, on which gold leaf has been put in the ordinary way, with oil gold size, and I think they illustrate well that even where it is desired to paint or gild iron to be placed in exposed situations, it is very desirable to have it first treated by my process; both the specimens have been out of doors for two months, exposed to rain and snow; for some days they were completely buried in snow.

I regret to say, gentlemen, that I cannot speak very definitely as to the cost of the process. I do not wish to delude any one by a statement that it can be done for so much per ton. It is simply impossible to do this, as you will see. Hollow goods—such as saucepans, &c.—will take up a much larger space per ton than a ton of 56 lbs. weight, and this shows how fallacious any general statement on this head must be. My experiments have not been conducted with special regard to economy, but to efficiency, and, having settled this point, economy must now be inquired into. This is rather the work of the manufacturer than mine; but this much I can say for your guidance, that, even with my means, the cost for light articles does not exceed that of galvanising. But I do think that, if the treatment gives a permanent protection to the iron, that articles treated by it should command a higher price than those which have been treated by a less enduring process. My experience tells me that different kinds of cast iron behave differently under treatment. Some kinds require longer exposure to the action of the superheated steam than others. Why this is I cannot as yet find out. Of this class is the iron in which the carbon is in more perfect chemical combination as a carbide. This iron is whiter than the other kinds. There are before you specimens which have resisted the rusting action of air in the presence of water. The two statues exhibited and other articles are of a different kind of iron; they required a shorter exposure, and have stood equally well. I have not yet met with any sample of cast iron which could not be properly treated. Wrought iron reqires a somewhat different treatment; a lower temperature, about 900 deg. Fahr., suits it best, and steel also. It is not well to expose articles very different in bulk at the same time; all that are put into the muffle should be pretty nearly equal in bulk. I mean that very heavy articles, such as 56 lbs. weight, should not be treated with these gutter spouts. Cast and wrought iron should not be treated together; but all these are matters which a little experience will regulate perfectly. Sometimes the sand from the mould adheres to cast iron; this is often the case inside pipes; it is of no moment, for the sand itself gets so firmly fixed on the coating of black oxide, that it assists in protecting the iron. I have proved this by severe experiments. In clearing off the rust from iron before it is submitted to the action of superheated steam, the usual method is employed, it is immersed in dilute oil of vitriol, and after washing is put into

some bran water; this last operation is to remove any basic sulphate of iron from the surface. If this basic sulphate is not completely taken away when the iron is heated, it is reduced, and red oxide of iron is left on the surface, which has the color of the red oxide used for paints, and you will see some articles so colored before you; this red oxide does not prevent the formation of the black oxide beneath it, and does not interfere with its stability, it is therefore of no importance, except to the appearance of the articles.

REPORTS OF ENGINEERING SOCIETIES.

The American Society of Civil Engineers.—At a regular meeting of the Society held on the 2d of April, Major-Gen. J. G. Barnard read a paper by the late Gen. B. S. S. Alexander, descriptive of the construction of the *Minots Ledge Light-House*, off Cohasset, ten miles south of entrance to Boston harbor. After mentioning several of the most noted and dangerous rocks throughout the world, upon which light-houses have been built, the paper relates in a most interesting way the first visit of the author to the Minots Ledge, the delays encountered before a reconnoisance upon the rock could be effected, the plan of operations devised while waiting; then the date of commencement, on April 1, 1853; what was done in preparing the foundation, cutting the stones, and laying and securing them; the number of hours worked per annum, ranging from 130 to 377 in the seven successive years that were spent in the construction of the light-house; the rules adopted for doing the most work in the time, and at the least risk of drowning, and the cost $300,000. It was completed June 29, 1860, and the first light was shown on Nov. 15 of the same year. The paper was exceedingly interesting throughout, and was fully illustrated by diagrams, showing the excavation of the foundation, and the plan of laying the stones and securing them against failure.

Gen. Barnard made some remarks upon the settlement of Pier No. 2 of the South Street bridge, Philadelphia, a paper upon which, by D. Mc.N. Stauffer, C.E., Philadelphia, was read before the Society, Sept. 4, 1878, and published in the transactions of the same month. General Barnard illustrated his remarks by reference to his own experience with the foundations of Fort Livingston at the mouth of the Barataria River (or bayou) La., and the settlement of the Fort, which, when examined a few years ago by a commission of officers, showed a sinking of nearly four feet. Fort Bienvenu, built in one of the worst marshes in the vicinity of New Orleans, has sunk three or feet feet. Fort Jackson, on the Mississippi River, below New Orleans, had settled perhaps one foot by the year 1840. It was a very heavy work; the parapet and coping were not laid, and, being abandoned for many years previous to the late war in the South, bushes and tall weeds grew up and hid it, which the raising of the levee further aided, so that river pilots used to say that the old fort was sinking out of sight. The sinking of the pier of the South street bridge, General Barnard considered, was occasioned by the escape laterally of the soft mud in the pocket which existed under one corner of the foundation; that the settlement commenced from the date of loading the foundation, and that the lesson to be drawn was to make more numerous borings, and prevent lateral escape of the mud before laying a heavy and costly foundation.

Mr. Dresser remarked: "In the case of the construction of a large hospital in New York, it was supposed that there was solid rock foundation to build on. Excavations developed the fact that under a portion of the building a pocket filled with mud or clay existed, into which the rock sloped suddenly. The question was to so join the foundations upon the solid and the compressible bottom that the settlement in the latter case would not crack the superstructure where compression ceased and solid rock commenced. This was done by excavating a trench in the rock and filling it to the depth of four or five feet with the material taken from the 'pocket.' The structure was completed upon this, and no crack shows itself up to the present time."

Mr. Collingwood gave an account of the removal of pockets of material under the caissons of the East River Bridge, and also of the foundations of piers etc., in the approaches in the "Swamp," as follows: In preparing the bed for the New York caisson, much the same thing was done as was mentioned by the last speaker (Mr Dresser). The bed-rock was quite uneven, and it was struck at several points at about two feet above the level finally fixed upon as the stopping place of the edge of the caisson. Over all other parts the sand and gravel varied in depth from one to sixteen feet, as found by borings. The high points in the rock were cut away, so as to leave a space of at least a foot everywhere between the caisson and the rock, and loose sand was thrown into the space. At other points there was found in pockets some fine sand, which we were fearful might move. The sand was taken out from a trench along the edge of the caisson, and this was filled with concrete, so as to cut it off and prevent possibility of movement. No great settlement was expected, but these precautions were taken so as to insure uniformity in settlement, and, as far as we could judge, they were entirely successful.

In founding the New York approach across the "swamp" (so called), considerable difficulty was anticipated, and we were assured by persons acquainted with the locality that piling would be necessary. Careful preliminary borings showed that a sound, clean, sandy bottom could be everywhere reached, with a maximum depth of eleven feet below the level of mean high tide. I decided, therefore, to make an open cut, and remove all compressible material.

Fortunately, there was a thin bed of clay over the sand, and by removing this over a

portion of the bed only at a time, and then concreting it, we had no great difficulty in keeping the water down.

In the last pit the water broke in with considerable force at one spot, but by putting in an interior line of sheeting, and leaving an open way for the water to the pumps, we were able to build up the other parts of the masonry for a few feet, and then by pumping rapidly and inclosing cement in bagging, to get the foundation in over the small uncovered portion. No settlement has since occurred.

PROCEEDINGS OF THE ENGINEERS' CLUB OF PHILADEDPHIA.—*March 15th*, 1879.—Mr. C. E. Buzby exhibited a model of Travers' Iron Railway Tie. It is in use on the Philadelphia & Baltimore Central Railroad, near Lamokin. The device dispenses with all spikes, bolts, nuts or fish plates, and drilling or punching the rails, avoiding fractures from such causes.

The iron tie, it is claimed, will outlast twelve renewals of the ordinary tie at one-half the cost to keep in repair.

Each tie is recessed under its rails, and along the bottom of the recesses wedge-shaped pieces are cast transversely. At the sides of each recess are creosoted blocks, which form a cushion, and a fulcrum for two clamps, which grasp the flange and web of the rail above, bearing upon opposite faces of the wedge below. The weight of the train forces the clamps upon the wedge, spreads them at the bottom, and grips the rail. The first cost is somewhat greater than the wooden tie, but is said to offset this in durability.

Percival Roberts, Junior, announced the death of the Vice-President of the club, Mr. J. B. Knight, and offered resolutions of respect, which were adopted.

O. B. Colton submitted a plan of the contemplated Water Works at the Middle Penitentiary at Huntingdon. Mr. W. A. Cooper read an interesting article on Richardson's method of sharpening and re-cutting files by the sand blast process as used by Trump Brothers, of Wilmington, Delaware. This process, it is claimed, will renew completely worn-out files and rasps at a nominal cost, without the necessity of softening and re-hardening, thereby retaining the original temper. Samples of the work done by this process were exhibited.

At a recent meeting of the Engineers' Club of Philadelphia, at their rooms, No. 10 North Merrick Street, Mr. Charles A. Young read a paper on the Intra-Conglomorate Coals shown on the New River, West Virginia, describing the general geological and topographical features of the district, and presenting sections, analysis of the coal, and a table showing its calorific value were included in the paper. In tee discussion of this paper, Mr. Hardin gave an account of the operations at Sewell Station, Chesapeake and Ohio Railroad, by Messrs. Firmstone and Pardee, descriptive of the means used to carry the coal from the mouth of the mine to railroad grade. Mr. Charles A. Ashburner, through the kindness of Messrs. P. & T. Collins, contractors of the Madeira and Marmore Railway, Brazil, presented to the club an abstract of the report of progress made by C. O. D'Invillieis, Chief Engineer, together with tracings of the maps and profiles of the preliminary and located lines:

"SAN ANTONIO, October 10th, 1878.

"MESSRS. T. AND P. COLLINS:

"*Gentlemen*:—According to your request, that I should, as soon as possible, submit a record of our work and a report of the country we have surveyed, I have had plans made of all our surveys up to date, which comprise the country between San Antonio and Calderao do Infierno, which I respectfully submit in connection with this report.

"Our examinations during the first two or three months, confined principally to the territory between here and Macacos, a distance of five miles by river, developed a country much broken by deep ravines, at the heads of which we found a plateau or ridge dividing the waters of the Jamari from those of the Madeira, which, at different elevations, continues to exist as far as the Jaci Parana River. The ground along the river's edge is flat and good, and, with a low embankment, would be available for railroad purposes, except from the fact that abrupt hills come flush to the river's edge at Macacos, Theotonia Falls and other points.

"Between the river and the dividing ridge there is no alternative but to climb to the plateau at a maximum elevation of 200 feet above high water.

"From San Antonio to the end of the sixth mile there is generally rough and comparatively heavy work; from there to the end of the forty-third mile, or six miles this side of the Jaci Parana River, the country is flat and gently rolling, at an average elevation of 400 feet above high water, over which a very cheap and generally favorable line can be obtained.

"Between the ridge and the Madeira River are numerous detached hills, many of them quite high, into which we ran with some of our trial lines. These hills are generally in the neighborhood of the rapids. Calling high water 200, the maximum elevation of the ridge is 400 feet, that of the detached hills 500 feet.

"Our survey along the Jaci Parana River shows us a very crooked stream from 350 to 400 feet wide during low water, with generally low banks from 2 to 8 feet below high water, and in some places high bluffs on one side and lower ground on the other. The highest and best ground on the west side is that which we have chosen for the crossing; it is about 8 miles from the mouth of the river by the river, and four miles in a direct line.

"From the crossing toward Santonio the ground rises gradually about 2 to 3 feet per hundred until it reaches the plateau, which is 100 feet above high water, being level in a direction up and down the Jaci Parana.

"Regarding water ways between San Antonio and Calderao, we shall require nothing in the bridge line except over a creek one-half mile from San Antonio and the Jaci Parana River. The creek is 150 feet wide at high water. Over the river from 350 to 400 feet of bridging will be ample. The necessary cul-

verts will average one in every two miles. As for timber, we can get tie timber anywhere. The timber is the heaviest on the first twenty miles, and gets gradually lighter as we approach the Jaci Parana River. Adjacent to the river small palms mostly abound, few of the trees being more than 10 inches through. The brush is everywhere very dense, making it impossible to investigate the country on either side of the line without running cut lines and levels.

"We have been much thrown back on account of sickness among the corps, resulting not so much from the unhealthiness of the climate as from the difficulty of keeping up the supplies of proper food. My own experience has been, that beyond a slight bilious tendency, the climate is not unhealthy, except, perhaps, through the months of July and August.

"There has been no suffering from the heat by those in the woods, although the temperature ranges at about 100° Fah. (max.) in the shade, and 120° (max.) in the sun. As to the geological features, I am inclined to think we shall find no solid rock, except at stations 110 and 200. In nearly every cut in the first six miles we have found a decomposed granite cemented with clay or iron—in many cases, granite boulders. That composing the Cachuelas is undoubtedly of volcanic origin.

"Regarding the location of our line to the Madeira River, after passing Macacos, where we are 2006 feet in the closest point, is at the sixteenth mile post, or two miles above Rosstown, where we are about two miles in. At Theotonio Falls, ten miles from here, we are six miles in; at San Carlos, twenty miles from here, three miles in; at the thirty first mile post, we are most distant, being nine miles inland; at Jaci Parana River, four miles; at Calderao, two and one-half miles; at Tres Irmaos we shall probably be on the river bank.

"I am, gentlemen, your obedient servant,

"C. S. D'Invilliers, *C. E.*"

"November 25, 1878.

"Messrs. P. and T. Collins:

"*Gentlemen*:—I send you this month maps of the located line of railway from San Antonio to station 1100, or about twenty-one and one-half miles, and projected location from station 1100 to Calderao do Infierno or station 3530. The locating party are now about station 1700, and making very rapid progress.

"At the time of writing, the track is laid to station 208, or four miles, and the grading from there to the end of the six miles of heavy work more than half finished. Considerable of the track is laid on temporary line at an expense of about $8,000.

"There have been nine trestles erected (two on temporary line) of a total length of 1453 feet, which have cost $7,500. There are twenty-one miles cleared of timber.

"In order to obtain the information we have been obliged to run about 160 miles of trial line.

"The locating party under Mr. Byers is composed principally of Bolivian Indians, there being seven whites and twenty-five Indians. These do the duty of axemen, move the camp, and transport provisions from San Antonio.

"On the first six miles there will be 158,500 cubic yards of excavation, about two-thirds of which will be paid for as rock or shale, and 175,000 cubic yards of embankment. On the remaining distance the quantities will probably average from 10,000 to 15,000 cubic yards of material to be moved per mile. There will probably be required, on the average, one culvert per mile, for which I should propose using iron pipe.

"The only bridging we are sure of is 150 lin. feet at station 30, and 350 lin. feet at station 2740, over the Jaci Parana River, though we may have to bridge some of the smaller streams —at most, three or four of them. There is but little stone to be obtained anywhere, except at San Antonio and Theotonio Falls. At the latter place the stone is already cut and bedded by nature.—Your obedient servant,

"C. S. D'Invilliers, *Chief Engineer.*"

"Mr. George Burnham, Jr., read a paper on the subject of some Features of Ancient Engineering." Modern research has developed the fact that nearly all the materials (in a very wide sense of the word) of modern civilization originated in antiquity; the peculiar province of our time being to ring the changes of variety upon these elements and give them an immense diffusion.

The textile fabrics of wool, cotton, flax and silk were known to the Egyptians of three or four thousand years ago, but the cotton-gin, the power-loom and the steam-engine have greatly increased their variety and put them into the hands of every one. The same thing is true of the Engineering art for, if we except iron framing the ancients originated nearly all the typical forms we now employ. They were acquainted with the constructive uses of wood; carried stone construction to a point that we have never since reached and probably never shall; their *brick* work dates from the very earliest times, and they constructed canals and aqueducts for irrigation, water supply and inland navigation as well as elaborate drainage systems long before their civilization culminated.

The Chaldean structures, dating from 2200 to 1500 B.C., were built of small sun dried bricks laid in bitumen and faced with kiln dried bricks stamped with the name of the king. These temples were built on elevated platforms of beaten clay in some instances cased with massive walls of stone the object being to raise them above the level of the plain for architectural effect and to avoid inundation. A brick burial vault at Mugheir exhibits a rudimentary arch. The vault is seven feet long, five feet high, and three feet seven inches wide. The sides slope gently outwards until the springing line is reached when the successive courses are pushed towards each other until they meet at the top.

Similar arches are found in early Greek work at Phigalia, Messene, and other places.

The old notion that the round arch was of Roman and the pointed arch of Gothic origin has been dissipated by the spade of the archæologist. Both of these varieties are found in Assyrian work. They are usually of brick and occur in underground construction as

drains and vaults. The brick arch existed in Egypt as early as 1540 B.C. and a stone arch has been found dating from 600 B.C.

The masonry of the past is, of course, identical with ours since we have simply adopted the methods of the ancients. We find in Egypt and Western Asia smooth and rock faced ashlar, rubble, and irregular range work essentially like that of to-day. The Assyrian and Egyptian bas-reliefs indicate their method of moving heavy masses. Sledges were used drawn by large bodies of men. Rollers were placed under the sledge and the piece was carefully "guyed" by parties of men with appropriate ropes and props.

The Roman military roads crossed mountains and valleys without regard to the nature of the ground; tunnels, open cuts, embankments, and bridges frequently occurring. Place cross ties and steel rails upon a Roman road and suppose the grade not too steep, and the points of approach and divergence of modern and ancient engineering are at once apparent. Substantially the substructure was the same as that of a modern rail road, but in place of the pedestrian or the ox team we have the locomotive with its "Fast Express" or heavily laden freight train.

Prof. L. M. Haupt read a paper explaining the *modus operandi* of the "Holly System of Steam Heating" following it by some remarks concerning its economy, general application and rapid introduction.

In his introduction he laid great stress upon the necessity for providing more perfect ventilation and the difficulties presented by the prevailing system of hot air furnaces.

The plant of the Holly Company is very similar to that generally used for the distribution of other fluids, as gas or water, that is there are generators or reservoirs where the steam is made under a pressure of about sixty pounds. Thence it is conveyed by mains to the house or shops where it is to be used at a very low pressure. It is registered by a meter in each house and can be turned on or off at pleasure. New radiators have been introduced open at the bottom so that the air is kept sufficiently moist and the racket occasioned by the condensation entirely avoided.

It eliminates the necessity of keeping up so many different fires, avoids all dust and removal of ashes, and is far more economical than the present system. It has been used with great success to warm schools, churches, halls and dwellings, to run mills and machinery, to extinguish fires, for cooking, heating water and for many other purposes.

The ingenuity displayed by Mr. Holly in so successfully solving the many mechanical difficulties connected with the transmission of steam for general use places him in the foremost rank of inventors and makes him a public benefactor.

The system has been in practical operation for two years in Lockport and organizations are now rapidly making for its introduction in many other places. It is in use in Buffalo, Detroit, Utica, Rochester and other cities and is just being introduced into New York City, Troy and elsewhere.

April 5th, 1879.—Prof. L. M. Haupt read a paper on the "Nomenclature and Classification of Masonry."

Various discrepancies exist in the use and meanings of the terms employed to designate the classes of work, and to remedy this Mr. Haupt attempts to remove the ambiguities by comparing all the authorities and arranging them in such order that they can be more readily referred to and understood.

Mr. Cleeman spoke on the subject of "the proper amount of water-way for Culverts," presenting the formula of Major E. T. D. Meyers of Richmond, Va., which in practice on some railroads recently built in that state appears to be more applicable than the formula heretofore used.

Prof. Marks, University of Pennsylvania, exhibited his adaptation of Peaucellier's Compound Compass, to the drawing of circles of any radius, and mathematically straight lines. This compass was built by Messrs. W. J. Young & Sons, Philadelphia, for the State Geological Survey.

IRON AND STEEL NOTES.

PROPOSED GENERAL CLASSIFICATION OF IRON AND STEEL.—This proposition emanates from a sub-committee of the German Railway Union, appointed for the purpose, and takes the form of a report presented to the general meeting of the Union at the Hague, July 1877. The meeting agreed to act on this report, both by bringing influence to bear on various governments, with the view of having such a classification definitely fixed and agreed upon, and also by instituting further experiments in order to render the classification more exact.

The sub-committee's report first enlarges on the desirability of such a classification, and especially mentions the great inequalities now found to exist among different specimens of iron and steel, supposed to be of similar make, owing to an ignorance of the laws under which differences of quality are produced. It observes that such a classification might require revision from time to time, and then proceeds to give a table which it considers to be as perfect as can be attained in the present state of knowledge. The two qualities specified throughout are strength, as represented by the tensional breaking strain in kilogrammes per square centimeter, and ductility, as represented by the percentage of contraction of area at the breaking point.

The table is as follows:

(*See Table on following page.*)

The above classification would not of course do anything towards solving the question which class of material is the most fitting to use for any particular purpose. This requires a much more extensive range of experience and of tests, and the report concludes by strongly recommending that an Office of Research (Versuchsanstalt) should be established by government with the special object of investigating such matters.

	Tensional Strength. Kilogramme per Square Centimeter. = .00635 Ton per Square inch.	Ductility. Contraction per cent.
A. Bessemer steel, cast steel, or Siemens-Martin steels used as a material of construction for rails, axles, tires, &c.		
1st Quality — Class *a.* Hard....	6,500	25
1st Quality — " *b.* Medium.	5,500	35
1st Quality — " *c.* Soft....	4,500	45
Fracture to be uniform, and no cracks visible in the broken ends.		
2nd Quality — Class *a.* Hard..	5,500	20
2nd Quality — " *b.* Soft...	4,500	30
Fracture, &c., as above.		
B. Bar iron.		
1st Quality......	3,800	40
2nd "	3,500	25
C. Plate iron.		
1st Quality — with the grain...	3,600	25
1st Quality — across " ..	3,200	15
2nd Quality — with the grain...	3,300	15
2nd Quality — across " ..	3,000	9

All material which falls below these limits it is proposed to distinguish as "non-classified" material.

RAILWAY NOTES.

The State of Guatemala, the largest in extent of territory and population in Central America, has recently granted a concession for the construction of a railroad from the port of St. Jose, on the Pacific Coast, to the city of Escuintla, in the interior. Escuintla is a town of some 8,000 inhabitants, situate about thirty miles from the port of St. Jose on the high road to the capital, from which it is nearly an equal distance. Guatemala, the capital of the State, contains a population of 50,000 to 60,000, is a well-built, old established city, and is the residence of the foreign ministers to the Central American States. It is intended to continue the construction of this line to the capital city of Guatemala, and eventually to the Atlantic port of Santa Tomas, and, if thus extended, this line must become a new inter-oceanic route. In this relation the Guatemala line will, says the *San Francisco Bulletin*, have the advantage of a shorter sea route between San Francisco and the Atlantic cities, of from 1,500 to 2,000 miles, and also a large local trade in which the Panama Railroad line is altogether deficient.

An electro-magnetic railway ticket counter, to indicate the number of passengers going by a particular train, and so the number of carriages of each class required, has recently been adopted at the station of the Kaiser Ferdinand's Nordbahn, in Vienna. The date press of each of the three ticket boxes is furnished with an electric contact, actuated each time a ticket is pushed in to be stamped. Three lines connect the presses with a clock-shaped counter in the office, where the train is arranged, having discs with numbers and pointers, so that the number of passengers booked can be seen at once. The arrangement conduces to promptness in ordinary circumstances, and where great crowds are travelling successive trains can be quickly despatched without overloading with passengers. Messrs. Meyer & Wolf of Vienna, supplied the apparatus.

We take from a detailed review of the railways constructed in the United States in 1878, published in the *Railroad Gazette*, the following condensed statement of the mileage of new railroad constructed in each state and territory during the year 1878:—Alabama, 22; Arizona, 30; Arkansas, 7; California, 71½; Colorado, 193¼; Dakota, 15; Delaware, 6; Georgia, 62; Idaho, 126; Illinois, 103; Indiana, 74; Iowa, 255½; Kansas, 169¼; Kentucky, 20; Maryland, 5½; Massachusetts, 6; Michigan, 110½; Minnesota, 338¼; Mississippi, 26; Missouria, 209; Nebraska, 55; New Hampshire, 35; New Jersey, 3; New York, 129½; North Carolina, 16; Ohio, 97; Oregon, 36; Pennsylvania, 188½; South Carolina, 16½; Tennesse, 10; Texas, 118½; Virginia. 16½; Washington T. 15; West Virginia, 16½; Wisconsin, 83¾; or a total of 2688 miles compared with 7340 in 1872, 3883 in 1873, 2025 in 1874, 1561 in 1875, 2450 in 1876, and 2281 in 1877.

The fall of a railway bridge took place recently on the Manchester, Sheffield, and Lincolnshire Railway. The bridge, situated at Chalk Hill, was considered to be in great danger of falling, and it was decided to take it down. Last Sunday night was appointed for the work, as no week-day could be spared in consequence of the great traffic at that place. Mr. Fisher, the resident engineer, was on the spot, and at nine o'clock operations commenced. The recent frost had done considerable damage to the earthwork, and the brickwork of the bridge had been also, to a large extent, dislodged. It was decided, if possible, to blow up the bridge, and no less than seventeen shots were placed thereon without making any perceptible effect. It was then determined to take it down piecemeal. The workmen worked on a temporary stage a few feet above the bridge proper, and were perfectly safe. About half past three o'clock on Monday morning, a little less than half of the key of the arch had been removed, when without a moment's warning the bridge fell. Some of the men had unfortunately, contrary to orders, been standing on the key of the arch a little before, and, as a matter of course, they fell with the bridge and were buried in the *debris*. Great efforts were made to get them out alive, but four of them were quite

dead when found. Twelve other men were more or less injured, and three were taken to the Grimsby Hospital, and the remainder to their homes.

EUROPEAN MOUNTAIN RAILWAYS OPERATED BY LOCOMOTIVES WITH COG-WHEEL DRIVERS.—The Journal of the Hanover Society, for last year, contains the report of a lecture by one of its members, Mr. Grove, on the several mountain railways built in Austria and Switzerland within the past eight years. The list (adding the Mount Washington Railway for comparison) is as follows:

Date.	Name.	Gauge.	Length. Kilometer.	Maximum grade. Rise in 1000	Least radius of Curvature. Meter.
1870	[illegible] near Bern	Common.	0 56-100	100	No curves.
1871	[illegible]	"	7	[illegible]	180
1874	[illegible]	"	5 16-100	100	190
1874	Schwabenberg, near Pesth	"	3	102 5-10	180
1875	Arth Rigi	"	9 8-10	212 6-10	180
1875	Rorschach-Heisden	"	5 5-10	[illegible]	240
1876	Wasseralfingen	1 meter.	8 25-100	78 5-10	400
1877	Ruti	Common.	—	—	—
1869	Mount Washington	"	5 3-10	375	151 5-10

The rack is made of two channel irons, spaced 5 inches, in which plays the cog-wheel of the engine, 4-inch face, and 4-inch pitch of cogs. The Wasseralfingen road alone has a $3\frac{7}{16}$-inch pitch. The cogs are shaped as evolutes of a circle, so that a variation in the depth to which they take into the rack does not affect their regular action. On the Rorschach-Heisden road the rack is elevated above the ties on two stringers, leaving the space open under the rounds of the ladder-like central rail, so that snow may fall through or be pushed through by the cog-wheel. This seems to have worked well, even solid ice having been thus pushed out from between the cogs. The latest roads have the rack fastened to the ties by suitable castings instead of by longitudinal stringers. Switches were at first built after the manner of turntable, but have recently been constructed similar to other switches by Mr. Klose, of Rorshach. The circumstance of the cog-rail being elevated above the other rails, simplified the construction of this switch materially. The common rails are switched as ordinarily, and the cog-rail is gradually widened, the rounds being bent at the same time, so as to remain normal to each side bar at their ends until the rail has attained double its usual width, when it is continued as two separate cog-rails. The engines were at first built with vertical boilers; next with boilers that were level on an average grade (agreeing in these two steps with those followed on the Mount Washington Railway), but are now built with horizontal boilers, same as ordinary locomotives. Various methods have been devised for enabling the locomotives to work by adhesion of their smooth wheels, as well as by means of their cog-wheel drivers, and by means of either at will. No one of these has been permanently successful, however, so that the proper construction of a double engine of this sort is still a matter of experimental inquiry.

ENGINEERING STRUCTURES.

DESCRIPTION OF A PAPER DOME FOR AN ASTRONOMICAL OBSERVATORY. BY PROFESSOR DASCOM GREENE, TROY, NEW YORK.—An astronomical observatory has recently been erected for the Rensselaer Polytechnic Institute, through the liberality of Mr. E. Proudfit of this city. In maturing the plans, and supervising the erection of the building, I have introduced an improved method of constructing revolving domes, a brief account of which may not be without interest.

While making the preliminary inquiries, I ascertained that a dome of the dimensions required, constructed in any of the methods in common use, would weigh from five to tons, and require the aid of cumbersome machinery to revolve it. It therefore occurred to me to obviate this objection by making the framework of wood, of the greatest lightness consistent with the requisite strength, and covering it with paper of a quality similar to that used in the manufacture of paper boats; the principal advantages in the use of these materials being that they admit of great perfection of form and finish, and give extreme lightness, strength, and stiffness in the structure—prime qualities in a movable dome. A contract was accordingly made with Messrs. E. Waters & Sons, of this city, the well-known builders of paper boats, for the construction of the dome, and they have carried out the undertaking with great skill and success.

The dome is a hemisphere with an outside diameter of twenty-nine feet. The framework consists primarily of a circular sill which

forms the base, and two semi-circular arch girders set parallel to each other, four feet apart in the clear, and spanning the entire dome. These are firmly attached to the sill, and kept in a vertical position by means of knee-braces. The sill and girders are of seasoned pine, the former being 8½ inches wide by 3½ thick, and the latter each 4½ by 3 inches.

The paper covering of the dome is made in sixteen equal sections, such that when set up side by side, their bases on the sill, and their extremities meeting at the top, they form a complete hemispherical surface. The frame-work of each section consists of three vertical ribs of pine, each 3¾ inches in width and ¾ of an inch thick, one at each side, and one mid-way between, and meeting at the apex. The paper was stretched over this frame-work as follows:

A wooden model of full size being made of that portion of the dome included within one of the sections, with a surface truly spherical, the frame-work of a section was placed in its proper position on the model, so that its outer edges formed part of the same spherical surface, and covered with shellac where it was to be in contact with the paper. The sheet of paper cut in the proper form was then laid on the model while moist, the edges turned down over the side ribs, and the whole placed in a hot chamber and left until thoroughly dry. In this way the several sections were dried off in succession over the same model. The paper used is of a very superior quality, manufactured expressly for the purpose by Messrs. Crane Brothers, of Westfield, Mass. Its thickness after drying is about one-sixth of an inch, and it has a structure as compact as that of the hardest wood, which it greatly excels in strength, toughness and freedom from any liability to fracture.

After being thoroughly painted, the several sections were ready to be set up side by side on the sill, and connected together by bolting through the adjacent ribs. The space between the arch girders being left uncovered on one side from the sill to a distance of two feet beyond the zenith, the upper ends of the sections required to be cut off and accurately fitted to the girders. The joints between sections were made weather proof by inserting a double thickness of heavy cotton cloth saturated with white lead paint. The adjacent side ribs were then bolted firmly together through the paper and cloth, the lower ends attached to the sill by angle irons, the upper ends bolted to the girders, and the lower edge of the paper turned under the sill and securely nailed. The joints were afterwards painted over on the outside. As the entire surface exposed is free from nail holes or other abrasions in the paper the structure promises, with an occasional coat of paint, to last for many years, and to form an effective and serviceable roof.

The four feet opening between the arch girders is covered by a shutter which is also of paper stretched over a wooden frame. With the exception of about two feet at the lower extremity this shutter is in a single piece. Attached to its sides are a series of iron rollers which run on a railway track of band iron laid down on the girders, by which means the shutter can be moved over to the opposite side of the dome. The wooden sides of the shutter have iron flanges attached to their lower edges, which project under the railway tracks, making the whole weather proof. The shutter is opened and closed by means of a windlass and wire rope.

The weight of the dome and its appurtenances is about 4,000 pounds. It is supported on six 8-inch balls which roll between grooved iron tracks, and can be easily revolved by a moderate pressure applied directly, without the aid of machinery.

ORDNANCE AND NAVAL.

Repeating-Rifles for the French Navy.—The French authorities have recently made a careful trial of repeating arms with a view to adoption should one be found which, while serviceable in other respects, fulfilled the following conditions, which were put forth in March, 1877:—(1) To fire the regulation metallic cartridge of the army. (2) To have the same trajectory and the same accuracy as the rifle model, 1874. (3) So constructed as to be used as an ordinary single shot arm, or, in other words, to admit of passing quickly and simply from single shot loading and firing to repeating and *vice versa*. (4) To be strong, not requiring too tender care, not to be exposed, from a breaking down of the repeating mechanism, to unserviceableness as a single shooter, to be dismounted, cleaned, and remounted without difficulty. On March 28th, 1877, the minister approved of this programme, and on September 14th he sent orders to Cherbourg to experiment with three types of repeaters, with detailed instructions as to the trials. These three arms were:—(1) The Hotchkiss; (2) the Kropatschek; (3) the Krag. To these three the board confined themselves.

The result of these trials showed that the magazine of the Hotchkiss was most quickly charged. The Hotchkiss also fires most rapidly; both in repeating and single shot fire the Kropatschek was not far behind it. The Krag does not seem to have been well understood and manipulated by the men.

The Kropatschek—modified—with eight cartridges in its magazine beat the Hotchkiss which had only six, while the Krag with nine cartridges was best of all. The time necessary to discharge this latter arm's magazine of nine rounds was 24.85 seconds, in which time the Kropatschek had on an average fired 8.9 cartridges per arm, and the Hotchkiss 7.9 starting with the magazine closed: with the magazine open 25 seconds were occupied, in which time the Krag fired 9, the Kropatschek 9.3 and the Hotchkiss 8.25 rounds on an average. Single shot fire proved better than recharging the magazine and repeating continually. The minimum times taken to fire off the magazines, at the conclusion of the experiments, when the men were expert, were as follows:—Hotchkiss—6 rounds—in 10 seconds; Kropatschek, modified—8 rounds—in 14 seconds; Krag—9 rounds—in 17 seconds; giving an average time

per round of 1.66, 1.75, and 1.88 seconds respectively.

Finally, it was concluded that the Hotchkiss rifle is the easiest and quickest in charging the magazines; then the Kropatschek; and last, Krag. As to rapidity of fire, the Hotchkiss and Kropatschek are about equal. Large magazines have a great advantage; the magazine once empty. it is best not to attempt to refill it till leisure gives the opportunity.—*Engineer.*

BOOK NOTICES.

Specimen Book of One-Hundred Architectural Designs, showing Plans, Elevations, Views, and including Bills of Materials. New York: A. J. Bicknell & Co. For sale by D. Van Nostrand. Price, $1.00.

The illustrations of this book are selections from various works on Architecture. The views are well printed on good paper, and constitute the book. There is no descriptive text worth mentioning.

The Study of Rocks. By Frank Rutley, F.G.S. New York: D. Appleton & Co. For sale by D. Van Nostrand. Price $1.75.

This is a late addition to the series of "Text-Books of Science."

It is adapted for beginners in Petrology and only supposes the merest rudimentary knowledge of Geology, and a fair amount of mechanical skill to be applied to the preparation of specimens for the microscope.

The method of the author is logical and clear, such that the ordinary student can follow with profit without requiring the aid of an instructor, provided only that he can detect the common minerals by the ordinary processes, and can also work the microscope.

Some diagram "schemes" of deviations from well known rocks are excellent. Other illustrations of the book are good and fairly numerous.

Experiments on Repeating-Rifles by a Board of French Naval Officers. Translated by Lieut. Theo. B. M. Mason. New York: Office of *Army and Navy Journal.*

The question of superiority of the repeating over the single short rifle has been the subject, in Europe especially, of long discussions.

The French Navy decided to arm the sailors with a magazine gun, and began for that purpose a systematic course of experiments, of which the report is a full account.

The Report first presents the reasons for using repeating rifles, quoting from eminent military authorities. The description of the arms experimented with next follows. These rifles were the Hotchkiss (American) Krag (Norwegian) and Kropatshick (Austrian). The reasons for finally adopting the latter are fully set forth.

Many readers who are not in any degree inclined to military tastes will read this report with interest.

Filtration of Potable Water. By Wm. Ripley Nichols. New York: For sale by D. Van Nostrand. Price $1.50.

The filtration of water on a large scale, has been but little practiced in this country. The necessity of such a treatment in the case of many sources of supply is very generally recognized, but the practice is but little more than a rude kind of straining.

Professor Nichols has clearly specified the circumstances which render filtering advisable, and has described with exceptional fullness the methods by which it may be accomplished.

The illustrations of working filters, both for town and for family supply, are given with an attention to detail which makes them working plans.

The work is indispensable to engineers who manage or project water supply systems, and will prove equally valuable to those who regulate such supply with reference to sanitary precaution.

Les Ponts de l'Amerique du Nord. Par L. Ant. Comolli. Ingenieur. Paris: Ambroise Lefevre. For sale by D. Van Nostrand. Price, $18.00.

This is a quarto volume of 450 pages, accompanied by an atlas of 54 plates.

The text is divided into two nearly equal parts. The first gives the theory of bridge building as presented by American writers, a large portion of Shreve's excellent work occupying a prominent place. Whipple and Merrill are also freely quoted. The second part presents examples of American Bridges, and in this part also well known American books including descriptive papers from several sources.

Hildebrand's work on "Cable Making for Suspension Bridges," published originally in this Magazine, is given entire.

The Atlas of Plates is an excellent collection of plans of American bridges with well drawn details. To engineers who would like native engineering literature done into French the opportunity is a good one.

MISCELLANEOUS.

Two great but frequently-recurring canal questions are again said to be occupying much attention in America. It is once more said that the route for the Panama Canal has been fixed. Lake Nicaragua is to be utilized, and a cut is to be made through the mountain chain between Nicaragua and Brito into the lake. From the northeastern part of the lake exit will probably be through a cut parallel to the river San Juan into the Gorgon Bay. The other canal is the long-talked-of ship canal to connect Chesapeake Bay with Delaware Bay, and shorten the water route from Baltimore to New York and Europe some 225 miles. The canal will be seventeen miles long, 100 feet wide and 25 feet deep, and the estimated cost is stated as £800,000. The promoters state that the present commerce of Baltimore would give to the canal an income of $160,000 from the authorized rate of toll—10d. per ton. The canal is to follow the valley of the Sassafrs, and be without locks.

VAN NOSTRAND'S

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THERMODYNAMICS.

By HENRY T. EDDY, C. E., Ph. D., University of Cincinnati.

Written for VAN NOSTRAND'S MAGAZINE.

IV.

39. MIXTURE OF A LIQUID AND ITS VAPOR. —A vapor confined in the presence of the liquid from which it evaporates, soon arrives at its maximum tension and it is then said to be a *saturated* vapor; but a vapor removed from its liquid is, in general, superheated.

The thermodynamic behavior of superheated vapors has been included, at least to a close degree of approximation, in the equations and discussions of the behavior of perfect gases; for superheated vapors are to be regarded as gases which are but slightly more imperfect than the so-called permanent gases.

The physical properties of liquids and their vapors are well understood in a general way from the familiar example of water and steam which may be taken as the type of the mixture to be treated.

The equations for mixtures which we shall arrive at, apply however, not only to mixtures of a liquid and its vapor during evaporation, but they also apply equally to a mixture of any solid and is liquid during liquefaction, such as melting ice and water, or to any chemical decomposition in progress or to the phenomena of dissociation.

The substances heretofore considered have not been regarded as undergoing a change of physical state which is of such a discontinuous nature as vaporization, liquefaction or decomposition, but it has been assumed that the substance under consideration was such that the volume (or any other physical property) of a unit of the given substance was a definite, though perhaps unknown function of its temperature and pressure. But with a mixture of a liquid and its vapor it is otherwise, for in that case the pressure depends directly upon the temperature, so that

$$p=f(t) \quad . \quad . \quad . \quad . \quad . \quad (212)$$

in which f denotes a form of function which can be determined by experiment.

The volume or any other physical property of the mixture evidently cannot in this case be expressed as a function of p and t, for by the help of (212) it would then be possible to eliminate one of these variables and express the volume, etc., in terms of either single variable p or t. The physical state of any substance can not be thus determinately expressed as a function of one independent variable, but requires, as has been previously stated, two independent variables in order to determine its state. Hence any of the previous general equations which can be reduced by help of (212) in such a manner that it will contain but one in-

dependent variable will evidently be useless as applied to a mixture as a whole, though still true as applied to each of the separate homogeneous substances which compose the mixture. We may notice among others that (31), (51), (60), (88) and (94) are useless as applied to mixtures.

40. LATENT HEAT OF EVAPORATION.—

Let l=latent heat of evaporation per unit of weight of liquid at temperature t.

Let x=weight of the vapor contained in a unit of weight of the mixture.

$\therefore$ $1-x$=weight of the liquid in a unit of the mixture.

$$\therefore \quad l=\left(\frac{dh}{dx}\right)_{pt}=t\left(\frac{de}{dx}\right)_{pt} \quad . \quad . \quad . \quad (213)$$

in which the subscripts, as before, denote the constants.

Let v=volume of a unit of weight of the mixture at temperature t.

Let v'=volume of a unit of liquid at t.

Let v''=volume of a unit of vapor at t.

$$\therefore \quad v=(1-x)v'+xv'' \quad . \quad . \quad (214)$$

in which x is independent of t.

$$\therefore \quad \frac{dv}{dx}=v''-v' \quad . \quad . \quad . \quad . \quad (215)$$

in which there is no restriction as to temperature or pressure, hence (215) is also true in case t and p are constant.

$$\therefore \text{ by (213), } \quad \left(\frac{de}{dv}\right)_{pt}\left(\frac{dv}{dx}\right)_{pt}=\frac{l}{t} \quad (216)$$

$$\therefore \text{ by (215), } \quad (v''-v')\left(\frac{de}{dv}\right)_{pt}=\frac{l}{t} \quad (217)$$

$$\text{By (42), } \quad \left(\frac{de}{dv}\right)_{t}=\left(\frac{dp}{dt}\right)_{v}=\frac{dp}{dt} \quad . \quad (218)$$

in case of a mixture. We are at liberty to remove the restriction of constant volume from this last differential co-efficient, because from (211) we shall obtain the same value for it when it is unrestricted as when it is restricted.

$$\therefore \text{ by (217), } \quad (v''-v')\frac{dp}{dt}=\frac{l}{t} \quad . \quad . \quad (219)$$

in which l is the latent heat of evaporation and $v''-v'$ is the increase of volume of a unit of liquid during evaporation. Equation (219) is of great importance in the discussion of mixtures, and in it the value of $\frac{dp}{dt}$ is to be derived from (212) the form of which has been determined with great accuracy by the experiments of Regnault.

As a result of these experiments it appears that the latent heat of steam may be expressed with sufficient accuracy for practical purposes by the formula

$$l=\mathrm{a}-\mathrm{b}t \quad . \quad . \quad . \quad . \quad (220)$$

which may be used where convenience demands it, instead of the exact formula (219). In case l is expressed in foot lbs.

$$\mathrm{a}=1109550, \quad \mathrm{b}=540.4.$$

We shall now obtain another equation involving the latent heat l, and the specific heats of the liquid and its vapor.

Let c'=the specific heat of the liquid.

Let c''=the specific heat of the vapor.

Then, by (60), the increments of heat imparted to a unit of the liquid and a unit of its vapor are respectively

$$\frac{dh'}{dt}=\left(k'_p+l'_p\frac{dp}{dt}\right)=c' \ldots (221)$$

$$\frac{dh''}{dt}=\left(k''_p+l''_p\frac{dp}{dt}\right)=c'' \ldots (222)$$

in which the primes refer to the liquid and the seconds to its vapor.

During the evaporation there may be, in the general case, also a change of temperature, hence, the total increment of heat imparted to a unit of the mixture is the sum of the increments imparted to the liquid and vapor respectively, added to the latent heat

$$\therefore \; dh=c'(1-x)dt+c''xdt+ldx \ldots\ldots (223)$$

In art. 14 it is shown that t^{-1} is an integrating of every value of dh such as (223)

$$\therefore \; \frac{dh}{t}=de=[c'+(c''-c')x]\frac{dt}{t}+\frac{ldx}{t} \ldots (224)$$

is an exact differential, and fulfills the equation of condition of integrability, which may be expressed as follows:—

$$\frac{c''-c'}{t}=\frac{d}{dt}\left(\frac{l}{t}\right), \text{ or, } c''-c'=\frac{dl}{dt}-\frac{l}{t} \ldots (225)$$

which is independent of x and is the equation involving the latent and specific heats.

It is known that $c'=772$ foot lbs. nearly, hence by obtaining the terms in the last member of (225) from the approximate equation (220), we find the following approximate value of the specific heat of saturated steam, per degree F.

$$c''=c'-\frac{a}{t}=\frac{772t-1109550}{t} \quad . \; . \; . \; (226)$$

which is a negative quantity at all pressures under a point somewhat above ten atmospheres.

This remarkable result, which has important practical bearings, may be stated thus: When saturated steam is compressed adiabatically, it becomes superheated, the external work of compression being more than sufficient to raise its temperature to the point of saturation due to this higher pressure; but when saturated steam expands adiabatically, some of it condenses, the external work it performs being more than equivalent to the heat emitted from the steam alone during its decrease in temperature.

The specific heat of certain vapors, as ether, are positive, and it appears probable that there is a temperature for each vapor above which its specific heat is positive but below it is negative.

41. Entropy Imparted to a Mixture, etc.—Eliminate $c''-c'$ between (224) and (225)

$$\therefore \; de=\left\{\frac{d}{dt}\left(\frac{l}{t}\right)+\frac{c'}{t}\right\}dt+\frac{l}{t}dx \quad . \; . \quad (227)$$

$$\therefore \; de=d\left(\frac{lx}{t}\right)+c'\frac{dt}{t} \quad . \; . \; . \; . \; . \quad (228)$$

$$\therefore \; e_2-e_1=\frac{l_2x_2}{t_2}-\frac{l_1x_1}{t_1}+\int_{t_1}^{t_2} c'\frac{dt}{t} \quad . \quad (229)$$

which is the general expression for the entropy imparted to a unit of the mixture in order to carry it from the state 1 defined by the variables t_1 x_1 to the state 2 defined by the variables t_2 x_2. The last term of (229) cannot be integrated until it is known how c', the specific heat of the liquid, is related to these variables. In the case of water c' is so nearly constant that the result obtained by so regarding it will not lead to sensible error in practice.

$$\therefore \; e_2-e_1=\frac{l_2x_2}{t_2}-\frac{l_1x_1}{t_1}+c'\log_e\frac{t_2}{t_1} \quad . \; . \quad (230)$$

is sensibly exact for a mixture of water and steam.

The last term in (230) being the integral of the last term of (229) expresses the entropy which would be imparted to the unit of water in raising it from t_1 to t_2, or which would be given out by the water in falling from t_2 to t_1. Hence we may consider that this term is the entropy of the sensible heat imparted which causes the change in temperature, and which by the third law is unaffected by the physical condition of the water.

We are enabled by this consideration to find the entropy which must be imparted to a mixture of steam and water in contact with a regenerator or in contact with any metallic conductor which may be regarded as having, by reason of this contact, the same temperature at all times as the mixture. Let the weight of metal in contact with each pound of mixture be y, and its specific heat be c_1,

$$\therefore \; e_2-e_1=\frac{l_2x_2}{t_2}-\frac{l_1x_1}{t_1}+(c'+c_1y)\log_e\frac{t_2}{t_1} \; (231)$$

is the entropy imparted to the mixture in contact with the metal.

Any equation for steam and water can be corrected so as to apply to the same in contact with metal by substituting $c'+c_1y$ instead of c'.

We may apply (230) to various special cases of expansion as follows:

1°. During isothermal expansion $t_2=t_1=t$ and by (219) or (220) we see that $l_2=l_1=l$

$$\therefore \text{ by (230), } h_2-h_1=t(e_2-e_1)=l(x_2-x_1) \, . \, (232)$$

is the total heat imparted during the isothermal expansion of a unit of mixture of steam and water between the states 1 and 2.

2°. During adiabatic expansion $e_2=e_1=e$, and if the state 2 is any state on the adiabatic we may conveniently omit the subscripts 2,

$$\therefore \text{ by (230), } \frac{lx}{t}-\frac{l_1x_1}{t_1}+c'\log_e\frac{t}{t_1}=0 \, . \, . \, (233)$$

is the equation of an adiabatic curve for a unit of mixture of water and steam, in terms of the variables tx, in which t_1x_1 is one point of the adiabatic.

3°. During expansion of the mixture without evaporation or condensation $x_2=x_1=x$,

$$\therefore \text{ by (230), } e_2 - e_1 = x\left(\frac{l_2}{t_2} - \frac{l_1}{t_1}\right) + c' \log \frac{t_2}{t_1} \quad \ldots \quad (234)$$

is the entropy imparted along this curve of no evaporation from t_1 to t_2.

When $x=1$, (234) refers to a unit of steam which is saturated and dry, and the curve expressing its successive states may be called the saturation curve.

42. External Work of a Mixture during Expansion.—If for convenience we let

$$v''-v'=u \quad \ldots \quad (235)$$

$$\therefore \text{ by (219), } u\frac{dp}{dt}=\frac{l}{t} \quad \ldots \quad (236)$$

$$\therefore \text{ by (214) } \frac{dv}{dt}=\frac{dv'}{dt}+\frac{d(ux)}{dt}=\frac{d(ux)}{dt} \quad . \quad (237)$$

for v' may be taken as sensibly constant in case of water.

$$\therefore \text{ by (10), } \frac{dw}{dt}=p\frac{dv}{dt}=p\frac{d(ux)}{dt} \quad \ldots \quad (238)$$

$$\therefore \quad \frac{dw}{dt}=\frac{d(pux)}{dt}-ux\frac{dp}{dt} \quad \ldots \quad (239)$$

$$\therefore \text{ by (236), } \frac{dw}{dt}=\frac{d(pux)}{.dt}-\frac{lx}{t} \quad \ldots \quad (240)$$

Either (238), (239) or (240) may be integrated to find the external work.

1°. In case the expansion is isothermal, p and u are constant

$$\therefore \text{ by (238), } w_2-w_1=pu(x_2-x_1) \ldots (241)$$

2°. In case the expansion is adiabatic, we have by (228)

$$c'+t\frac{d}{dt}\left(\frac{lx}{t}\right)=0 \therefore c'+\frac{d(lx)}{dt}=\frac{lx}{t} \ldots (242)$$

$$\therefore \text{ by (240), } dw=d(pux)-d(lx)-c'dt \, . \, (243)$$

$$\therefore w_2-w_1=p_2u_2x_2-p_1u_1x_1-l_2x_2 + l_1x_1-c'(t_2-t_1) \ldots (244)$$

in which each of the terms in the last member are known in terms of t_2 and t_1 by help of (212), (220), (233) and (236).

3°. Let x be constant during the expansion,

$$\therefore \text{ by (240), } dw=x\left(d[pu]-\frac{l}{t}dt\right) \, . \, (245)$$

$$\therefore \text{ by (220), } dw=x\left(d[pu]-\frac{\mathrm{a}}{t}dt+\mathrm{b}dt\right) \quad \ldots \quad (246)$$

$$\therefore w_2-w_1=x\left(p_2u_2-p_1u_1-\mathrm{a}\log_e\frac{t_2}{t_1}+\mathrm{b}[t_2-t_1]\right) \quad \ldots \quad (247)$$

from which we obtain the work along the saturation curve by making $x=1$.

43. Internal Energy and Total Heat Imparted to a Mixture.—

$$\text{By (228), } dh=tde=c'dt+td\left(\frac{lx}{t}\right) \, . \, (248)$$

$$\therefore \; dh=c'dt+d(lx)-\frac{lx}{t}dt \, . \; . \quad (249)$$

$$\text{But by (236), } \frac{lx}{t}dt=uxdp=d(pux)-d(ux) \quad \ldots \quad (250)$$

$$\therefore \text{ by (237), } \frac{lx}{t}dt=d(pux)-pdv \, . \, (251)$$

$$\therefore \text{ by (249), } dh=c'dt+d(lx)-d(pux)+pdv \quad \ldots \quad (252)$$

Either (248), (249) or (252) may be integrated to obtain the total heat imparted, which can also be found from (223).

Equation (4) may be written by (10) in the form $dh=di+pdv$, and by comparing this with (252), we have

$$di=c'dt+d(lx)-d(pux) \quad . \quad (253)$$

$$\therefore \; i_2-i_1=c'(t_2-t_1)+l_2x_2-l_1x_1-p_2u_2x_2 + p_1u_1x_1 \quad \ldots \quad (254)$$

which is independent of the kind of expansion, as it should be by art. 15.

The total heat imparted is to be obtained in any case by adding (254) to the external work obtained in art. 42.

The total heat imparted during expanpansion along a curve of no evaporation is so important in its practical bearings and so simple in its expression that we shall now obtain it. In this case $x_2=x_1=x$. Substitute the value of l taken from (220) in (248)

$$\therefore \; dh=c'dt-\mathrm{a}x\frac{dt}{t} \, . \; . \quad (255)$$

$$\therefore \; h_2-h_1=c'(t_2-t_1)-\mathrm{a}x\log_e\frac{t_2}{t_1} \quad (256)$$

from which by making $h_2-h_1=0$, we can find the value of x such that no heat need be supplied in order that x may have the same value at t_1 as at t_2.

44. Empirical Formulae for Steam.—It appears from the indicator cards of steam engines and other experiments on the expansion of steam that the curves of expansion can be represented with sufficient exactness for practical purposes by an equation of the following form:

$$pv^n=c \quad \ldots \quad (257)$$

which has the form previously employed in case of perfect gases, but the constants in (257) have values for steam which differ from those of perfect gases.

In case of superheated steam it has been shown by experiment that even to the point of saturation.

$$n=\tfrac{4}{3}=1.3333 \quad . \quad . \quad (258)$$

is a close approximation. We wish to direct especial attention to this important experimental result since it has been widely misstated, and hence shall quote authority for it at some length by translating a paragraph found on p. 448 of *Théorie Méchanique de la Chaleur*, par G. Zeuner, traduit de l'Allemand, par Arnthal et Cazin, Paris, 1869. This paragraph is found in an addition made by these translators—"Zeuner supposes, *a priori*, that the equation of the adiabatic curve for superheated steam, coincides with that of gases; only he replaces the constant $n=1.41$, which applies to gases by $n=\frac{4}{3}$. He regards this hypothesis as sufficiently justified by the exactness of the results which he deduces from it. We are constrained to say that it agrees sufficiently well with the recent experiments which M. Hirn has made in conjunction with one of us.* Though our experiments have not shown that n is rigorously constant, yet the numbers which we have deduced for that quantity vary so little from $\frac{4}{3}$ that Zeuner's constant is acceptable in technical applications."

Rankine refers to the same experiments on p. 320 of his *Steam Engine* as authority for $n=1.3$ for steam in a perfectly gaseous state.

McCulloch on p. 234 of his *Treatise on the Mechanical Theory of Heat* makes the statement that $n=\frac{4}{3}$ for saturated steam, referring to the same experiments as authority for the statement, which taken in connection with the rest of the paragraph has been taken to mean that $n=\frac{4}{3}$ for a mixture of steam and water during adiabatic expansion. This is a mistake, as Zeuner shows on p. 332 of the book above referred to, that we are justified in taking as a mean value,

$$n=1.135 \quad . \quad . \quad . \quad . \quad (259)$$

for adiabatic expansion when the steam is saturated and dry at the beginning of the expansion and expands within the limits usual in steam engines.

In case of a mixture of steam and water, Zeuner further shows on p. 335, that for adiabatic expansion

$$n=1.035+\tfrac{1}{10}\,x_1 \quad . \quad . \quad . \quad . \quad (260)$$

in which x_1 denotes, as previously, the fraction of the unit of mixture which at the beginning of the expansion is in the form of steam.

It is seen that (259) is a case of (260). Rankine was the first who proposed the use of a formula of the form of (257), and he proposed in ordinary practice to let

$$n=\tfrac{10}{9}=1.1111 \quad . \quad . \quad . \quad . \quad (261)$$

by comparing which with (260) it will be found that it corresponds to the case in which $x_1=\frac{3}{4}$ nearly.

Besides adiabatic expansion, the case most necessary to treat is that in which the steam is kept saturated and dry. The importance of this case arises from the fact that in a jacketed steam cylinder while it may be very possible to supply heat so as to prevent condensation during the stroke, it would be hardly possible to super-heat the steam in course of the stroke owing to the fact that steam is a poor conductor of heat. Zeuner shows on p. 301 of the book before quoted that

$$n=1.0646 \quad . \quad . \quad . \quad . \quad (262)$$

in case the steam is kept saturated and dry during expansion.

Rankine proposed in his *Steam Engine* for expansion in jacketed cylinders

$$n=\tfrac{17}{16}=1.0625 \quad . \quad . \quad . \quad . \quad (263)$$

which differs little from (262).

To recapitulate these results approximately,

$n=1.41=\frac{7}{5}$, for adiabatic of perfect gases,
$n=1.333=\frac{4}{3}$, for adiabatic of steam gases,
$n=1.135=\frac{17}{15}$, for adiabatic of wet steam,
$n=1.0645=\frac{17}{16}$, for saturation curve of steam,
$n=1.000$, for isothermal of perfect gases.

The relation between these curves is shown in Fig. 6, in which it is seen that the greater the value of n the steeper the slope of the curve. The curves so nearly coincide when drawn to scale that strict

* *Mémoire sur la détente de la vapeur d'eau surchauffée*, par MM. Hirn et Cazin. (*Annales de Chemie et de Physique*, 4e série, tome X).

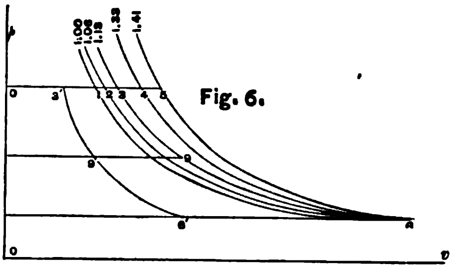


Fig. 6.

accuracy has not been sought in the figure.

Suppose for example that a volume of dry and saturated steam represented by 03 expands adiabatically in a cylinder to 6; then as previously stated in art. 40, part of it is condensed during the expansion. This fact has been observed experimentally by Hirn, and is now universally admitted. The work performed during the admission of steam from the boiler, and during its expansion is represented by the area 0367o. If the cylinder should be employed as a compressor, the water which was condensed during expansion would be re-evaporated during the compression along 63, and the total work required to drive the steam into the boiler is also represented by the area 0367o. But suppose we should, instead of driving back the entire mixture, separate the condensed water at 6 from the steam, and drive back the steam only. Neglecting the volume of the water so separated, the steam will on compression become superheated (as it has no water to re-evaporate), and will be compressed along 64, so that the work to be performed in forcing it back into the boiler will be represented by 0467o, which is an amount larger than was performed during the admission of the steam to the cylinder and during its expansion.

Certain new types of steam engines have been recently proposed by Prof. Thurston* apparently on the supposition that the second law of thermodynamics does not apply to steam. As no known facts are at variance with this law, such a supposition is inadmissible.

In particular, it was contemplated that one of these types of steam engines should have essentially the cycle above described, the proposer quoting certain authorities to show that the curve 64 lies on the other side of 63 from that shown in Fig. 6.

If it is possible to accomplish the result proposed in these engines, then the second law, which is apparently well grounded, is not tenable, and we must not only set aside the results of that most accurate of modern experimentalists, Regnault, as untrustworthy, but we must also regard all who have made experiments upon the specific heat of steam as having arrived at erroneous results.

45. Adiabatics for Steam. — These curves are given by (233), or by (257) and (260), but these equations are somewhat complex for practical purposes. A practical simplification has been effected which, together with other matters of importance, will now be detailed.

Suppose that the three adiabatics be drawn for a unit of mixed steam and water starting from the temperature t_1, in which x_1, x_1', x_1'' are fractions expressing the initial proportions of steam in the three cases. Then by (233),

$$\frac{lx}{t}-\frac{l_1x_1}{t_1}=c'\log_e\frac{t_1}{t}$$

$$\frac{lx'}{t}-\frac{l_1x_1'}{t_1}=c'\log_e\frac{t_1}{t} \quad . \quad . \quad (264)$$

$$\frac{lx''}{t}=\frac{lx_1''}{t}=c'\log_e\frac{t_1}{t}.$$

$$\therefore \frac{x-x'}{x''-x'}=\frac{x_1-x_1'}{x_1''-x_1'} \quad . \quad (265)$$

which is a relation between any three adiabatics. In order to render this of practical use, let $x_1'=0$, in which case the mixture initially contains no stream, and the adiabatic sensibly coincides with the axis of no volume; also let $x_1''=1$, in which case the adiabatic is that of steam initially saturated and dry (259),

$$\therefore \text{ by (265), } \frac{x-x'}{x''-x'}=x_1 \quad . \quad . \quad (266)$$

Now by (214), $v=w+ux$

$$v'=w+ux' \quad . \quad . \quad (267)$$
$$v''=w+ux''$$

$$\therefore \text{ by (266), } \frac{v-v'}{v''-v'}=x_1 \quad . \quad . \quad (268)$$

in which w is the volume of a unit of

* Journal of the Franklin Institute, Vol. 104, pp. 252, 313, 389.
Proceedings of the Am. Assoc. Advance Science, 1877, p. 85.
Hist. of the Growth of the Steam Engine. New York, 1878, p. 484, *et seq.*

water, v, v' v'' are the volumes of the respective mixtures at any given temperature t, or what by (212) is the same thing at any given pressure p. Hence to construct the adiabatic corresponding to any initial value of x_1: Suppose, as in Fig. 6, that the adiabatic 396 for steam initially saturated and dry has been constructed, as also the adiabatic for water, which may be taken as approximately vertical, then by (268), we have approximately,

$$v=x_1 v'' \quad . \quad . \quad . \quad . \quad (269)$$

In Fig. 6 the adiabatic 3′9′6′ is drawn for the case in which $x_1=\frac{1}{2}$.

The same relation holds between the various curves of no evaporation, as appears by combining equations (267) together; so that to obtain the curve for which x has a given value, it is only necessary to draw that for which $x'=1$, as obtained by (262), and then subdivide the abscissas in the manner expressed by (269).

46. Work of Expansion of Steam.—This can be found for the different cases by art. 42, but the convenience of the empirical formulae (257), etc., in art. 44, have led to the frequent use of expressions for the work derived from them.

By (10) and (257), $dw=pdv=cv^{-n}dv$

$$. \quad . \quad . \quad . \quad (270)$$

$$\therefore\ w_2-w_1=\frac{c}{1-n}(v_2^{1-n}-v_1^{1-n})$$

$$=\frac{p_1 v_1(1-r)}{n-1} \quad . \quad . \quad . \quad . \quad (271)$$

in which by (186) $v_2^{1-n}=rv_1^{1-n}$.

When, however, $n=1$, the curve is a rectangular hyperbola, and we have from (270)

$$w_2-w_1=p_1 v_1 \log\frac{v_2}{v_1} \quad . \quad . \quad . \quad (272)$$

The work performed along the different curves does not differ greatly, and hence (272) is frequently used for computing the external work, especially any leakage through the valves causes this result to be more nearly correct.

The total heat supplied along these different empirical curves is, however, quite different, and it cannot be computed from the curves as they appear on the indicator card.

47. Adiabatic Flow of Mixed Steam and Water.—

By (10) $dw=pdv=d(pv)-vdp \ . \ . \ . \ (273)$

$$\therefore\ \text{by (131)},\ d\left(\frac{v^2}{2g}\right)=-vdp=dw-d(pv)$$

$$. \quad . \quad . \quad . \quad (274)$$

$$\therefore\ \text{by (243)},\ d\left(\frac{v^2}{2g}\right)=d(pux)-d(lx)$$

$$-c'dt-d(pv) \quad . \quad . \quad . \quad (275)$$

Integrate between the states 1 and 2, and reduce by the following equations derived from (214)

$$v_1=v'+u_1x_1,\quad v_2=v'+u_2x_2,$$

$$\therefore\ \frac{v^2}{2g}=c'(t_1-t_2)+l_1x_1-l_2x_2+v'(p_1-p_2)$$

$$. \quad . \quad . \quad . \quad (276)$$

which result after neglecting the term $v'(p_1-p_2)$ can be still further reduced,

$$\text{for by (233)},\ l_2x_2-\frac{t_2}{t_1}l_1x_1-c't_2\log\frac{t_2}{t_1}$$

$$. \quad . \quad . \quad . \quad (277)$$

$$\therefore\ \frac{v^2}{2g}=\left(c'+\frac{l_1x_1}{t_1}\right)(t_1-t_2)+c't_2\log_e\frac{t_2}{t_1}$$

$$. \quad . \quad . \quad . \quad (278)$$

which is the fundamental equation of adiabatic flow for the mixture.

In case $x_1=0$ we have the expression for the flow of water from a boiler, in which case $t_2=212°$ F, and the term $v'(p_2-p_1)$ must not be neglected, as it must not be whenever x_1 is small.

In order to determine the state of steam after efflux we must find the total increment of internal energy after the energy of motion has been transformed into heat, which may be done in several ways, but most readily from the consideration that since no heat is received during efflux,

$$di+dw+vdp=0 \quad . \quad . \quad . \quad (279)$$

$$\therefore\ \text{by (273)},\ di+d(pv)=0 \quad . \quad . \quad . \quad (280)$$

But, $v=v'+ux \quad \therefore\ d(pv)=d(pux)$ (281)

$$\therefore\ \text{by (253) and (280)},\ c'dt+d(lx)=0 \quad (282)$$

$$\therefore\ c'+x\frac{dl}{dt}=-l\frac{dx}{dt} \quad . \quad . \quad . \quad . \quad (283)$$

$$\therefore\ \text{by (220)},\ 772+540x=-l\frac{dx}{dt} \quad . \quad (284)$$

from which it appears by (220), that as the temperature decreases evaporation occurs, *i.e.* that when dt is negative dx must be positive.

Equations (279) to (280) are the equations of the isodynamic curves of steam. Zeuner states that the isodynamic curves

of wet steam are represented by (257) when

$$n=1.0456 \quad . \quad . \quad . \quad . \quad (285)$$

Hirn deduces from his experiments that in case of superheated steam $n=1$. A comparison of these curves with the curves of no evaporation and saturation curves shows that the result deduced from (284) is correct, and that in case the steam is saturated before efflux it is superheated after efflux, although it is not so within the orifice, before its energy of motion has been transformed into heat. This result can be observed, as a conical surface separating the cloudy (wet) steam from the transparent (superheated) steam can be seen at an orifice when steam is discharged at a high initial pressure.

48. Steam Injectors and Ejectors.—A steam injector consists essentially of a condensation chamber into which flows a jet of steam and also a supply of water, and out of which flows a jet of water.

Let the steam jet flow from a boiler in which the state is denoted by subscript 1. Let the state of the water at its entrance into the chamber be denoted by the subscript 0, and let 2 denote the state of the jet of water flowing from the chamber. The velocity of the water flowing into the chamber may be neglected, and its pressure p_0 should be computed by taking the algebraic sum of the atmospheric pressure and the pressure due to the head of the reservoir from which the supply comes, above or below the chamber.

Consider first the jet of wet steam supplied by the boiler, which is condensed in the chamber and flows out in the jet of water. If v is the velocity of jet of water, then the total energy lost, per unit of steam supplied, in its passage through the chamber is evidently

$$i_1-i_2+p_1v_1-p_2v_2-\frac{v^2}{2g} \quad . \quad (286)$$

for the steam carries with it into the chamber its initial internal energy i_1, and the external work p_1v_1, which is expended upon by the boiler in forcing it into the chamber, and in going out of the chamber it carries out internal energy i_2 and does external work p_2v_2, besides its energy of motion due to the velocity v. The loss of energy expressed by (286) is that imparted to the water it meets in the chamber. The energy gained, by each unit of the water supplied, between its entrance into the chamber and its efflux from the chamber is expressed similarly by

$$i_0-i_2+p_0v_0-p_2v_2-\frac{v^2}{2g} \quad . \quad (287)$$

Now suppose that each unit of water flowing out of the chamber is composed of a fraction of a unit y which came from the boiler, and the rest of the unit $1-y$, which came from the reservoir. Hence (286) multiplied by y and added to (287) multiplied by $1-y$ is equal to the total heat per unit of efflux of water imparted to or taken from the chamber by conduction or otherwise than through the water and steam. This under ordinary circumstances may be taken as zero,

$$\therefore \frac{v^2}{2g}=y(i_1-i_0+p_1v_1-p_0v_0)$$
$$-(i_2-i_0+p_2v_2-p_0v_0) \quad . \quad (288)$$

By (254), $i_1-i_0=c'(t_1-t_0)+l_1x_1-p_1u_1x_1$ (289)

for $x_0=0$; and similarly in i_2-i_0, $x_2=0$,

$$\therefore \frac{v^2}{2g}=y[c'(t_1-t_0)+l_1x_1+(p_1-p_0)v']$$
$$-c'(t_2-t_0)+(p_2-p_0)v' \quad . \quad (290)$$

This is the fundamental equation of the injector, and in it all the quantities except v, y and t_2 may be considered to be fixed in any given case by circumstances over which we have no immediate control. Indeed t_2 is also partially independent of control for the injector will not work, *i.e.* complete condensation will not take place, if t_2 is near boiling point at the pressure inside the chamber. But over y it would at first appear that we have complete control, for the proportions of steam and water can be regulated by suitable cocks, but such is not in reality the case, for by sufficiently increasing y, t_2 will be increased beyond the boiling point within the chamber. It then appears that t_2 is dependent on y.

Were it possible to find what function t_2 is of y, we should be able to determine what value of y would, in any given case, give a maximum value to v. But it is evident that it is impossible to find out what this function is in general, because it depends upon the construction, arrangement, relative size and position of the tubes conveying steam and water.

It would seem possible to solve the problem in the case when the greatest velocity is given to the water which can theoretically be communicated to it, but the solution would be of no practical importance in case the injector is employed, as it usually is, to supply the feed water to a steam boiler. For in this case all of the energy which is drawn from the boiler in order to work the injector is restored again to the boiler in the internal energy of heat and motion of the feed water, except that expended in raising the feed water from the level of the reservoir to that of the boiler, as is evident from the fact that no energy has been expended except upon the feed water. Hence it is of no consequence how much steam is used in driving the injector since it all returns to the boiler. It thus appears that a boiler injector is a machine whose efficiency is unity, and hence it is impossible that any other apparatus can work more efficiently. If, however, the injector be used simply as a pump for raising water from one level to another, it is a matter of prime importance to investigate the relation of t_2 to y, for in this case the energy expended in heating the water is lost, and the efficiency depends upon the energy of motion communicated to the water. In any case (290) enables us to compute the energy of motion imparted to the water when we have observed t_2 the temperature of the effluent water.

The action of the steam injector is more prompt and energetic when the temperatures t_0 and t_2 are sufficiently low to insure rapid condensation.

To determine the limiting value of t_2, above which it is impossible for the injector to work, we must find the temperature t' of saturated steam at the pressure p_0, which is the hydrostatic pressure within the condensation chamber, $\therefore\ t_2<t'$. In practice it is found that t_2 must be from 12° to 20° F. below t' in order to insure a sufficiently rapid condensation.

The efficiency of the pumps which have thus far been constructed on the principle of the injector has been small, but there is another way in which this apparatus is employed, viz: as an ejector condenser for condensing engines in which the results obtained are more remarkable than when employed as an injector. The ejector condenser forces the water of condensation to heights due to the difference between atmospheric pressure and the vacuum in the condenser, and also ejects the whole volume of condensing and condensed water against atmospheric pressure, out of the necessarily rejected heat of the steam.

This is the total work ordinarily performed in working the pump to supply the water of condensation, and in working the air pump to preserve the vacuum. The whole of this work is performed by the ejector condenser without expenditure of useful energy by the engine. In other words the ejector converts a high pressure engine into a condensing engine without drawback, thereby increasing the efficiency of the engine through changing at once the temperature of its refrigerator from 212° to 175° or possibly to 150° F. Such a diminution of the temperature of the refrigerator adds notably to the efficiency of the engine as appears from the concluding paragraph of art. 19. As at present constructed, the energy due to the velocity with which the water reaches the ejector should not be neglected, and can be introduced into the preceding formulæ by introducing into (287), etc., instead of i_0 the expression

$$i_0+\frac{v_0^2}{2g} \quad . \quad . \quad . \quad . \quad (291)$$

in which v_0 is the velocity of approach.

Two Pennsylvania gentlemen have returned from China, whither they went about a year ago to examine, for the Chinese Government, the oil grounds of the Island of Formosa. They report that a well was drilled through soapstone 396 feet; then 136 feet of drill pipe were put in and 265 feet of casing. No more casing could be got in owing to the caving in of the rock. At 348 feet depth a large vein of salt water was struck, and it was found impossible to go more than 48 feet deeper. Fifty barrels of oil, says the *Scientific American*, were pumped in ten days. The oil is very light in color and gravity, and was burned in lamps without refining. The property belongs to the Chinese Government.

THE TURBINE-WHEEL DISCUSSION.

A REVIEW OF THE REPLY OF PROFESSOR TROWBRIDGE.

By Prof. WM. H. BURR.

Written for Van Nostrand's Engineering Magazine.

As Professor Trowbridge refrains from giving any detailed proof regarding the main points of this discussion, he leaves them in essentially the same positions in which my April article placed them. Proof of a little more analytical character of what was therein shown will, however, be given in this article. Two or three points, also, of his last communication will be examined.

The superabundance of his general assertions can only, and simply, be met by other assertions equally general, equally futile, and, consequently, equally valueless.

Such assertions, therefore, are entitled to no consideration and will receive none.

He seems to the writer to raise a cloud of "confusion" with no apparent object, unless, perhaps, the purpose of discreetly taking refuge within it.

In view of the different statements in the two articles of Professor Trowbridge, it becomes a matter of no little interest to determine just what that is to which he objects, in "the theoretical investigations of the turbine wheel, as given by Rankine, Weisbach, Bresse and others."

In his March article occurs the following paragraph:—

"The theorem or axiom insisted upon by these writers was that the 'water must enter the wheel *without shock*, and hence, the mathematical condition that the *tangential velocity of the wheel, where it receives the water, and the corresponding component of the velocity of the entering water* must be equal, the effect of which is to prevent all *impulsive* effects of the entering water."

Does Professor Trowbridge mean by "*corresponding component*" a rectangular or oblique component? If he means the former then, truly, all impulsive effects of the entering water are prevented. But neither Weishbach nor Bresse *even state* that the rectangular component of the entering water, tangent to the wheel circumference, must be equal to the tangential velocity of the wheel; it is an oblique component which they make equal to that velocity.

So far as the formulæ of those writers are concerned the rectangular component of the entering water, tangent to the wheel circumference may be greater than, equal to, or less than, the tangential velocity of the wheel; and in the first of these three cases an impulsive effect of the water will be assured.

The objections of Professor Trowbridge do not, therefore, hold if his "corresponding component" means "rectangular component."

It may be supposed, therefore, that he objects to the oblique tangential component being made equal to the tangential velocity of the wheel. But these authors *advise* and "insist upon" this equality (without in any way affecting their formulae for best velocity and efficiency), in order that the water may enter the wheel tangentially to the bucket, and thus avoid the loss of energy due to what they call "shock," *i.e.*, impact of water in water.

It is important to notice in passing that this elimination of "shock" does not prevent "impulsive action" of the water.

From the paragraph quoted above it is seen that Professor Trowbridge objects to the "theorem or axiom" that "water must enter the wheel *without shock*." This taken in connection with what was said above in reference to the "oblique component," forces on to the conclusion that he objects to the water entering the wheel tangentially to the bucket. On the first page of his May article, however, he says: "I do not 'object' to the authors named insisting on the principle that the water must enter the wheel without 'shock.'"

Consequently he does not, after all, object to the water entering along the bucket, and therefore to the equality of the oblique tangential component and tangential wheel velocity.

What then does the quoted paragraph mean, if it means anything?

It is idle for him to make reference to the "*reaction* wheel," for it was shown above in a general way, and will be shown in detail hereafter, that impulsive action of the water is not prevented by such a condition of entrance.

In fact as a general principle the greatest impulsive effect will be obtained, other things being equal, by water passing on the surface, against which it impinges, in a tangential direction; since in that case all loss in eddies caused by impact of water in water will be avoided. This difference does not appear in the ordinary formulae, simply because the loss spoken of is omitted in them.

Since, therefore, Weisbach and Bresse do not in any way suppose the rectangular component of the velocity of the entering water to be equal to the tangential velocity of the wheel at the same point, and since in his May article Prof. Trowbridge states that he does not object to the water entering tangentially to the bucket, his objections against those authors are contradictory and futile.

He admits that there can be no objection against Rankine on the basis of what he does in reference to the oblique component of the entering water, or the bucket, for he (Rankine) deliberately ignores the whole of the bucket or blade, except that part of it which gives direction to the effluent water, and he thereby admits all I claimed for Rankine's formulae. I simply held the point that (omitting all loss caused by eddies consequent upon impact of water in water, or shock) the formulae of Rankine, as well as those of Weisbach and Bresse, are not dependent on the angle at which the bucket cuts the wheel circumference at the point of entrance.

I used the word "shock" in the same sense in which Bresse and Weisbach use it in the same connection, *i.e.*, as indicating the violent disturbance manifested by eddies, and not as indicating impulsive effect of the water. The two things are very different.

In order, however, to avoid all confusion on this point, the word "shock" will be used not to indicate impulsive effect, but, as heretofore, to indicate that "violent disturbance," as Bresse calls it, in which energy is lost by eddies, &c.

I readily agree with Prof. Trowbridge that it is necessary for Rankine to make $v=ar$ in order to get the greatest efficiency from his formulæ when friction is neglected. His method of investigation (which is considerably different from that of Weisbach and Bresse) renders such a substitution necessary.

But Professor Trowbridge has neglected to follow Rankine a little farther. In Art. 177 of his "Steam Engine," Rankine discusses the "efficiency of turbines, allowing for friction." He goes back, for the outward flow turbine, to the general expression for efficiency in Art. 174, introduces a proper term for the friction and then writes the whole expression in terms of $z=\dfrac{ar}{\sqrt{2gh_1}}$, in which h_1 is the total head. He then proceeds by the ordinary methods of the calculus to determine what value of z will make the expression for the efficiency a maximum. *He does not do it by making* $v=ar$, but in speaking of friction, states, "this cause of loss of work not only diminishes the efficiency of the turbine, but *diminishes very considerably the speed of greatest efficiency*." The italics are my own.

Taking Professor Trowbridge, therefore, on his own ground, it is seen that his objections against Rankine's general formula for included friction for outward flow turbines do not exist.

Omitting friction, Rankine's general value of best efficiency takes either one of the following forms:—

$$\varepsilon=\frac{2}{2+n^2\tan^2\beta}=\frac{2}{2+\dfrac{\tan^2\alpha}{n^2}}\quad\ldots\ldots(1)$$

α is the angle at which the guides cut the wheel circumference at the point of entrance into it, and β is the angle at which the buckets cut the wheel circumference at the point of exit from it. If β is zero, the efficiency is unity; as with the formula of Prof. Trowbridge. Eq.(1) will be used hereafter.

There are two methods of showing in detail that the formulæ of Weisbach and Bresse do not depend upon the angle at which the bucket cuts the wheel circumference at the point of entrance.

Proceeding with Rankine, all of the bucket except that part which gives direction to the effluent water may be ignored, or omitted entirely from consideration. Or the angle at which the bucket cuts the wheel circumference at the point of entrance may be given a different value from any that can possibly enter the desired formulæ.

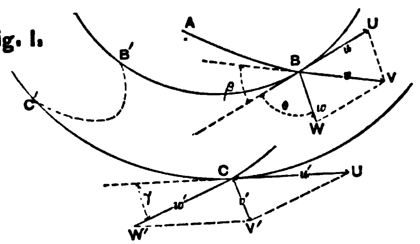
The first method will be used with the equations of Bresse and the second with those of Weisbach.

Before proceeding with Bresse's formulæ there is one point to be noticed.

In his May article, page 373, Professor Trowbridge states that his Fig. 1 (given in that page) is copied from Mahan's translation of Bresse; but it differs from that figure in one particular very important to this discussion. In his Fig. 1, which he states he copied, the angle between w and u is 90°, while in Mahan's Bresse (edition of 1869) that angle is about 101°, as nearly as can be measured. The importance of this point will be at once perceived, if it be remembered that Professor Trowbridge is endeavoring to show that the rectangular component of the entering velocity of the water is equal to the tangential velocity of the wheel, although Bresse lays down no such condition. Consequently the changed figure would apparently make Bresse indicate, through it, that which Professor Trowbridge, only, holds.

The true figure is the Fig. 1 of this article, with this exception: the end only of the bucket which directs the effluent water is retained; the rest of it is omitted.

Fig. 1.



The notation of Bresse will be used as seen in the figure. It is unnecessary to explain it here. As distinctly stated by Bresse, *all* losses of head are neglected except that due to the final absolute velocity v'.

The two following equations may then be written:

$$v^2=2g\left(H+h+\frac{p_a-p}{\pi}\right) \quad . \quad . \quad . \quad (2)$$

$$w'^2-w^2=2g\left(h'+\frac{p-p'}{\pi}\right)+u'^2-u^2 \quad . \quad (3)$$

The velocity w is simply the velocity relative to the wheel at point of entrance into it. Its magnitude and directions depend entirely upon the relative proportions and directions of u and v, or, in other words, upon u, v and β, and nothing else. In the figure θ is simply the angle which w makes with a tangent at B. Having thus determined w, w' is at once known in magnitude from (3). As is well known, the "fictitious" head $\frac{u'^2-u^2}{2g}$ is entirely independent of the path described and dependent only upon the angular velocity and distances from the center. It is to be noticed that, thus far, w' is only given in magnitude: its direction is known as soon as γ is given; its magnitude does not in any way depend upon that angle.

A simple law of hydrostatics gives—

$$\frac{p_a}{\pi}+h+h'=\frac{p'}{\pi} \quad . \quad . \quad . \quad . \quad (4)$$

The law of continuity gives—

$$brv \sin\beta=b'r'w' \sin\gamma \quad . \quad . \quad (5)$$

The triangle in the figure gives—

$$w^2=u^2+v^2-2uv\cos\beta \quad : \quad . \quad (6)$$

$$v'^2=u'^2+w'^2-2u'w'\cos\gamma \quad . \quad (7)$$

Also evidently—

$$u'r=ur' \quad . \quad . \quad . \quad . \quad (8)$$

Eq. (6) might have been written immediately after eq. (2) but they are taken in the same order with Bresse. By supposing the angle γ small Bresse writes:

$$u'=w' \quad . \quad . \quad . \quad . \quad (9)$$

It is now important to notice that the equations (2) to (9), inclusive, have been determined without any regard to the bucket at B in Fig. 1; after having omitted the consideration of all resistances, they are in no sense dependent upon it. They hold whether that part of the bucket exists or not.

Now, the equations (2) to (9) are *the only ones* which Bresse uses in determining v'^2.

There is no necessity of here following through the operations by which he obtains that quantity; he simply combines

eqs. (2) to (9) and no other. Any one can verify this statement by turning to pages 85 and 86 of Mahon's Bresse. By the mentioned combination Bresse obtains—

$$v'^2=2gH\frac{b}{b'}\frac{\tan\beta}{\sin\gamma}(1-\cos\gamma) \quad . \quad (10)$$

He then goes on to treat of some other matters, after which, page 89, he says:

"Again it might be proposed, for a turbine known to be working with a maximum effective delivery, to seek this delivery as well as the dynamic effect. As we are supposing that all losses of head, other than that due to the velocity of exit v', may be disregarded, the head that is turned to account will be

$$H-\frac{v'^2}{2g}$$

and consequently the productive force μ will be expressed by

$$\mu=1-\frac{v'^2}{2gH}\text{"} \quad . \quad . \quad . \quad (11)$$

It is seen therefore that μ, or the efficiency, in eq. (11) is written by Bresse at once; without any regard to any preceding equation whatever. Hence it does not depend upon the angle at which the bucket cuts the circumference at the point of entrance into it, directly or indirectly, implicitly or explicitly, or in any other manner known to the human understanding. Whenever, therefore, Prof. Trowbridge states that it is so dependent, he is simply in error. He is also in error when he states in his last article, page 374, that w depends on the bucket angle at B, Fig. 1. For eq. (6) shows that w depends only on u, v and β, and that bucket angle may have any value whatever (or that part of the bucket may not exist at all, so far as the equations are concerned) while those quantities remain the same. w is, therefore, entirely independent of the angle in question.

Now it has been assumed that there is no loss of energy by "shock," but such a loss would exist if the buckets, at the point of entrance, made an angle different from that of the relative component w, hence, in an actual case, in order to get the best results, that angle should be the same with that of w, for then the water will glide along the bucket without disturbance. Let θ be this angle, then from Fig. 1 there is easily deduced:

$$\frac{u}{v}=\frac{\sin(\theta+\beta)}{\sin\theta} \quad . \quad . \quad . \quad . \quad (12)$$

Bresse writes this equation between those two which I have numbered (8) and (9), but I have omitted it until the present in order to make prominent the fact that *it has absolutely no connection whatever with any of the equations from which v'^2 is deduced;* and it evidently has no connection with equation (11). There is no need of bidding its influence "out," for it is never "in." Bresse writes it, it is true, and states in connection with it that the water must enter tangentially for the best results (just what I explicitly stated in my previous paper), but he does not combine it with any equation which precedes or follows it for the determination of v'^2 or the efficiency. This statement may also be verified by consulting Bresse.

The statement of Prof. Trowbridge, therefore, to the effect that Bresse combines eq. (12) with others to obtain v'^2 and the efficiency is, to put it very mildly, somewhat extraordinary.

He simply falls into the error of supposing that the supplement of the angle which w makes with u is necessarily the angle which the bucket makes with the tangent at the same point, whereas so far as the equations are concerned, there is no connection between them; the former is fixed when u, v and β are known, but the latter may have any value whatever. Again it is important to observe that the equations are based on no particular relation between the rectangular tangential component of v, and u; much less is equality any where assumed. So far as the equations are concerned, that component of v may be greater or less than, or equal to, u. The formulae are then perfectly general in this respect.

Introducing v'^2 from eq. (10), in eq. (11), there results:—

$$\mu=1-\frac{b}{b'}\frac{\tan\beta}{\sin\gamma}(1-\cos\gamma) \quad . \quad (13)$$

Eq. (13) will be used again.

The preceding results might also have been shown by taking any bucket angle different from θ, and then by establishing the equations without any regard to it; this method will now be applied to Weisbach's equations.

Fig. 2 is the figure to be used; it is

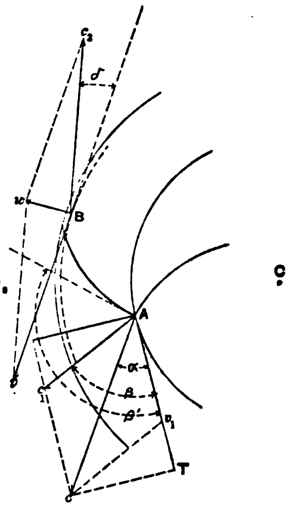


Fig. 2.

very similar to the one found in Art. 250 of Du Bois' Weisbach. The notation is the same as that of Weisbach (and does not need to be explained here) with this exception: β' will represent the angle made by the bucket with the circumference of the wheel at the point of entrance into it, while β *represents the angle which the relative component* c_1 *makes the circumference at the same point.* This distinction is important.

From the triangle c A v_1 Weisbach (Art. 250) writes,

$$c_1^2 = c^2 + v_1^2 - 2cv_1 \cos\alpha \quad \ldots \quad (14)$$

By the aid of this equation, the fictitious head $\frac{v^2 - v_1^2}{2g}$ (see remarks following eq. (3) and the actual heads h_1 and h_2, he then writes,

$$\frac{c_2^2}{2g} = h_1 - h_2 - (1+\zeta)\frac{c^2}{2g} + \frac{c^2}{2g} + \frac{v^2}{2g} - \frac{2cv_1\cos\alpha}{2g} \quad \ldots \quad (15)$$

Writing h for $h_1 - h_2$ and adding a term $\frac{\zeta_1 c_2^2}{2g}$, to represent the loss of head by friction in the wheel, there results from (15),

$$(1+\zeta_1)c_2^2 = 2gh + v^2 - 2cv_1\cos\alpha - \zeta c^2 \; . \; (16)$$

With Weisbach let Q be the discharge from the wheel; F the sum of all exit orifices from the guide curves, and F_2 the sum of the exit orifices from the wheel. Then by the law of continuity,

$$Q = Fc = F_2 c_2 \;\therefore\; c = \frac{F_2}{F} c_2;$$

also, evidently,

$$v_1 = \frac{r_1}{r} v.$$

Assuming that δ is small Weisbach takes $v = c_2$.

Making the substitutions in eq. (16);

$$v = \sqrt{\frac{2gh}{2\frac{F_2}{F}\frac{r_1}{r}\cos\alpha + \zeta\left(\frac{F_2}{F}\right)^2 + \zeta_1}} \quad \ldots \quad (17)$$

This is Weisbach's expression for best velocity and is found in Art. 251 of Du Bois' translation. It will be observed that $v = c_2$ is the same condition that Bresse uses.

From Fig. 2 there are easily deduced the following relations:

$$\frac{F_2}{F} = \frac{r \sin\delta}{r_1 \sin\alpha} = \frac{r_1 \sin\beta}{r \sin(\beta - \alpha)}$$

Introducing these conditions successively in (17) there result the two values of v_1:

$$v_1 = \frac{r_1}{r} v = \sqrt{\frac{2gh}{2\left(\frac{r}{r_1}\right)^2 \frac{\sin\delta}{\sin\alpha}\cos\alpha + \zeta\left(\frac{r}{r_1}\right)^4\left(\frac{\sin\delta}{\sin\alpha}\right)^2 + \zeta_1\left(\frac{r}{r_1}\right)^2}} \quad \ldots \quad (18)$$

$$v_1 = \frac{r_1}{r} v = \sqrt{\frac{2gh}{2\frac{\sin\beta\cos\alpha}{\sin(\beta-\alpha)} + \zeta\left(\frac{\sin\beta}{\sin(\beta-\alpha)}\right)^2 + \zeta_1\left(\frac{r}{r_1}\right)^2}} \quad \ldots \quad (19)$$

It is seen, now, that these two values of v_1 are identical and entirely independent of β', in fact would hold if that end of the bucket did not exist. The only condition involved is simply $v = c_2$. In applying therefore eq. (19) to an actual case, β *should not be taken as the angle which the bucket makes with the circumference of the wheel at the point of entrance into it,* but should be calculated by the simple formula for a triangle, knowing c, v_1 and α; or it may be determined from the equation which immediately precedes eq. (18).

Here I will state what, by an oversight, I omitted to state in my April article.

In that article the values for v_1 were calculated from eq. (19) taking β directly from the wheel. This I did to save the

labor of estimating them as directed above, knowing, since they are excellent examples, that the values would differ little from estimated ones. In the Boott Center-Vent and Westerly Swain wheels (my 3d example) the differences were a little greater than I expected, but not enough to make any essential difference in the results.

In the Center-Vent Boott wheel, for instance, v_1 by eq. (18) becomes

$$0.758\sqrt{2gh}=22.2 \text{ feet.}$$

But experiment 27 of the same series, from which I took my example, gives a little higher efficiency than experiment 30, which I took, and v_1 in that experiment was found to be 20.8 feet. The agreement is therefore as close as before. In the Westerly Swain wheel (my third example) eq. (18) gives $v_1=20.78$ feet, while experiment gave $v_1=21.48$ feet; an agreement considerably closer than before.

I explicitly stated the value of the empirical co-efficient I used as 0.075. In art. 260 of DuBois' translation Weisbach states where Professor Trowbridge may find an account of the experiments by which the values "$\zeta=\zeta_1=0.05$ to 0.10" were fixed.

In art. 260 of DuBois' translation is found Weisbach's expression for efficiency. Putting L_1 for the work performed and, as before, h for the total fall, there may at once be written;

$$L_1=\left(h-\frac{\zeta c^2+\zeta_1 c_1^2+w^2}{2g}\right)Q\gamma \quad . \quad (20)$$

Using the relations between c, c_1 and v, already given, there results for the efficiency, after taking v from eq. (18);

$$\eta=1-\frac{\zeta\left\{1+\left(\dfrac{r\sin\delta}{r_1\sin a}\right)\right\}+4\left(\sin\dfrac{\delta}{2}\right)^2}{\zeta\left\{1+\left(\dfrac{r\sin\delta}{r_1\sin a}\right)^2\right\}+2\cot a\sin\delta} \quad (21)$$

Eq. (21) will be recognized at once as the one used in my previous communication.

It will now be at once perceived that eqs. (18) or (19) and (21) are entirely independent of the angle β'; it is not possible for it to affect them. If possible, it is still more absurd to suppose that either (18) or (19), or (21) is based on the assumption that the tangential rectangular component of c, Fig. 2, is equal to v_1. The figure was purposely so drawn as to represent that component as greater than v_1, as does Weisbach, in which case the water would have an impulsive effect on the wheel depending upon the excess v_1 T.

Weisbach's formulæ for best velocity and efficiency, therefore, are *not* based either upon the "axiom" that the water should enter tangentially to the bucket, or upon the axiom that the water shall not act by impulse.

They are based upon the condition $c_2=v$, but that is all. They do not involve any term which represents the loss by shock any more than they involve terms which represent loss arising from appreciable thickness of buckets and guide curves, or fluid friction in the flume or penstock, and any objection which is brought against the formulae on the basis of any one of these omissions holds equally on the basis of the omission of all the others.

The pseudo-reasoning of Professor Trowbridge, in his last article in the vicinity of his Fig. 2, is too deeply involved in hopeless absurdity to be worthy of serious attention.

In his last article, page 375, Professor Trowbridge states essentially that the formulæ of Weisbach and Bresse do not indicate that a small value of a (Weisbach's notation) or β (Bresse's notation) should be taken. In my last article I showed that Weisbach's formula for efficiency calls for a small value of a, and took occasion to criticise his advice in regard to making it unnecessarily large. Bresse's eq. (13) shows that the smaller is the angle β the greater is the efficiency; and Rankine's eq. (1) shows the same thing (his a is the same as Bresse's β). The small guide curve angles of the Lowell wheels are, therefore, nothing more than those which the formulae of Weisbach, Bresse and Rankine indicate ought to exist. It is nothing to the point to quote the *opinions* of those authors, for in his first article (see its heading) Professor Trowbridge set out to show "the inapplicability of the theoretical investigations of the turbine wheel as given by Rankine, Weisbach, Bresse and others." It is a matter of little consequence what their opinions were.

In Art. 262, however, Wiesbach advises that *α be made from* 20° *to* 30°. In the same article he advises that the exit angle of the wheel be less than 20°.

In his last article Professor Trowbridge states that the "best modern wheels, introduced by Boyden and Francis . . . . are *impulse*-and-reaction wheels," and, in the paragraph following my quotation, he speaks in particular of the inward flow turbines. He thus states that the water acts in those wheels partly by impulse. The experiments of Francis and Mills, however, show the contrary. Figs. 3, 4, 5 and 6 are the velocity diagrams belonging to the four examples given in my April article; in fact, Figs. 3 and 4 are reproduced from that article.

The notation is that of Weisbach, c being the exit velocity from the guides and v_1 the tangential wheel velocity at the same point.

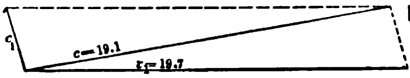


Fig. 3.

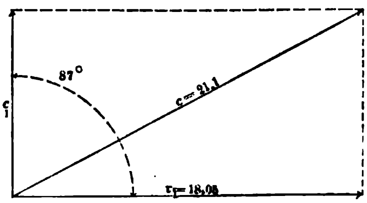


Fig. 4.

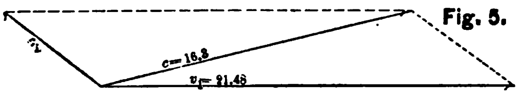


Fig. 5.

Fig. 6.

The velocities c and v_1 are both experimental in the four cases, and may be easily verified by any one taking the trouble to look up the data given in my previous article. Figs. 3, 5 and 6 belong to inward flow turbines, and in each of those cases, since the component of c in the direction of v_1 is less than v_1, *it is impossible for the water to do any work by impulse*; that is, they are simply reaction wheels.

Fig. 4 belongs to the Tremont wheel. In that case, since the angle between c_1 and v_1 is only 87°, there is a very little work done by impulse; but since that little is due to 3° only, it is essentially none.

If Prof. Trowbridge will read my previous article with a little more care, he will see that what I called an "admirable arrangement" was simply the means adopted to prevent leakage in the Lowell wheels, which I took for examples. Also that I did not speak favorably of the idea of Bresse and Weisbach of so proportioning α and β, as to prevent unbalanced pressure outward between wheel and guides.

Perhaps the most remarkable characteristic of Prof. Trowbridge's last article is his claim to originality in the investigation of his first paper. He has given, he says, the theory of impulse-and-reaction wheels "for the first time" ! ! !

Let any one turn to Fig. 1 of his March article and the page following of his text. The general expression for work which he deduces is:

$$W=M(xu+v_r\cos\gamma.u)\quad . \quad . \qquad (22)$$

From his figure $v_r=\sqrt{v_1^2+u^2 2uv_1-\cos\alpha}$ and $x=v_1\cos\alpha-u$; these values in eq. (22) give:

$$W=M\left(v_1\cos\alpha+\sqrt{v_1^2+u^2-2v_1u\cos\alpha}\cos\gamma-u\right)u \qquad (23)$$

Now let any one turn to Art. 231 of Du Bois' translation, where Weisbach gives the theory of the turbines of Borda and Bondin. About the middle of page 345, Weisbach has the formula.

$$L=\frac{Q\gamma}{g}\left(c\cos\alpha+\sqrt{c^2+v^2-2cv\cos\alpha}.\cos\delta-v\right)v\,. \qquad (24)$$

In eq. (24) L, $\frac{Q\gamma}{g}$, c, v and δ are exactly the same quantities as W, M, v_1, u and γ, respectively, in eq. (23). Consequently, the equation of Professor Trowbridge is identical with that of Weisbach.

Weisbach then makes $\cos\delta=1$; Prof. Trowbridge does the same with $\cos\gamma$. Weisbach next seeks by the method of the calculus what value of v will make L a maximum and finds $v=\frac{c}{2\cos\alpha}$; Professor Trowbridge does the same with W in respect to u and finds

$$u=v_r,\left(\text{Weisbach's } \frac{c}{2\cos a}\right).$$

Finally, Weisbach puts $v=\frac{1}{2}\frac{c}{\cos a}$ in his L and finds an efficiency of unity; with great fidelity Professor Trowbridge puts $u=v_r$ in his W and also finds an efficiency of unity. The identity of the formula and operations of Weisbach and Professor Trowbridge and the great degree of similarity in the figures they use are quite remarkable, and render any comment superfluous.

A NEW CENTER OF GRAVITY FORMULA OF GENERAL APPLICABILITY.

By J. WOODBRIDGE DAVIS, C.E.

Written for VAN NOSTRAND'S MAGAZINE.

The formula

$$\tfrac{1}{2}D+\frac{(B-A)D^2}{12V}, \qquad (1)$$

wherein A is the area of the generatrix of a space, in its initial position, B, its area in its final position, D is the distance between these positions, and V is the amount of space between, is an expression representing a distance, or linear measurement, like in kind to D, because the second term is a product of four dimensions divided by a product of three dimensions of like kind. If the space be a plane, the second term is still, as well as the first, an expression of a single dimension. This expression has been found to represent the distance of the center of magnitude from the initial end, of each of a number of shapes, or the center of gravity of a mass of uniform density, which fills the space.*

To determine the extent of its applicability, let us first consider two spaces,

$$y=f_1(x)_0^D, \qquad y=f_2(x)_0^D. \qquad (2)$$

Let A_1, B_1, be the end areas of first, V_1, its volume, D, the distance between these limits, and D_1, the distance, from first-end, of the center of its magnitude. Let A_2, B_2, V_2, D, D_2, be the similar values of the second space, both lying between same limiting planes, as indicated in (2).

If formula (1) apply to each of these spaces, then

$$D_1=\tfrac{1}{2}D+\frac{(B_1-A_1)D^2}{12V_1},$$

$$D_2=\tfrac{1}{2}D+\frac{(B_2-A_2)D^2}{12V_2}.$$

The distance, D_3, of the center of gravity of the two spaces from the initial plane, is

$$D_3=\frac{D_1V_1+D_2V_2}{V_1+V_2}=$$

$$\frac{\tfrac{1}{2}DV_1+\tfrac{1}{12}(B_1-A_1)D^2+\tfrac{1}{2}DV_2+\tfrac{1}{12}(B_2-A_2)D^2}{V_1+V_2}$$

$$=\tfrac{1}{2}D+\frac{(B_1+B_2-A_1-A_2)D^2}{12(V_1+V_2)}=\tfrac{1}{2}D+\frac{(B_3-A_3)D^2}{12V_3}$$

wherein A_3, B_3, V_3, are the areas of ends, and volume of combined space. Therefore, formula (1) applies to the space

$$y=[f_1(x)+f_2(x)]_0^D=f_3(x)_0^D.$$

In the same manner it may be shown that the formula applies to the space

$$y=[f_3(x)+f_4(x)]_D^D,$$

if it apply to space, $y=f_4(x)_0$, and so on. Consequently, *if formula* (1) *apply to each of any number of spaces between same limits, it applies to their sum.*

From a similar course of reasoning it follows that *If formula* (1) *apply to all but one of any number of spaces between same limits, it does not apply to their sum.* Also, *if the formula apply to certain spaces between same limits and do not apply to certain other spaces, more than one, between the same limits as before, the formula does not apply to the sum of all the spaces, except in special cases, when its error for some of the spaces is balanced by its error, with opposite sign, for the rest.*

Now let us consider the expression for

* Formulæ for R. R. Earthwork, Second Edition, p. 105

all spaces, $y=F(x)$.* Each term of this is of the form Kx^n; and the distance of the center of gravity, from the first end, of the space represented by $y=Kx^n$, is

$$\frac{\int_0^D Kx^{n+1}dx}{\int_0^D Kx^n\,dx}=\frac{n+1}{n+2}D.$$

Formula (1) applies to this space when

$$\tfrac{1}{2}D+\frac{\frac{(KD^n-K(0)^n)D^2}{12}}{\frac{1}{n+1}KD^{n+1}}-\frac{n+1}{n+2}D=0. \quad (3)$$

This is true when $n=0$. For all other values of n, (3) becomes

$$\tfrac{1}{2}D+\frac{n+1}{12}D-\frac{n+1}{n+2}D=0. \quad (4)$$

From this $n^2-3n+2=0$;

whence $n=\frac{3}{2}\pm\frac{1}{2}=2$ or 1.

It follows from this and the rules written in italics, that the center of gravity formula applies, between any limits, to those spaces only which are represented by the equation

$$y=a+bx+cx^2; \quad (5)$$

and it applies to special cases only, [*i. e.*, between special limits,] of all other spaces.

By comparison of (5) with similar equation, [eq. (11), p. 412, May No. of this Magazine,] representing the limit of the prismoidal formula's applicability, which equation is

$$y=a+bx+cx^2+dx^3, \quad (6)$$

it is seen that the center of gravity formula is, *practically*, co-extensive with the prismoidal formula; because the generatrices of very few, if any, practical shapes vary as cubic functions of the path. This formula will, therefore, serve in the same way as the prismoidal, as a widely general rule, which renders unnecessary the demonstration, recollection and use of a large number of special rules.

Thus, the chapter on center of gravity in a treatise on mechanics can be much abbreviated by use of this formula. The fact that the generatrix of a space, expressed in terms of its distance from initial end of that space, or from any position whatever, since to shift the axis of Y does not change the degree of eq. (5), is all that requires demonstration. The position of the center of magnitude in that space is, then, immediately indicated by formula (1), which may be reduced for particular cases.

For instances: Because the areas of generatrices of triangle, pyramid, cone, and paraboloid, in terms of distance from their vertices, are, respectively, ax, $b(cx)^2$, $\pi(ex)^2$ and πfx, formula (1) applies to all of these and to all of their frusta. Therefore, for triangle, the distance of center of gravity from vertex is

$$\tfrac{1}{2}D+\frac{(B-0)D^2}{12(\frac{1}{2}DB)}=\tfrac{2}{3}D.$$

For trapezoid;

$$\text{dist. c. of g.}=\tfrac{1}{2}D+\frac{(B-A)D^2}{12(B+A)\frac{1}{2}D}=\frac{(2B-A)D}{3(B+A)}.$$

For pyramid or cone when B is area of base,

$$\text{dist. c. of g.}=\tfrac{1}{2}D+\frac{BD^2}{12(\frac{1}{3}BD)}=\tfrac{3}{4}D.$$

For frustum of this:

$$\text{dist. c. of g.}=\tfrac{1}{2}D+\frac{(B-A)D^2}{12(A+\sqrt{AB}+B)\frac{1}{3}D}=$$

$$\tfrac{1}{2}D+\frac{(B-A)D}{4(A+\sqrt{AB}+B)}.$$

The last result, which is formula (1) only changed by cancelation of the common factor D, is at once in the simplest form. Dr. Weisbach requires a page and a quarter of laborious demonstration to reach this result. See pp. 233-4, Eckley Coxe's edition of *Theoretical Mechanics*.

For paraboloid, the radius of whose base is r,

$$\text{dist. c. of g.}=\tfrac{1}{2}D+\frac{\pi r^2D^2}{12\pi r^2\frac{1}{2}D}=\tfrac{2}{3}D.$$

For frustum of this we obtain an expression similar to that for trapezoid, since its generatrix varies according to a function exactly similar. It is

$$\text{dist. c. of g.}=\tfrac{1}{2}D+\frac{\pi(r_2^2-r_1^2)D^2}{12\pi(r_2^2+r_1^2)\frac{1}{2}D}=$$

$$\frac{(2r_2^2-r_1^2)D}{3(r_2^2+r_1^2)}$$

Formula (1) applies to the sphere. because, with center as origin, the magnitude of generating circle varies as

* See introduction to article entitled *Prismoidal Formula* in May number of this Magazine.

$$\pi(r^2-x^2).$$

Hence, for hemisphere, measured from center,

$$\text{dist. c. of g.}=\tfrac{1}{2}r-\frac{\pi r^4}{12(\frac{2}{3}\pi r^3)}=\tfrac{3}{8}r.$$

Because the generatrix of the complete rectangular prismoid [Fig. 2, p. 414, May No. of this Magazine] varies as

$$a+bx+cx^2,$$

formula (1) applies to all rectangular prismoids and wedges, in direction perpendicular to bases.

$$\therefore\ \text{dist. c. of g.}=\tfrac{1}{2}\text{D}+\frac{(\text{B}-\text{A})\text{D}^2}{12(\text{A}+4\text{M}+\text{B})\frac{1}{6}\text{D}}=$$

$$\tfrac{1}{2}\text{D}\left\{1+\frac{\text{B}-\text{A}}{\text{A}+4\text{M}+\text{B}}\right\}=\frac{(\text{B}+2\text{M})\text{D}}{\text{A}+4\text{M}+\text{B}}.$$

For wedge, measured from edge;

$$\text{dist. c. of g.}=\tfrac{1}{2}\text{D}\left\{1+\frac{\text{B}}{\text{B}+4\text{M}}\right\}=\frac{(\text{B}+2\text{M})\text{D}}{\text{B}+4\text{M}}.$$

To reach results similar to these for wedge and prismoid requires more than a page of Weisbach's Mechanics. By calculus, the demonstration is nearly as long.

In a manner similar and quite as easy it may be shown that formula (1) applies to any segment of common parabola, in direction perpendicular to axis, to all parallelograms, prisms, cylinders, prismoids, cylindroids, spheroids, hyperboloids, all segments of these between parallel planes, and to the class of shapes whose lateral boundaries are right line surfaces the type of which is illustrated in Fig. 6, p. 418, of May No. of this Magazine. The last class includes alone, columns, chimneys, piers, abutments, warped-faced wing-walls, banks, retaining walls, dams, earthwork solids, and numerous other practical shapes.

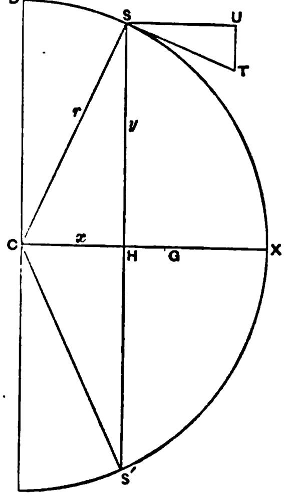
If the extremity, S, of the generatrix of a circle, Fig. 1, generate a length, dx, of the circumference in an infinitesimal time, dt, while it passes through center, then, at any distance, x, it will in same time generate a length equal to

$$dx.\frac{\text{ST}}{\text{SU}}=dx.\frac{r}{y}=dx.\frac{r}{\sqrt{r^2-x^2}}.$$

Hence formula (1) does not apply to arcs of circles.

But, if the circumference be revolved

Fig. I.



about CX, then the circumference of the generating circle whose radius is y, generates in the time, dt,

$$dx.\frac{r}{y}.2\pi y=2\pi r dx,$$

which is the form of eq. (5), when $b=o$, $c=o$. Hence formula (1) applies to any zone of sphere, and the position of center of magnitude is, when h is altitude of zone,

$$\tfrac{1}{2}h+\frac{(2\pi r dx-2\pi r\,dx)h^2}{12\text{ V}}=\tfrac{1}{2}h.$$

Because formula (1) applies to the cone CSS′, and to the segment SXS′, as has already been shown, it may be used to determine the positions of the centers of gravity of these; and the c. of g. of sector may then be found by composition of moments.

Formula (1) applies directly to the spherical sector when D is understood to be the distance passed over by the center of magnitude of generatrix, a varying, concentric zone, and V to be the amount of space described by a plane generatrix, varying as the same function of, and passing over, the same path.

This is true, because the distribution of magnitude along the path D is identical for both modes of generation, and because, since formula (1) applies to one of these shapes, the variation of generatrix being as cx^2, it also applies to the other.

Thus is obtained for sector of sphere,

$$\text{dist. c. of g.}=\tfrac{1}{2}d+\frac{Bd^2}{12(\frac{1}{8}Bd)}=\tfrac{2}{3}d$$
$$=\tfrac{2}{3}(r-\tfrac{1}{2}h) \quad . \quad . \quad (7)$$

where B is area of spherical circle, d is distance of its c. of g. from center of sphere, h is its altitude, and r is radius of sphere.

In this sense formula (1) applies to frusta of spherical sectors, [*i. e.*, between concentric zones], whether the zones be terminal or annular; also, in direction of sector's axis, to all divisions of these included each by two planes passed through axis; to circular sectors between any limits, to all portions of regular polygons and polyhedrons, which can be inscribed in circular and spherical sectors, respectively, between any limits, and to every shape, each part of whose generatrix varies as the same function of its distance from initial end.

Thus, it applies to the square diagonally, when the initial end is one vertex, and the generatrix is the broken line composed of the opposite two sides.

Applied in this way, formula (1) generally requires, preliminarily, the aid of some other method to determine the c. of g. of generatrix itself. In case of spherical sector, it applies both to the generatrix and to the space described thereby.

It will be observed from the foregoing that the only familiar simple shapes, such as are made examples in works on mechanics and engineering, to which formula (1) does not apply *directly*, are the circular arc, sector, segment and spandrel; and to latter three of these it applies either indirectly or with aid of composition of moments.

Formula (1) applies to all lines for which

$$\sqrt{\frac{dy^2}{dx^2}+1}=a+bx+cx^2, \qquad (8)$$

where a, b, c, are arbitrary. If $c=o$, dy is always in integrable shape; but the integration is tedious. Eq. (8), however, shows at once that the only familiar line to which formula (1) applies is the straight line. To obtain another example, without much labor in the integration, make a and c zero and square, subtract unity, extract square root, and multiply by dx, each member of eq. (8). By integration the equation of a line, subject to formula (1) is found to be

$$y=\tfrac{1}{2}bx\sqrt{x^2-\frac{1}{b^2}}-\frac{1}{2b}\text{ nap. log.}$$
$$\left\{x+\sqrt{x^2-\frac{1}{b^2}}\right\}+\text{const.}$$

Formula (1) applies in direction of axis to all surfaces formed by revolution of the lines for which

$$y\sqrt{\frac{dy^2}{dx^2}+1}=a+bx+cx^2, \qquad (9)$$

where a, b, c, are arbitrary. The only familiar surfaces of revolution for which eq. (9) is true are the surfaces of cylinder, cone and sphere.

Formula (1) applies, as a center of pressure formula, to the common cases of hydrostatic pressure. The equation of any immersed plane figure, referred to intersection of its plane with surface of fluid, as the axis of Y, and to any line in its plane, perpendicular to this intersection, as the axis of X, is

$$y=F(x).$$

If θ be the angle of deviation of its plane from plane of fluid surface, and γ be the specific gravity of the fluid, supposed to be uniform in density, the pressure on the figure at a distance x is

$$y'=F(x)\gamma\sin\theta\, x=F'(x).$$

In order that formula (1) shall apply,

$$F'(x)=a+bx+cx^2.$$

$$\therefore F(x)=\frac{a+bx+cx^2}{\gamma\sin\theta.x}=\frac{a'}{x}+b'+c'x. \ (10)$$

Fortunately, the last member of eq. (10), when we make $a'=o$, represents nearly all the usual submerged plane shapes, since these are seldom other than the rectangle, triangle and trapezoid, with bases parallel to fluid surface. Because the pressure at every point of submerged surface is normal thereto, and equal in intensity to $\gamma\sin\theta\, x$, it may be represented by a line perpendicular to surface and varying in length as the same constant multiple of x. Therefore, the equation of entire pressure,

$$y=(b'+c'x)\gamma\sin\theta.\, x=bx+cx^2,$$

is also the equation of a space, whose generatrix is a rectangle varying as y in last equation, its sides remaining constant in direction. This shape can, consequently, be only a frustum of the rectangular pyramid, prism or prismoid.

We may, therefore, confine our attention wholly to these shapes, because the resultant of pressure passes through the center of magnitude of each.

Of course, as shown by expression (6), the prismoidal formula also applies to these shapes, and, in consequence, to the pressure. Indeed, it would seem that this is the simplest method of determining the total pressure. The following is the

RULE.

To find the total pressure on a submerged rectangle, triangle or trapezoid, whose bases are parallel to fluid surface,

Multiply top-width of shape by its distance beneath surface; call this A. *Multiply bottom width by its distance beneath surface; call this* B. *Multiply mid-width by its distance beneath surface; call this* M. *Then total pressure is* $\frac{1}{6}$ *of distance between end-widths, multiplied by the specific gravity of fluid, and again by*

$$A+4M+B.$$

When top-width is w_1, and its distance beneath surface is h_1; and w_2, h_2, correspond to lower width, while D is distance between, the formula is

$$P=\tfrac{1}{6}D\gamma[w_1h_1+(w_1+w_2)(h_1+h_2)+w_2h_2] \quad . \quad . \quad (11)$$

If it be desired to determine the component of pressure in any direction, use, instead of D, the projection of D normal to that direction.

The formula determining the distance of center of pressure, measured from fluid surface along submerged plane, is

$$d_1+\tfrac{1}{2}D+\frac{(B-A)D^2\gamma}{12P}, \quad . \quad . \quad (12)$$

where d_1 is distance from fluid surface to top base, measured on plane of figure, and the other symbols are as before. (12) may be written

$$d_1+\tfrac{1}{2}D+\frac{(B-A)D}{2(B+4M+A)}, \quad (13)$$

or

$$d_1+\tfrac{1}{2}D+\frac{(w_2h_2-w_1h_1)D}{2[w_1h_1+(w_1+w_2)(h_1+h_2)+w_2h_2]} \quad . \quad . \quad . \quad (13a)$$

When, instead of h_1, h_2, we know the distances d_1, d_2, along submerged plane, formula (11) becomes

$$P=\tfrac{1}{6}D\gamma\sin\theta[w_1d_1+(w_1+w_2)(d_1+d_2)+w_2d_2] \quad . \quad . \quad . \quad (14)$$

and formula (13*a*) becomes

$$d_1+\tfrac{1}{2}D+\frac{(w_2d_2-w_1d_1)D}{2[w_1d_1+(w_1+w_2)(d_1+d_2)+w_2d_2]} \quad (15)$$

as simple as before.

To find distance of center of pressure below fluid surface, multiply (15) by $\sin\theta$, or use in (15), instead of d_1, d_2, D, their vertical projections, if these be known

EXAMPLES.

The submerged figure is a triangle whose base is at fluid surface. Here, d_1 and w_2 are zero; consequently, the distance of center of pressure is $\frac{1}{2}$D. The pressure is

$$\tfrac{1}{6}D\gamma\sin\theta\, w_1d_2 \text{ or } \tfrac{1}{6}D\gamma\, w_1h_2.$$

This corresponds to the case of the entire middle branch of the complete prismoid, Fig. 2, p. 414, May No. of this Magazine.

The vertex of the triangle is at surface, and the base is parallel thereto. Here w_1 and d_1 are zero; and, in consequence, formula (15) reduces to $\frac{2}{3}$ D. Then,

$$P=\tfrac{1}{3}D\gamma\sin\theta.\, w_2d_2, \text{ or } \tfrac{1}{3}D\gamma\, w_2h_2.$$

The figure is a rectangle. Here $w_1=w_2$, and the formula (15) becomes

$$d_1+\tfrac{1}{2}D+\frac{(d_2-d_1)D}{6(d_2+d_1)}.$$

$$P=\tfrac{1}{2}D\gamma\sin\theta.\, w_1(d_1+d_2).$$

The distribution of pressure on submerged trapezoids corresponds to the distribution of magnitude in the various segments of the complete rectangular prismoid, as wedges, etc.

If the plane of submerged figure be parallel to fluid surface, formula (13*a*) shows that the center of pressure is coincident with the center of magnitude of the plane shape itself. When, now, $w_1=w_2$, we have the case corresponding to that of the rectangular prism.

While defining the rectangular prismoid on p. 414, May number of this magazine, as a shape generated by a moving rectangle, the product and quotient of whose two dimensions vary, we noticed two other shapes, in one of which—the pyramid—the product is variable and the quotient constant, and in the other of which—the prism—both are constant; also, a fourth shape was de-

scribed for which the product is constant and the quotient variable. All these cases occur in hydrostatics. The only one not mentioned is that of the figure whose generatrix varies in magnitude as $\frac{a'}{x}$, the first term of last member of eq. (10), while the pressure varies as $a'x$.

Formula (13), or its equivalent, formula (15), in same manner as formula (1), saves the calculator the inconvenience of remembering numerous rules, because the reduction for special cases, a few of which have been illustrated, can be instantly effected before he proceeds to the numerical part of the work. The same may be said of the prismoidal formula, as applied to the summation of these pressures.

The only other submerged figure mentioned in treatises on hydrostatics is the circle, which is usually the surface of a valve. Formula (15) does not apply to this. If r be its radius, and h the depth of centre below surface, then

$$P=h\pi r^2\gamma;$$

and dist. cen. of press. $=h+\frac{r^2}{4h}$.

The statical moment of a material shape, represented by eq. (5), referred to an axis parallel to plane of generatrix, and at a distance d_1 beyond its initial position, is the product of V by the sum of d_1 and expression (1).

$$\therefore \text{ Statical moment}=(d_1+\tfrac{1}{2}D)V+\tfrac{1}{12}D^2(B-A). \qquad (16)$$

This has exactly same advantages as (1).

Formula (16) is very convenient when we would determine the center of gravity, or the statical moment of a compound figure, every part of which is a shape represented by eq. (5). The first term of second member shows that the *c.* of *g.* of each part may be assumed to be half way between its ends; whereafter, by composition, very easily a false statical moment of the whole figure can be obtained. This should be corrected by the second term of second member applied to each part.

The statical moment of the pressure, whose resultant occupies the position indicated by (15), is, referred to intersection of submerged plane and fluid surface, the product of P and expression (15).

To find the moment of the horizontal component of P, referred to fluid surface, which is the moment usually required, multiply the former moment by $(\sin\theta)^2$.

This is useful when we would find the position of center of pressure of a compound figure. Such a figure may be divided into triangles, trapezoids and rectangles, whose bases are parallel to fluid surface; and the moment of each may be found as above.

The moment which is most often required is that of the horizontal component of P, referred to the lower base. This is the product of $\sin\theta$. P and $\sin\theta$. $[d_2-(15)]$, which is

$$\tfrac{1}{2}D\sin^2\theta.P-\tfrac{1}{12}D^2\gamma\sin^2\theta.(w_2d_2-w_1d_1), \quad \text{or}$$
$$\tfrac{1}{12}D^2\sin^2\theta.\gamma[2w_1d_1+(w_1+w_2)\ (d_1+d_2)]. \qquad (17)$$

This is the moment which tends to overturn the solid whose surface receives the pressure.

Formula (1) will prove useful to the practical engineer, since few shapes to which it is inapplicable come under his consideration. In his service it will be especially simple, for the reason that he will be very likely to have already calculated, for other purposes, the contents of the shapes he deals with, and will, consequently, know at the outset the value of the denominator of second term. For instance, as often occurs, if it be required to find the position of the c. of g. of a piece of iron or timber of known volume or weight and of prismoidal shape, for the purpose of hoisting it, loading it upon a vehicle, or because it is a member of a structure or machine, the formula

$$\frac{(B-A)D^2}{12V},$$

indicating its distance from mid-section toward larger end, is exceedingly simple. If γ be the specific gravity of the material and W the weight of piece, the expression becomes

$$\frac{(B-A)D^2\gamma}{12W}.$$

Even when neither the weight nor volume are known, the practical calculator will find formula (1) very convenient, because it can so easily be reduced to the simplest possible form for special

cases, by mere cancelation of symbols, before the numerical part of the work be commenced.

The calculator can, in most cases, recognize at sight the shapes to which formula (1) applies. For doubtful cases formula (5) is the criterion, and it is easily used. When the shape satisfies this, it is known that both the prismoidal and the center of gravity formulæ apply.

The best practical application of formula (1) is to the determination of the *mean distance*, which the material of a cutting has been hauled to form an embankment, since here it apparently satisfies the greatest want. *Haulage* is the product of the *quantity of material* and the *average haul* or mean distance which it has been transported. *The unit of haulage is one cubic yard hauled one hundred feet.* On this basis the price for haulage is fixed.

After having calculated one factor of haulage, the *quantity*, it remains for us to find the other, or *average haul*. To do this pass a plane anywhere between cut and fill normal to route of haul. Suppose the magnitude of cutting to be generated by a limited plane, whose variable area is represented by y. Let x denote the variable distance of y from the secant plane. However irregular may be the shape of cutting, we know that y varies as a function of x. Hence ydx, the elementary volume, multiplied by x, which produces the elementary amount of haulage, is integrable.

$$\therefore \int_{d_1}^{d_2} xydx,$$

where d_1, d_2, are distances of ends of cutting, is the haulage of cut to plane, and, when x' is the average haul so far as to the plane, the following equation is true.

$$x' = \frac{\int_{d_1}^{d_2} xydx}{\int_{d_1}^{d_2} ydx}. \qquad (18)$$

The first member of this equation is, by definition, the average haul to plane; the second member is, by principles of mechanics, the distance of c. of g. of material from plane, and the equation shows that these are equal.

By similar reasoning it is proven that the average haul from same plane to fill is equal to the distance of c. of g. of fill from plane.

Therefore, *the average haul of a piece of excavation is the distance between the center of gravity of the material as found and its center of gravity as deposited.*

This is as it is stated in works which touch upon the subject. But the practical computation of this theoretical result has been found to be a far more tedious task. It is evident that, first, the centers of gravity of the component solids must be severally ascertained, since the cut or bank is measured as a compound shape. But the application of formula (18) to each of these produces a very intricate expression, involving about double the labor necessary to calculate the true content of the solid by means of the prismoidal formula in crudest shape. After this the several moments must be compounded.

To avoid this some calculators have been in the habit of dividing the excavation into two parts of equal volume by a plane normal to center line, and establishing this as the initial point of the average haul. A plane similarly fixed in the embankment marks the terminal point of same distance. But this plane is always nearer the larger end of shape than the c. of g. is, as may be illustrated upon the cone, triangle or any shape of unequal end dimensions.

For instance, if the first five 100 ft. solids of a railroad cutting have been transported to a bank, and the generatrix commence with an area zero at beginning of cut, and reach, at the end of considered part, an area whose center height is 20 ft., road bed width 20 ft. and side slopes $1\frac{1}{2}$ to 1, the total volume is about 10,000 cubic yards, and, consequently, a difference of 1 ft. in distance makes a difference of one dollar in money. But the difference in the cut is 20 ft., and, if the bank be of same form, 20 ft. is there added to average haul distance. The average haul of other end of same cutting is likely to be also 40 ft. too long. The error of this method, then, makes a total error of eighty dollars for that cutting, which is invariably at the expense of the railroad company. In the time that a calculator would, by this method, compute the haulage of a division of ten miles he would be likely to cost his em-

ployers an amount equal to a year's salary.

To divide the cutting into numerous small parts, and find the sum of their moments, or to determine the c. of g. by a mental estimate merely, are methods either laborious or liable to error.

Because the earthwork solid belongs to the class of shapes bounded laterally by straight line surfaces, formula (1) applies thereto. Let A, B, C, ... K, be the areas of cross-sections, a constant distance apart, 100 feet, or D, in a piece of railroad excavation, whose material is all carried in same direction to form an embankment. Assume an axis, outside of cutting, a distance d_1, equal to 50 feet, beyond A. Then, according to formula (16), the statical moment of first volume, referred to this axis, or the haulage of its material to this axis, is, since $d_1+\frac{1}{2}D=100$,

$$100V_1+\tfrac{1}{12}D^2(B-A), \qquad (19)$$

The haulage of second volume is, evidently,

$$200\,V_2+\tfrac{1}{12}D^2(C-B). \qquad (19a)$$

The haulage of each remaining volume is represented by a similar expression, except that the coefficient of first term is always the product of 100, or D, by the ordinal number of the volume. Thus, for last, or n^{th}, volume, the haulage is

$$100\,n\,V_n+\tfrac{1}{12}D^2(K-J) \qquad (19b)$$

The sum of these expressions is the total haulage to the axis. But the sum of all the second terms is

$$\tfrac{1}{12}D^2(K-A). \qquad (20)$$

Therefore, an exceedingly simple rule for determination of haulage can be constructed.

Before stating this rule let us make a further reduction in the formula. The unit of expressions (19), (20), is a cubic foot hauled a linear foot. To reduce this to the haulage unit, divide by 27D.=2700. Let $Vol_1=\frac{1}{27}V_1$, etc. . . . . $Vol_n=\frac{1}{27}V_n$, that is, let the abbreviations represent the number of cubic yards instead of cubic feet. This, it happens, is the denomination used in dealing with these quantities, and is, therefore, the denomination in which these quantities are presented to us when we commence to calculate the haulage. Now, divide the sum of expressions (19), (20) by 27D, using the abreviations.

$$\left.\begin{array}{c}\text{Partial}\\ \text{haulage}\end{array}\right\}=Vol_1+2Vol_2+\;\ldots\; nVol_n+\tfrac{100}{324}(K-A) \qquad (21)$$

Obviously, the axis may be established anywhere. It is merely convenient to place it midway between the 100 ft. stations. It might occupy the position midway between A and B. Then the moment, or haulage thereto, would be expressed by formula (21), with each coefficient of all but final term [called the *correction term*] decreased by unity. So the first term, Vol_1, would vanish.

Since this is so, the embankment can be referred to the same axis, wherever that may be with respect to the bank. In short, all the terms, except the correction term, express the operation, *to multiply the number of cubic yards in each volume by the number of hundred feet that the mid section of that volume is removed from the axis.*

It is well to place the axis between cut and fill, or, if they overlap, as nearly so as possible, in order to avoid negative moments or haulage. For instance, the axis might be established half way between B and C. Then the partial haulage would be

$$-Vol_1+Vol_3+2Vol_4\;\ldots\;+(n-2)Vol_n+\frac{100}{9\times6\times6}(K-A). \qquad (22)$$

This is less than (21), but the difference in defect, 2 Vol, where Vol is total volume in cubic yards of material removed, is exactly balanced by the same difference in excess, which is created when the haulage from axis to fill is considered.

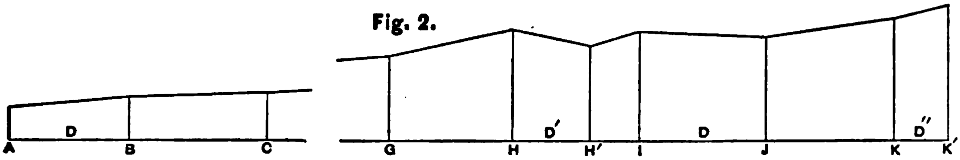
If the irregularity of the ground surface make it requisite to sub-divide one or more of the 100 ft. volumes, as the volume between H and I in Fig. 2, where, also, a plus or intermediate station, K′, instead of a full station, terminates the portion of cutting carried one way; the same rule expressed in italics, second paragraph above, holds, but the correction formula is, for example above,

$$\tfrac{1}{32400}[D^2(H+K-A-I)+D'^2(H'-H)+(D-D')^2(I-H')+D''^2(K'-K)], \qquad (23)$$

which is simply a combination of the correction formulæ for the volumes of different lengths.

The haulage from axis to fill is deter-

Fig. 2.



mined in same manner. But the result must be multiplied by

$$\frac{\textit{Excavation Vol.}}{\textit{Embankment Vol.}},$$

for the reason that the material does not occupy same space in embankment as in cut. Ordinary earths become compressed to various degrees. Solid rock fills more space in the bank. The result of division by *Embankment* Vol. is the number of hundred feet from axis to c. of g. of embankment, or the true average haul between. This should be multiplied by *Excavation* Vol., because the amount of material should be measured in the excavation. *The sum of the haulage from cut to axis and from axis to the fill is the total haulage from cut to fill.*

The foregoing may be condensed into the following systematic

RULE.

To find the haulage of material from a piece of railroad excavation to the embankment built therewith, in terms of the haulage unit, 1 cu. yd. hauled 100 ft.

Consider a plane to be passed midway between two consecutive full stations, as nearly as possible between cut and bank.

Multiply the number of cubic yards in each volume of full [100 *ft.*] *or minor* [*less than* 100 *ft.*] *length, in cutting by the number of hundred feet its mid-section is removed from the plane. If any such mid-section be on the side of plane toward the fill, its product must be taken as negative. The sum of these products is approximately the haulage from the cut to the plane.*

To correct this add the expression

$$\frac{D}{9\times6\times6}(K-A),$$

once for every series of consecutive volumes of equal length, in the cutting, D *being the length of each such volume,* K, *the area of end cross-section of the series, farther from the plane, and* A, *the area of end cross-section nearer the plane.*

Determine in exactly same manner the haulage from the assumed plane to the fill; but divide result by number of cubic yards in bank, and multiply quotient by number of cubic yards in cutting. The sum of these is the total haulage required.

This rule, although wonderfully simple, in view of what might be expected from so irregular a solid as a railroad cutting, is absolutely correct. It only remains to multiply its result by the price paid per unit of haulage to find the price to be paid for the work.

The position of c. of g. of cut may be determined in same manner described above for the bank.

It frequently occurs that portions of excessive cuttings are transported to spoil-banks near at hand. Often the entire top is taken off by scrapers. The accurate final estimate does not distinguish between these portions. In such a case the haulage from cutting to fill, as found by rule, is not the true haulage. Let *Vol'* be the amount in cubic yards wasted, as recorded in monthly estimates. Let H′ be the haulage of this amount to waste pile, as determined in monthly estimates. The position of c. of g. of this portion of cutting must be known. It can be found, according to method of last paragraph, at the time when that material is measured, and its position should be recorded. Let the distance of this position from the c. of g. of fill be L′. Now, the error in the haulage, as first calculated, is the result of the operation founded on the supposition that V*ol'* was moved to the fill instead of to the waste bank. The correction is, in consequence,

$$H'-L'\times \mathrm{Vol}'. \qquad (24)$$

Quite as often it happens that portions of the embankment are built of material from borrow-pits at hand. Let V*ol″* be the number of cubic yards borrowed, as measured in fill, H″ the haulage thereof, and L″ the distance of c. of g. of this portion of fill from the plane. Let the number of cubic yards in entire part of bank to which material from cut has been hauled be V*ol‴*. This is equivalent with expression, *Embankment* V*ol*, used above. Then (V*ol‴*−V*ol″*) is the portion of bank brought from cut; and its c. of g.,—not

the c. of g. of Vol''', as calculated in rule, —is the terminus of average haul distance from cutting. If L''' be the distance of c. of g. of fill from plane as determined in rule, and L^{iv}, the distance therefrom of c. of g. of $(Vol'''-Vol'')$, then

$$Z^{iv}=\frac{L'''Vol'''-L''Vol''}{Vol'''-Vol''}. \qquad (25)$$

The correction to be made in mean haul distance from cutting is $(L^{iv}-L''')$; and the correction to be made in haulage is, when a portion Vol' of cut, has been wasted,

$$(L^{iv}-L''')\ (Vol-Vol') \qquad (26)$$

$$=\frac{(L'''-L'')Vol''}{Vol'''-Vol''}(Vol-Vol') \qquad (27)$$

When nothing has been wasted, Vol' in (26), (27) is zero.

The haulage H'' from borrow-pit is usually kept separate.

Formulæ (26), (27) are correct on the supposition that the correction (24) has already been made.

It appears from the foregoing that in cases where parts of cuttings are wasted, or parts of embankments are borrowed, not only should the quantities and haulage of such parts be estimated from monthly measurements, as is always done, but also the centers of gravity of these parts should be established and recorded at such times, for use in determination of total haulage finally.

The same methods apply, of course, to the borrow-pits and waste-banks. The method can be readily modified to suit all practical cases.

For another method, also depending upon formula (1), of determining haulage, when the cuts and banks have been calculated entire, that is, when the contents of single volumes are unknown; also, for graphical methods of solving the problems just presented, and for details concerning the terminal solids of the banks and cuts, the reader is referred to the chapter on *Average Haul in Formulæ for Railroad Earthwork, Quantities and Average Haul*, since these depend partially upon formulæ foreign to the nature of this article.

The plan of basing contracts upon excavation and haulage prices, seems to be preferred to that which considers excavation and embankment prices. The advantage of the former is that any difference in labor, occasioned by change in amount of haulage, so often made necessary during progress of work, is directly provided for in the original agreement.

The paragraphs containing expressions (19), (20), show that the statical moment of a series of consecutive, equally long shapes, each of which is represented by some of the forms of a quadratic function, we may find by assuming the c. of g. of each shape to be half way between its ends, then compounding the several moments, and finally correcting by the expression

$$\tfrac{1}{12}D^2(K-A). \qquad (20)$$

To determine the c. of g. of series, divide foregoing result by V. Therefore, find the c. of g. of series on supposition that the c. of g. of each shape is in its mid-section, and correct by adding to the distance, of the point thus found, from A the following:

$$\frac{D^2(K-A)}{12V}. \qquad (28)$$

Thus, formula (1) reduces the problem to this simple one: to find the resultant of a system of parallel forces in one plane, whose intensities and positions are given, the position of this to be corrected by expression (28). The singular advantage of formula (1) is, that its second or correction-term, (28), remains as simple for any number of shapes in the series as for one.

It is evident, in consequence, that the error of the assumption that the c. of g. of each shape is in its mid-section, is comparatively less as the series is longer; also, that no error whatever results from this assumption, when the end areas are equal.

For instance, to find c. of g. of a spherical sector whose component cone and segment have equal altitude, it may be assumed that the c. of g. of each is half way between its bases.

To find c. of g. of a series of trapezoids such as represented in Fig. 2, assume as before the c. of g. of each to be midway its length, and correct the resulting position of c. of g. of series by

$$\frac{D^2(H+K-A-I)+D'^2(H'-H)+(D-D')^2(I-H')+D''^2(K'-K)}{12\,V}. \qquad (29)$$

The series of trapezoids may have such arrangement that some are negative, as

illustrated in Fig. 2, p. 291, April No. of this Magazine. When the last ordinate is coincident with first, the algebraic sum of trapezoids is the area of a polygon, included by the top lines of the trapezoids, and the c. of g. or statical moment of this polygon can be found by application of formula (1) or (16).

Suppose the polygon to be ABCDEA. Let a, b, etc., be the ordinates of the vertices; and let a', b', etc., be the corresponding abscissæ of the same. Since we may at first assume the c. of g. of each trapezoid to be midway its length, the several moments of these figures, when the lever arm lies in the direction of abscissæ, are,

for first,

$$\tfrac{1}{2}(a+b)(b'-a')\tfrac{1}{2}(b'+a') = \tfrac{1}{4}(a+b)(b'^2-a'^2), \qquad (30)$$

for second,

$$\tfrac{1}{4}(b+c)(c'^2-b'^2), \qquad (30a)$$

. . . .

for last,

$$\tfrac{1}{4}(e+a)(a'^2-e'^2). \qquad (30b)$$

The correction term, for first trapezoid, is

$$\tfrac{1}{12}(b'-a')^2(b-a); \qquad (31)$$

. . . .

for last,

$$\tfrac{1}{12}(a'-e')^2(a-e). \qquad (31a)$$

The sum of expressions (30), (31), is the statical moment of polygon, when the lever arm is parallel to abscissæ. This may be arranged as follows:

$$\tfrac{1}{6}[a(b'-e')(e'+a'+b')+b(c'-a')(a'+b'+c')\ldots+e(a'-d')(d'+e'+a')]. \qquad (32)$$

To obtain abscissa of c. of g., divide (32) by area of polygon, as expressed by rule B, p. 293, April No. of this Magazine. Accordingly, distance c. of g. of polygon is

$$\frac{\tfrac{1}{6}[a(b'-e')(e'+a'+b')+b(c'-a')(a'+b'+c')\ldots+e(a'-d')(d'+e'+a')]}{\tfrac{1}{2}[a(b'-e')+b(c'-a')\ldots\ldots+e(a'-d')]} \qquad (33)$$

Each term of the statical moment is the product of three factors, two of which are the factors of a term of the area. These two factors need be used once only. This makes the determination easy.

Always, when choice may be had, place the origin at one vertex, as at A. Then $a=o$, $a'=o$. In consequence, one term in the numerator, and one in the denominator, of fraction, vanish, and some of the remaining terms become reduced.

Each term of the statical moment is the continued product of the ordinate of each vertex, the difference between the abscissæ of the two adjacent vertices, (the subtraction being made always in same direction around polygon,) and the sum of the abscissæ of the vertex itself and the two adjacent vertices.

To construct the formula for the ordinate of c. of g., simply change, in formula (33), *a, b, etc., to a′, b′, etc., and a′, b′, etc., to a, b, etc.*

Professor Weisbach in article 112 of his *Theoretical Mechanics*, Eckley Coxe's translation, demonstrates a method of determining c. of g. of polygons. The one above presented is, however, decidedly shorter when one co-ordinate only of the c. of g. is required. When both co-ordinates are sought, there is little preference between them, this being in favor of Weisbach's method.

The first two columns of the following example are quoted from the article in *Theoretical Mechanics* above referred to. The problem is solved by the method of this paper.

a'	a	Double Area.	Sextuple Stat. Moment.
24	11	× 11=121	× 49= 5929
7	21	× 40=840	× 15=12600
-16	15	× 19=285	× -21=-5985
-12	-9	× -34=306	× -10=-3060
18	-12	× -36=432	× 30=12960
		1984	22444

$$x_1 = \tfrac{1}{3}\cdot\tfrac{22444}{1984} = 3.771.$$

To find y_1, substitute a for a', and a' for a, etc.; then proceed as in above example.

Applied to a triangle ABC, when the origin is placed at vertex A, formula (33) becomes

$$\frac{\tfrac{1}{6}(bc'-cb')(b'+c')}{\tfrac{1}{2}(bc'-cb')} = \frac{\text{Stat. Mom.}}{\text{Area.}}$$

$$\left.\begin{aligned} &= \tfrac{1}{3}(b'+c') = x_1 \\ \text{Likewise,}\quad &\tfrac{1}{3}(b+c) = y_1 \end{aligned}\right\} \qquad (34)$$

The abscissa and ordinate of c. of g.

of the retaining wall, Fig. 3, the origin being at the left end of lower base, are:

$$x_1 = \frac{\frac{1}{6}[hd'(d+d') + h'(w'-d)(d+d'+w')]}{\frac{1}{2}[hd' + h'(w'-d)]} \tag{35}$$

$$y_1 = \frac{\frac{1}{6}[-dh'(h+h') + d'h(h+h') + w'h'^2)]}{\frac{1}{2}[-dh' + d'h + w'h']} \tag{36}$$

The numerator of (36) can be reduced.

For Fig. 4,

$$x_1 = \frac{\frac{h}{6}[d'(d+d') + (w'-d)(d+d'+w')]}{\frac{h}{2}(w+w')} \tag{37}$$

According to formula (1),

$$y_1 = \tfrac{1}{3}h + \frac{(w-w')h^2}{12V}; \tag{38}$$

or, from (36), when $h=h'$,

$$y_1 = \frac{2w+w'}{w+w'}\cdot\frac{h}{3} \tag{38a}$$

Fig. 3.

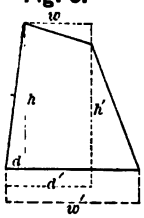


Fig. 4.

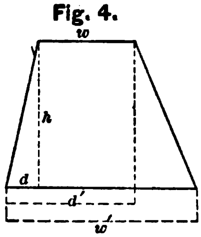
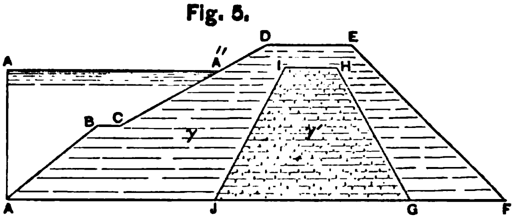


Fig. 5.



To determine the abscissa of the c. of g. of the embankment, Fig. 5, which has a core of different density from that of the covering material, we may find the abscissa of polygon ABCDEFGHIJA, and then that of polygon JIHGJ, and compound the moments, having regard to the unequal densities. But the problem can be solved more simply in one operation. Suppose γ to be the heaviness of covering material, and γ' the heaviness of core material. Now consider at once the polygon ABCDEFGHI JIHGA, regarding the ordinates of I and H, the second time used, as $\frac{\gamma'}{\gamma}i$, $\frac{\gamma'}{\gamma}h$.

To obtain the ordinate of c. of g. of whole dam, consider the same complete polygon, but regard the abscissæ, when used second time, of all the vertices belonging to the core [J, I, H, G], as if they were longer in ratio of γ' to γ.

If it be desired to ascertain the common c. of g. of the water resting on slope, and the dam itself, consider the polygon, AA'A''CBA, BCDEFGHIJ, JI HGA, and regard, when determining the abscissa, the ordinates between first and second commas as multiplied by γ, and those after second comma, as multiplied by γ'. When determining the ordinate of c. of g., multiply the corresponding abscissæ by same quantities.

For the determination of the statical moment, or the c. of g., of any series of trapezoids, formula (32) or (33) is preferable to formula (16) or (1), when the lengths are all different. This is very apt to be the case in that particular arrangement of trapezoids which forms the polygon. (32), (33) are, therefore, eminently suitable to this shape. Whenever a majority of the trapezoids have equal lengths, as in Fig. 2, formula (1) is to be preferred.

As it usually happens that the areas and volumes of practical shapes are calculated for other purposes in advance of the determination of centers of gravity, formula (33) is generally in simplest form. When the area of polygon is neither known nor required, expression (33) can be always reduced by cancelation of common factor $\frac{1}{2}$, and frequently can be much further simplified, as in case of triangle, and to form expressions (37), (38*a*).

The denominator of (33), when that formula expresses the abscissa, is equal with in value, though different in form from, the denominator of (33) when it expresses the ordinate of c. of g. Therefore, the denominator need be calculated once only. Thus the denominators of (35), (36) are the same in value, each representing the area of the polygon.

THE ÆSTHETICS OF ENGINEERING.

By RICHARD GERNER, M. E.

Written for VAN NOSTRAND'S MAGAZINE.

By way of repeating a frequently quoted remark, with a view of impressing its significance upon the reader's mind, be it stated that there is no profession which is so interwoven, not only with other professions, but also with the more ideal and oftentimes fanciful walks of the human mind, as that of the mechanical engineer. We see the engineer's handiwork in that of the architect, the builder, and, in brief, the representative of almost every industry, trade and profession. This is only natural, for wherever ingenuity is called into play, wherever primitive nature is to be reconstructed for the practical comforts of mankind, wherever necessity or ambition drives the human mind into the realms of mechanical genius, the engineer is at his station,

> "Placed on this isthmus of a middle state,
> A being darkly wise, and rudely great."

There was a time when his services were required solely in the solution of practical life problems—as simply to span the river with a bridge, and myriads of similar questions.

> "Great nature spoke; observant man obeyed,"

and civilization grew out of primitiveness.

But now that we are established, now that our wants are supplied, now that we are at leisure to turn from the merely problematic to the consideration of work of improvement, let us lend an ear to the dictates of the mind, the heart and the soul; and while continuing in our work to minister to the bodily wants of mankind, see if we cannot apply our work to the aggrandizement of the beautiful and the sublime, or, vice versa, the flights of art, imagination and poetry, to the elevation of our work.

You would not have, rising on a foaming, craggy promontory, a huge, rude tripod, supporting a torch to warn the storm-tossed mariner off the breakers, or a few planks thrown carelessly across a stream to afford a transit, or a clumsy pipe sticking out from a basin to represent the fountain of a garden, or a bell tolling from an uncouth gallows to call the pious to their devotions.

Each serves the purpose for which it was intended, it is true; mariners would not be safer, transit less difficult, jets of water less brilliant or bell peals less inspiring, were it otherwise. But it is an outrage upon our sense of the beautiful.

What you want is to behold a Bedloe's Island Gift of France rising from every surging shore, a Girard Avenue Bridge over every stream, a Loreli or Neptune in bronze splashing from every fountain basin, and a Trinity Church steeple everywhere to proclaim the hour of devotion.

How uncivilized they would feel in Philadelphia, had they to miss their Fairmount Water Works, and their water to be taken from the Schuylkill and the Delaware, borne in barrels on mules through the city and sold for a penny a gallon to the inhabitants, as is now practised in the far Orient, and the land of the Southern Cross; how unprogressive we should feel, in New York, had we to want our gas works, and to light up our proud metropolis with small oil lamps, secured on wooden posts, as now yet practised half the civilized world over.

How odious such comparisons. And yet, those who do not possess these advantages cannot appreciate what it is to want them; people never miss what they never had; they feel comfortable in their misery.

> "See the blind beggar dance, the cripple sing,
> The sot a hero, lunatic a king;
> The starving chemist in his golden views
> Supremely blest, the poet in his muse."

And again, do you doubt that our posterity will make such comparisons between their state of civilization and ours? Will the Captain Nemos and the Professor Wises of those days not smile in derision upon the diving bell and the balloon ascensions of the nineteenth century? Will the engineers and builders of the great unknown future of mankind not jest over the primitiveness of this age, in contemplating the colossi of New York, San Francisco, Yokohama, London and where not? The Island statuary Gift of Greece to France, of Italy to Germany, of Spain to England, etc.? The Miltonian tunnels through the An

des, the Himalayas, the Rockys and the Alps, resembling gates to the Infernal Regions, on whose hinges you might fancy hearing

> "— grate
> Harsh thunder, that the lowest bottom shook
> Of Erebus"

for electric locomotives to flash through? the stupendous structures over the rivers, as the Eagle spreading its bronze wings over the Hudson; the Crescent stretching its arc over the Dardanelles, the Lion leaping the Thames, or the Tea Leaf Bower gracefully spanning the Yangste-Kiang? Shall we not suffer ridicule and humiliation?

Shall that ever be with us? No! Then teach your engineers the legends of all nations and climes, mount where fancy guides, study the dreams of our poets, observe the thoughts of our artists, follow where history leads, and lift your gaze into the clouds, so that Alad din may be seen standing on some rocky isle, illuminating with his wonderful lamp the gloom of the dangerous waters, and Lohengrin ascending the Hudson, bearing a pleasure party in his shell, so that dragons may fly over the rails, on festive occasions, and ice bears spit the sparkling soda; so that our cities may be graced with Vulcans supplying us with gas, and Rebeccas with water, and so that the splendors of the empires of romance and reality may be symbolized to beautify our works of to-day.

Why should things be done clumsily when they might just as well be done handsomely? A nation in whom the sense of the beautiful is deadened, is little more than a colony of barbarians.

But it must be remembered that there is a marked line between going to work in a reasonable manner and going into extremes. There are instances where the old epigram, "beauty unadorned, adorns the most," finds a remarkably good application.

Take the steam engine of to-day. Observe as it exists in its height of perfection, how graceful its proportions, how beautiful its curves, and how tasty its form. How proudly the engineer gazes upon that child of his brain. Would he feel the better, do you suppose, were the fantastic dreamer to seize upon its symmetry and simple charm, and transform the piston-rod into the sword of the Duke of Glo'ster, and the cylinder into the aged breast of Henry VI? Not while such designers as Mr. Babcock or Mr. Corliss are living to give life to such engines as we have seen displayed at our Centennial Exhibition and elsewhere. What exquisite regularity and grace of motion; there is almost poetry in it. There is as much food for the mind of the dreamer in the engine-room of a steamer as in a romantic wilderness, where, in the former case, he is tranquilized and wrapped up into a delicious brown study in the observation of the regular, alternate, up-and-down stroke of the steel bars in their journals, and in the latter case, perchance, in the dripping of successive crystal drops from out of a rock into a tiny pool.

Who is there that can stand within a leap of a railroad track and not feel stirred into the innermost confines of his feelings, with the sublime grandeur that takes momentary possession of his entire self when the locomotive dashes by?

Who is there so hardened to fancy that he cannot penetrate into a gloomy tunnel and not imagine himself on his way to fantastic abodes: of the throne of the monarch of the dwarfish miners, of the old Teutonic legends, of the chamber of the Queen of night, or of the realms of Satan himself?

But it were purposeless to cite further instances; go where you will, into any department of engineering, you will encounter appeals to your admiration, your wonder, your emotions, and, perhaps, your passions.

Whence we observe that, of all things, engineering is less incompatable with the beautiful and the sublime than many believe, and is more indispensable to the furtherance of the beautiful, the sublime and the poetical than many know.

Of this latter we have a striking exemplification near at hand. All the splendor, the beauty, the wonder and the grandeur of the spectacle of *Baba*, lately on the boards of the New York stage, where would it be but for the engineer? People go there and praise the author for the charming fairy tale he has told us, and the artist for the magnificent illustratory scenes he has painted us, without taking into consideration that, but for the genius of Mr. Sherwood, the designing engineer, the literature and

the art of the play would all be in vain, towards approximating to the results attained.

It is in this manner that we see engineering play its part in every department of human activity; even in the activity of the imagination, and we see the utility of the engineer being versed in art, romance, etc., no less than the artist or the scholar. He cannot be too thoroughly acquainted with any department of human learning; his profession is at once an art, a science, and an industry.

It should be remembered that it is not so much the detail beauty of a structure which wins a reputation for the author, but the boldness of its conception; and and boldness in this regard is an attribute of that mind which has roamed unrestricted through all the confines of knowledge.

Decoration in detail is very good in some instances, and has won a name for many a designer, but it will be observed that the greatest triumph is reserved for him who at once conceives an ornament in the construction of the whole.

As, for instance, to construct a bridge in a mountainous or rocky region after the model of a bridge of the middle ages, one of those immense, gaunt, grim structures, which look so well on canvas, and which would be so proper on some rivers adapted by nature to be associated, in its surroundings, with such a bridge.

Beauty in structures to-day depends somewhat more upon style and taste than on fanciful conception. Everything is aimed at simplicity; designs lose the intricacies of those of the sixteenth and seventeenth centuries; surfaces are smooth to-day in contrast with the heavy decoration of those days; in many cases, the simplicity has become almost to severe.

This is but an era of style, as there have been so many eras, and so many as there are yet to come, and the simplicity of to-day, while it may find resurrection in some future age, may be replaced to-morrow by that ponderous ornament which *it* replaced yesterday.

The engineering fashions of to-day seem very much in accordance with the general intellectual levity of the age; the outward *tout ensemble* of our metropolis speaks of easy elegance rather than of solid magnificence. As an example may be cited the Girard avenue bridge in Philadelphia, and also the latter-day park and illumination improvements in New York. They are entirely in harmony with the character of gay flashy capitals. London, Paris, Vienna, Berlin and St. Petersburg look to-day as they did a century or two ago, as much as the gas fixtures of those days look like the gas fixtures of to-day.

The Fairmount Bridge at Philadelphia is an attempt at a contrast, and was built on an entirely different conception. Unfortunately, it is partly a failure. Mr. A. P. Boller, in his work on "Iron Highway Bridges," calls it a "huge architectural fraud," and with justice, for "instead," Mr. Boller says, "of letting the enormous trusses stand in all their grandeur, depending wholly upon judicious painting and the design of the cornices and railing for their æsthetic effect, thousands of dollars have been spent in actually covering up the trusses, to a great extent, with sheet-iron, forming an arcade, as it were, of great massiveness, by arching between the posts of the trusses, the arches springing from large Roman sheet-iron capitals, about halfway down the posts! The result is that, at a little distance, the spectator beholds an arcade, without any visible means of support, for 340 feet. To be thoroughly consistent, the architect of this constructed decoration should have at least sanded his sheet-iron when painted, and marked in strong lines the joints that masonry of similar forms suggests."

The bridge is built in violation of the Ruskinian axiom: "Decorate the construction, but do not construct decoration." "Such a principle conscientiously kept in view," continues Mr. Boller, "can only result in good work. Its violation, in a majority of cases, results in a demoralization of the taste of the community. Public works, in a sense, play a part in the education of a people, and their authors and builders consequently have a responsibility in addition to the mere utilitarian idea of endurance and safety."

The axiom just enumerated can only violated under peculiar circumstances, and then only by men who have their wits about them. An error is fatal, as

in the case of the Fairmount Bridge. In all other respects it "is a monument to its designer and an honor to American engineering."

Ruskin, be it stated, in his axiom, did not mean to say that a thing was not to be constructed because it was an *ornament*, such as a statuary light-house. The terms *ornament* and *decoration* are not convertible; *decoration* is that which serves to beautify, adorn or embellish something else, such as a structure; an *ornament* is a *decoration* in itself, or, perhaps, a *decoration* of the general surroundings; thus, a frescoed ceiling is a *decoration*, while a fountain is an *ornament; an ornament* may be *decorated*, but never a *decoration ornamented*. Ruskin referred strictly to *decorations*. Webster does not make this distinction, but I think I shall be sustained in making it here.

Another departure from the style of the age is rustic work, and the specimens in the Central Park of New York furnish excellent examples of the beauty and practicability of such work in bridges, steps, cottages and railings.

Style, in construction, depends in a great measure upon the materials at the command of the engineer. The somberness of the style of the middle ages was due as much to the stone then at their command as to anything else. And so it was in ancient history. Rome was built essentially of marble and granite, and hence, in great massiveness. We do not depend upon stone only to-day. Our progress in chemical and physical science has given us the metals and other material, and the result is necessarily lightness in construction. They have aided us in conceiving of bolder engineering projects, but they have not by any means reached the maximum of their adaptability. Only when they are utilized in the repetition of the engineering wonders of the ancients, and when our cities partake of the magnificence and grandeur of the capitals of Nero, Sardanapalus and others, shall this have been attained. And with our facilities, we shall be able to remodel the appearance of our surroundings in a manner that will outvie the fondest conceptions of the boldest and most extravagant dreamer.

The Blue Process of Copying Tracings. —In a short paper read before the Institute of Mining engineers Mr. P. Barnes gave the following summary of the manipulations required in this process: 1. Provide a flat board as large as the tracing which is to be copied. 2. Lay on this board two or three thicknesses of common blanket, or its equivalent, to give a slightly yielding backing for the paper. 3. Lay on the blanket the prepared paper with the sensitive side uppermost. 4. Lay on this paper the tracing, smoothing it out as perfectly as possible so as to insure a perfect contact wiih the paper. 5. Lay on the tracing a plate of clear glass, which should be heavy enough to press the tracing close down upon the paper. Ordihary plate-glass of ⅜ in. thickness is quite sufficient. 6. Expose the wholp to a clear sunlight, by pushing it out on a shelf from an ordinary window, or in any other convenient way, for six to ten minutes. If a clear skylight only can be had, the exposure must be continued for thirty or forty-five minutes, and under a cloudy sky sixty to ninety minutes may be needed. 7. Remove the prepared paper and drench it freely for one or two minutes in clean water, and hang it up by one corner to dry. Any good hard paper may be employed (from even a leaf from a press copy-book up to Bristol board) which will bear the necessarry wetting. For the sensitizing solution take 1⅞ oz. citrate of iron and ammonia and 8 oz. clean water; and also, 1¼ oz. red prussiate of potash and 8 oz. clean water; dissolve these separately and mix them. keeping the solution in a yellow glass bottle, or carefully protected from the light. The paper may be very conveniently coated with a sponge of four inches diameter, with one flat side. The paper may be gone over once with the sponge quite moist with the solution, and a second time with the sponge squeezed very dry. The sheet should then be laid away to dry in a dark place, as in a drawer, and must be shielded from the light until it is to be used. When dry the paper is of a full yellow or bronze color; after the exposure to the light the surface becomes a darker bronze, and the lines of the tracing appear as still darker on the surface. Upon washing the paper the characteristic blue tint appears, with the lines of the tracing in vivid contrast.

The Zabiaka, the fourth and last of the Russian cruisers, built at Philadelphia, is reported to have averaged over fifteen knots an hour on her trial trip. The Zabiaka is 220ft. long, has 30ft. beam, moulded, 17ft. 6in. depth of hold, and has a mean draught of 11ft. 9in. She is bark rigged and carries a great deal of canvas; is about the same length as the Alabama, and is said to look like that famous privateer externally. The battery consists of six rifled breech-loading guns of Russian manufacture, sent from St. Petersburg, and made of cast steel. Four are of 4in. bore and are broadsides, the other two are of the 6in. bore and are pivot guns. All are to be mounted on the spar deck, and the pivot guns will train in any direction. The interior arrangements of the Zabiaka are—according to the American *Army and Navy Journal*—models of fine workmanship and convenience.

ENGINEERING GEOLOGY.

By W. H. PENNING, F.G.S.

From "The Engineer."

I.

In the execution of engineering works, however scientific in design and clever in workmanship, failure has frequently usurped the place of success, because due attention has not been paid to geological phenomena. The same may be said of building operations; whilst it is notorious that vast sums of money have been thrown away in mining speculations which would at once have been characterized as hopeless by any one possessing the slightest acquaintance with geology. Although a knowledge of this science is undoubtedly a great acquisition, which affords both pleasure and profit to the possessor, it is not possible, nor even desirable, for all professional men to become proficient geologists. Those for whom these papers are more especially intended have too many claims upon their time and attention to bestow either upon a study of abstract principles, laws, and theories, which do not relate to their own particular science, art, or occupation; but they may with advantage avail themselves of the labors of others, when the results of those labors bear directly and in a very important degree upon the stability or success of the works whereon they are engaged.

The engineer certainly should make himself acquainted with the geology of a district through which a line of railway is to be constructed from his designs and under his superintendence. He should ascertain the nature of the various rocks that will be met with, not only at and near the surface of the ground, but for a considerable distance below; their relation to each other, and the consequent influence they will exert over the works in contemplation. "Trial-holes" are generally dug for this purpose, but these are simply pits excavated to a depth of a few feet, and afford information which, although valuable in itself, extends only to the superficial deposits. Borings are sometimes made, but are, in most cases, too costly; and however numerous these, or trial-holes may be, both fall far short of what can be achieved in the same direction through the methods employed by the field-geologist. By these he determines not only the kind of rocks occurring at or near the surface, but also their position in regard to each other; and is enabled to indicate with reasonable accuracy what strata will be met with, and in what successsion, to a depth, it may be, of several hundred feet; and these results of his labors include not merely a knowledge of what beds would be pierced in sinking a well, or in excavating trenches for foundations, such as would be afforded equally by trial-holes or borings of sufficient depth; but they embrace also the important points of the "lie" of the beds, the order of their superposition, their outcrop, dip, and consequent water-bearing properties; by all of which the stability and durability of engineering works are greatly affected. One cannot fail to perceive how differently placed or constructed would have been many of the most important works, such as fortifications, railway-cuttings, embankments, tunnels, and even sewers, had those who designed them been acquainted with the principles, methods, and results of field-geology; or how much capital might have been usefully instead of fruitlessly expended, or how many catastrophes would have been averted. Mention has already been made of costly sinkings for minerals, where they could not possibly have been found; large sums of money have also been wasted in equally fruitless searches for water. Yet water-supply is as amenable to known laws as any other phenomenon of Nature, and within certain limits it may be determined without experiment. Although the divining rod has not even yet ceased to be a power amongst foolish people in certain localities, its days are surely numbered; men must, sooner or later, come to see that springs are merely water finding its own level, and that for water to issue forth at one part of the earth's surface, it must

have been absorbed at another. When the conditions affecting its absorption by and passage through the strata of a district are known or can be discovered, the existence of springs, their depth from the surface, and the height to which they will ascend, can be approximately, if not with extreme accuracy, determined. In his "Rudimentary Geology," Major-General Portlock has truly and eloquently said, "Geology is now a true science, being founded on facts and reduced to the dominion of definite laws, and in consequence has become a sure guide to the practical man. The miner finds in it a torch to guide him, in his subterranean passage, to the stratum where he may expect to find coal or iron, or to the recovery of the mineral vein which he has suddenly lost; the engineer is guided by it in tracing out his roads or canals, as it tells him at once the firmest stratum for supporting the one, and the easiest to cut through for the other, and makes him acquainted with the qualities of the materials he should use in his constructions, and the localities where he should seek them; the geographer finds his inquiries facilitated by learning from geology the influence of the mineral masses on the form and magnitude of the mountains and valleys, and on the course of rivers; the agriculturist is taught the influence of the mineral strata on vegetable and animal life, and the statesman discovers in the effects of that influence a force which stimulates or retards population; the soldier also may find in geology a most valuable guide in tracing his lines both of attack and defense; and it is thus that a science rich in the highest objects of philosophical research is at the same time capable of the widest and most practical application."

In the following papers rules and methods relating to stratigraphical geology only are given; as the geological conditions which affect engineering and similar works are the extent of the various strata, their lithological character, and order of succession. It matters not what may have been the forms of Life during the ages when the strata were deposited, what their relations to those older or more recent, or what the order of their appearance in time; although the evidence upon these questions is as strong and as interesting as any upon which is based the science of geology. The rocks will be treated merely as stones, clays, and sands of varying quality; some possessing commercial value and great utility; others having qualities to be guarded against in all mechanical operations; some only exhibiting water-bearing properties; but all worthy of study, independently of the old-world histories which they contain. The names of places are given only in particular instances, such as those of mines, important quarries, notable sections, and so on, it having been considered advisable not otherwise to refer to localities in the description of the rocks. These are mentioned generally, and under specific denominations geological maps indicating much more readily the formation at any particular spot than a lengthy reference to the many places which must otherwise have been mentioned as situated on an extended outcrop. The main object of these papers is to enable the observer to discover and for all practical purposes to trace out for himself the nature and extent of the rocks with which he is concerned. The "drifts" are usually omitted from the maps; these are a series of superficial deposits not shown on any of the older geological charts, and noticed on only a few of the more recent official publications. They consist chiefly of clays and gravels of peculiar character, which are found here and there upon the older rocks, on hills and in valleys, with no very definite mode of occurrence. A section is devoted to a brief description of these deposits, with the methods of tracing and mapping them, as they must be treated from a practical point of view, in the same way as the older and more important formations.

Geological Strata: Their Nature, Relation, and Bearing upon Practical Works.—The earth's crust consists of a great number of alternating rocky layers, various in kind, thickness, and extent, but always in regular, if not in constant, sequence. The uppermost have been formed in a great measure from the waste of those beneath, in the same way as the material now being deposited on the bottom of the sea has been derived from the denudation of the existing dry land. These layers are but rarely hori-

zontal, and they bear evidence of having been subjected to an upheaving force which has acted at various times, unequally and with different degrees of intensity, beneath every portion of the earth's surface. There have been corresponding, and on the whole nearly equal, movements of depression, and all areas have frequently been dry land, again to be covered by the waters of the ocean. It is owing to this inequality in the upheaval of the beds, and to their consequent partial destruction by the sea, that the lower and older strata are now exposed at the surface of the ground, and that we are enabled to classify the rocks and to decipher their ancient history. The formations, of which the denuded edges are thus bared and thrown open to our inspection, are indicated by different tints upon geological maps. If it be borne in mind that each of the areas thus distinguished represents, as a general rule, the edge and not the surface of a formation, the proper understanding of a geological map is much facilitated. It is evident that were these variously-colored portions each indicative of an original surface, the rocks so depicted would generally be the newest, as overlying those which are hidden beneath. But their edges only being exposed and portrayed on the map, the planes of bedding must be either in a vertical position, or inclined from the surface in some direction, and the rocks, as a matter of course, must then pass in under some of those that are contiguous. Any geological map shows that, in this country, by far the larger proportion of these edges, or lines of outcrop, follow a nearly north and south direction, therefore the beds must dip, if at all, either to the east or to the west. The general dip of the formations in these islands is towards the south-east; consequently those on the north-west are the oldest, and the lowest in the geological scale; those on the south-east are the highest in the scale, therefore the most modern. All the beds of which the various formations are composed are termed "rocks," whether they be hard or soft, of aqueous or igneous origin. The following remarks upon them have been as far as practicable classified under three headings—(*a*) The nature of the rocks; (*b*) Their relation to each other; (*c*) The bearing of the nature and relation of the rocks upon practical operations.

(*a*) *The Nature of the Rocks.*—The rocks of which the known crust of the earth has been built up, in successive layers, are of infinite variety as regards texture, color, hardness, and other peculiarities. All are made up of minerals either in a crystalline or fragmentary form, or of mineral matter in a state of comminution. Some rocks contain metals, either free, or, as ores, in combination with oxygen, and but comparatively few are without metallic coloration. All rocks may be divided into two great classes:—(1) The igneous or unstratified, which were formed below the surface, and were by volcanic or similar force erupted through or intruded into the pre-existing formations. (2) The aqueous, or stratified, which were deposited from water as sediment, or in some cases as chemical precipitate. There are rocks which are otherwise formed, and some which have been altered from their original condition by heat or pressure, or by both agencies combined; but in a work of this nature the scientific differences may be disregarded. (1) The igneous rocks are the granites, dolomites, diorites, felspars, basalts, trachytes, and tuffas. (2) The aqueous are sandstones, clays, limestones, flints, and gravels. The altered, or metamorphic rocks may have been either igneous or aqueous, but were principally the latter, and are now found as gneiss, quartzites, clay slates, schists, and altered limestones. The class to which rock belongs is practically important, on account of the difference in the normal mode of its occurrence. The stratified rocks lie evenly, the one upon the other, and preserve a regular but sometimes interrupted sequence; the unstratified follow no definite lines, but suddenly break through older rocks and disappear in an equally abrupt manner. In both classes the rocks of every kind present many varieties and gradations towards each other, but on the whole they possess broad characteristics by which they may be fairly determined. It may be noted that, generally, but not without exceptions, the older stratified and the altered rocks are more crystalline and compact than those of more recent date. Those that were by

an old classification designated primary, consist of slaty and crystalline strata, such as gneiss, and mica-schist, marble, and clay slate; transition, of slaty and siliceous sandstones and calcareous shales; secondary, of chalk, limestones, red sandstones, marls, and clays; tertiary, of sands and clays; recent, of sands, gravels, silt, peat, and alluvium. The loose and friable beds are the most recent, overlying others more consolidated of secondary age, which in turn rest upon the older and more crystalline strata. All were once in the same unsolidified condition, but some have become hardened during the ages which have elapsed since their accumulation.

(*b*) *The Relation of the Rocks to Each Other.*—For mechanical purposes, the relation of the rocks of a district is quite as important as their individual nature, but not so readily ascertained, unless the proper methods of procedure be understood. Their relative thickness, of course, rules the extent of the ground occupied by each, but it must be studied, also, in connection with their dip, which exercises a powerful influence on the shape of the country where they come to the surface. Where the bedding of rocks is horizontal, or nearly so, the surface will be much more flat and spreading than where the dip is sharp, which will produce a rugged and rapidly-alternating landscape. This fact is well worthy of notice, because we may reason conversely that if a country be flat, the beds are tolerably level, and extend some distance in any direction; but if it be much broken, that they have a high angle of inclination. Upon the dip other properties of the beds depend; and it will be seen that it involves many important consequences to works constructed on their outcrop. The relative elevation of varying deposits bears directly upon the flow of surface water from one area to another; therefore it affects the land springs, and the dryness or dampness of any given locality. This leads to the more extensive and intricate question of relative permeability, upon which depend the all-important points of the power of absorption of water by the beds, and the nature and origin of deep-seated springs. These points influence not merely the supply to artesian wells, but the liability to landslips, and the varying pressures by which engineering works are especially affected. The phenomena of deep-seated springs depend not altogether upon permeability—although this is one of the chief elements in their production—but also upon the relative position of pervious and impervious strata. These may succeed each other in the simplest way, by being in regular sequence the higher beds resting evenly upon the lower, each possessing the same angle of dip; or in a more complicated manner, which is described as "unconformable." This term is applied to beds, or to sets of beds, which at any particular spot possess different degress of inclination. It is evident that in such a case the uppermost beds rest upon the edges, and not upon the surface of those beneath, and that before the higher were deposited, the lower had been cut off by some process of denudation. Occasionally beds overlap each other, without being exactly unconformable; and sometimes those which are known to be so, do nevertheless rest evenly upon each other, and with the same dip; but this is a merely local, and may be considered an accidental occurrence. When strata are unconformable, of course the continuity of the lower beds beneath the surface is broken, and this must affect the flow of water through them in any direction. The underground extension of rocks is likely to be interrupted also by "faults," or other dislocations, by which portions of them are displaced, sometimes several hundred feet, and which may extend horizontally for a short distance only, or for several miles. Such faults or breaks must in all cases be discovered, and their influence estimated, in the consideration of problems in engineering geology.

(*c*) *The Bearing of the Nature* (*a*) *and Relation* (*b*) *of the Rocks upon Practical Works.*—The rocks which form the surface of the earth, whatever their nature may be in any particular locality, of course form the base of all engineering and architectural works. The same remark applies equally to mining and well-boring operations, and to agricultural pursuits; the rocks must exercise over them all a permanent influence, of necessity greatly conducive either to their success or to their failure; and this not only by their inherent properties or peculiarities, but also, as we

have seen, by the relations which they bear towards one another. These relations, and the broader characteristics of the strata, have been briefly mentioned above—*a* and *b*—and are more fully described further on. The ways in which their influence is exercised may be here enumerated, as indicating the necessity for all available geological information being secured, and calculations based thereupon, before works of any importance are commenced or even designed. This influence is exerted in two distinct ways, (*a*) Through the nature of the rocks; (*b*) Through their relation; but the two, although distinct, are sometimes blended, and always so intimately associated that a partial separation can only be attempted.

(*a*) The trial-holes usually made before the commencement of railways, and other engineering works, expose the kinds of rock near to the surface along a given line, or over a given area. But these holes, by themselves, afford no indication of the thickness of any deposit, and in this respect may actually mislead, unless other conditions be ascertained; a knowledge of which points may, however, be derived from the trial-holes, when once the method of utilizing their indications be understood. For instance, a line of such holes over a hill may be, almost without exception, in clay, one or perhaps two of them on the flank of the hill being sunk in a hard rock. The presumption would probably be that a cutting made through this hill must pass entirely through clay, except where the hard rock was exposed; the reality, that nearly the whole of the work has to be carried on through the harder stratum at a great additional cost. For the edge only of the bed was touched by the one or two holes where it comes to the surface, with a narrow outcrop, along the steepest part of the hill. This sort of thing has repeatedly happened, and more frequently still, a bed of gravel or sand, only a few feet in thickness, but spread over an extended area, or on the slope of a hill, has led to the conclusion that the whole of the cutting would be, as all the trial-holes were, in sand or gravel. It may turn out to be hard and intractable rock, removable only by blasting operations, but covered with gravel just thick enough to reach below the few feet exposed in the trial holes. Or the converse may be the case; the engineer unacquainted with geological methods, feels sure, from his trial holes, that he will obtain from such and such a cutting, building-stone sufficient for all the bridges, or gravel enough to ballast the line; but as the work progresses, and the deeper strata come into view, he meets with serious disappointment. These instances, selected from many, are sufficient to show the necessity for the nature of the rocks being ascertained, not merely at the surface, but to a depth below, certainly not less than that of the deepest cuttings.

If such information be necessary for cuttings, it is much more required for tunnelling through the rocks, where the work is far more costly, and where, consequently, much greater saving may be effected by a previous knowledge of what strata will, or will not, be passed through in any line at a given level. Yet here the indications from trial holes must be still more meager seeing that tunnels are seldom made except where the hills are too lofty to be passed through by open cuttings, therefore through strata at a greater distance from the surface. But the evidence so obtained, if treated by geological methods, may be made equally reliable, and in proportion far more valuable. Many tunnels have been made in places which would have been avoided had the geological phenomena been previously ascertained; others where, by diverting the line a short distance to one side or the other, they might have been made with a saving of more than half the cost of construction.

The nature of the rocks to be passed through in tunnels or cuttings affects also the calculations for the necessary slopes, and consequent widths, of embankments. For main-drainage works, trial holes afford ample information regarding the kind of strata along which the sewers are to be laid, but not as to their relation—a much more important point in this particular class of work, and one which is again referred to further on. The remark is applicable also to excavations for docks, foundations for dock walls and sills, water towers, and similar works. The evidences of the solidity of the rock, its liability to slip, or to squeeze outwards under pressure, being obtaina-

ble from trial holes and borings, need not here be considered.

In many cases the nature of the rock upon which bridges or culverts are to be built can be very well ascertained by trial holes; but not by any means in all.

For it happens that many of the largest and most important structures, such as bridges and viaducts, are required in the lower-lying parts of a district, that is, in its valleys; and here the evidence thus obtained is apt to be misleading. The smaller valleys are, in places, filled to a depth of many feet with a wash from the neighboring hills, composed perhaps of sand or clay; which wash so closely resembles the rock whence it has been derived, that it is, except to the experienced eye, a part of, and continuous with, the rock of which the high ground consists. But beneath this wash there may be a treacherous bed of peat, or even of quicksand, the evil influence of which is perhaps only discovered when the new structure is sufficiently advanced for its weight to cause an ugly settlement. In such places, it may be urged, borings are resorted to rather than trial holes; even where such is the case similar results may occur, should the misleading characters be repeated. And it often does happen that in alluvial flats there is a great number of rapidly alternating ancient river deposits, which may consist of peat, silt, gravel, sand, or clay. The solid substratum may not be reached perhaps for fifty, sixty, or even a hundred feet. if the spot in question be situated over an old course of the river, sometimes a long way from its present channel, and of which there is nothing on the marshy plain to indicate the existence.

A very important matter, in regard to the cost of construction in all engineering and building works, is the material which the rocks of the neighborhood will afford, and this of course varies according to the nature of the rocks themselves. The local quarries, lime-kilns, brick-yards, &c., will almost certainly be on the outcrop of beds most prolific in building materials. But a geological knowledge is nevertheless requisite to guide the engineer in laying out his works so that they may strike the more valuable strata to the best advantage. Building material is frequently brought long distances, when that which is as good, or even better, occurs—but perhaps hidden by a few feet of drift—in the vicinity, and, it may be, in abundance. A railway cutting or a tunnel may be judiciously set out so as to follow exactly the course of a useful stratum, even to a considerable depth from the surface, probably to rail-level. On the other hand, it may be planned so as to miss the bed, except just at the surface, or possibly altogether; for the point depends upon the direction of the dip of the stratum, its consequent strike, and the actual amount of its inclination.

The drift gravels occur in a more irregular manner than any other series of deposits, but if they be previously mapped, and the work be designed accordingly, a great saving may be effected; the labor of excavating a cutting, for instance, may perhaps be made to yield the additional result of affording ballast or road metalling. The extent and thickness of the gravels must be ascertained, also their mode of occurrence, whether as capping a ridge, filling an old channel, or resting on the sloping flank of a hill. Where gravels are scarce, or altogether absent, ballast may be obtained from the more solid rocks, broken up small for that purpose, and in some districts these are sufficiently plentiful. Even in many thick deposits of clay there are found occasional beds of hard septaria, or thin bands of limestone, suitable for the purpose; the line of these, when their outcrop has been traced, may frequently be followed with advantage.

Nearly, if not quite all the geological formations, yield some one or more forms of building material, and many consist almost entirely of rocks that can be utilized in construction. The varieties and qualities are numerous, but nearly all fall under the general terms limestone, sandstone, and brick-earth, and under these headings they will be dealt with further on, with reference to typical localities. But viewing the deposits on a large scale, one or two points may be noted that are worthy of remembrance. There are many series of strata which, being either all limestones, or all sandstones, or a mixture of both, with perhaps intervening clays, are grouped under some comprehensive term. These general in-

clusive denominations are convenient rather than strictly accurate; as instances may be mentioned the Lower Oolite limestone and the Caradoc sandstone. In every series there are some beds of more especial value for particular purposes than others above and below them, although the difference may not be at once apparent. There are beds also which for building works should be scrupulously avoided, in consequence of being liable to crumble on exposure, or possessing some other detrimental peculiarity of composition. As beds vary rapidly, that one which is good in every respect in one district being worthless in another, no general description can accurately apply to all localities; therefore these matters should receive careful and local investigation.

The minerals and metals which occur so abundantly in these islands, and the cost of mining for them, are also dependent directly upon the nature of the rocks with which they are associated, and the conditions by which those rocks have been affected since their deposition. In the most important instances they exist as an integral part of the formations in which they are found, as coal, ironstones, &c., but these are not, strictly speaking, either minerals or metals. Pure minerals and native metals are comparatively rare, but the terms are conveniently, if somewhat loosely, extended to include rock-masses, or portions of rock-masses, of which certain mineral matters, or metallic ores, form a characteristic part. The "minerals" and "metals" included in the terms thus qualified will be spoken of in another place, and for all practical purposes they may be treated in the same way as any other of the rocks in which they are enclosed, or with which they are interstratified. Their existence in any given area can be ascertained in a similar manner, their outcrop surveyed, their extent determined, and their value approximately estimated.

Agricultural pursuits are affected, to a degree, much greater than is perhaps generally understood, by the nature of the solid rocks beneath the surface soil, on which the success of farming operations is admitted to depend. For all soil, or mold, has been produced, during the lapse of many years, by the atmospheric disintegration of the surface of the strata which form the base or subsoil. It has been increased in depth and somewhat modified, but its constituents have not been materially altered, by the annual growth and decay of vegetable matter. The process has been assisted by the apparently trifling, but still ceaseless action of earth worms working into and turning up the subsoil, thus constantly adding new material of similar character. It is evident that the nature of the soils of any district must therefore vary as the subsoils or strata from which they have been derived, and in a corresponding degree; and that the geology of a place being known, the subsoils and soils are equally understood. A base of gravel or sand produces a light soil, abounding in silica, that substance not unfrequently forming four-fifths of its whole weight. This will vary from a fine sandy mold to a stony soil, as the particles of the rock beneath are fine and uniform in size, or coarse and irregular. Clay gives rise to a stiff, heavy, and sometimes tenacious soil, containing from 10 to 30 per cent. of alumina, and varying in quality perhaps more than any other kind, but being as a rule more productive. Limestones produce light soils, variable, and sometimes full of detached lumps of the rock, but generally yielding good returns for high cultivation.

There are certain natural causes which modify, to some extent, the normal characters of the soils thus derived from subærial disintegration of the rocks of a locality. The result of the influence exerted by these causes may not be extensive, but they are locally important. First the rain-wash, which removes the lighter particles of the rocks from higher to lower ground. In a flat country, the effects of this action are not very perceptible, but where the surface is broken by hills and small valleys, accumulations of this material may often be seen several feet in thickness. Then, where the downward progress of rain-wash has been arrested by a wall or fence on a hill-side, the result is, after a time, very evident, in the ground being unduly higher on the upper side. Again, the growth of peat and the accumulation of marsh-clay are agencies which give rise to soils very different from what they

would otherwise have been at the spots beneath such influences. The proportion of decaying vegetable matter, so useful a constituent generally, is in peat so excessive as to render the soil almost worthless for purposes of cultivation. The soil of which a marshy plain is composed is due to the rocks somewhat further up the valley, the particles having been brought down in suspension by the waters and deposited, thus forming a flat, when the stream has at times overflowed its banks. Further, where two indifferent soils meet, which have been formed from decomposition of contiguous rocks, that which occurs along the line of junction is generally found to be of better quality, owing to admixture. Some soils are rich in fattening properties and excellent for grazing, but through want of lime, without which no bone can be formed, young stock do not thrive upon their produce. The deficiency can often be supplied at a small cost, and the value of the land be thereby much enhanced; this is frequently the case in low marshy situations. Other soils, having had much of their productive properties removed by excessive croppings, may be considerably renovated by a surface dressing of the parent subsoil. The same object may be effected by deeper ploughing, the subsoil being of course gradually incorporated with the surface mould. But for this and similar operations, certain particulars must be obtained, or the labor so expended will perhaps have been thrown away. These are the chemical composition of the subsoil, and the constituents required to be added to the soil itself; details which are readily obtainable through a small expenditure of time or money by chemical, but not necessarily quantitative, analysis.

As the subsoil varies, so does the necessity for draining the land, and the facility with which the operation can be performed. Land-drainage, as ordinarily understood, is a simple matter, but there are some geological considerations respecting it to which attention may be briefly directed. As the strata affect the soils and sub-soils, so they must of necessity exert an influence upon the natural drainage, and upon the means best adapted to that of an artificial character. The springs of one locality being but the natural outlet of water from another, the strata that now throw out springs would, if occurring at a different level, act as the channels for draining water away from the surface to the interior, to be afterwards thrown out elsewhere. And it sometimes happens that the subsoils, or underlying strata, may be by some artificial aid, made available for purposes of drainage where they would not so act without that assistance. In other words, a plan of combined natural and artificial drainage can sometimes be easily carried out, where a natural system does not exist, and where an entirely artificial scheme can be adopted only with very considerable trouble and expense.

Although the last few years have witnessed a great, and, on the whole, beneficial change in the methods of disposal of town-sewage, the difficult problem is still far from being solved. In many cases plans of irrigation have been adopted, and these are always affected by the nature of the rocks upon which the sewage farms are situated. Opinion is greatly divided, not only as to the respective merits of the methods of precipitation and irrigation, but also, when the latter plan is in question, regarding the kind of soil best suited to the purpose. Doubtless much may be said on behalf both of the light and heavy soils, of the gravels and the clays, but a point that should not be lost sight of is the ultimate disposition of the water holding sewage particles in solution and suspension. A heavy soil will frequently yield enormous crops when judiciously irrigated and in favorable seasons, but beyond the mechanical deposition of its suspended particles, the water is not clarified; it runs off the land almost as chemically impure as when pumped or discharged from the reservoir. Very little of the liquid percolates down into a typical clay, and the benefit derived by the crops seems to be mainly owing to the moistening of the surface when it would otherwise be dry and parched. It is probable that pure water would have almost as good an effect as liquid sewage upon heavy land. Light soils, on the other hand, absorb and, for a while, retain a great deal of the moisture, giving it out again to the crops in a more equable manner; they may yield less produce, but this is, in some measure, through their containing within themselves a

smaller quantity of the elements of fertility. It seems reasonable to suppose that as the liquid is considerably filtered by its passage over or through sandy soils, the ingredients removed from it are ready for absorption by vegetation. A light soil, owing to its permeability, will at all times take more sewage liquid, acre for acre, than a heavy one, and in some seasons—during a period of floods, for example—this property may prove of great advantage. But considering the phenomenon referred to in connection with land drainage, that the springs of one part are but the drainage of another, it is questionable how far we are justified in saturating the sands of any locality with tainted water.

There are, of course, such soils as loam, intermediate between sand and clay, combining the characters of both in proportion to the quantity of each which occurs in their composition; and this kind of soil may ultimately be found the best for sewage irrigation. Or, what is more probable, an area partly on impervious clay and partly on pervious sand or gravel will offer the greatest advantages. For in addition to being somewhat more independent of the seasons, farms so situated would admit of the water, after running over the clay and there depositing its suspended matter, passing, by gravitation if possible on to the pervious beds. In its passage over or through these it would be more or less filtered, and the effluent water thus rendered, perhaps, sufficiently pure to be allowed to flow into a stream or river with impunity.

It has been shown that the "minerals" and "metals" for which mining is carried on depend upon the nature of the rocks with which they are associated. Upon this depends also in a great measure the kind and the cost of preliminary borings, of the main shafts or pits, and of the actual mining operations; but much more are these influenced by the relation of the rocks in and beneath which these are performed. For instance, "in no respect do collieries differ more from each other than in the quantities of water which they encounter, either in the mining or in the subsequent working of their mineral. In one case a retentive clay cover may prevent the access of surface water which in another may pass in abundance through a sandy or a gravel alluvium. In certain districts water-bearing measures of an almost fluid consistency must be passed through, whilst in others the comparatively tight coal measures may at once be entered. Frequently the strata above and below the coal are so compact as to render the workings actually too dusty and dry; but instances are common enough in which water makes its way through the roof stone, or through the coal itself, and adds difficulties and expense to the whole of the operations. When the measures through which the pit is sunk consist of stony rock, they are often allowed to stand open, but when shales preponderate it has to be walled with brick or stone, to which in some cases, as against the influx of water, wood or cast iron may be preferred. But when the measures are covered by other and more absorbent strata, saturated with water, the winning of a colliery becomes a most serious undertaking, tasking the energies of the best men, and sometimes collapsing after a ruinous outlay. Examples of these difficulties are afforded by surface beds of sand and gravel, and by the well-known red sand under the Magnesian Limestone. One of the most serious questions to be solved by the coal-viewer in the very outset is the system by which he means to work his mineral; and in order to form a judgment upon this head it is important that he should not only be acquainted with the various modes in use elsewhere, but should have acquired a knowledge of the peculiarities of the seams in his own district. Where the beds have a definite dip in one direction, the working pits are usually placed as far towards the deep as it is convenient to go, so that underground the coal may be brought down hill to the pit-bottom. Should the strata lie in a trough, the pits may advantageously be placed in its middle line, so as to command the coal on both sides."—See "Coal and Coal Mining," Smyth, 1872.

The relation of two or more rocks to each other may also affect engineering works even to a greater degree than the actual quality of the rocks on which such works are situated. This relation consists in their respective dips, their position in regard to each other, their

various qualities of permeability, and so on. In nothing is it more evident than in railway works that the relation of two beds—to take a simple case—differing from each other in kind, but the same in dip, may be such as to increase the cost of any work upon them, if risk to its stability is to be avoided. Let us suppose a railway cutting of moderate depth to traverse two beds of different character, one a water-bearing sand resting evenly upon a tenacious clay, both dipping at an angle of 3 deg. The slope on the higher side of the cutting will be scored by a series of weeping springs along the line of junction, which will surely, although perhaps slowly, cause serious slips unless means be taken for their prevention. Should the dip lie in the same direction as the fall of the ground—which is, however, unusual—the flow of water will be quicker at some times than at others, perhaps intermittent. Any structures, such as bridges over the cutting, must then have their footings well down into the clay, not on it, for its surface, even many feet from the ground-level and under an equal thickness of solid-looking rock, would be absolutely unsafe as a foundation. If the above simple instance demands precautionary measures, much more must the frequently intricate relations of the rocks receive careful consideration. The beds may dip rapidly, the water-bearing strata may be numerous, and the geological structure may be complicated by faults or by local unconformity.

In tunneling, a previous knowledge of the relations of strata to each other is still more desirable; indeed, it would be impossible to insist too strongly on the necessity for all geological details being known in regard to a hill to be pierced in that manner. Not only might the style of working be varied, but the form or strength of the tunnel itself might perhaps be altered with advantage, to suit either the varying pressure or the peculiarities of rocks known to occur in the center, different to those exposed in the cuttings at either end. Even the gradients might require to be modified according to the existence or non-existence of faults or of springs in the body of the hill, which if not previously detected would be discovered when too late to make any alteration. If the surface of a hill be carefully examined, the boundary lines of the beds of which it is composed be accurately surveyed, their dip ascertained, and the lines of all faults laid down, the position of the rocks within the hill—and consequently the points at which they will be met with—can be accurately determined. Even when the all-important point of the amount of dip cannot be obtained from actual sections, it can be worked out—by a method to be hereafter explained—from the boundaries, or other definite lines, if these have been laid down on the plan with precision. The highest of the beds in a series passed through by a tunnel generally occur near the center of the hill, unless they are inclined in one direction only. This is owing not merely to the fact of the rail level rising from each end towards the interior, but to a well-known geological phenomenon. As a general rule beds dip from each side into a hill or ridge; the statement being limited to hills and ridges as such, and not to include escarpments. Therefore, as the tunnel proceeds, beds are pierced higher and higher in the series, until the uppermost of all met with is somewhere near the middle.

Banks.—At first sight the relation of the rocks beneath the surface may not seem to have any direct bearing upon railway embankments, and similar artificial accumulations of material. But there are ways in which it does now and then greatly affect the cost of such works, and, what is equally important, that of bridges and culverts erected beneath them. A line of railway does not usually run in the direction of dip of the strata, but rather at right angles thereto, nearer to that of the strike of the beds, as it follows the contour of the country. The dip, whether great or small, therefore is generally away from the railway, either to the right hand or to the left as the case may be. If it be of any amount, say exceeding 5 deg., the bank, when it attains to any height, will be very likely to force the beds beneath it over each other along the planes of bedding, and thus give rise to slips, sometimes of great extent and involving much loss of material. These slips are usually sudden and liable to repetition, causing great expenditure for piling and other preventive measures. But if the

liability to slip, owing to the dip of the beds—which usually are, in such cases, alternating clays and dis-similar deposits —be previously entertained, it may be minimised by "running the tip ahead," and working backwards with the bulk of the material. Sometimes lines of railway follow the direction of dip for a short distance; if this be into the hill whence the material comes, there is no risk of slipping, but it is occasionally the other way, when slips are sure to occur. If these slips be quite forward they are sometimes of slight consequence, as the bank is then advancing in the right direction. But should any bridge or culvert have been built in their path, ready for backing up, the consequences may be, and frequently are, serious, for such structures are then almost sure to be overthrown, unless the bulk of the work be done in the backward manner mentioned above.

The preceding remarks apply equally to the construction of water reservoirs and canals, but it may be added, in regard to them, that the geological structure of a country must greatly affect the supply of water to canal-feeders and its retention in natural reservoirs. The "head" of water that can be maintained in such places, generally secured by a dam across some minor valley, and supplemented by pumping, is limited by the springs which may occur within its area; not by the amount they are capable of yielding, but through other phenomena yet to be described, which may, and frequently do, render utterly useless large expenditure for pumping. The attempt is, in fact, made to obtain a head of water which the geological conditions, left to themselves, make absolutely impossible; therefore, if these be understood, and measures taken accordingly, much cost and useless trouble may often be saved.

A smaller, but not unimportant matter, is the difficulty sometimes experienced in carrying out main-drainage works, owing to the surface springs and others tapped by the excavations. Several instances have occurred where, owing to the quantity of water thus met with, the plans, after commencement, have had to be altered, at great disadvantage; others, also, in which the pipes have been, through this difficulty, improperly laid, or have afterwards settled in the sands, so that the joints have ever after been imperfect. In consequence, they have admitted the spring waters, thus adding several hundred pounds a year to the cost of pumping, besides deteriorating the value of the sewage for irrigation or precipitation. These results have happened, in some cases beyond hope of remedy, not because the geological details, including a knowledge of the springs, could not have been ascertained in time, but simply because they have been ignored. With the remedy for such a state of things, where one is practicable, we shall deal in a succeeding impression, but it must always be adopted within certain limitations, and indeed, should only be allowed under official supervision.

One observation may be made with regard to foundations, whether of dock works, bridges, or buildings; it is that however solid the stratum may apparently be in which the excavations are made for foundations, the calculations as to stability are incomplete and liable to error—as the works are to unforeseen catastrophe—unless all the relations as well as the nature of the rocks beneath have been ascertained and taken into consideration.

An improvement in the water supply of a district is one of the practical results that may be expected to arise from the working out of its geological structure—that is, from the knowledge of its rocks and of their relation to each other. The supply of water in any given locality is not by any means proportionate to its rainfall; for the widely-spread water-bearing beds are great distributors, and by them it is to a great extent equalized. The supply to be obtained by boring down to deep-seated springs is practically inexhaustible, being scarcely, if at all, affected by drought, and these springs form the only source on which can be placed a full reliance. The phenomena of springs, and of the sources of supply to artesian wells, both of which are practically important, are entirely dependent on stratigraphical and physical features. An explanation of them will be given, as well as of the reason why sometimes salt waters occur far inland, and fresh water springs beneath the sea; why some waters are

chemically pure, whilst others are saturated with mineral salts.

Another point affected by similar conditions is the dampness of a locality, which sometimes renders almost uninhabitable what would otherwise be a desirable and healthy situation. This may arise either from the physical conditions of elevation, situation, rainfall, and so on, or from those of a purely geological nature. It is frequently owing to the saturated condition of the water-bearing beds immediately beneath, and probably close to the surface; in other words, by the proximity to the ground level of the general water line of the district. This is a question that should influence the choice of situation for all public buildings, and, indeed, for private houses also, where circumstances are such as to admit of selection. This will be fully considered in an article on the subject of "Sites," as one which merits, but seldom receives, much attention.

A knowledge of the undoubted relation existing between subsoil and disease must also be beneficial to those who are seeking for themselves a new home. The whole question cannot be entered on here, but the results of certain official inquries into one branch of the subject are appended. In the "Report of the Medical Officer of the Privy Council," for 1867, pp. 14-17, and 57-110, is discussed, from several points of view, the interesting question of the connection between the geological structure and the consumption death-rate of a district. After careful consideration of all the facts and statistics adduced, the following suggestive and valuable conclusions were arrived at, and may be considered as fairly well established. (*a*) That on pervious soils there is less consumption than on impervious soils. (*b*) That on high-lying pervious soils there is less consumption than on low-lying pervious soils. (*c*) That on sloping impervious soils there is less consumption than on flat impervious soils. (*d*) These inferences must be put along with the other fact, that artificial removal of subsoil water, alone, of various sanitary works, has largely decreased consumption. From which follows the general inference, that wetness of soil is a great cause of consumption. If this one disease can be so influenced that its ravages in a district may be, as they have been, lessened one half by a simple draining of the land, it may reasonably be assumed, that the power of other diseases also is more or less dependent on certain physical conditions, which are susceptible of natural or artificial modification.

ENGINEERING AND ART.

BY MR. CHARLES H. DRIVER, F.R.I.B.A.

From "The Builder."

"ENGINEERING," as the word is commonly understood, may be considered to be the science of "construction," and an engineer is likewise understood to be one who practically applies the theory and science of construction to the every-day wants and requirements of our lives.

"Engineering" is a very comprehensive word, including, rather widely, all matters relating to the formation of roads, bridges, canals, docks, harbors, lighthouses, mines, drainage, waterworks, sewers, fortifications, building, machinery in general, &c. Properly speaking, engineering is divided into two classes, viz., civil and mechanical engineering. In the first are comprised road-making, bridge-building, canals, docks, harbors, waterworks, drainage, mines, &c., and all the other works which may be classed under the head of railway, hydraulic, and mining engineering; whilst the second is principally connected with the manufacture and use of machinery, the working of metals, the construction of railway plant, steamships, guns, armor, plates, &c. In works of the first-class the "contractor" plays an important part, as it is he who executes the work from the designs of the engineer, and on his ability and good management the success of many undertakings very materially depends. In the second class, however, the case is somewhat different, as the

mechanical engineer is very generally both designer and executant of the work he undertakes. Thus much for the engineering of the present day, which may be said to have taken its stand, as a distinct profession in England, about the middle of the last century, and has since, by its varied achievements, done so much for the world at large,—much for comfort, much for luxury, much for wealth, but little, alas! for "art." All must admire the wondrous powers of such magicians as Watt, Telford, the Stephensons, Brunel, &c., and all will admit that they and their works have wrought an enormous amount of good; yet we must admit that though their works contain a vast amount of the *utile*, they have little of the *dulce* about them. Such, however, was not the case with the engineers of antiquity, who not only constructed works of pure utility, such as the harbors of the Phœnicians and of ancient Greece, the bridges of boats make by Xerxes to transport his army into Europe, the canal across the isthmus of the peninsula of Mount Athos, the aqueducts, roads, and bridges of ancient Rome, but they constructed, or assisted in the construction of works which are now looked upon as the triumphs of art (architecturally), such as the temples and pyramids of ancient Egypt, the Palaces of Nineveh, the splendid buildings of Greece and Rome, and in latter times the glorious cathedrals of Europe and of our own country.

"Art" is the result of the endeavor of the human mind to achieve the perfection of beauty, whether it be in form, color, or sound, and, like engineering, it is universal; but, unlike engineering, it is strictly confined to man, and is the result of that power of selection that man, of all animals, alone possesses. Many animals, birds, and insects execute work which rivals and excels the best and most delicate work of man, but which, although artistic in result as to form or color, is not art in itself, and birds or insects cannot be credited with "*art*," as they only follow a line of action they have no will or power to alter. The word "art," as generally understood, is applied to all such matters as music, painting, sculpture, architecture, medicine, agriculture, &c., and may be divided into two classes, —viz., "fine art" and "useful art." Music, painting, sculpture, architecture, &c., are specifically termed fine art. Medicine, agriculture, &c., belong to useful art. It is difficult, however, to define exactly where "fine art" ends and "useful art" begins, the two being, as they ought to be, so closely united. "Art," in the abstract, may be considered to be that which gives pleasure to the purely mental faculties as opposed to the purely animal passions. In this sense we view and accept the mental pleasures afforded to us by music, painting, sculpture, or literature, in contrast to the bodily pleasure we obtain by warmth or coolness, eating and drinking, rest, &c.

Again, the pleasures afforded by art are not confined to individuals or nations, but are universal, whilst those which are corporeal are for the most part personal and selfish. The beauties of nature, such as are seen in fine landscapes, glowing sunsets, the flowers, the songs of birds, &c., yield pleasure to all, and of them there is no monopoly, and therefore they belong, in the truest sense, to art; and it is in this sense, and with this aim, viz., that of giving the greatest amount of pleasure to all, that the painter, the poet, the musician, and the architect should work. The mental pleasures which are embodied in the word "art" reach us chiefly through the eye and ear, the organs of smelling, tasting, and feeling ministering more to our bodily pleasures. Of the two which we may call the artistic senses, viz., seeing and hearing, the one which now concerns us at present is that of "seeing," for it is by means of sight that we learn to appreciate those attributes of "art" that are most nearly connected with works of construction, viz., "sublimity," beauty, grace, harmony, picturesqueness, proportion, order, and fitness; as, for example, the agreeable effect designated by "fitness" is an artistic pleasure which may be called the æsthetic of the useful; as, when a work is not only done effectually, but done with the appearance of ease, or the total absence of restraint, difficulty, and pain, we experience a delight quite different from the mere satisfaction growing out of the end obtained. Much of the pleasure of architectural support is referable to this source. Among the pleasures that are afforded by artistic arrangements may be noticed the sense of "unity

in multitude" arising when a great number of things are brought under a comprehensive design, as when a row of pillars is crowned by a pediment. The use of simple figures,—the triangle, circle, square, &c.,—for enclosing and arranging a host of individual parts has a tendency to make an easily apprehended whole out of a numerous host of particulars. In all large works abounding in detail, we crave for some such comprehensive plan whereby we may retain the total while surveying the parts. A building, a poem, a dissertation, or a speech should have a discernible principle of order throughout, the discernment of which gives an artictic pleasure even to works of pure utility.

Let us now proceed to consider my third point, viz., are engineering and art of service to each other, and can they be united? I think, perhaps, it will be as well to sub-divide the question by seeing, firstly, whether engineering and art are of service to each other; and, secondly, can they be united? Let us, then, begin by considering how art is benefited by engineering. Engineering benefits art when engineering is in itself good and right, and when it does not benefit art, it is because it has been wrongfully and improperly applied, and I must ask you to take this remark at what it is worth, for time will not permit me to go fully into the point; but, as an instance of the good that accrues to "art" from engineering, I may refer to printing, and its attendent belongings. How much does art owe to it? Truly, in the days when printing was unknown, there were then, as now, poets and philosophers who expressed their thoughts in poetic language, and chronicled in strong prose the actions of their fellow men. But to how limited a number was the pleasure that their works afforded confined, as compared with the countless thousands who now delight in their genius? Then every copy had to be painfully transcribed, each copy taking months, perhaps, to complete. Now, by the aid of engineering (machinery), hundreds of copies can be supplied daily. Engraving, again, is an art that has greatly benefited by engineering science; for when it was, as it used to be, confined to copper plates, comparatively few copies could be produced. The copies thus produced were necessarily so costly, that only the wealthy could obtain them. Now, by using steel plates and the electrotype process, the copies are so multiplied, and thereby cheapened, that they are within the reach of all. Engineering affords great facilities for reproducing beautiful form and material, as, for instance, in such matters as porcelain and glass ware, iron and bronze work, textile fabrics, &c., thereby affording pleasure to many. But it may be,—as it has been said,—that all this tends to vulgarize art, and that machinery produced articles, however beautiful, are not artistic, *because* they have been produced by machinery. This argument (I speak with all deference) is, I think, very fallacious. If a vase or a cup is in itself artistic in design, and is good art, so are equally so the 20,000 copies of it, provided they are exact copies, such as machinery has the power of making. And, again, a gardener produces from a chance seed, or, by care in cultivation, a flower or plant that has some especial point of beauty, and for which he obtains a very large price. By-and-by, from cuttings or other means of propagation, the plant or flower becomes common, and can be bought for as many pence as it cost pounds before. The plant or flower has not changed, it has just the same beauty as it ever had, but instead of pleasing only a few it pleases many. And it is on the ground that they give mental pleasure to many, that I claim for the multiplied copies of an artistic original the right to be themselves considered artistic. Engineering itself, however, is not artistic, nor does it directly produce art, but it *does* disseminate it, and, therefore, engineers may very fairly be said to benefit art.

But now let us consider how art reciprocates the benefits she receives from Engineering. In the earlier days of engineering, art certainly was of great service to engineering by teaching those who practiced it to clothe their works with beauty; it taught them, when they wanted a vertical support, to give it a tapering form, to give the gentle swell of the entasis to the column, thus satisfying the eye's sense of beauty and grace; to build the temple, the palace, and the cathedral all with proper fitness, and all with beauty. Though engineering and art are, and have been, co-existent, art is the master, directing and guiding engi-

neering into the right paths; and while engineering acknowledges this mastership of art all is well; but when, as is the case in these modern times, engineering strives to obtain the mastery, the result is chaos as regards art. Modern engineering no longer pays allegiance to art, and arrogantly considers that it can do quite well without it, and hence it is that we have gone on with an increasing loss of beauty in the works of our modern engineering (this remark, however applying only to the civil, and not to the mechanical engineer). The modern engineer seems utterly to ignore beauty of form; if he has to use a vertical support, he will probably make it of the same thickness all the way up; if he wants a buttress, he will put a great lump of masonry or brickwork in, without thought as to whether it will look ill or well, but calculated, I grant you, to do to an ounce what it has to do. There is an immense amount of thought in their work, but no mind,—I mean in an artistic sense; and even the thought they give to their work is of a sordid kind; for while they study how to use their material economically, they omit at the same time to study how to make their work pleasing, thus leaving half their work undone. I admit that engineering possesses a strong point of affinity to art in its *truth*, and by this I mean the honest construction employed by engineers in their work, never disguising or hiding it, but letting it be plainly visible to all; and there is a good honest purpose in what they do, and the sentiment of reality and truth, as opposed to fiction and falsehood, appealing as it does to our practical urgencies, disposes us to assign a high value to every work in which truth is strongly aimed at, and to derive an additional satisfaction in work in which fidelity of rendering is induced upon the charms peculiar to art. But while we admit and admire the truth of the engineers' work, and give them their due meed of praise for what they do, we must not forget at the same time to blame them, in that they leave so important a part of their work undone; viz., the making of it artistic. To my mind it is very nearly of equal importance that a building, a bridge, or whatever it may be, should be of good form and pleasing to the eye, as to be strong. There is an impertinence and brutality and want of regard for the feelings of others in many of the erections of late years,—the work of modern engineers. Consider how the Thames at London has been maltreated, and I ask you, as reasonable men, what right have we to inflict on ourselves and future generations such awful examples of the selfish disregard of all that is beautiful as have there been perpetrated? Our forefathers left us, their successors, works of beauty. What shall we leave our successors?

What remedy is there for this? Can Engineering and art be united, and if so, how? My answer is, yes, and by means of Architecture; for though, as I have said, Architecture is to be considered as the child of Engineering, yet it is through the graces of that child that we must hope to again reconcile and unite Art with engineering. I say "again reconcile," for in old days Art and Engineering were united, and Architecture was their offspring. We moderns have divorced them; let us re-unite them. But how will Architecture unite the two? Engineering is the science of construction. Architecture the art. Engineer is the hard matter-of-fact, uneducated man of business. Architect is the cultured and polished gentleman. Architecture is educated engineering. A man may be honest, bold, truthful, and business-like, but hard, selfish, and inconsiderate, with no knowledge or care for the beautiful. He is the modern engineer. Another may be honest, but timorous, uncertain, and unbusiness-like, kindly considerate with respect to the feelings of others, and with a great desire for the beautiful, often failing from lack of power. He is the modern architect.

Perhaps you will think I do not draw a flattering picture of the representatives and practisers of that art by which, as I have already told you, art and engineering can be united. Well, that may be true, but it is only true as regards the modern architect, as my first picture is of the modern engineer. The modern architect is as the times, his own want of pluck, and the modern engineer have made him; he, if I may say so, has been bounced and bullied out of his proper position by the engineer; and much that of late years has been done by engineers should have been done, and would have been better done, by architects, if they

had but retained their proper position by keeping themselves abreast of the times. I have given you a figurative sketch of the modern engineer and the modern architect. I will give you one as to the past. Here we have a man who is honest, bold, and truthful, certain and business-like, with kindly regard for the feelings of others, an intense love and knowledge of the beautiful, with a desire that all should participate in the pleasures it affords, and having the will to do good work, has also the power to execute it; he is the architect and engineer of old, the architect and engineer being one. . . .

The difference in the way an engineer and an architect would set about the same piece of work, if they had it to do, would, I think, be as follows, viz:—The engineer would begin by studying the purpose of the intended structure, then consider as to the best materials of which to erect it, and calculate to a nicety how little of each material he could use. . If he wants a door, he puts in one; if he wants a window, a window is put in; but he never studies or thinks for one moment whether he can, by placing his door or window in some other position, improve the appearance without altering the utility of his work, and the result is, his work is as bald and ugly as possible. He then, perhaps, proceeds to what he calls "ornament" it, by sticking on here and there a moulding, which as likely as not he places upside down, thus destroying the only grace his work possesses, viz., truth; and then says, with satisfaction, "See what we engineers can do: we want no architects with styles and orders to teach us what to do." On the other hand, the architect (I speak of the true architect) would first, like the engineer, consider well the purpose of his intended building, and, like him, consider as to the best material to build with, and how best to use it. He also would place his doors and windows in their most useful positions; but, in addition, he would, from the first commencement of his work, have in mental view before him a certain effect, which he would strive to produce, and thus, while so planning his building as best to suit the requirements of the work, he at the same time makes it pleasing and artistic; and when the work is finished, it needs not the extraneous aid of moulding or applied ornament, for all that is wanted in that way forms part of the structure itself. It is common to find engineers twitting architects with their want of knowledge of scientific construction; but I think I can safely say of architects that they, as a body, have a far greater knowledge of practical engineering than engineers have of architecture,—as their works show,—the architect's failures in "construction" being as nothing to the engineer's failures in "art."

THE INJURIOUS EFFECTS OF THE AIR OF LARGE TOWNS ON ANIMAL AND VEGETABLE LIFE, AND METHODS PROPOSED FOR SECURING A SALUBRIOUS AIR.

By WILLIAM THOMSON, F.R.S.E., F.C.S.

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In considering these questions it is important that we should have clear ideas—first, as to what is pure air; secondly, for comparison, what alteration in the constituent parts of air would make it incapable of supporting life (say, for instance, air in which a candle could not burn); and, thirdly, between this wide gulf, what may be termed impure or unhealthy air.

Air is composed principally of a mixture of two gases, oxygen and nitrogen, in the proportion of about 21 volumes of the former, and 79 volumes of the latter. Oxygen is the constituent which supports life and combustion; nitrogen is an inert gas, which was doubtless intended by the Creator to dilute the active, or life-and-combustion sustaining-gas, oxygen.

The following may be regarded as an analysis of the purest air which can be obtained:

	Per cent. by volume.
Oxygen	20.999
Nitrogen	77.568
Carbonic acid gas	.033
Water vapor	1.400
	100.000

Dr. Angus Smith found that if he abstracted about 2½ per cent. of oxygen from pure air, it would not support the combustion of a candle and it would be impossible for any one to live long in such an atmosphere; so that the air, so far as life is concerned, may be compared to a reservoir with pipes placed near the top—the water under the level of the pipes being useless, except as affording a support for the top stratum, which can be utilized by being drawn off; so we have 18.5 per cent. of the oxygen of the air of no greater value, as a life supporter, than the water which lies under the level of the pipes of our assumed reservoir, and we may take it, therefore, that only one-eighth part of the volume of the oxygen of the air is of practical value.

We must, then, look upon air which has even a very minute fraction less of oxygen than that given in the above analysis, as more or less objectionable or bad, because the part of the total amount of oxygen contained in the atmosphere available for supporting life is so small.

The following analysis made by Dr. Angus Smith, shows the actual proportions of oxygen contained in air from different places, the last four of which cannot be regarded as otherwise than unhealthy:

	Per cent. by volume of oxygen.
Tops of hills (Scotland)	20.980
In the outer circle of Manchester (not raining)	20.947
Low parts of Perth	20.935
In a sitting-room which felt close but not excessively so	20.890
Pit of a theatre, 11.30 p. m.	20.740
About backs of houses and closets	20.700

It will be observed, then, that judging from the deficiency of oxygen alone in the air from the pit of a theatre, for instance, a draught of over 10 per cent. has been made upon the total available oxygen. Judging, however, from this point of view, we are assuming that the place of the oxygen abstracted is filled by an innocuous gas, such as nitrogen, which is not the case. When the oxygen is consumed by man, it combines with the carbon from his lungs, forming carbonic acid gas, which may be regarded as a positive poison, and if we take the amount of this noxious gas found in a theatre, we see by Dr. Smith's analysis, that it amounts to 0.320 per cent., being 10 times greater than that found in pure air. In Manchester streets, in ordinary weather, the air was found to contain 0.403 per cent. of carbonic acid gas, and although this is a very small fraction, it is yet about 20 per cent. greater than that found in pure air.

When we consider that a healthy man will pass through his lungs about 2,000 gallons of air per day of 24 hours, we see that he will during that time inhale 5¼ pints of carbonic acid gas in the country, and 6½ pints in the streets of Manchester, and about 51 pints in an atmosphere such as that found in the pit of a theatre.

These, however, together with organic emanations, are the impurities which are found in air polluted by the breathing of animals, but these are not the only impurities found in the air of large towns; others more irritating and poisonous proceed from the burning of coal, to which I will refer later on; but to confine ourselves to the physiological effects of air charged with large quantities of carbonic acid, such as that found in a theatre, I will refer to certain statistics. It is well known that children who are kept after birth in badly ventilated rooms, die in large numbers from what were termed "nine-day fits." In 1783, Dr. Joseph Clarke, master of the Rotunda Hospital in Dublin, recorded that during 25 years in that hospital, when the ventilation was bad, no fewer than 3,000 out of 18,000 children born there died within the first fortnight of their birth. The number of these cases at once rapidly diminished as soon as the better ventilation of the wards was attended to, so that, in the following 28 years, out of 15,072 children born, only 550 died, being 1 in every 104, instead of 1 in 6, as was the case previous to the better ventilation; and since, Dr. Mc Clintock, one of the successors of Dr. Clarke, reported in 1861, that further improvements having been made in ventilation, the disease was then almost unknown.

Again, in the Mediteranean squadron, a fearful lung disease made its appearance among the men in 1861, which was found to be due to the want of proper ventilation in the lower decks of the ships where the men slept. The lung is a most delicate organ, in which the blood meets with the air which we breathe. The air passes into the lungs, through air tubes, which divide and subdivide, like the stem and branches of a tree, each branchlet ending in a small air cell or leaf, which is formed of a delicate membrane, not more than the thickness of a soap bubble ; and the lungs of a full-grown man contains six hundred millions of these leaflets or air-bags. The membrane of these delicate air-bags is covered with fine, hair-like blood vessels, through which the blood flows, and thus spreading over an enormous surface it becomes aërated. The blood from one side of the heart, charged with the impurities, etc., which it has received from the body, passes across the lungs through the tiny blood vessels on the delicate membrane, and there meets with the air, the oxygen contained in which burns or oxidizes the impurities in the blood, carrying them away as carbonic acid gas, which we exhale, along with the nitrogen and excess of oxygen. This delicate organ is protected from injury by a variety of means. If, for instance, a particle of solid matter or liquid, for instance, touches the air passages as is sometimes the case in eating or drinking when the liquid or food is said to "pass into the wrong throat," a reflex action is produced, involuntary coughing takes place, and the intruding matter is expelled. If, however, the particles of matter be not sufficiently large to produce violent coughing, it produces a rapid secretion of mucus from the mucus membrane of the air passages, in the same way as a particle of dust coming in contact with the eye produces a flow of tears, which eventually washes away the irritating particle. The mucus, or living membrane, or skin of the air passages, is also provided with another most ingenious appliance; it is covered with myriads of minute hairs or cilia, which are in constant motion, like a field of corn in the breeze, so that when the mucus secretion, which is a glairy fluid, has taken hold of the offending particle, the movement of the small hairs passes it slowly along, till it comes near to the top of the trachea, or wind-pipe, when the mixture of foreign matter and mucus may be finally dislodged by coughing.

The results of this process can be observed when we breathe some time in the open air during a fog in a large town. The air then contains a copious amount of soot in suspension, which finds its way into the air passages, and which is ultimately expelled mixed with phlegm, the whole being sometimes of a dark or even black color.

If, however, the air passages are constantly being irritated by small particles of foreign matter, the mucus membrane gradually loses the power of performing its normal functions, and becomes diseased, and this diseased condition soon extends to the lung itself. Remarkable examples are to be found of diseases originating directly from this cause in the men employed at certain manufactories, such as knife and fork grinders, where the fine particles of steel float in the air, and are inhaled by the men. The lives of the men engaged in such occupations are extraordinarily short, a knife or fork grinder being considered an old man at the age of thirty, and if he has worked at his trade from boyhood without using fan appliances to take away the steel dust, as most of them do, having a strong prejudice against any such apparatus, they often commence at the age of twenty-five or twenty-six to die of a long and painfull illness, resulting from lung disease.

This may serve as an example of how the lives of men may be much shortened, by the irritation resulting from inhaling minute particles of a hard and angular dust, but the minute particles of carbon which float in the air of our large towns, and which may be found adhering to the sides of the air passages of the nostrils, for instance, after one has spent some few hours in a town, assuredly have an injurious effect upon the health and lives of the inhabitants. The same kind of irritation may, however, be produced by the inhaling of noxious gases, which are also found in town air.

Many other cases of consumption being produced in strong, healthy men, by breathing impure air, may be given; but, as one more instance of the pernicious

effects of bad air, we may refer to the average life-time of the people living at different places, which will speak for itself.

In London, the average life-time is	29	years.
" Manchester "	21	"
" Surrey "	34	"
" France "	34	"
" England "	29	"

Supposing a man to take pure air into his lungs, containing, as it does, about 21 per cent. of oxygen and $\frac{3}{100}$ths of a per cent. of carbonic acid gas when he exhales it, the carbonic acid gas would be found to have increased to 5 per cent., and the oxygen to have diminished to about 13 per cent. If he were compelled to breathe again the same air, death would ensue in a few minutes; this air must, therefore, be largely diluted with pure air, and different hygienic authorities hold that each volume of air exhaled should be mixed or diluted at once with from 130 to 200 volumes of pure air, and that whilst a healthy man cannot pass through his lungs more than about 72 gallons of air per hour, he should be supplied with, at least, 10,000 gallons, that the impurities exhaled may not prove poisonous or deleterious to him.

The above facts show us the vast necessity of being supplied with pure air, and yet, strange to say, this is one of the points which by the generality of men least attention is paid. Millions of money are being spent to obtain for our towns liberal supply of pure water, which, is no doubt, most important for our welfare, but of much less importance than the supply of pure air. What a lamentable sight is presented to us, when we go to the top of a hill near to one of our large towns and look down upon it. We observe that it is enveloped in one vast cloud of smoke, and yet in such atmospheres thousands are living, suffering, and prematurely dying; and still more marked to the eye is the injurious effects which such atmospheres have upon plant life. Few plants or trees can grow in or near our large towns, and those which do stand better these injurious influences become weaker and weaker every year, until they ultimately succumb.

We ask what is the cause of this, and the answer is undeniable. It is owing to the gases produced from the burning of coal. When coal is consumed, a large number of different products of combustion are formed, which escape up the chimney and enter the outside air. Coal is burned in large towns for two purposes, the one for the processes of manufacturing, the other for heating our rooms and for culinary purposes. Up to the present time, wealth has almost entirely ignored the nuisance produced from the house smoke, but has acted vigorously against that produced from manufactures; and although this is a step in the right direction, it is unfortunate that it has only attacked by far the lesser of the two nuisances.

The smoke or gases produced from coal used in manufacturing are of a different composition from those produced by coal burnt in the ordinary room or kitchen grates. In the former, the coal and its products are more completely burned than in the latter; that this is so may be observed by looking at a room-fire in comparison with the fire under a large boiler. The general mode of keeping up the fire in an ordinary room grate is to put some pieces of coal on to it when necessary, either as large pieces when the fire is intended to last for some considerable time, or as small pieces, or as a mixture of both, when a good fire is required; in any case we may observe, when this is done, that a distillation takes place, and a yellow vapor begins to flow slowly up the chimney, and later on these products become ignited; on the other hand, the gases evolved from the coal by distillation in the front are completely consumed in passing over the red-hot fuel in the back part of the furnace. The smoke, therefore, from domestic fires is of a more complicated nature, and more injurious to the health of men and animals than the thoroughly burned products which escape from furnace fires, and not only so, but the smoke from domestic fires is distributed all over our towns, and allowed to escape into the air generally not more than a few yards from the level of the streets, and it is wafted down by the wind or air currents, contaminating the air which we have to breathe, and making it look as if a faint haze existed everywhere in our town atmospheres; indeed, in my opinion, the sanitary conditions would not be materially improved if all the manufacturing industries at once ceased to exist. This

is borne out by a paper read before the Manchester Philosophical Society by Mr. Peter Spence in 1869, in which he describes the results of a series of experiments made at his own house with blue litmus paper. These papers were changed morning and evening for some time, until he became an expert judge of changes in color produced by the air blowing from different directions. He found, as a rule, that on Saturdays, when the manufactories were not working, more acidity was observed in the air than on any other day or night in the week.

The principal impurities found in the air of large towns are soot, hydrocarbons, sulphide of ammonium, carbonic acid, sulphurous acid, carbonic oxide, and probably very minute quantities of arsenic expelled from the pyrites existing in the coal. All these are due to coal smoke, others exist such as emanations from putrid or decomposing organic matter, but by far the greatest quantity of atmospheric impurities in large towns can be traced to the emanations from burning coal. The sulphur compounds contained in the different atmospheres is regarded by Dr. Angus Smith as a measure of the pollution of the air from coal smoke and other causes, such as putrefaction and decomposition, but principally from the former, and these compounds, such as sulphurous and sulphuric acids, &c., he has estimated; and, calculating the sulphur compounds into sulphuric acid, he has arranged the following interesting table of results. Taking the sulphates in the air found in Valentia, in Ireland, as 100, we have the following amounts found in various places throughout the United Kingdom:

Scotland.—Sea-coast country places, west	132.2
England.—Inland country places	202.2
Scotland.—Towns (Glasgow not included	604.4
London, 1869	750.5
English towns	1255.3
Manchester, 1869	1526.
" 1870	1757.8
Glasgow	2591.

Again, Dr. Smith compares the amount of acidity, or free acid in the air of different places. The free acid, as a rule, comes directly from the burning of coal, and may be regarded specially as the plant-destroying ingredient of town air. He gives the amount of ammonia which is eliminated principally from the burning of coal, and the following gives some of these results. In the case of acidity, the air of Valentia contains none, and cannot, therefore, be taken as a standard of comparison; but, for ammonia, the amount found therein is taken as unity. Here follows the table:—

	Acidity.	Ammonia.
Ireland.—Valentia	None.	1
Scotland.—Sea-coast country places, west	1	2.69
England —Inland country places	None	5.94
London, 1869	27.97	19.17
Manchester, average of 1869 and 1870	73.44	35.94
Ditto, 1870	86.76	36.54
Glasgow	109.16	50.55

The question has often been raised as to whether black smoke did more damage to animals and plants than the products of the complete combustion of coal, and some held the view that black smoke was innocuous, whilst others held that it was highly deleterious.

It has been shown that if a plant be completely covered with pure carbon, the plant will grow luxuriantly, but it has also been shown that soot is not pure carbon, but is carbon saturated with sulphurous and sulphuric acids, tarry matters, ammonia salts, &c., and it has been equally clearly shown that soot or black smoke is highly destructive to plant life. The soot, or black smoke, no doubt acts injuriously on plants in two ways, the first mechanically, by the tarry ingredients interfering with the free action of the stomata, or breathing orifices of the leaves; the second chemically, by the acids, salts, and other substances which would be washed from the smoke deposited on the leaves by the dew or rain which comes in contact with it, these substances would be absorbed into the substance of the leaf, and, coming in contact with the juices of the plant, would decompose it, and so render it incapable of imparting nourishment. It is remarkable to observe that the leaves of plants and shrubs in large towns are generally so thickly covered with smoke, that, if they be rubbed between the fingers, the latter will be much soiled or completely blackened. The effect of the air of large towns is to prevent the early budding of the leaves of the trees and plants, and to make the leaves fall early

in the autumn, or before it sets in. It is clear, then, that it would be a great advantage, to both animal and plant life, if black smoke were not emitted from chimneys, because it falls rapidly on to the plants, bringing with it, and consequently acting as a vehicle for carrying, the really noxious ingredients of the smoke. It also falls on and blackens the magnificent public buildings which most English towns possess; and, not only so, but injures the stone and carving exposed to it, by reason of the dirt being washed from it into the stone, many kinds of which ultimately undergo a crumbling process from that cause. It is evident, then, that it is important that, if possible, coal should be burned thoroughly, and no black smoke produced, because, as I have mentioned, it injures, not only plant and animal life and buildings, but soils everything which the good housewife considers worthy of being kept free from stain. If, then, it be found that cleanliness can only be obtained or retained at an immense cost of labor, many will lose heart, and become contented to remain dirty; and so the emission of black smoke from chimneys should and does prove a great social evil. If, however, the smoke were completely consumed whenever coal is used, the evil would not be stamped out, because the rain which falls in large towns dissolves from the atmosphere the impurities which it brings down with it, and which prove highly deleterious or poisonous to the plant. It is calculated that about 1½ millions of tons of ammonia, or salts of ammonia, are dissolved per annum from the atmosphere by the rain which falls on the globe, and this furnishes the plants with a most valuable food—the ammonia being formed in nature by the electric discharge, or lightning, or by the decomposition of animals and plants. What a vast difference is presented by the rain water which falls in and near towns. If a vessel be exposed in a town the rain water falls into it, but so does the black smoke, and after a few days if the water and soot which has fallen be mixed together, the liquid will present an inky appearance, and will change the color of blue litmus to red, whilst a vessel left for the same length of time in a country place will be free from sooty or noxious ingredients.

Lastly, I may mention that it has been calculated that about two millions of pounds worth of property are destroyed in London and its vicinity alone per annum by the emanations from burning coal. I do not vouch for the accuracy or otherwise of the last calculation, but, doubtless, we have sufficient proof that an immense amount of property is annually destroyed by allowing coal smoke to enter our atmosphere, as it does at the present time in England; and this is one of the points which weighs heavily in the scale on one side to show that, even if some scheme were devised and worked, even at great cost, it is probable that, looked at from a commercial point of view, from depreciation by damage to property alone, the scheme might prove a success; but this is one of the least important items in the problem. One which stands much higher is the increased degree of health which even the strongest of town inhabitants would experience, even by a small mitigation of the amount of impurities thrown into the air. I think that any one who works in the vitiated atmospheres of towns must feel, if he has tried it, that he can do more work, and can do it more easily and comfortably, in the atmosphere of a country place than in that of a town, and further, that he requires less food when working in a pure atmosphere than when employed in a vitiated one, the wear and tear on the system being much greater in the latter.

If, then, it were possible to secure a fine atmosphere in towns, what a vast benefit would be conferred upon the inhabitants. It would tap thousands of springs of energy which at present are kept dormant, and that energy, if directed towards the development of the country would bring back to it much of the prosperity of which it at present so much needs; but, above all, it would give an increased supply of good health and spirit, and would prevent many of our unfortunate citizens from wasting their money and destroying themselves mentally, morally, and physically, by excessive indulgence in stimulants.

So much, then, for the benefits which would be gained by improved sanitary conditions of our atmospheres; and I come now to the question whether it be practicable to prevent much of the con-

taminations which at present find their way into our atmospheres, and I think there there can be no doubt that it is. Schemes have been discussed of draining away the noxious gases from our chimneys, and passing them along the sewers, and ultimately allowing them to escape into the atmosphere from the tops of tall chimneys, but nothing was brought forward of a definite nature till Mr. Peter Spence, F.C.S., of Manchester, the well-known manufacturing chemist, made the necessary calculations, and embodied his scheme in a paper read before the Manchester Philosophical Society, 22 years ago. To give his ideas in as few words as possible he says that it is practicable to build a chimney 600 feet high, 140 feet external diameter at the bottom, and 100 feet internal diameter at the top, at a cost of about £40,000, and calculating the amount of air required for the complete combustion of the whole of the coal burned in Manchester, assuming that that air would enter the tall chimney at a temperature of 100° Fah. (about the temperature of the human body), the column of air would ascend at the rate of about 40 feet per second, and would be capable of carrying away about ten times as much air as that required for the combustion of all the coal employed in Manchester.

We may, assume, however, that if such a chimney, were built, and subsequently found incapable of doing the necessary drainage work, it is possible that a large fan could be erected in connection with the chimney, and the draught so aided by mechanical power.

The scheme has many interesting points connected with it. First, it combines the liquid and gaseous sewage, and makes the one in many respects neutralize the other. The sulphurous acid, hydrocarbon, &c., from the coal smoke, would prevent the decomposition of the liquid sewage, whilst the liquid sewage would condense much of these noxious substances from the smoke which now contaminate our atmosphere. Again there would be a constant draught through the sewers, and, consequently, it would be impossible for any foul air to escape from them; on the contrary, if there should happen to be any leak in the sewers, the pure air would flow into them, instead of, as is the case at the present time, the foul air of the sewers finding its way out from every crack into our houses; because it seems almost practically impossible to keep sewers so tight that objectionable smells will not proceed from them—when one place is made tight the bad odors make themselves evident from another. Mr. Spence relates an interesting method by which he prevented the sewer gases from entering his house. He found that one of the bedrooms in a house which he had occupied for a short time could not be used, owing to the bad smells which evidently found their way through some cracks in the wall of the room, and he hit upon the plan of joining a 4-inch iron pipe with the sewer at one end, and placing the other end a few feet up the kitchen chimney. The draught from the chimney so far ventilated the flue that the smell from the bedroom, almost immediately after this arrangement was completed, ceased to exist. This scheme might materially help the solving of the liquid sewage problem, inasmuch as less water would be required to be passed along the sewers, and the slightly diluted sewage would, in its passage along, meet and dissolve from the coal smoke the ammoniacal salts which it contains, and the liquid might afterwards be treated directly for the recovery of these valuable products, which might be converted into the products richer in plant nourishment than the richest guanos.

Mr. Spence calculated that, by the burning of two millions of tons of coal in Manchester and Salford, about 20,000 tons of sulphate of ammonia could be produced, all of which, by his scheme, would be deposited in the tunnels and added to the sewage; and taking its average value at £15 per ton, would be worth £300,000, a heavy item to be thrown away uselessly and harmfully into the atmosphere, and, taking this in combination with the manurial value of the sewage, he considers that about £800,000 per annum might be realized by its sale. This, then, is another point which adds to the prospect of the financial success of such a scheme.

Other schemes have been suggested for the prevention of the pollution of our atmosphere, such as the preparation of purified coal gas from the coal, and using that as fuel. We are, however, a conservative people, and any material improve-

ments which are made must be done slowly and cautiously. I understand that Mr. Spence's system has been adopted at the Assize Courts in Manchester, where all the fires are ventilated into the same chimney, and that it will be adopted in the New Law Courts of London; whilst I believe that the full scheme advocated by Mr. Spence and others will ultimately be adopted in all large towns. Mr. Spence, who has spent much time during his life in the study of those matters, and who has a thorough knowledge of what work can be done by large chimneys, &c., informed me a few days ago, that he saw no reason, since he first brought forward that scheme, twenty-two years ago, of altering his opinion as to its practicability. The point, however, which I wish to be most clearly brought forward in this paper, is that, considering the vast and growing importance of the subject, some means should at once be taken to mitigate the nuisance, and reduce the impurities which we are now compelled to breathe in our large towns, and this may be done with certainty, simplicity and little expense, by joining all the chimneys in each block or line of buildings together, for instance, and connecting them into one chimney, which should stand considerably above the tops of the houses. Much of the impurities would be condensed and deposited in the flues before the smoke reached the chimney, and it would allow of greater diffusion of the noxious gases which escaped from its increased height, so that we should require to inhale less of them, and further it would increase the draught in every chimney so joined. The draught could then be easily regulated by dampers; and, lastly, it would be preparing the way to centralize the points at which the smoke should be allowed to escape, until lastly a chimney of immense height would take the smoke and fumes sufficiently high to prevent their ever reaching our level.

Weighed against the comparatively small expense which the carrying out of some such plan would entail, the immense benefit which would be derived from it, it certainly seems astonishing that something has not before been attempted in this direction. Each fraction of a percentage of impurity which we are compelled to breathe adds considerably more to the drag which is thus put on the wheels of human energy, and it kills the plants and trees which were provided by nature to purify the air from the necessary contamination produced from the breathing of animals, and to refresh and delight the eye and the mind by their beauty. The smoke also prevents the free entrance to our towns of the rays of the health-giving sun.

The benefits which might be derived from having a pure atmosphere in our large towns can scarcely be estimated, and I trust that this Society will spare some of its talent and energy in the future in trying to improve the sanitary condition of the air of our large towns.

RIVER CONTROL AND MANAGEMENT.*

By J. CLARKE HAWKSHAW.

From "Engineering."

The question of the management of the rivers of this country is one that is yearly assuming more importance. Royal Commissions and Committees of both Houses of Parliament have been appointed of late years to consider the best means of preventing pollution, of preventing floods, and of providing for the control and management of rivers. Many facts have been accumulated and some valuable recommendations have been made which would go far to remedy the existing evils. It is to be hoped that decisive legislation will not be long deferred.

During the last three years I have had occasion to examine the greater part of three river basins, those of the Clyde, the Witham, and the Upper Thames, for the purpose of obtaining detailed in-

* Read before Section G of the British Association: Dublin meeting.

formation for my father who has reported on them. In the case of the Cylde he was appointed Royal Commissioner to inquire into the best means of preventing the pollution of that river, in the other cases he was called upon to report on the best means of preventing floods. This being the case (and seeing that the question of water supply has formed the leading topic of the president's address in this section) I have thought that a few remarks on river control may prove of some interest to members of the section. In this country, although so many great engineering works have been carried out, we have little cause to congratulate ourselves on the condition of our rivers. Periodically thousands of acres of land are flooded, and crops on them are destroyed and villages and towns are inundated. In January, 1877, 40,000 acres of land above Lincoln and between Lincoln and Boston were flooded by the River Witham. Two months later I saw the water still standing on parts of the fens, and the people only then returning to their homes from which they had been driven at a moment's notice. In some parts the water had been from 5 feet to 6 feet deep. By the same flood nearly 100 acres of the city of Lincoln were flooded to a depth in places of 3 feet. Twice in the previous year the River Trent flooded the town of Burton, rendering many of the principal streets impassable owing to the strong current running 3 feet deep through them. The banks of the Thames from near its source as far as London, and including parts of London itself, are liable to be flooded. These cases have come under my own notice, but each river valley could supply similar or even more striking ones. Yet I venture to think that engineers are not to blame for this state of things. If they cannot cope with the rivers of this country what hope is there that the great rivers of India or of other parts of the world will be controlled. The Mississippi drains an area of 1,147,000 square miles, which is more than 180 times as large as the area drained by the Thames, and discharges on an average 800,000 cubic feet per second during the flood period which lasts six months. A maximum flood discharge of the Thames might reach a fortieth part of the above amount for a few hours in the course of twenty years. Yet American enterprise has not shrunk from attempting to control the waters of the Mississippi; embankments were made at New Orleans 150 years ago, but the Americans are now wisely beginning to deal with the mouth of the river, a course which has been too often neglected in this country. By making jetties, one of which is now more than two miles long, formed of piles and matrasses of fascine work, they are constraining the waters of one of the main outlets of the river to flow in a deep straight course to the sea. Already the depth of water on the bar has been increased from 8 feet to 19 feet.

I will not further describe this great work, though I am much tempted to do so. Its progress should be watched by all those interested in river engineering. Although much still remains to be done before even the great rivers of Europe will be brought under control, much progress has been made where local interests have been made subservient to those of the community in general. The fault in this country most frequently lies in the existence of numerous authorities each with limited jurisdiction over the same river. Rivers can rarely be dealt with to any good purpose piecemeal, but should more often be dealt with as a whole from their source to the sea. And, moreover, whatever is done to them should be done with a view to making them as efficient as possible for the three purposes which they should serve, viz., for drainage, for navigation, and for water supply. The first of these three is the most important, for on it the existence of a river depends. It need not necessarily serve the purpose of navigation; water may be obtained for the population within its basin from other sources, such as wells, or from districts without its basin, but a river cannot escape, being the natural and proper outlet for the land drainage of its basin. I shall attempt to discuss these three functions of a river separately, though I believe they cannot fitly be separated, whether we regard the works which are proper to them or the organization which will best lead to their being well performed. I have said that I do not think engineers are to blame for the condition of our rivers. This is certainly

not the case in respect of floods. A flood occurs and an engineer is called in by a local board or committee of landowners, and is asked to report and suggest a remedy for the evil. He has generally to consider but a small portion of the river, and that only in so far as it affects the interests of his employers, who have possibly no jurisdiction or at most only a partial jurisdiction over it. Proper surveys are not to be had, nor observations on the flow of water in the river, and they often have to be dispensed with in a great measure owing to the time it would take to make them and the cost. He is limited as to expenditure, yet a partial scheme of improvement must generally be costly. It can hardly fail to benefit some people higher up the river who are not called on to pay. Very often it will injure others lower down. While the report is being prepared, the ardor to remedy the evil cools. Some are not much injured by the flood, some not at all. The prospect of having to pay at once seems more distasteful than to risk the chance of having to suffer the same loss and discomfort at some uncertain future time. So the report is received and is discussed. The cost of the proposed works is too great, and there the matter ends till another bad flood comes and the whole process is gone through again.

Engineers have made more reports on drainage than on almost any other subject. These reports lead to much unprofitable expenditure when special observations and special surveys have to be made for them. Such work can easily be so thoroughly done as to be available a second time. For a succeeding report it often has to be done over again.

The Ordnance survey maps on the 6-inch scale supply much of the information that is wanted, but for many districts no maps have been published on the 6-inch scale, and for some we have nothing but the 1-inch scale maps 30 or 40 years old.

Much of the district drained by the Witham is not correctly shown on any map. The Ordnance survey of it was made in 1824, and the appearance of the fen land has totally changed since then. When the Thames Valley Drainage Commissioners were appointed in 1871, by Act of Parliament, to deal with the rivers and flooded parts of the Upper Thames Valley, they could not properly perform the duties imposed upon them without a correct map of the district. The Ordnance survey of the district was made in 1828, on the scale of one inch to a mile. The commissioners were forced to have a new survey made which took some years to make. The work was done by the Ordnance Survey Department, to whom the Commissioners paid several thousand pounds for it, so that in this case a special tax was laid on the landowners of the district under the commissioners for the benefit of the Ordnance Survey Department. This is one of many cases in which the want of system in the management of our rivers has led to a limited district having to bear burdens which should rightly have been borne by larger districts, or, in this case, by the whole community.

In England, alone, the extent of land which directly depends for its drainage on the state in which the main river channels are maintained is very great. The fens of the Eastern Counties extend over 830,000 acres, and this forms but a portion of the land throughout the country, some of which would be relieved from disastrous floods, and all of which would be better drained if the river channels were dealt with on some systematic plan, and were not, as at present, either entirely neglected or dealt with piecemeal.

Much of the fen land has no natural drainage, but no less does the cost of draining it by mechanical or other means depend on the state in which the river channels are maintained. To drain the fen lands in the first instance it was necessary that considerable areas should be dealt with at once, and this led to the formation of numberless drainage districts. Each of these provides for its own drainage as best it can with regard to its own interests only, and often to the detriment of adjoining interests. Occasionally several of them may unite for some special purpose, but they rarely co-operate heartily. They have not always control over the main channel into which they drain, so that they are liable to be flooded from causes beyond their own control. Before the advent of railways river channels were valuable as means of inland transport, and were

maintained for that purpose. But traffic on rivers has fallen off, and the old conservancy boards have not funds to maintain the channels, though they still have jurisdiction over them, and drainage suffers in consequence.

The River Witham may be taken as a good example to illustrate the variety of jurisdictions to which some of our rivers are subject. The Witham is between 60 miles and 70 miles long, and drains an area of 1079 square miles. It flows past the towns of Grantham, Lincoln, and Boston, entering the Wash about seven miles below Lincoln. For the last 40 miles below Lincoln it is embanked on both sides to prevent its flooding the low land adjoining. Parts of its tributaries, both above and below Lincoln, are likewise embanked for the same purpose. For some miles below Grantham the Witham is under the Grantham Sewers Board, from thence to Lincoln it is under the Lincoln Sewers Board. These boards have rating powers over the lands adjoining the river; the Lincoln authority rates all lands below flood level and less than 2 feet above that level. The part of the basin of the Witham which lies to the north-west of Lincoln is drained by a tributary the Till, which does not, however, discharge directly into the Witham but into the Fossdyke, a navigation connecting the Witham at Lincoln with the Trent at Torksey. The Till is under the Lincoln Sewers Board, but the Fossdyke is under the Great Northern Railway Company. The lands adjoining the Fossdyke, which are below the level of its waters, are divided into two districts, the north and south-west districts, under separate Boards. The only outlet for the drainage of these districts, 7,000 acres in extent, is through a small culvert under the Witham at Lincoln, which is not large enough in time of floods. Besides this, these districts are liable to be overflowed by the waters of the Till, the Fossdyke, and the Witham by the failure of banks over which they have no control. Between Lincoln and Boston, the Witham is a navigation subject to the Great Northern Railway, and neither the Lincoln Sewers Board nor the West District Drainages have any control over it, but the jurisdiction of the Lincoln Sewers Board extends over the Lanworth river, the principal tributary of the Witham between Lincoln and Boston, and likewise over a cut called the Sincil Dyke, which passes through Lincoln and connects the Witham with the South Delph, which is under the Great Northern Railway, and which enters the Witham about seven miles below Lincoln. The Lincoln Sewers Board are prohibited by Act of Parliament from making a proper outlet for the Witham waters into the Sincil Dyke, and as their only other outlet, the navigable channel of the Witham, is not under their jurisdiction and is insufficient in time of floods, this arrangement insures a large tract of land above Lincoln, and part of the town itself being flooded at times. From Lincoln to Boston the Witham is under the Great Northern Railway for navigation purposes, but it is under another authority called the Witham Commissioners for drainage purposes. If the Witham banks were to fail at any point as they have done the adjoining lands are flooded. These banks, more than 60 miles in length, are maintained partly by the Great Northern Railway, partly by the Witham Commissioners, and partly by the adjoining landowners. From Boston to the sea the Witham is partly under the jurisdiction of the Boston Harbor Commissioners and partly under the Witham Commissioners. Below Lincoln the lands adjoining the Witham are divided for drainage purposes into seven large districts which are again divided into many smaller ones, each of which has its own body of commissioners and other officers, and power to rate for drainage purposes. Lastly, there are two affluents of the Witham below Lincoln, which have been made into navigations under separate authorities, the Horncastle navigation and Heaford Canal.

Thus there are no less than seventeen sets of Commissioners or other authorities having jurisdiction over the Witham, and for drainage purposes over the lands adjoining it between Grantham and the sea, exclusive of the commissioners of the smaller drainage districts. If we included them the above number would be doubled. In the first large district below Lincoln there are eight sets of commissioners. The ratepayers and others whom they represent all suffer periodically, some to a disastrous extent, from

the present state of the river. Yet these heterogeneous bodies never have united for the purpose of making the main channels, in which they are all interested, contribute to their common welfare, instead of being, as now, a constant source of danger and loss.

It is only by a long series of observations made at different points along a river channel that we can ascertain the maximum quantities of water which the different parts of the channel may have to convey. The quantity varies from day to day and from year to year, being, in the first instance, dependent on the rainfall. The rate of flow is liable to be altered by the operations of man in tilling the land. The cutting of open drains and ditches and subsoil drainage for agricultural purposes, as well as the destruction of waste and woodland, all tend to alter the rate at which the water falling on the land is conveyed to the sea. The observations required to ascertain the rate of flow are not likely to be made by local bodies elected for drainage purposes with partial jurisdiction, local interests, and limited funds. Until our rivers are placed under bodies who have wider interest and larger funds, and who will make systematic observations, we shall not know as nearly as we ought the quantity of water which the different parts of the river channels should be made to convey, and yet this is among one of the first things which we ought to know. I only know of one extensive set of observations on the flow of water in the River Thames, and they were made for the purposes of water supply, and at one place only.

The channel of a river, whether it be a channel excavated beneath the natural surface of the land, or a channel partly so formed and partly by banks rising above the surface of the land, should be large enough to convey all the water which systematic observation has shown it may have to convey. Some riparian owners are of opinion that floods should not be prevented or only partially prevented, and as long as any part of our rivers are under the control of those who hold such opinions, their channels will not be made as perfect as they should be for drainage purposes. It is generally held that grass lands are benefited by being laid under water at times, but there must be a wrong time and a right time for so treating them. As long as it is left to a chance flood to flow over the meadows, there will be a possibility of its happening at a wrong time. Given the water it will rarely be beyond the power of the engineer to distribute it by gravitation when it is wanted over any of the lands which are now ready to trust to floods for their supply.

In considering the works necessary for the prevention of floods, the question arises whether the extra sectional area required in a river channel should be obtained wholly by increasing the section of the channel below the surface of the land, or partly by that means and partly by embankments. When the fall is small, the latter plan must often be resorted to, and also when the flow in flood time is very large, as compared with the ordinary flow. As there is often some saving in making use of embankments, there will be a tendency to adopt them, more especially among local drainage authorities having small means. But unless they are well maintained, the first saving is soon swept away by the damage done by floods to which their neglect gives rise. As a rule, the larger the jurisdiction of the authorities having to maintain the banks, the better are they maintained. This is noticeable on the Witham, where the worst banks are those kept up by small local drainages.

Although it is safer to depend on a channel excavated below the surface of the land which lowers the flood line, embankments might be economically used, and without the danger which so often now attends them, if rivers were under wider jurisdictions. Formerly river channels throughout a great part of their length were maintained under navigation authorities by the tolls derived from the traffic on them. As the inland navigation tolls have dwindled away since the construction of railways, so the channels inland are no longer maintained. But although navigation authorities now may pay little attention to their inland waterways, large sums are still spent in many cases in maintaining the lower or tidal portions of rivers, and while they do this only with a view of navigation requirements, they at the same time render the channels efficient

for drainage purposes. It is fortunate indeed for those interested in land drainage that this is so, for they have often no jurisdiction over the tidal parts of their rivers.

Some districts, however, are not so fortunate. On the River Witham not only has the inland navigation fallen off, but Boston has ceased to be one of the chief ports of the country. The river channel through and below Boston is now often choked up with silt and mud, which increases the difficulties and cost of draining the lands above. There is sometimes 12 feet of silt against the sea-side of the doors of the Grand Sluice at Boston at the end of summer, so that the outlet to the Witham is completely blocked up. When the autumn rains come suddenly, a flood is almost sure to occur between Lincoln and Boston, owing to this state of things. If Boston were a thriving port, the river would never be allowed to remain in the state in which it is. The drainage authorities have only a partial jurisdiction over this part of the river, and the funds of the harbor authorities are so limited that between the two little is done.

At the mouth of a river the requirements of drainage and navigation are much the same. For drainage purposes the low water level in the river from the sea upwards should be kept as low as possible. Generally this end will be attained by straightening the river's course or by enlarging and regulating the channel. But this will at the same time increase the rate of tidal propagation and prolong its duration, which will benefit navigation. The Eau brink cut on the lower part of the Ouse was made for drainage purposes. It lowered the low-water level at Denver Sluice 7 feet, which was an immense gain for drainage, but it was also a great benefit to the navigation. Many cases could be given where the low-water level has been lowered far up a river as on the Tay, the Ribble, the Lune, by works carried out for navigation purposes, and I think it will generally be found that the works required for navigation may be so designed as to improve a river at the same time for drainage purposes.

Moreover, although it has been found unprofitable to maintain many rivers in a condition suitable for navigation, if rivers were once again made to fulfill efficiently the agricultural requirements of the adjoining lands, they could, with very little additional expenditure in many cases, be made available for navigation, so much is there in common between the works required for the two purposes. The difficulty of finding a good and sufficient supply of water for centers of population throughout the country is constantly increasing. Water from rivers, except when obtained from near their source, can rarely be used, owing to the filthy condition to which they have been reduced by the increase of population and extension of manufactures uncontroled by legislation. In some districts, as I found to be the case in the Clyde valley, manufactories have been established high up the purest streams, which are now polluted almost from their source.

Much improvement cannot be looked for as long as it is left to voluntary action to effect it. There is little encouragement to try and prevent the pollution of water which flows past your door if you are powerless to prevent its being polluted to a greater extent before it comes there.

One of the principal difficulties which stand in the way of legislation is the absence of a body to enforce any enactments for the prevention of pollution of rivers. If, however, a body of Commissioners were appointed for each river basin, this duty is one which might be imposed on them in connection with their other duties.

In the case of the River Thames some stringent measures were rendered necessary, owing to the difficulty of finding a source of water supply other than the Thames for so large a population as that of London. Advantage was taken of the existence of the Thames Conservancy, a navigation authority having jurisdiction over the greater part of the Thames but none over its tributaries. Their jurisdiction for the prevention of pollution was at first, confined to the part of the Thames for which they were the navigation authority, and to the tributaries of that part for a distance of three miles from the river. This distance was afterwards extended to five miles, and this year to ten miles from the river. The Thames Conservancy at once prohibited certain

towns on the banks of the Thames from discharging their sewage into it, and they have taken other steps to improve the quality of the water. This is good so far as it goes; without it the Thames would soon become unfit to supply London with drinking water. But there is still a large population living on the banks of its tributaries of the Thames which is not prohibited from discharging sewage into them, but is permitted to pollute the water which is drunk by three millions of people in London.

In order to provide the Thames Conservancy with funds to be used by them in carrying out the new duties imposed upon them of preventing pollution, London has been taxed through the water-works companies, who yearly pay a large sum to the Conservancy, so that London now bears the whole burden of enforcing the partial purification of the Thames.

The purity of river water is directly affected by its depth. Weeds, which by their decay pollute the water, do not greatly flourish in depths over five feet. Navigation is rarely compatible with a depth of much less than five feet, so that any works which may be necessary, as is generally the case inland, to maintain that depth or a greater one for navigation purposes, will tend to benefit the water supply; the cleansing of channels from mud and weeds is also necessary for land drainage purposes. Each watercourse if neglected may become a source of pollution, so that well-regulated land drainage works tend to some extent to improve the quality of the water.

Quality is not all that is required in the water of our rivers regarded as sources of water supply. Quantity is also necessary. Weirs for providing the depth of water required for navigation tend to maintain a large volume of water in the river in dry seasons. And generally it will be found, I think, that the works required to make a river serve satisfactorily the purposes of land drainage and navigation will tend to improve the quality of the water, and in many cases increase its available quantity by increasing the storage capacity of the river.

I have endeavored so far, in a somewhat fragmentary manner I am afraid, to show the connection there is between the works required to make a river fulfill the purposes of drainage, navigation and water supply, and some of the evils which arise when such works are carried out, as is now the case, by numerous separate authorities having only limited jurisdiction. I will now state what appears to me to be some of the requirements for improving the existing state of things. Generally I think that Parliament should make it compulsory for each river basin to appoint a body of commissioners who should have jurisdiction over a river from its source to the sea, and over all its tributaries and sources of water supply. Their powers should be exercised for the purpose of drainage, navigation, and for the preservation of the water and maintenance of its purity for the purposes of water supply. These commissioners could divide the drainage area into districts, so that local matters might be referred to local commissioners, but it is essential that all works should be carried out under the general supervision of the central authority.

The question remains as to how the money should be raised to carry out the works to be done by the commissioners for each river basin. When large or thickly populated districts have to be dealt with, any rate or tax must press more on some than on others. But it ought to be possible to find some means of adjusting the cost of maintaining river channels in an efficient state that would be fairer to all than the present method.

Every acre of land in a river basin either directly contributes its supply of water to the river channels or in some way affects the supply from adjoining acres. As we cannot ascertain the effect of each acre on the supply of water, it appears to me it would be most just to tax each acre according to its value, that is, to levy a tax on all the land within the river basin in proportion to its annual value for whatever purpose it is used, and whether it be covered by buildings or not.

But as some lands are now subject to floods and would be at once improved in value by their prevention, such lands should, in the first case, be taxed at a higher rate, but that when the works necessary to prevent such damage by floods were once done and paid for, the general fund for the maintenance of the

river channels should be raised by an equal rate levied on all land according to its annual value.

A small rate levied over all the acreage of a river basin would produce a large sum. Taking the area of the Thames basin at 6160 square miles, or close upon four million acres, and assuming the average annual value to be £2 per acre, which is probably too low, a rate of one penny in the pound would produce £33,000 a year.

When a river was once made efficient the navigation as far as purely navigation works are concerned would probably more than maintain itself, as the river channels would have to be maintained for drainage works alone. Any surplus might go to the general fund. As regards the use of the water the river channels should be free to all for the purposes of recreation, and as all districts would pay to maintain the quantity and purity of the water, all should have a right to use the water subject to not prejudicing others. Lastly, as regards pollution, no claim for compensation for desisting from it should be admitted. The principle of dealing summarily with offenders has already been adopted on the Thames and elsewhere. Those still offending should be similarly dealt with.

DRAINING AND IMPROVING 1,500,000 ACRES OF DESERT LAND.

By GEORGE WILSON, M. Inst. C. E.

From Proceedings of the Institution of Civil Engineers.

In the west of France, close to the important town of Bordeaux, there existed, previously to the year 1850, an immense territory of uninhabited desert lands, comprising nearly 2,000,000 acres superficial area, and designated "Les Landes de Gascogne." The territory is triangular in shape. The base, commencing at a point close to the mouth of the river La Gironde, extends to the south of that river along the shores of the Bay of Biscay for a distance of about 140 miles; while one of the sides, passing by and about 5 miles distant from Bordeaux, is approximately parallel to the course of the river La Gironde for a distance of about 120 miles from its mouth.

The desert lands of this territory resembled a vast and nearly horizontal plain at an average altitude of about 330 feet above the level of the sea. The surface of the ground consists of poor sandy soil, without any trace of clay or calcareous matter in its composition, varying in depth from 12 to 20 inches. Under the surface soil there is an impermeable layer of silicious sand, agglomerated by vegetable matter, varying in thickness from 16 to 20 inches, forming a sort of organic cement, which is known in the country under the name of "alios." The substratum under this layer of "alios" consists of compact white sand impregnated with water. There does not exist on the surface of the plain any spring, nor is there any trace of water during the summer. During the winter, however, previously to the execution of the drainage works, the abundant rain of that coast, experienced for more than six months, falling on the plain, could neither flow off its surface, nor percolate sufficiently through crevices of the impermeable layer of alios into the substrata. The water remained stagnant until it was evaporated by the heat of summer. Thus the constant inundation during the winter, and the dryness of the hot sandy soil during the summer, rendered the ground sterile for any kind of cultivation, and extremely unhealthy for the inhabitants and for animals to exist or work on it.

Previous to 1850, several attempts had been made, and much money expended, in endeavoring to improve and drain portions of this desert, all of which, however, resulted in failures, and caused the impression to prevail, that the soil was of little value, certainly not worthy the expenditure necessary to drain and improve it for the purpose of cultivation. In

1849 a report appeared by M. Chambrelent, Ingénieur en Chef des Ponts et Chaussées, describing the results of complete studies, surveys, levels, borings, &c., of the desert, which he was officially commissioned to carry out, extending over seven years. From these studies the exact physical features of the desert were developed, and at once indicated the mode of improving and rendering it profitable for cultivation. It was shown that, upon the whole extent of the apparently horizontal plain, there exists from the summit towards the valleys extremely regular slopes, but so small that the least irregularities of the ground hinder the water from following the normal declivities.

The average inclination of the ground is about 1 in 1,000, but in some parts it does not exceeed 1 in 2,000, and continues always in the same directions towards the valleys without any noticeable change or break. The small irregularities on the surface, which hinder the flow of the water, are never more than 12 to 16 inches above or below the normal slopes of the plain. Therefore a drain 20 to 24 inches deep, with its bottom parallel with the general slope of the ground, can be made along its entire required length without necessitating a cutting of more than from 24 to 28 inches, in order to allow the water from rainfall to pass perfectly away. Moreover, a ditch extending along such sandy permeable ground will drain the surface to a considerable distance on each side of it, and as its average inclination will be about 1 in 1,000, the water will percolate slowly and regularly to it without causing injury to the sides. In consequence of the permeability of the surface soil very few drains are required.

It had been noticed that there existed in a few isolated places of the desert clusters of fir trees, locally termed "Pignades," which flourished remarkably well; also that the ground at such places was slightly higher than the normal inclination of the plain, and was therefore drained naturally, and that both the surface soil and the subsoil where the trees flourished were physically and chemically, as shown by analyses, similar to the ground of the entire plain. It was thus evident that the desert only required to be properly drained in order to cultivate it for a forest, and for it to produce trees similar to the clusters or "Pignades" actually growing. There were, however, at the time, numerous objections against these deductions as to the practicability of the drainage and improvement scheme. It was contended that drains cut through such sandy ground would soon be silted up, from the effects of the wind on the sandy plain, and be trodden in by cattle, or destroyed by the drainage water percolating through their sides. A further objection was urged, that the drains would have to be made of such great lengths that the water could not be caused to flow along them to the distant valleys.

To meet these objections, and to test the practicability of the scheme, it was decided to make a trial. A superficial area was selected of about 1,250 acres of the desert, nearly on the summit, and in the middle of one of the most sterile parts, being in every sense under the most unfavorable conditions for draining the soil. At the time of commencing operations the ground was covered with stagnant water, so that it was necessary to move about on the stilts used in the country. The effects of the drains was immediately satisfactory. The ground was so well drained that during the heaviest winter rain no stagnant water existed on it, and the water flowed away abundantly and with great regularity. All the rain water percolated immediately through the soil into the drains, and none was noticed to flow along the surface. At present, although it is more than twenty-seven years since these drains were executed, yet such is the slowness and regularity with which the water percolates into and along them, that they have not silted up nor have their sides been injured.

Shortly after this trial ground was drained, it was sown with fir seeds and acorns, and the results were so satisfactory that in the following year numerous proprietors of portions of the desert carried out similar works, and obtained an equal success; and in less than five years an area of more than 50,000 acres was drained and cultivated for forests. As regards the first sowing on the trial ground, in five years fir and oak trees had grown over the whole area to a height of nearly 12 feet, and each tree

was about 12 inches in circumference above the ground.

After proving the practicability of draining the territory, the important question arose as to the possibility of obtaining wholesome drinking-water for the inhabitants. There are no springs of water, and the potable water hitherto obtained was derived from wells dug through the "alios" to a depth of about 4 feet below the surface of the ground. The bad quality of the water caused even worse fevers and other diseases than the unhealthiness arising from the marshy nature of the soil. Borings were made to various depths below the surface through the "alios" into the substratum of compact white sand, and on analyzing the water, it was shown that the water increased in purity with the depth. At a depth of about thirteen feet from the surface the water was practically fit for drinking purposes provided it was caused to filter at the bottom of the well, by ascending through an artificial bed of broken calcareous small stones and clay gravel. Wells have therefore been sunk, wherever required over the territory, to a depth of about thirteen feet, and the sides built with masonry laid in cement so as to be perfectly and permanently watertight. The water can thus only ascend the bottom of each well after filtering through an artificial layer of broken stones (generally the *débris* from dressing the stones for lining the well) and clay gravel. The organic matter found in the water of the old wells was as much as $\frac{1}{29412}$ part by weight, or about 2.4 grains per gallon, while the water drawn from the new wells only contained organic matter equal to $\frac{1}{500000}$ the weight of the water, or 0.14 grain per gallon.

Extensive portions of the deserts belonged to one hundred and sixty-two communes, and were so intermingled with the property of private owners as to be a hinderance to the latter carrying out their drainage works effectually until such portions were drained. To facilitate and ensure the communal possessions being drained, and to construct the necessary roads of communication over such a large territory, the Government, in 1857, made the following special laws: "The waste lands of the communes of the two departments, La Gironde and Les Landes, shall be drained and improved at the expense of the communes. In case of inability, or refusal on the part of the communes, to carry out these works, the same shall be done at the expense of the State, to be repaid the advance made in principal and interest out of the produce and cultivation." The law also provided for the construction of the roads of communication by the State.

So soon as the above law was promulgated, a general plan was prepared for draining the waste lands belonging to the communes, comprising about 720,000 acres, irrespective of about 864,000 acres belonging to private individuals, which remained to be drained. The plan comprised the execution of the principal canals necessary for receiving and carrying away the water flowing from the land drains. The fixing of these canals over ground almost flat presented many difficulties. The whole country had to be again carefully leveled, which showed that the inclination of some portions of the plain was not more than one in 2,000. The inclinations of the canals had to be calculated in such a manner so as to be sufficient for the proper drainage of the water resulting from a varying rainfall, and yet not such as would cause a current injurious to the canals constructed through ground of such a sandy nature. The main canals in the department of La Gironde were 13 feet to 16.4 feet wide at the bottom, and their total lengths amounted to 1,365 miles. The canal between the pools or lakes Lacanau and Langourarde, bordering the sand dunes on the coast (which had to be constructed for a distance of $\frac{5}{8}$ mile through a marsh always covered with water), is 39.4 feet wide at the bottom, 6.3 miles in length, and has an inclination of 1 in 4,000; while the canal made between the pools or lakes d'Hourtins and Lacanau, through the middle of a marsh for a distance of 5.2 miles, is 23 feet wide at the bottom. The two last-mentioned canals receive the drainage water of 200,000 acres superficial area.

In 1865 the drainage works of the communal lands were completed at a cost of about 1*s*. 10*d*. per acre drained. Out of the total area of communal lands comprising 720,000 acres, 466,000 acres have been sold from time to time to meet expenses, so that the funds of the State

were not drawn upon. Some portions, previous to being drained, were disposed of for a sum of for a sum of £537,278, which sum was applied for the following purposes:

	£
Draining	35,739
Sowing land with seed	27,272
Construction of new and restoration of old churches	95,660
Construction of new and restoration of old parsonages	27,082
Construction of new and restoration of old town-halls and school-houses	65,455
Subscriptions, subventions and allowances for the construction of roads	79,489
Various expenses—construction of wells, removing cemeteries from market towns and villages	32,471
Communal funds invested in rentes of the State	174,110
Total	537,278

With the exception of lands situated in various parts of the plain, amounting to about 70,876 acres, respecting which there are, amongst other causes, disputes as to the boundaries, the whole of this formerly desert territory is now under forest cultivation, consisting mainly of fir trees. Oak trees, although growing equally well, and more profitable in the course of time, are nevertheless more expensive to cultivate during the first few years, and consequently the ground has only been to a limited extent used for their cultivation. Everywhere and in every respect the results of forest cultivation are highly satisfactory, showing the most vigorous vegetation and growth, and developing equally well each successive year. The only cause for anxiety arises from accidents and losses by fire. From 1865 to 1870 serious losses occurred from that cause, destroying 25,000 acres of forest. Since 1870 such fires have somewhat diminished, and the Government is adopting special measures to prevent these disasters. It might be considered that the production from forests, comprising an area of 1,500,000 acres, would so much diminish the price of timber, that profitable markets could not be found. Time and experience have, however, proved that no fears need be entertained on this question. Each year the purposes and the appliances for the production of such forests increase, and the selling prices augment accordingly. The timber of these forests is not only sold in France, but is exported to England and Scotland, and even to America and Spain. In England large quantities are sold for props, &c., used in coalpits, in pieces each about 6½ feet long and 3½ inches in diameter. The younger trees and branches are sold for the fuel throughout France. Large quantities are used for making wine cases and barrels. A mill has recently been successfully set to work, making 1,000 tons of pulp per annum, for the manufacture of paper direct from the fir tree, which industry it is anticipated will rapidly increase, inasmuch as the pulp produced is of brilliant whiteness. Another industry is being developed in manufacturing, by the aid of recent discoveries, rectified essence of turpentine from fir trees, for the purpose of illumination, the light produced being considered superior to, and more economical than that, from any of the oils or from petroleum. The above are some of the objects for which the production of the forests are used up to the first twenty-five years of their growth, after which the timber is sufficiently large to be applied to more extended purposes, such as general carpenters' work, sleepers for railways, telegraph posts, &c. For the latter purpose large quantities are already being used in France, and exported to Algeria and Spain. In 1877 the timber on a plot of ground of 11 acres superficial area, which was in its twenty-eighth year of growth, was sold for £192, equal to £17 9*s.* per acre; but this plot of timber was favorably situated for being carried away.

At an average age of thirty years the present forests, comprising 1,514,110 acres, are estimated to produce timber worth at a minimum price £9 15*s.*, and in seventy years £32 10*s.* per acre, respectively equivalent to a total value of about 14¾ and 49¼ millions of pounds sterling for the produce of those lands which were recently a desert. This valuation is based on the probable casualties by fire, and would consequently be considerably higher if the dread of losses from that cause did not exist.

Independently of these satisfactory results, the drainage of the desert, and the construction of d eper wells for the supply of wholesome drinking water, have changed the district into one of the healthiest in France; whereas previously, the

inhabitants suffered constantly, from fevers and other fatal diseases, causing the death-rate to be exceedingly high.

The maintenance of the main canals, of a total length of 1,365 miles, is under the inspection of the administrations of the two departments, La Gironde and Les Landes, who give orders to the mayors of the communes and proprietors of lands bordering the canals to clean them out when necessary, or to remove impediments in them from whatever cause, inasmuch as it is of importance that canals of such great lengths, small inclinations, and dug in sandy soil, should always be maintained in good order.

ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION.

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V.

The next case of vortex motion that we shall consider is that in which the vortex lines are circles having their centers in the axis of z. The direction of the axis of rotation of each fluid particle will lie in a plane at right angles to z and be parallel to the tangent to the vortex line at that point. The reader will do well to observe the motion of a ring of tobacco smoke; he will see that the ring seems to be turning inside out, each particle moving in a plane passing through the axis of the ring and revolving in a circle whose axis is in the direction of the tangent to the ring. He will also observe that there is no motion around the axis of the ring. Now if we introduce polar co-ordinates ρ and ϑ we have, evidently,

$$x=\rho\cos\vartheta \quad y=\rho\sin\vartheta$$

and for the rotations we may obviously write

$$\xi=-\lambda\sin\vartheta,\ \eta=\lambda\cos\vartheta,\ \zeta=o,$$

when λ is not a function of ϑ. The equation of the path of the particle is evidently one between x, y, z where x and y are connected by the relation

$$\rho=\sqrt{x^2+y^2}$$

so that the equation of the path can be made to depend only upon ρ and z. Resuming now our equations

$$u=\frac{dW}{dy}-\frac{dV}{dz},\ v=\frac{dU}{dz}-\frac{dW}{dx},$$
$$w=\frac{dV}{dx}-\frac{dU}{dy}$$

and observing that

$$W=\frac{1}{2\pi}\int\frac{\zeta d\tau}{r}$$

these give

$$W=o,\ u=-\frac{dV}{dz},\ v=\frac{dU}{dz}\quad w=\frac{dV}{dx}-\frac{dU}{dy}$$

If now we assume an element $d\tau'$ given by the co-ordinates ϑ', ρ', z' and for which λ becomes λ'; also denote by r its distance from the point (ρ, ϑ, z) or $(x\,y\,z)$. Now our equations for U and V are

$$U=\frac{1}{2u}\int\frac{\xi' d\tau'}{r}=-\frac{1}{2\pi}\int\frac{\lambda'\sin\vartheta' d\tau'}{r}$$

$$V=\frac{1}{2\pi}\int\frac{\eta' d\tau'}{r}=\frac{1}{2\pi}\int\frac{\lambda'\cos\vartheta' d\tau'}{r}$$

When

$$d\tau'=dx'\,dy'\,dz'=\rho' d\rho'\,dz'\,d\vartheta'$$

and

$$r=\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos(\vartheta'-\vartheta)}$$

Now make for convenience

$$\vartheta'-\vartheta=\varphi$$

this gives $\vartheta'=\varphi+\vartheta$ and $d\vartheta'=d\varphi$ therefore we have again for U and V,

$$U=-\frac{1}{2\pi}\int\rho' d\rho'\int dz'\int\frac{\lambda'[\sin\varphi\cos\vartheta+\cos\varphi\sin\vartheta]d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

$$V=\frac{1}{2\pi}\int\rho' d\rho'\int dz'\int\frac{\lambda'[\sin\varphi\sin\vartheta-\cos\varphi\cos\vartheta]d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

In both cases we have the integral

$$\int \frac{\sin\varphi\, d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

this is to be taken between o and 2π, these being the limits of $\vartheta'-\vartheta$,

$$\int_0^{2\pi} \frac{\sin\varphi\, d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

$$=\log\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}\Big]_0^{2\pi}$$

$$=\frac{1}{2}\log\frac{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}=o$$

and our expressions are thus reduced to

$$U=-\frac{1}{2\pi}\int\rho'd\rho'\int dz'\int\frac{\lambda'\cos\varphi\sin\vartheta\, d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

$$V=\frac{1}{2\pi}\int\rho'd\rho'\int dz'\int\frac{\lambda'\cos\varphi\cos\vartheta\, d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

Denoting the common integral by Φ we have

$$U=-\frac{1}{2\pi}\int\rho'd\rho'\int dz'\,\lambda'\,\Phi\sin\vartheta$$

$$V=\frac{1}{2\pi}\int\rho'd\rho'\int dz'\,\lambda'\,\Phi\cos\vartheta$$

Φ will clearly be of the form

$$\Phi(z'-z,\rho,\rho').$$

U and V now differ only by a factor, in fact we have

$$V=-U\tan\vartheta.$$

So if we write

$$P=\frac{1}{2\pi}\int\int\rho'd\rho'dz'\lambda'\Phi$$

we will have briefly

$$U=-P\sin\vartheta,$$
$$V=\ \ P\cos\vartheta.$$

The value of the function Φ is not difficult to obtain; we have

$$\Phi=\int_0^{2\pi}\frac{\cos\varphi\, d\varphi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2-2\rho\rho'\cos\varphi}}$$

We will find the integration much simplified by the introduction of a new variable Ψ defined by the equation

$$\Psi=\frac{\pi-\varphi}{2}$$

Then we have

$$\cos\varphi=-[1-2\sin^2\psi]$$
$$\cos\varphi d\varphi=2[1-2\sin^2\psi]d\psi$$

and for the limits we have when $\varphi=(o,2\pi),\psi=\left(\frac{\pi}{2},-\frac{\pi}{2}\right)$. Making this transformation we have

$$\Phi=2\int_{+\frac{\pi}{2}}^{-\frac{\pi}{2}}\frac{[1-2\sin^2\psi]d\psi}{\sqrt{(z'-z)^2+\rho^2+\rho'^2+2\rho\rho'-4\rho\rho\sin^2\psi}}$$

or,

$$\Phi=-4\int_0^{\frac{\pi}{2}}\frac{[1-2\sin^2\Psi]d\Psi}{\sqrt{(z'-z)^2+(\rho+\rho')^2-4\rho\rho'\cos\varphi}}$$

$$=-4\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{(z'-z)^2+(\rho'+\rho)^2}\sqrt{1-\frac{4\rho\rho'\sin\Psi}{(z'-z)^2+(\rho'+\rho)^2}}}$$

$$+4\int_0^{\frac{\pi}{2}}\frac{2\sin^2\psi\, d\psi}{\sqrt{(z'-z)^2+(\rho'+\rho)^2}\sqrt{1-\frac{4\rho\rho'\sin^2\Psi}{(z'-z)^2+(\rho'+\rho)^2}}}$$

Make,

$$\frac{4\rho\rho'}{(z'-z)^2+(\rho'+\rho)^2}=k^2$$

then

$$\frac{1}{\sqrt{z'-z)^2+(\rho'+\rho^2)}}=\frac{k}{2\sqrt{\rho\rho'}}$$

then

$$\Phi=-\frac{2k}{\sqrt{\rho\rho'}}\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}$$

$$+\frac{2k}{\sqrt{\rho\rho'}}\int_0^{\frac{\pi}{2}}\frac{2\sin^2\psi\, d\psi}{\sqrt{1-k^2\sin^2\psi}}$$

The first of these is the complete elliptic integral of the first kind; we will, as

usual, denote it by K. Examine now the second integral; we have on multiplying it numerator and denominator by k

$$\frac{4}{k\sqrt{\rho\rho'}}\int_o^{\frac{\pi}{2}}\frac{k^2 \sin^2\psi d\psi}{\sqrt{1-k^2 \sin^2 \Psi}}$$

$$=\frac{4}{k\sqrt{\rho\rho'}}\int_o^{\frac{\pi}{2}}\frac{1-(1-k^2\sin^2\psi)}{\sqrt{1-k^2\sin^2\psi}}d\psi$$

$$=\frac{4}{k\sqrt{\rho\rho'}}\int_o^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-k^2 \sin^2 \psi}}$$

$$-\frac{4}{k\sqrt{\rho\rho'}}\int_o^{\frac{\pi}{2}}\sqrt{1-k^2 \sin^2 \psi}d\psi$$

The second of these is the elliptic integral of the second kind and denoted by E; we have then finally for Φ,

$$\Phi=-\frac{2k}{\sqrt{\rho\rho'}}\text{ K}+\frac{4}{k\sqrt{\rho\rho'}}\text{K}-\frac{4}{k\sqrt{\rho\rho'}}\text{ E}$$

or

$$\Phi=\frac{2}{\sqrt{\rho\rho'}}\left\{\left(\frac{2}{k}-k\right)\text{K}-\frac{2}{k}\text{ E}\right\}$$

and consequently

$$\text{P}=\frac{1}{\pi}\int\int\lambda'\sqrt{\frac{\rho'}{\rho}}\left\{\left(\frac{2}{k}-k\right)\text{K}-\frac{2}{k}\text{ E}\right\}d\rho' dz',$$

$$\text{U}=-\frac{1}{\pi}\int\int\lambda'\sqrt{\frac{\rho'}{\rho}}\left\{\left(\frac{2}{k}-k\right)\text{K}-\frac{2}{k}\text{ E}\right\}\sin\vartheta\, d\rho' dz',$$

$$\text{V}=\frac{1}{\pi}\int\int\lambda'\sqrt{\frac{\rho'}{\rho}}\left\{\left(\frac{2}{k}-k\right)\text{K}-\frac{2}{k}\text{E}\right\}\cos\vartheta\, d\rho' dz'.$$

In the function that we have denoted by Φ we see that the derivatives taken for z and z' have the same absolute values but opposite signs; consequently

$$\int\int\lambda\lambda'\frac{d\Phi}{dz}\rho\rho' d\sigma\, d\sigma'=o$$

when $d\sigma=d\rho dz$. But we have also

$$\frac{d\text{P}}{dz}=\frac{1}{2\pi}\int\lambda'\rho'\frac{d\Phi}{dz}d\sigma'$$

therefore

$$\int\rho\frac{d\text{P}}{dz}\lambda d\sigma=o.$$

But we have for the energy T the equation

$$\text{T}=\int d\tau(\text{U}\xi+\text{V}\eta)$$

Substituting for U, V, ξ and η their values this becomes,

$$\text{T}=\int\text{P}\lambda\, d\tau=\int\int\int\text{P}\lambda\rho d\rho dz d\vartheta$$

integrating with respect to ϑ from o to 2π, and writing again $d\rho dz=d\sigma$,

$$\text{T}=2\pi\int\text{P}\rho\lambda d\sigma.$$

Substituting for P its value we have

$$\text{T}=\int\Phi\rho\rho'\lambda\lambda'\, d\sigma\, d\sigma'$$

when of course $d\sigma$ and $d\sigma'$ denote the cross sections of the vortex filaments under consideration. Let S denote the component of velocity in which ρ increases:

$$\text{S}=\sqrt{u^2+v^2}$$

which is the same as

$$u=\text{S}\cos\vartheta \qquad v=\text{S}\sin\vartheta.$$

But

$$u=-\frac{d\text{V}}{dz}=-\frac{d\text{P}}{dz}\cos\vartheta$$

$$v=\frac{d\text{U}}{dz}=-\frac{d\text{P}}{dz}\sin\vartheta$$

therefore

$$\text{S}=-\frac{d\text{P}}{dz}.$$

Now for w we have

$$w=\frac{d\text{V}}{dx}-\frac{d\text{U}}{dy}$$

and substituting for U and V their values.

This gives

$$w=\frac{d\text{P}}{d\rho}+\frac{\text{P}}{\rho}$$

or

$$w\rho=\frac{d(\text{P}\rho)}{d\rho}$$

and we may also write

$$s\rho=-\frac{d(\text{P}\rho)}{dz}\text{ since }\frac{d\rho}{dz}=o.$$

From these we have the equation $\int\rho\frac{d\text{P}}{dz}\lambda\, d\sigma=o$ in the form

$$\int\rho s\lambda d\sigma=o$$

and also

$$s=\frac{d\rho}{dt} \text{ and } w=\frac{dz}{dt}$$

therefore

$$\int \rho \frac{d\rho}{dt} \lambda\, d\sigma = o$$

and since for each vortex filament $\lambda d\sigma$ is constant this gives

$$\int \rho^2 \lambda d\sigma = \text{const.}$$

Some other interesting forms may be given before we proceed to the examination of a special case. We had

$$k^2=\frac{4\rho'\rho}{(z'-z)^2+(\rho'+\rho)^2}$$

Taking logarithms this becomes

$2 \log k = \log \rho\rho' - \log\left[(z'-z)^2+(\rho'+\rho)^2\right]$

Differentiating with respect to ρ and z we obtain

$$\frac{2}{k}\rho\frac{dk}{d\rho}=\frac{(z'-z)^2+\rho'^2-\rho^2}{(z'-z)^2+(\rho'+\rho)^2}$$

$$\frac{2}{k}z\frac{dk}{dz}=\frac{zz(z'-z)}{(z'-z)^2+(\rho'+\rho)^2}$$

consequently,

$$\frac{2}{k}\left\{\rho\frac{dk}{d\rho}+z\frac{dk}{dz}\right\}=\frac{z'^2-z^2+\rho'^2-\rho^2}{(z'-z)^2+(\rho'+\rho)^2}$$

The denominator of this second member does not change by the interchange of accented and unaccented letters, but the numerator does change its sign, also k does not change by making the same transfer, therefore

$$\rho\frac{dk}{d\rho}+z\frac{dk}{dz}$$

assumes the opposite value by writing the accented letters in the place of the unaccented and vice versa. If from the value of Φ in terms of K and E we obtain $\frac{d\Phi}{dk}$ we will see that this quantity does not alter by the interchange of accented and unaccented letters, consequently the quantity

$$\frac{d\Phi}{dk}\left(\rho\frac{dk}{d\rho}+z\frac{dk}{dz}\right)$$

assumes the opposite value after the interchange. We have by partial differentiation of Φ

$$\frac{d\Phi}{d\rho}=\frac{d\Phi}{d\rho}+\frac{d\Phi}{dk}\frac{dk}{d\rho}$$

$$=\frac{d\Phi}{dk}\cdot\frac{dk}{d\rho}-\frac{1}{z}\frac{\Phi}{\rho}$$

$$\frac{d\Phi}{dz}=\frac{d\Phi}{dk}\frac{dk}{dz}$$

Consequently the above quantity assumes the form

$$\rho\frac{d\Phi}{d\rho}+z\frac{d\Phi}{dz}+\frac{1}{z}\Phi$$

And by virtue of the property proved for this quantity we know that

$$o=\int\int\left(\rho\frac{d\Phi}{d\rho}+z\frac{d\Phi}{dz}+\frac{1}{z}\Phi\right)\rho\rho'\lambda\lambda'\, d\sigma d\sigma'$$

But we have

$$P=\frac{1}{2\pi}\int\int\lambda'\rho'd\rho'dz'\Phi$$

therefore

$$\int\left(\rho\frac{dP}{d\rho}+z\frac{dP}{dz}+\frac{1}{z}P\right)\rho\lambda d\sigma=o$$

Since now

$$\rho\frac{dP}{d\rho}=\frac{d(P\rho)}{d\rho}-P$$

and

$$w\rho=\frac{d(P\rho)}{d\rho},\quad s=-\frac{dP}{dz}$$

this equation becomes

$$\int(w\rho-zs)\rho\lambda d\sigma-\frac{1}{z}\int P\rho\lambda d\sigma=o$$

or

$$\int(w\rho-zs)\rho\lambda d\sigma=\frac{T}{4\pi}.$$

We will now introduce the complementary modulus k' defined by

$$k'^2+k^2=1$$

we have for this modulus

$$k'^2=1-k^2=\frac{(z'-z)^2+(\rho'-\rho)^2}{(z'-z)^2+(\rho'+\rho)^2}$$

If k is very nearly equal to unity, that is, if k' is indefinitely small we see that Φ will be of the same order of magnitude as K, and again, that P will be of the same order as Φ. We will examine the order of K on the supposition that k' is infinitely small. Since k' is indefinitely small, we have, neglecting higher powers than the second,

$$k=\sqrt{1-k'^2}=1-\frac{1}{z}k'^2$$

Now

$$K=\int_o^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}$$

$$=\int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{1-\sin^2\psi+k'^2\sin^2\psi}}$$

$$=\int_0^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{\cos^2\psi+k'^2\sin^2\psi}}$$

or, by introducing an indefinitely small quantity ε which is, nevertheless, indefinitely large as regards k',

$$K=\int_{\frac{\pi}{2}-\varepsilon}^{\frac{\pi}{2}} \frac{d\psi}{\sqrt{\cos^2\psi+k'^2\sin^2\psi}}$$

$$+\int_0^{\frac{\pi}{2}-\varepsilon} \frac{d\psi}{\sqrt{\cos^2\psi+k^2\sin^2\psi}}$$

Write now in the first integral $\frac{\pi}{2}-\theta$ in in place of Ψ: since throughout the integral θ is small the integral becomes

$$\int_0^{\varepsilon} \frac{d\theta}{\sqrt{k'^2+k^2\theta}}$$

$$=\frac{1}{k}\log\frac{k\varepsilon+\sqrt{k'^2+k^2\varepsilon^2}}{k'}$$

or since k' is infinitely small with regard to $k\varepsilon$ this is $=\frac{1}{k}\log\frac{2k\varepsilon}{k'}$ or $=\log\frac{2\varepsilon}{k'}$.

In the second integral $k'\sin\psi$ is throughout small as regards $\cos\psi$ and this integral is

$$\int_0^{\frac{\pi}{2}-\varepsilon} \frac{d\psi}{\cos\psi}=\log\tan\left\{\frac{1}{2}\pi-\frac{1}{2}\varepsilon\right\}$$

or what is the same thing $=\log\frac{2}{\varepsilon}$.

Hence we have

$$K=\log\frac{2\varepsilon}{k'}+\log\frac{2}{\varepsilon}=\log\frac{4}{k'}$$

Consequently for k' indefinitely small we see that K is indefinitely large, and Φ and therefore P are indefinitely large of the order $\log k'$. Of course Φ does not depend on E as for k nearly equal to unity we have,

$$E=\int_0^{2\pi}\cos\psi\, d\psi=1.$$

Representing the elliptic integral of the first kind as is usual by F, we have

$$F=\int_0^{\psi}\frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}$$

and also

$$E=\int_0^{\psi}(1-k^2\sin^2\psi)^{\frac{1}{2}}\,d\psi.$$

Differentiating for k, and writing $\sqrt{1-k^2\sin^2\psi}=\Delta$

$$\frac{dF}{dk}=\int\frac{k\sin^2\psi d\psi}{\Delta^3}$$

$$\frac{dE}{dk}=-\int\frac{k\sin^2\psi d\psi}{\Delta}$$

Now writing $\sin^2\psi=\frac{1}{k^2}(1-\Delta^2)$ we see that these two integrals depend on

$$\int\frac{d\psi}{\Delta},\quad \int\Delta d\psi,\quad \int\frac{d\psi}{\Delta^3}:$$

the two first of these are F and E respectively; as regards the third, we have

$$\frac{d}{d\psi}\cdot\frac{\sin\psi\cos\psi}{\Delta}=\frac{1-z\sin^2\psi+k^2\sin^4\psi}{\Delta^3}$$

or

$$k^2\frac{d}{d\psi}\cdot\frac{\sin\psi\cos\psi}{\Delta}=\frac{\Delta^4-k'^2}{\Delta^3}=\Delta-\frac{k'^2}{\Delta^3}$$

and thence by integration,

$$\int\frac{d\psi}{\Delta^3}=\frac{1}{k'^2}E-\frac{K^2\sin\psi\cos\psi}{k'^2\Delta}$$

The expressions for $\frac{dE}{dk}$, and $\frac{dF}{dk}$, thus become,

$$\frac{dF}{dk}=\frac{1}{kk'^2}\left\{E-k'^2F\right\}-\frac{k\sin\psi\cos\psi}{dk'^2}$$

$$\frac{dE}{dk}=\frac{1}{k}\left\{E-F\right\}$$

and for the complete functions when $\psi=\frac{\pi}{2}$,

$$\frac{dE}{dk}=\frac{1}{kk'^2}[E-k'2F]$$

$$\frac{dE}{dk}=\frac{1}{k}[E-F]$$

When k' is indefinitely small the first of these is of the order $\frac{1}{k'^2}$ and the second by k'. Now we have obviously if k' be indefinitely small that $\frac{d\Phi}{dk}$ is of the order $\frac{1}{k'^2}$. We had also

$$\frac{dk}{dz}=\frac{k(z'-z)}{(z'-z)^2+(\rho'+\rho)^2}$$

$$\frac{dk}{d\rho}=\frac{k}{2\rho}\frac{(z'-z)^2+\rho'^2-\rho^2}{(z'-z)^2+(\rho'+\rho)^2}$$

These quantities are of the same order as k'. Therefore

$$\frac{d\Phi}{d\rho}-\frac{d\Phi}{dk}\frac{dk}{d\rho}\frac{1}{z}\frac{\Phi}{\rho}$$

and

$$\frac{d\Phi}{dz}=\frac{d\Phi}{dk}\frac{dk}{dz}$$

are of the same order as $\frac{1}{k'}$. By the aid of these preliminary investigations we will now proceed to the examination of the case when only one vortex ring exists in the fluid, and will further more suppose this ring to be of indefinitely small cross section and of the same order of magnitude as the indefinitely small quantity ε. We may again write as before

$$m=\int\lambda d\sigma$$

as m will be finite and as $d\sigma$ is of the order ε, λ must be of the order $\frac{1}{\varepsilon^2}$. Assume the equation of a circle such that the fluid elements of which it is composed shall lie indefinitely near the vortex filament. Let its equations be

$$\rho=\rho_0 \qquad z=z_0.$$

We had

$$P=\frac{1}{2\pi}\int\int\lambda'\rho'd'\rho dz'\Phi=\frac{m}{2\pi}\rho\rho_0\Phi$$
$$(z-z_0)\rho\rho_0)$$

for all points lying at a finite distance from the circle. For these points by aid of the equations

$$\Phi=\frac{z}{\sqrt{\rho\rho_0}}\left\{\left(\frac{2}{k}-k\right)K-\frac{z}{k}E\right\}$$

$$\rho s=-\frac{d(P\rho)}{dz},\ w\rho=\frac{d(P\rho)}{d\rho}$$

we can find the values of s and w. But a difficulty exists inasmuch as ρ_0 and z_0 are functions of the time and as such will have to be determined, and to that end, it is necessary to consider the points that lie indefinitely near the circle, or points in the vortex filament.

Suppose that the two circles (ρ, z), (ρ_0, z_0) are at a distance apart, that is of the same order as ε. If we call r the distance between them, we have

$$r^2=\rho^2+\rho_0^2-2\rho\rho_0\cos\vartheta+(z_0-z)^2$$

or since ϑ is indefinitely small

$$r^2=(\rho-\rho_0)^2+(z-z_0)^2$$

this is of the same order as k'^2 therefore k' is of the same order as ε—therefore by our preceding investigations we see that P is of the same order as Φ when k' is of the same order as ε. And also that P is of the order log. ε. We had for the energy

$$T=2\pi\int P\rho\lambda d\sigma$$

therefore T is of the same order as log. ε. The preceding investigations taken in connection with the equations

$$s\rho=-\frac{d(P\rho)}{dz},\quad w\rho=\frac{d(P\rho)}{d\rho}$$

show that inside the vortex ring s and w are of the same order as $\frac{1}{\varepsilon}$. We will now examine more closely ρ_0 and z_0. We have

$$\rho_0^2\int\lambda\sigma=\int\rho^2\lambda d\sigma$$
$$z_0\int\rho^2\lambda d\sigma=\int z\rho^2\lambda d\sigma$$

We assume that λ is constantly of the same sign, consequently the circle (ρ_0, z_0) lies either in or indefinitely near the vortex filament.

Now we found that, $\lambda d\sigma$ not varying with the time,

$$\int\rho^2\lambda d\sigma=\text{const.}$$

consequently ρ_0 does not vary with the time. That is, if only one vortex filament exists in the fluid its radius will remain unaltered, during the motion. Now to examine z_0. We have the equation

$$z_0\int\rho^2\lambda d\sigma=\int z\rho^2\lambda d\sigma$$

$$\int\lambda d\sigma=m$$

from these

$$m_0\rho^2 z_0 = \int z\rho^2 \lambda d\sigma$$

Differentiating with respect to t

$$m\rho_0{}^2 \frac{dz_0}{dt} = \int \rho^2 \frac{dz}{dt} \lambda d\sigma + 2\int z\rho \frac{d\rho}{dt} \lambda d\sigma$$

But we had also

$$\frac{1}{4\pi} T = \int (w\rho - sz)\rho\lambda d\sigma$$

$$= \int \rho^2 \frac{dz}{dt} \lambda d\sigma - \int \rho^2 \frac{d\rho}{dt} \lambda d\sigma$$

$$= m\rho_0{}^2 \frac{dz_0}{dt} - 3 \int z\rho \frac{d\rho}{dt} \lambda d\sigma$$

consequently

$$m\rho_0{}^2 \frac{dz_0}{dt} = \frac{1}{4\pi} T + 3 \int z\rho \frac{d\rho}{dx} \lambda d\sigma$$

T, we have seen, is constant and infinitely great, of the same order as log ε; the difference between the values of z, in the second member, we know to be of the same order as ε, and we have seen that $\frac{d\rho}{d\tau}$ is of the order of $\frac{1}{\varepsilon}$; consequently the second member of the right hand side of the equation is finite. Therefore, it follows that $\frac{dz_0}{dt}$ is infinitely great of the order of log ε, and, neglecting the finite term — is constant. Since T is positive, $\frac{dz_0}{dt}$ must have the same sign as m, *i. e.*, as λ. Thus, if only one vortex ring exist in the fluid, it will retain its radius unaltered during the motion, and will advance in the direction of the axis of z with the velocity $\frac{dz_0}{dt}$. Now, as this vortex motion implies motion in all the particles of the fluid we have, that all the fluid particles at a finite distance from the filament flow through the ring in the direction of z, or the reverse, according to the sign λ. If $\vartheta = o$, we have $\lambda = \eta$, and, according to the convention for positive rotation we have, that the motion in the direction of z will be positive, if, in the case $\theta = o$, η is positive. Therefore, it follows that the ring is moving in the same direction as the fluid particles are flowing. I will now give the concluding remarks of Helmhotz's great memoir as nearly as may be. We can now readily see in general how two vortex rings having the same axis will move with reference to each other, by observing that each will have its motion modified, due by the motion of the particles of fluid, due to the rotation of the other. Suppose that both rings have the same direction of rotation, then they will both move forward in the same direction, and the former will widen and move more slowly, while the latter will contract and move forward more rapidly, finally overtaking and passing through the former, when the same operation will be repeated, the rings continually changing position throughout the motion.

Suppose that the vortex rings have equal radii, the result is not changed in the case of the same direction of rotation existing for both. But now let them equal radii, and equal but opposite angular velocities; they will approach each other, and both will expand—approaching very near the effect of one upon the other is greatly increased—and they expand with constantly increasing velocity.

Now suppose that the rings having equal and opposite angular velocities, are symmetrical to each other. Then the motion in the direction of the axis of those particles that lie midway between the rings is o. We can conceive this surface in which these particles lie as a fixed boundary, and we have the case of vortex rings moving in contact with a fixed surface. These rings can be readily formed in water; or, rather, half rings can be formed, if we draw through the water rapidly, and for a short distance, a half immersed hemispherical vessel. Half rings will be formed in the water, having their axes in the fine surface of the fluid, which will move exactly as described in the theory. The free surface of the water will form a limiting plane, passing through the axes of the rings, and will not affect the motion. Rings of tobacco smoke have a rapid motion forwards in the direction of and due to the impulsive force which produced them; at the same time the ring flows through itself in the direction of the motion of translation.

It is very interesting to observe the motions of smoke rings, and for this purpose the following simple apparatus, which has been described in a great many places, will be found useful: A rough box, about ten inches long, and the same

height and width, is large enough; one end of the box to be open, and over this stitch a piece of cloth or rubber; make a hole, about three inches in diameter, in the opposite end of the box, and a number of slides having smaller holes in them, to be placed over the larger opening and concentric with it. Now place inside of the box a vessel containing salt, on which pour strong sulphuric acid; and also place in the box a piece of cotton saturated with ammonia; fumes of ammonium chloride will immediately fill the box. Now tap on the stretched membrane; rings will issue from the hole in the slide at the opposite end, and will move forward with velocities proportional to the force of the blow struck. A very light tap is all that is necessary, and, indeed, is all that can be given, if it is desired to investigate the motion, as, otherwise, the rings move forward with such velocity that they can scarcely be followed with the eye. If the rings are allowed to impinge upon a surface, the rotational velocity is suddenly increased very much, and the rings thus spread out over the surface.

The same effects will be noticed if two rings be allowed to meet each other in their motion through the air. If the orifice be elliptic, the rings will be seen to interchange rapidly their axes, vibrating about a mean circular position.

If bubbles of phosphuretted hydrogen be allowed to escape into the air, each bubble, as it breaks, forms a vortex ring of phosphoric anhydride, which is composed of a number of small rings.

The reader is advised to read, on the subject of vortex motion, Sir William Thomson's paper in the Edinburgh Transactions for 1869; also an article by D. Bobylew, in the *Mathematische Annalen*, Vol. VI., in which he shows that the equation $\omega k =$ const. is true not only for frictionless fluids, but also for those in which the friction has to be taken into account.

The following articles will also be found to contain much of interest: On the Motion of Water in a Rotating Rectangular Prism, A. G. Greenhill, *Quarterly Journal of Mathematics* for Nov., 1877; on Plane Vortex Motion, by the same author, and in the June number of the same journal for 1877. There are also several interesting articles in the *Messenger of Mathematics* for the year 1878, notably one of vortex motion in elliptic cylinders; and on the motion of a liquid in a rotating quadrantal cylinder.

SANITARY WARNINGS FROM HIGH PLACES.

From "The Builder."

In the destructive course of the cholera through India, it was remarked by some observers that the pestilence seemed to come from below. Not only were the inmates of houses at the foot of a hill attacked sooner than those towards the summit, but the fowls and other small animals, naturally abiding on the surface of the ground, sickened and died some days before dangerous symptoms broke out in the human inhabitants. If this statement be thoroughly reliable, it is of no little value, both as affording a warning, and as indicating in what direction the sanitary engineer should look in his precautions against the danger intimated. Exhalation, in one word, is, at all events, one sign or cause of danger, in local and epidemic diseases, and the remedy against mischievous exhalation is, not only drainage, but proper regulation of the entire water *régime*. Baking ground, in some cases, has proved even more sickly than swamps.

It is not, however, from the lower levels, either physically or socially regarded, that the most serious warnings to provide against future mischief now come. The remark which has more than once been made in our columns of the mode in which that subtle and dangerous malady which now takes the form of diphtheria, and now that of enteric or typhoid fever, chooses its victims among the very flower of society, has received fresh illustration. In the past few weeks fatal blows have been struck in two royal houses, in that of the Prince Royal of

Prussia and in that of the Duc de Montpensier. And the alarm thus caused is intensified by the recollection how many of those who could so ill be spared have been carried off within the last few years, from the palace and from the castle, by a disease which we profess to believe is preventible by the engineer.

It is to be hoped that the true lesson will be drawn from these repeated appearances of what we must regard as malarious diseases in the homes of luxury. It would not be unworthy of the Society of Arts to direct a special investigation to the subject. If we venture on a few hints, it is rather as indicating the line of inquiry that the public welfare seems to demand, than as at all anticipating the result of an inquiry of which the need has become very urgent.

Predisposition to attacks of this nature may prove to be either personal or local. It is not clear, but it may be suspected, that there is some element at work, in the present state of civilization, which renders the more gently nurtured, or more highly cultured, members of society specially unfitted to resist malarious influences. Connected with this must be borne in mind the manner in which the external atmosphere is more and more kept out from our houses. Doors and windows close better, draughts are more carefully excluded, than of old. Appliances are introduced for artificially warming the passages and vestibules, the natural function of which places is to afford a graduated transition from the warm atmosphere of a chamber to the external temperature. Clothing is much more complex than was formerly the case. In the time of our grandfathers a man was called a puppy if he wore an overcoat. What would those hardy gentlemen have said to the "Ulsters" of the present day? or the sealskin jackets and coats? Human habit is so much modified by circumstances, that the adoption of all these safegards against an occasional chill may have a direct tendency to lower the resisting power of the constitution. And there are well known facts that square with this view. Such is the influence on the constitution of the prolonged heat of tropical or sub-tropical countries. Even in Italy, where the thermometer rarely rises above 96° Fahr. in the middle of the summer, this effect is more perceptible. Those who can fly to mountain heights or cool spots invariably do so for about six weeks in the summer; those who cannot do so (we now speak of foreigners resident in Italy) disregard the first hot season, and laugh to see the natives slowly slinking from shade to shade, and never moving without an umbrella,—truly so called, when used not against rain, but against sun. The second season, however, becomes more serious. The third still more so; and a succession of summers, even as far north as Naples, is a very severe trial to an Englishman or a German. The body seems to dry up, the hair to become like hay, and old age to advance with untimely speed. No one who has had experience of the matter will deny that this is the general rule. The inference is not unnatural that the greater comfort, as we regard it, at all events the more sustained heat, which we are steadily giving to our abodes, is really tending to lower our constitutional power of resistance, not only to the great tonic, cold, but to those influences against which that tonic has the prime function of strengthening the frame.

When we note, further, that it is so often in the palace and the noble mansion that the outburst of the fatal malady occurs, this view of the case is yet further supported. The first point, therefore, for the inquiry of the scientific physician is, how far do greater habits of luxury, avoidance of cold, late hours, gas, carriage exercise in lieu of walking or of riding, and luxury of food, sap and undermine the vigor of the individual, and thus, in an increasing ratio, the vigor of the race? And if this prove to be the case, is there no Lycurgus to be found to raise the cry of "Health before ease and luxury?"

The second point regards the sanitary condition of our houses themselves. This is a different thing from the effect of luxury on the frame. It may be possible, or at all events conceivable, for a great house to be so built, and so tended, that a given temperature, let us say 66° Fahrenheit, may be constantly maintained through vestibules, corridors, staircases, and rooms; and at the same time that the ventilation shall be such as to change the air with sufficient rapidity, and to prevent the entrance of any poisonous gas. But in our variable

climate the cost of the provisions for this purpose would be great, and the necessity for constant supervision would demand attendants of a high degree of education to be detailed for this special service. As it is, warmth is too frequently secured at the cost of pure air. The products of the combustion of gas, and the minute quantities of gas (where this mode of lighting is allowed in sitting or sleeping room) that escape unburnt, either from the pipes, the fittings, or the burners, are constantly vitiating the atmosphere of the house. Our own experience is that plants will never thrive in a room in which gas is burnt. If such be the effect on the vegetable tissue, what must be the case with the more delicate tissue of the lungs? We know houses where sore throat is frequent, almost constant, during the winter, and from which we believe that it might be banished by cutting off the gas, and having the windows regularly opened in all weathers.

Then there is the great danger attendant on the water-closet system. We are not about to propose any Utopian remedy. We do not deny that in this system we have, all things considered, the best yet discovered for the removal of the *debris* of organic change from large cities. But it will not do to shut our eyes to the fact that the system is attended with great danger. The slightest irregularity—stoppage, leakage, frost, want of water, too much water, incurs the risk of turning a fatal exhalation of sewer gas into the house. Sometimes this arises from external causes alone, quite beyond the control of the residents. An irruption of gas may occur in offensive quantities. This, horrid as it is, is far less dangerous than a more subtle and gradual escape. Sometimes it is the very effort to ventilate the sewer that provides a stream of fatal gas to dribble down an unused chimney, or to lurk under the eaves of the attic stories. Sometimes, and that more especially in large hotels, it is sheer neglect. We know one instance in one of these buildings, which it would not be proper to particularize, of this nature. A child, heir and hope of his family, was staying with his parents at this place for a day or two. The mother discovered the neglected state of one of the closets, to which the child had been taken, and the family left the house forthwith. It was too late; the seeds of the malady had been sown, and diphtheria very soon carried off the only hope of his parents. We speak of one instance within our immediate circle of acquaintances, and within a very few months. How many deaths are due to the like cause? How many, for that matter, to this identical source of mischief?

In referring to this we may, it is true, be reverting to advice formerly given in our own pages. The story of sewer-gas poisoning is not new. But it is none the less important to bring the subject before our readers in its proper connection. We are attempting to account for a very threatening phenomenon. It is one that has for some time been denounced, but that seems to be increasing in virulence, from year to year. The evil must be either essential, or accidental. It must spring either from the increased power of mischief, or from the decreased power of resistance. In our opinion it is due to the combined influences of both causes. Habits of luxury, or, if we may even not go so far as that, habits of delicacy, are, we fear, impairing the constitutions, especially of the richer classes. The provision for those habits is affecting the salubrity of our dwellings. That both these influences are to some extent, at work, we think, is undeniable. Is it not high time that the matter should be looked in the face?

With regard to the question of house ventilation, the scheme has recently been put forward of a large and lofty chimney, connected with a block of houses, which by its draught should sweep and purify the entire block. There is something very fascinating in this idea, especially to those who are practically aware of how well chemical works can be kept pure by a lofty chimney. It may be well to look, for a few minutes, at the leading facts that would determine the size and number of such chimneys. We lately gave a brief account of the highest chimney in the world, one of 468 feet high, at the Port Dundas works, Glasgow. The circular area of the tube of this chimney, at the top, is under 88 square feet. This chimney was built with a taper, the propriety of which is questionable, as, in order to utilize the larger part of the diameter a higher velocity is requisite

through the narrower part of the chimney, while the disposition of the air will be to lose velocity as it cools. But, taking the 88 feet of area, and allowing a velocity of 33 feet per second (which is equal to 22½ miles per hour, or that of a stiff gale) to be attained, 118,800 cubic feet of air would pass through the aperture per minute. As a check on that calculation we take from Mr. D. K. Clark's valuable "Manual of Rules, Tables, and Data for Mechanical Engineers" (p. 927), the maximum quantity of air discharged per minute by a Guibal fan, working at Staveley Colliery, of 30 feet diameter and 10 feet wide, which is 110,005 cubic feet per minute. To attain the speed of 33 feet per second, a pressure of 15.5 lb. per square foot must be maintained. The quantity of atmospheric air required for the consumption of one ton of coal, of average quality, is 313,600 cubic feet; so that a chimney of the area indicated, provided such a draught could be maintained, would effect the consumption of a ton of coal in a little under three minutes, or 22 tons of coal per hour. Taking eighteen hours out of the twenty-four for the combustion, we obtain close upon 400 tons per day, or 146,000 tons of coal per annum. If we allow a ton of coal per unit of the population per annum, this would require from twenty-four to thirty such chimneys for the whole of London.

But when we come to look at the question of ventilation, for the purpose of respiration, it will at once be seen how inefficient this powerful draught would be to discharge the functions of our present wasteful and inconvenient chimneys. According to Mr. Bailey Denton, in his first letter on sanitary engineering (p. 2), 23,000 cubic feet of ordinary air are required per individual per twenty-four hours in order to keep the vitiation of the air below 1 per 1,000, at which point impurity becomes perceptible to the nose. Allowing only eighteen hours to be spent within doors, either in bedrooms, houses, or offices, we shall find that the 824 million cubic feet passing through the hypothetical chimney in that time would only provide for the ventilation requisite for a little over 47,000 human beings. Three times as much ventilating power, therefore, is required for keeping the chambers of a house pure, as is requisite for maintaining combustion of coal at the allowance of one ton per inhabitant per annum. And no provision is made for the products of combustion of gas or other artificial lights.

We need not add together the two quantities; as the partially vitiated air would to a great extent serve for combustion. But, all things considered, it would seem that a chimney possessing as high a power of draught as we have named would be requisite for, at least, from every 33,000 to 40,000 of population, supposing the whole of the ventilating arrangements to be brought under proper control by its means. As to the details of these arrangements, we cannot at present find space for discussion. But it may be mentioned that from 400 feet to 740 feet per minute is the rate of ventilation which is effected in mines; the upcast pit having the former rate of draught, and the velocity in the several channels varying inversely as their areas. The point where the above considerations practically bear on our present subject is this. The removal of air from a house that is requisite to ensure a due ventilation for sanitary purposes is so much more considerable than the removal which is due to, and effected by, the combustion of the amount of fuel proportioned to the number of inhabitants in the dwelling, that special attention should be directed to the subject in all large houses. Chiefly is this the case in those houses, whether large or small, in which it is the object of the housekeeper to maintain a high, or a moderate but invariable, temperature, through all parts of the house, in the winter season. This can only be done, with safety, by warming the air as it enters; and the question is, whether either this or any other mode of keeping up the temperature of the vestibules and corridors is not a direct invitation to disease. We suggest that it is so, in two modes. First, that the resisting power of the frame is lowered by the maintenance of an invariable temperature. While persons remain within the house, the aëration of the blood and the animal vigor will be abated; and on going out the cold will be more keenly felt, and very possibly more injuriously felt, than if the passages had been cool. The other,

and entirely independent question, is that of the risk that there is in all highly warmed houses of slighting the proper ventilation. The carbonic acid exhaled from the lungs is, of course, so much direct poison, unless removed. To this have to be added all other sources of impurity, products of combustion, gas resulting from the course of the organic functions, emanations from the pet birds or other animals, and, most dangerous of all, infiltration of sewer gas. A wholesome thorough draught is the only safeguard against this domestic malaria; and very few are the instances in which this is to be ensured without the utmost regularity and care as to freely-opened windows. Those who know what Belgian, German, and even French hotels are in this respect, will be likely to form the opinion that palaces and noble mansions may be made, and very frequently are made, far less safe and salubrious, especially for the young, than were the rude and draughty nooks in the halls and castles of our ancestors, or than our very modest cottages in the present day.

THE GERMAN NAVY.—The *personnel* of the German Navy is to be increased this year by 29 officers and 343 warrant officers, petty officers and men, although, according to the original plan for the establishment of the fleet, the increase should comprise only 168 officers and men of all ranks. The engineer department is also to be augmented by 95 petty officers and men, instead of but 73, the number prescribed in the plan. Altogether, since 1875, more than 1000 men have been added to the strength of the divisions of seamen, so that the German Navy now comprises 419 sea officers, 24 engineers, 5459 warrant officers, petty officers and men in the two divisions of seamen, besides 1001 petty officers and men in the engineer department. Of the two divisions of seamen, one belongs to each of the two naval stations, the Baltic and North Sea, the headquarters of the one being at Kiel, and of the other Wilhelmshafen. Each division is commanded by a post captain, and is divided into five detachments, one of which is composed solely of seamen gunners. Besides these divisions of seamen, there is also a "dockyard division" at the headquarters of each station, Kiel and Wilhelmshafen.

REPORT ON THE METRIC SYSTEM (ERRATA).—A couple of errors in the engraving on page 382 of our last issue are corrected in this cut.

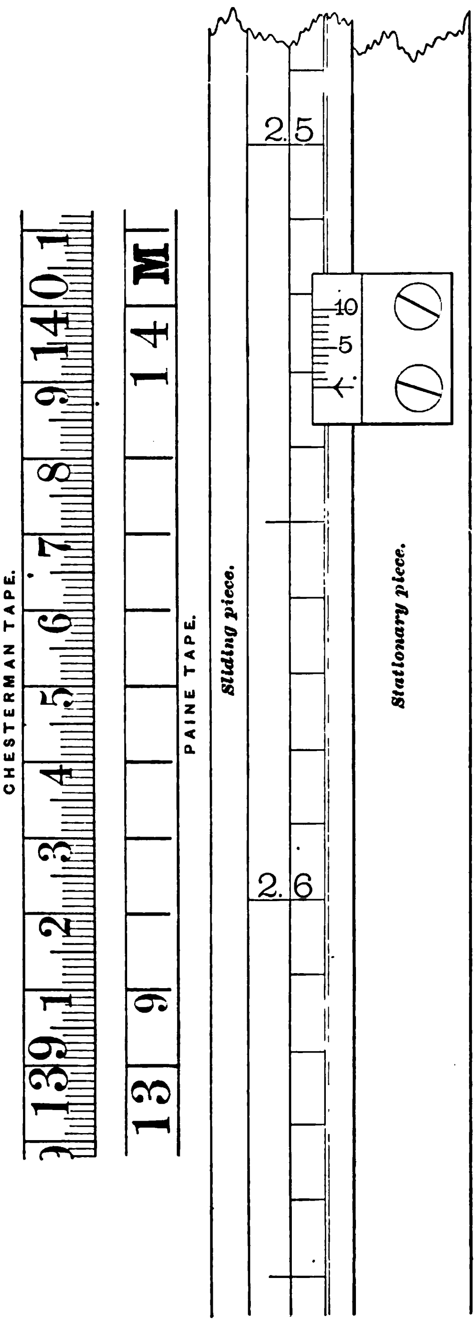
The reading of the slide rod should have been 2.532, as is here given.

The numbering on the Paine tape was incorrectly represented by the misplacing of the figure 9.

A SLIDE-ROD SCALE

Submitted by Charles H. Swan, C.E. Providence R.I.

READING 2.532



REPORTS OF ENGINEERING SOCIETIES.

AMERICAN SOCIETY OF CIVIL ENGINEERS.—The Eleventh Annual Convention of the Society will be held at Cleveland, beginning Tuesday, June 17th, 1879.

Sessions for the consideration of professional subjects, and one for the transaction of business will be held.

The details of the programme will be announced as soon as determined by the local committee.

The following is a list of topics to be considered with reference to papers published in Transactions during the preceding year:

American Engineering at International Exhibitions.—CLXXIV. American Engineering as illustrated at the Paris Exposition of 1878. George S. Morison, Edward P. North and John Bogart.

Bridges.—Discussion of Paper CXL. The Determination of Stresses in the Eye-Bar Head. De Volson Wood. Vol. VII, page 189. Discussion of Papers CXLIV and CXLIX. Relative Quantities of Material in Bridges of different kinds, of various heights. William H. Searles. Vol. VII., page 192.

Cements.—Discussion on Cements. Don J. Whittemore. Vol. VII, page 274. Discussion on Cements and Strength of Bricks. F. Collingwood. Vol. VII., page 280.

Foundations.—CLXXII. The use of Compressed Air in Tubular Foundations, and its application at South Street Bridge, Philadelphia, Pa. D. McN. Stauffer.

Hydraulics.—CLX. On the cause of the Maximum Velocity of Water flowing in Open Channels being below the Surface. James B. Francis. CLXI. The Flow of Water in Pipes under Pressure. Charles G. Darrach. Discussion on the Cause of the Maximum Velocity of Water flowing in Open Channels being below the Surface, and also on the Flow of Water in Pipes under Pressure. Theo. G. Ellis, C E. Emery, Clemens Herschel, De Volson Wood and John T. Fanning. Vol. VII., page 122. CLXVII. Distribution of Rain-fall during the great storm of October 3d and 4th, 1869. James B. Francis. CLXVIII. The Gauging of Streams. Clemens Herschel. CLXXV. The Flow of Water in Small Channels, after Ganguillet and Kutter, with Kutter's Diagram modified, and Graphical Tables with special reference to Sewer Calculations. R. Hering.

Masonry.—CLXX. Brick Arches for Large Sewers. R. Hering. Discussion on Brick Arches for Large Sewers. E S. Chesbrough, W. Milnor Roberts, R. Hering and F. Collingwood. Vol. VII., page 258. CLXXI. Fall of Western Arched Approach to South Street Bridge, Philadelphia, Pa. D. McN. Stauffer. Discussion on Nomenclature of Building Stones and Stone Masonry. J. Foster Flagg, J. J. R. Croes, J. P. Davis, F. Collingwood, J. Veazie and E P. North. Vol. VII., page 284.

Metals.—CLXIII. On a newly discovered relation between the Tenacity of Metals and their resistance to Torsion. Robert H. Thurston. CLXIV. Observations on the Stresses developed in Metallic Bars by Applied Forces. Theodore Cooper.

Preservation of Timber.—CLXXVI. The Permanent Way of Railways in Great Britain and Ireland, with special reference to the use of Timber, preserved and unpreserved. Compiled from information received from Engineers in charge of these Railways John Bogart.

Railroads—CLIX. On the Theoretical Resistance of Railroad Curves. S. Whinery. Discussion on the Resistance of Railroad Curves. O Chanute, Chas. E. Emery, E. Yardley, E. P. North, C. L. McAlpine, F. Collingwood and Wm. H. Paine. Vol. VII., page 97. CLXVI. Reminiscences and Experiences of Early Engineering Operations on Railroads, with especial reference to Steep Inclines. W. Milnor Roberts. Discussions on Inclined Planes for Railroads. O. Chanute and William H. Paine. Vol. VII., page 216.

Rivers and Harbors—CLXII. The South Pass Jetties. Descriptive and Incidental Notes and Memoranda E. L Corthell. Discussions on the South Pass Jetties. Charles W. Howell, E. L. Corthell, C. Shaler Smith, J. Foster Flagg. Vol. VII., page 159. CLXIX. The Dangers threatening the Navigation of the Mississippi River and the Reclamation of its Alluvial Lands. B. M. Harrod.

Steam Engines.—CLXV. Cushioning the Reciprocating Parts of Steam Engines. John W. Hill. Discussions on Steam Engine Economy. J. Foster Flagg and E D. Leavitt, Jr. Vol. VII., page 194.

Submarine Telephony.—CLXXIII. Submarine Telephoning. Charles Ward Raymond.

In addition to the above papers, it is expected that the following subjects will be presented by papers printed previous to the date of the Convention, or read at its meeting:

Engineering Questions involved in the Development of Electric Lighting. Stephen Chester.

Gelatine Dynamite and High Explosives. Julius H. Striedinger.

Light House Construction. J. G. Barnard

Memoir upon the Construction of the Minot's Ledge Light. B. S. Alexander.

The Construction and Maintenance of Roads. Edward P. North.

The Resistances of Railway Rolling Stock. A. M. Wellington.

The Railroad Crossing of the Allegheny Mountain. Moncure Robinson.

Notes on Early Railroad Engineering. Ashbel Welch.

Remarks on the Causes of Fall of the Western Arched Approach to South Street Bridge, Philadelphia, Pa. J. G. Barnard.

Parabolic Arches in Masonry. W. A. G. Emonts.

Notes on the Foundations of Piers of the East River Bridge. F. Collingwood.

Experiments with Cements and appliances for testing. Alfred Noble.

Comparison of Standard Measures, English, French and United States. Arthur S. C. Wurtele.

The Construction of Concrete Blocks at the end of the South Pass Jetties. Max E. Schmidt.

Notes as to construction and operation of the Railroad over the Ratou Mountains, Col., and the construction and performance of the Locomotives thereon. James D. Burr.

Design and Construction Tables for Egg-shaped Sewers. Cyrus G. Force, Jr.

Members of the Society are earnestly requested to furnish information or memoranda upon any of the subjects referred to. They are also invited and expected to take part in the discussions either in person or by sending to the Secretary notes for presentation.

In either case, it will assist the Committee in arranging the details for sessions of the Convention, if members expecting to take part in the discussions will notify the Secretary at once to that effect.

Invitation to visit Pittsburg and the Government Works for the improvement of the river at that place (Davis Island Dam) has been extended by James H. Harlow, member of the Society.

Society of Engineers.—At a meeting of the Society of Engineers, held April 21st, Westminster, a paper was read by Mr. J. L. Haddan on "The Essentials which should govern the Construction and Working of Tramways." The author observed that when tramways were first introduced, they were a great advance upon the ordinary roads, but that the modern improvement of roadways had, in the present day, led almost to a reversal of the relative positions of road and tramway. In America, the tramways were superior to the roads, because the latter were sacrified to the former. The tram-rail there, moreover, was available for the moderate ordinary traffic, whilst the speed of the tram service was about twenty per cent. greater than in this country and on the continent. Mr. Haddan alluded to the general tendency to employ wood in roadways, and he described a system of construction by which a road could be made with a perfectly flat surface, and yet be well drained, and which should have the tramway incorporated with it. The tram-rails, he said, would be of wood, and the roadway would be kept surfaced with grit, so that the wood would not form the actual wearing surface. This system of tram and roadway, the author observed, would be homogeneous, and would combine the best possible road for ordinary vehicles, with a perfect tramway for special carriages at less cost than the present method of construction. The author condemned the indiscriminate introduction of railway and omnibus principles into the construction and working of tramways, and described his proposed arrangement for meeting the requirements of a tramway service. This consists of a locomotive engine to be worked by steam and air, the steam being used for compressing, during the journey, its own supply of air as well as that which supplies the continuous motive power for propelling the cars. By reversing, the same driving mechanism acts as a continuous brake, and the same system is so arranged that the driver constantly feels the pull of the train upon a regulator handle. The withdrawal of his hand from this handle is to instantly cause the steam power to block the train. Thus the brake mechanism would always be in action, instead of lying dormant, as in ordinary continuous brakes. The author, in conclusion, stated what, in his opinion, were the technical, physical, and administrative requirements of mechanical traction on tramways generally.

Liverpool Engineering Society.—At the fortnightly meeting on Wednesday evening, 23rd April, Mr. Arthur J. Maginnis read a paper entitled, "A Few Years' Experience of the Screw Propeller, its Lessons and Results," for which he was awarded the Carlyle Gold Medal at the Institution of Naval Architects, session 1879. as being the best on the subject of screw propulsion. The first part of the paper dealt with the various methods of construction, the materials used for the bosses and blades of large propellers—for which recent experience has proved the old style of solid cast iron to be altogether unsuitable, although still occasionally used owing to their cheapness—and also gave some important particulars of the manner in which failures have occurred from time to time, pointing out how they are to be avoided in the future. The much vexed questions of the relative merits of large and small diameters, and of pitch, uniform, or varying, were next treated in an able manner, and interesting statistics compiled from the logs of some of the most important merchant steamers were given, which showed that moderate reductions of both diameter and pitch had given a reduction of about 8 per cent. in the consumption of fuel on a round voyage, and an increase of about 3 per cent. in speed, the same result being also shown in a marked degree by H. M. S. Iris, the important trials of that ship being also given in a concise table similarly arranged to those of the merchant steamers. Passing over the mode of attachment to the propeller shaft, and the loss occasioned by the corrosion of the blade, etc., the author dealt with the present disputed question of solid cast iron *versus* movable bladed propellers, and pointed out several events, showing the advantages to be greatly in favor of the latter. The various ways by which propellers may become disabled were also noted, some of the most remarkable being illustrated by drawings. After pointing out what experience had taught, as to how a large propeller should be constructed, the important question of the effect of the screw propeller on steering was reviewed at considerable length, and the strange effect produced by the sudden reversing of the propeller, from full speed ahead to full speed astern, was fully treated and explained by explicit diagrams, showing in a lucid manner the directions and flow of the currents of water round about the rudder and screw propeller which produced these effects. Appended to the paper was a neat tabular arrangement of the effect of the propeller on the steering of screw steamers under all circumstances for both right and left-handed propellers, and also a table giving the dimensions and particulars of propellers at present fitted on several of the most important Liverpool steamers.

IRON AND STEEL NOTES.

Manufacture of Steel Ingots.—With a view to economize the labor at present required in producing steel ingots, and also to improve the quality of the ingot by avoiding

the tendency of the metal to boil when run into the moulds, Messrs. Taylor & Wailes, of Panteg, Monmouth, propose to use moulds, which consist of metal troughs placed side by side or parallel to one another, and in rear therof they fit ways on which a traveling trough or gutter runs. This traveling trough is bent at right angles at its forward end to form a spout parallel to the moulds, and it serves to deliver into the moulds the molten metal, which is conducted to it from an inclined stationary gutter or gutters in connection with the furnace. The stationary gutter is so situated with respect to the traveling gutter that the latter will be free to run under the fixed gutter and receive therefrom a continuous supply of the molten metal while it is presenting its discharging mouth to the several moulds in succession. This arrangement not only facilitates the filling of the moulds, but by filling them from the top in a broad stream will remove all tendency which the metal has to boil in the moulds, and will also prevent the cutting of the moulds by the falling metal. The moulds, which will consist of open metallic troughs, may be made of any suitable length, and by an arched block or tile inserted in the moulds the metal poured therein will be divided up to form two ingots.

For facilitating the discharge of these ingots from the moulds they propose to place, transversely of these moulds and near their opposite ends, cross-bars of iron fitted with rough hooks or studs at equal distances apart, and so arranged as to enter the several moulds and bed into the molten metal. These bars will be connected at their opposite ends to a traveling crane, which, when the metal is sufficiently set to secure the connection of the bars with the ingots will be caused to lift the bars, and thereby draw out the ingots from the moulds. The strain thus put on the opposite ends of the castings will cause them to break in the line of the block or tile, and the ingots will then hang perpendicularly in the air, and may be delivered on to any suitable receptacle. In some cases they place a rectangular metal lifting strap with eyes at its extremities inside the moulds, and at such position that it will embrace the ingot to be cast at about the middle of its length. This strap fits close to the inside of the mould, and is covered over with clay, loam, brick, or other substance to prevent the fusion of the metal: this permits of the lifting out of the ingot horizontally by connecting lifting chains with the strap, and when the strap is disengaged from the chain it may readily be separated from the ingot.

In order to prevent any irregularity in the form of the ingot, the ingot mould may be recessed to admit the strap, and the joints may be made good by clay loam. In some cases they propose to make the moulds of the length of the ingot desired, and to set two of such moulds end to end, making good the butt joints by means of clay puddle. The metal cast in such jointed moulds will, when lifted out of the moulds by the means above described, break at the middle of their length or at a point coincident with the line of junction with the moulds, and the ingots will then hang from the lifting bars as explained. This mode of clearing the moulds of the ingots will greatly economize the labor at present required for performing such duty, as the handling of the moulds and the ingots for this purpose is entirely avoided.

RAILWAY NOTES.

Railway to India.—A paper was read before the Civil and Mechanical Engineers' Society by Mr. Haughton, C.E., on the subject of "A Railway to India."

Mr. Haughton exhibited a map showing the regions to be crossed, taking Constantinople and the town of Shikarpore, on the River Indus, as the terminal points of the railway; he spoke of the suggested Euphrates Valley Railway from Scanderoon or Swadia, on the mainland opposite the horn of the island of Cyprus to the head of the Persian Gulf, with its possible extension along the Mekran coast to Kurrachee, at the mouth of the Indus, as being quite out of the line of the terminal points named, and as being objectionable because of passing through a region destitute of towns after passing Bussorah; such a line, while forming a perhaps useful strategical road to India, entirely failed to tap the larger centers of population and trade of the countries intervening betweeu Europe and India, viz., Asia Minor and Persia. He preferred a railway passing through Sivas and Diarbekir, in Asia Minor (where it would cross the Taurus range of mouutains), Mosul, and Kefri, in Turkish Arabia, with a short branch to Bagdad, crossing the Gates of Zagros range on the Persian frontier, and going through the Persian towns of Kermanshah, Hamadan, Teheran, Shahrud, and Mushed, crossing the Afghanistan frontier, and passing through Herat and Candahar and through the Bolan Pass into Shikapore, a station on the Indus Valley Railway. Such a line would open up the most populous and traffically-valuable districts of Persia, towards which country, as well as towards India, Russia is hastening every day.

Colonel Sir Arnold Kembell wished to impress on the meeting the vastness of such an enterprise. He believed it would be of much importance in view of the interests of England and of India, and of great value in opening up the trade of Turkish Arabia.

Mr. Trelawney Saunders said the importance of such a line of railway could hardly be overestimated, and that he thought the route indicated on the map would be eventually that selected for a railway to India.

Inefficient Continuous Brakes. — A treacherous friend is more dangerous than an open enemy, and it is far better to be without a continuous brake than to place reliance on one which will fail when it is most wanted. A few gentlemen connected with the Great Northern Railway have for a long time held to the vacuum brake as being perfect, just as Mr. Webb and one or two London and North-Western Railway men claim that the chain brake is the best in the world. Constant practical failures have taught these men nothing. Perhaps because the failures did not entail actual loss of life or limb on a large scale. How

long they are to enjoy this immunity is, however, a doubtful question, and an accident which occurred on Friday afternoon to the Flying Scotchman on the Great Northern line, teaches a lesson which should not be overlooked As this train, which does not stop between York and Grantham, was passing through Bawtry station, it was noticed by some of the officials on the platform that one wheel of one carriage was off the line, and a telegram was immediately despatched to the next station to stop the train. The carriage, however, caught the bridge over the Scaftworth road, and carried away a portion of the stone coping, and was pulled up at about 200 yards beyond. By this time some six carriages had left the metals and considerable damage was done to the permanent way. The passengers were much shaken, and one lady was seriously injured. Two of the first-class carriages were almost on their sides, while the axle of another was broken. The communication cord was broken before it was noticed that a carriage had left the rails, and the vacuum brake was rendered useless in consequence of its having become disconnected. If the principal features of this accident be contrasted with that of certain others in which the train was fitted with the automatic brake, a vast difference in favor of the latter will be recognized. The vacuum brake will do the work of guards and firemen very fairly, but as a safety brake it is quite inefficient. This truth has long been recognized by everyone, save a few individuals who probably like the vacuum brake because it is the smallest remove from the old hand brake to be found; and if a change must be made, they give the preference to the least change possible. Railway companies require severe lessons to teach them to do what is right. The Great Northern Company narrowly escaped receiving such a lesson on Friday.

ENGINEERING STRUCTURES.

Tubular Piles.—The difficulty which attends driving long tubular piles is well known to all those who have attempted to drive them, especially when they are to be driven from the surface of water of considerable depth above the bed of the water course. Messrs. Le Grand and Sutcliffe, of Bunhill-row, so well known through their connection with tube wells, have devised a method of putting down tubular piles of any length. This consists in driving them by means of an elongated cylindrical monkey which works inside the tubular pile. The monkey is raised by means of rods or rope, and allowed to fall upon the flat head of the solid point. The pile thus forms its own guide for the monkey. One advantage of these piles is that they can be driven with facility, of very great length and in deep water. A wrought or cast iron pile, say 75 ft. long, is lowered through the water length by length, screwed together by strong barrel-shaped steel sockets until ground is reached and additional lengths are added as the pile is driven. They can also be filled with concrete well rammed to increase their strength. For some purposes the piles are made in a short length with a flange on the top, the length being sufficient to allow the flange to stand at the ground surface; a tubular or other post with a bottom flange can be fastened thereto permanently or temporarily. It might at first sight appear that a heavy ram falling upon the inside upper surface of the point of a long pile would tend to pull such a pile in two at the screw joints by reason of the inertia of rest of all above the point. It does not seem, however, that this is the case, and short cast iron piles have been driven a number of times, on some occasions into a macadamised road, without apparent injury. The method of driving secured great facility and simplicity, and the number of applications of the method is almost innumerable. For tube wells, piles, telegraph and signal posts, tent poles and plugs, it will be very valuable, and as a tool for making holes for wood telegraph posts, a short tubular pile and hand monkey will greatly simplify operations The War-office has instructed the Royal Engineer Committee to make a series of experiments with this pile in various soils immediately. The experiments will take the form of actual bridge construction.

The St. Gothard Tunnel.—In a paper just communicated to the French Academy, M. Colladon gives some interesting details of the progress of this great enterprise. Besides the excessive hardness of the banks of serpentine and quartz, and the insufficiency of hydraulic forces on the Airolo side (after the lowness of water in winter), there has been a very troublesome infiltration in the south portion, the volume of water having increased since the second year to more than 230 litres per second in the advance gallery. The difficulty of working here under jets which had often the force of those from a fire-engine pump, can be readily imagined. Another difficulty arises from a mass of decomposed felspar mixed with gypsum, found under the plains of Audermatt. This plastic material swells on contact with moist air, and exerts a pressure of terrible force, capable of crushing the strongest woodwork, and even arches of granite. In some of these parts the workers thought themselves happy when they were able to advance (with manual boring) about 1 metre in three or four days; whereas, through granite, with compressed air and mechanical perforation, a regular advance of 4 metres in 24 hours has been achieved through gneiss 6 metres, and more. As regards apparatus, M Colladon states that the volume of water from the Tremola (Airolo side) having been found insufficient, M. Favre brought water in an aqueduct, 3000 metres long, from the Tessin, to work new turbines of four compressors, on the same system as the others, but of greater value. These turbines are of cast iron. It is noticeable that the old and smaller bronze turbines (formed of one piece), which have made some 155,000,000 revolutions per annum, are in good preservation, after four, or even five years' service, and still work usefully, after slight renewals. On each side of the tunnel there are at present sixteen air compressors in action, serving both for aeration and for boring operations. They send into the tunnel air under a pressure of eight atmospheres, sufficient to drive eighteen to twenty drills, and effect good ven-

tilation of the part already bored, which is at present 6100 metres on the north side, and 5390 metres on the south side. On either side there are, night and day, several hundreds of workmen with lamps, and about 300 kilogrammes of dynamite are consumed. The compressors are found to suffice for good ventilation, rendering unnecessary two large exhaust vessels, placed two years ago at either mouth of the tunnel for drawing off smoke and vitiated air. The transport of materials is effected by horses in the more advanced part of the tunnel, and by compressed air locomotives in the portions near the mouths. To feed these locomotives eight of M. Colladon's compressors are placed at Airolo and Goeschenen. They draw air from the ventilating pipe, and force it, under a pressure of 12 or 14 atmospheres, into a pipe which passes along the part traversed by the locomotives. A great variety of rock drills have been used. Each year brings new improvements.

ORDNANCE AND NAVAL.

THE experimental cruise of the Royal Mail steamer Gallia, already celebrated on account of the part played in her construction by foreign workmen, took place on the 27th ult. On her trial she made the mile in three minutes and forty-five seconds, equal to sixteen knots an hour, or eighteen and one-quarter miles. She is bark-rigged and has nine iron bulkheads, of which seven are watertight, and run up to the spar deck. forming eight watertight compartments. There are two dining saloons for cabin passengers, one of spacious size, on the spar deck. It is lighted by a handsome cupola, and beautifully ornamented with Japanese panelling. The coal bunkers are well ventilated and constructed to contain 1,000 tons of fuel. The water tanks are made of galvanized iron, and are capable of holding 14,000 gallons. The fresh water condensers are capable of providing 4,000 gallons daily. Steam steering gear is fitted amidships, but to meet casees of emergency there is also a manual steering gear with wheel-house aft. There are iron lighthouses, entered from the deck below, allowing lights to be easily accessible even in the wildest weather. Suspended from davits are ten large lifeboats, with patent lowering apparatus. The Gallia is the largest and most complete of all the Cunard ships. She starts on her first voyage to New York to-morrow.

KRUPP'S GREAT STEEL GUN.—Herr Krupp is about to try at Meppen, Germany, the latest and largest steel gun turned out by his great works, and indeed the largest specimen of steel ordnance yet made. It weighs 72 tons, with a calibre of 15¾ in. The length of the gun is 32 ft. 8 in., and that of the bore 28 ft. 6 in. The English 80-ton gun has a calibre of 16 in., a total length of 27 ft., and a bore 24 ft. long. The charge for Krupp's monster gun is to be 385 lb. of prismatic powder, the projectile being a chilled iron shell, weighing 1,660 lb., and having a bursting charge of 22 lb. of powder. The force of the shot on leaving the gun is estimated at 31,000 foot-tons, and it is calculated that the gun can throw its projectile a distance of 15 miles. The forthcoming trials will take place on a range 11 miles long, and the targets will have to be placed at such a distance that the gun will have to be directed by other means than the visibility of the object to be struck. The material of which the Krupp gun is composed is steel throughout. The core of the gun consists of a tube running its entire length, as in the Woolwich gun, but open at the rear, the loading being at the breach instead of the muzzle. The tube of this large weapon being of such great length, it has been made in two portions, the joint being secured in a peculiar manner. Over the tube are four "jackets" or cylinders of various lengths, supplemented by a ring over the breech portion. The cylinders are much less massive than in the Fraser gun, and approximate more to the pattern of the Armstrong ordnance. The gun is chambered, that is to say, the powder chamber has a greater diameter than the bore. The form given to the powder prisms, and the adjustment of the cartridge in the bore, allow altogether an amount of space which gives 40 per cent. of air to the powder actually composing the charge. The rifling is polygonal with a uniform twist, and the shot is rotated by means of a copper ring around its circumference near the base, the Vavasseur system in short The closing of the breech is effected by the well-known Broadwell system adopted for all his ordnance by Mr. Krupp.

BOOK NOTICES

MODEL YACHT BUILDING AND SAILING. London: Charles Wilson. For sale by D. Van Nostrand. Price $2.00.

This subject possesses rare attractions for a large class of people who delight to pursue a diversion with a systematic plan and scientific precision.

This is a complete instruction book for the amateur model yacht builder and includes directions for all details. It is also supplemented with an illustrated vocabulary of nautical technology.

The author has given his instruction in a singularly felicitous straightforward way which will surely prove acceptable to all interested in the subject.

ELECTRIC LIGHTING AND ITS PRACTICAL APPLICATION By J. N. SHOOLBRED, M. I. C. E. London: Hardwicke & Boyne. For sale by D. Van Nostrand. Price 2 50.

This is a very compact summary of descriptions of present modes of electric lighting, and affords a fair estimate of the relative value of machines and lights which are now before the public.

The chapters treat severally of topics as follows:

I. Historical and Descriptive. II. Electric Light Machines. III. Lamps and Regulators. IV. Carbons and Conducting Wires. V. Motive Power. VI. Luminous Intensity. VII. Applications and Economic Results. VIII. Prospects of Electric Lighting.

The illustrations are sufficiently numerous and of fair quality,

The Student's Text-Book of Electricity. By Henry M. Noad, F. R. S. A new edition with additional chapters. By W. H. Preece, M. I. C. E. London: Crosby, Lockwood & Co. For sale by D. Van Nostrand. Price $5.00.

This new edition of an excellent work is quite timely. Nothing better than the earlier edition has ever appeared to meet the wants of those who apply electricity. Nothing else in English is so full of practical electrical science.

This new edition is enriched by the additions of Mr. Preece, who has given a summary of dynamo-electric inventions and electric illumination, down to the beginning of the present year.

The remarkable abundance of illustrations which characterized the former edition is exhibited quite as fully in the new chapters.

Coal Mines Inspection—Its History and Results. By R. Nelson Boyd, F. R. G. S. London: W. H. Allen & Co. For sale by D. Van Nostrand.

This is a history of the legislation in Great Britain relating to the inspection of coal mines, giving a full account of the improvements in methods and machinery adopted from time to time, whereby the danger to life in the mines has been steadily lessened.

It appears from the statistics given quite fully in the appendix, that whereas, in 1851, the mining of one million tons of coal cost 19.35 human lives, in 1876 the same amount of coal cost but 6.95 lives, and that the decrease was steadily maintained during this period.

To the English collier, or the mine owner, the book is full of interest.

Les Metaux a l'Exposition Universelle de 1878. Par H. Sebasteur Paris: Dunod. For sale by D. Van Nostrand. Price $5 00.

This is a large quarto volume of 300 pages of text and 16 pages of plates.

The metals treated of are the various irons and steels together with such alloys of copper as have been proved to possess an exceptional strength.

The resisting powers of the metals is treated at considerable length as is also the method of determining such resistances by the various testing machines. Most of the illustrations are of these machines and include the prominent and familiar American forms.

The work is a valuable compendium of information on the subject of metals and in engineering constructions.

A Practical Treatise on the Combustion of Coal. By Wm. M. Barr. Indianapolis: Yohn Brothers. For sale by D. Van Nostrand. Price $2.50.

This work aims to present within a moderate compass the theory of the combustion of coal, with a view of adapting it to the needs of that large body of men whose interests are identified with the use of coal, and to whom a correct knowledge of the subject is especially desirable, but who, on account of the abstruse style of the present treatises, cannot readily obtain the required information.

All that the author claims is the adaptation of the science of the higher treatises to readers who find difficulty in studying the higher standard works.

Considerable space is devoted to descriptions of special devices for burning fuel of different kinds. The illustrations are of fair quality, and the general typography is excellent.

The Transmission of Power by Compressed Air. By Robert Zahner, M.E. New York: D. Van Nostrand. (Van Nostrand's Science Series, No. 40). Price 50 cts.

The opinion is now very general that the maximum of economy in the use of rock-drilling machinery is to be looked for less in the further improvement of the drills themselves than in the perfection of the air-compressing machinery, and this perfection can only be arrived at by careful attention to the various scientific considerations connected with the subject. The whole question is ably discussed in a recent volume of Van Nostrand's Science Series, by Mr. Robert Zahner, and the information he supplies is so valuable that it should be generally studied by all who have occasion to use compressed air. The subject of compressed air and compressed-air machinery offers, as Mr. Zahner states, a wide field for useful investigation. Compressed air has become a most efficient and powerful agent in the hands of the modern engineer. Its applications are rapidly growing both in extent and importance. There can be no doubt that the great waste of energy that to-day accompanies the use of compressed air is due not only to sickly design and faulty construction of machines, but very largely also to the general ignorance of the principles of thermodynamics. In his historical notice Mr. Zahner points out that the application of compressed air to industrial purposes dates from the close of the last century. Long before this, indeed, we find isolated attempts made to apply it in a variety of ways, but its final success must be ascribed to the present age—the age of mechanic arts—an age inaugurated in so splendid a manner by the genius of Watt, and which has been so wonderfully productive in good to mankind. Cubitt and Brunel, between 1851 and 1854, first applied compressed air in its statical application to the sinking of bridge caissons. Prof. David Colladon, of Geneva, in 1852, first conceived and suggested the idea of employing it in the proposed tunneling of the Alps, and finally Sommeiller first practically realized and applied Colladon's idea in the boring of the Mont Cenis Tunnel.

For transmitting power to great distances shafts, belts, friction wheels, and gearing are clearly out of the question The practical incompressibility and want of elasticity of water renders the hydraulic method unfit for transmitting regularly a constant amount of power; it can be used to advantage only where motive power acting continuously is to be accumulated and applied at intervals, as for raising weights, operating punches, compressing, forging, and other work of an intermittent character requiring a great force acting through a small distance. Compressed air is the only general mode

of transmitting power; the only one that is always and in every case possible, no matter how great the distance nor how the power is to be distributed and applied. No doubt as a means of utilizing distant yet hitherto unavailable sources of power the importance of this medium can hardly be overestimated. But compressed air is also a storer of power, for we can accumulate any desired pressure in a reservoir situated at any distance from the source, and draw upon this store of energy at any time, which is not possible either in the case of steam, water, or wire-rope. But compressed air is especially adapted to underground work; steam is here entirely excluded, for the confined character of the situation and the difficulty of providing an adequate ventilation renders its use impossible. Compressed air, besides being free from the objectionable features of steam, possesses properties that render its employment conducive to coolness and purity in the atmosphere into which it is exhausted. The boring of such tunnels as the Mont Cenis and St. Gothard would have been impossible without it. Its easy conveyance to any point of the underground workings, its ready application at any point, the improvement it produces in the ventilating currents, the complete absence of heat in the conducting pipes, the ease with which it is distributed when it is necessary to employ many machines whose positions are daily changing, such as hauling engines, coal-cutting machines, and portable rock drills; these and many other advantages when contrasted with steam under like conditions give compressed air a value which the engineer will fully appreciate.

There is every reason to believe that compressed air is to receive a still more extensive application. The diminished cost of motive power when generated on a large scale compared with that of a number of separate steam engines and boilers distributed over manufacturing districts, and the expense and danger of maintaining an independent steam power for each separate establishment where power is used, are strong reasons for generating and distributing compressed air through mains and pipes laid below the surface of the streets, in the same way as gas and water are now supplied. Especially in large cities would the value of such a system be invaluable; no more disastrous boiler explosions in shops filled with hundreds of working men and women; the danger of fire greatly reduced; a corresponding reduction in insurance rates; an important saving of space; cleanliness, convenience, and economy. As affording a means of dispensing with animal power on our tramways compressed air has been proposed as the motor. It has already met with some success in this direction, and to-day there are eminent French, English, and American engineers at work upon this interesting problem. Mr. Zahner treats first of the conditions modifying efficiency in the use of compressed air—loss of energy, methods of cooling, conditions most favorable to economy in the use of compressed air, efficiency attained in practice, and losses of transmission. In treating of the physical properties and laws of air Mr. Zahner deals with the subject in a manner which will make it intelligible to every practical man, and the same may be said of the subsequent chapters—thermodynamic principles and formulæ, thermodynamic equations applied to permanent gases, thermodynamic laws applied to the action of compressed air, the efficiency theoretically attainable, the effects of moisture, of the injection of water, and of the conduction of heat; American and European air compressors; and examples from practice. There is, probably, no other book containing the same amount of useful information in a similar space, a circumstance which will suffice to commend it to the attention of every engineer who uses or could use compressed air as a motor.—*London Mining Journal*

MISCELLANEOUS.

Mean Depth of the Sea. — A large amount of material for arriving at some approximately correct notion of the mean depth of the sea, has accumulated in recent years. In a note to the Göttingen Academy, Dr. Krümmel has lately attempted this, in view of the vague and variable statements on the subject in text-books. Soundings were wanting for the Antartic and a part of the North Polar Sea, *i e.*, about 475,000 square miles, or 7 per cent. of the entire sea-surface, so that he gives his estimate only as a closer approximation. He estimates, then, the mean depth of the sea as 1,877 fathoms, or 3,432 meters, or 0.4624 geographical miles. It was natural to compare the mean height of dry land above the sea level. Humboldt's estimate of 308 meters is regarded as quite out of date. Leipoldt has since estimated the mean height of Europe as 300 meters. Accepting this number for Europe, 500 for Asia and Africa, 330 for America, and 250 for Australia, Dr. Krümmel obtains the mean of 420 meters, or 0,0566 miles. The surface-ratio of land to water being considered 1:2.75, the volume of all dry land above the sea-level is inferred to be 140,086 cubic miles, and the volume of the sea 3,138,000 cubic miles. Thus the ratio of the volumes of land and water is 1:22.4. That is, the continents, so far as they are above the sea-level, might be contained 22.4 times over in the sea-basin. Reckoning, however, the mass of solid land from the level of the sea-bottom, the former would be contained only 2.443 times in the sea-space. Dr. Krümmel also compares the masses (taking recent data); he finds that of the sea 3,229,700 cubic miles, and that of the solid land 3,211,310 (a small difference). If the specific gravity of the land were raised merely from 2.5 to 2.51432, we should thus have perfect equilibrium. Such equilibrium is probably the fact.—*Nature.*

One of the latest attempts to correct the irregular flight of war rockets consists of fixing an additional head to the missile with holes in the neck for the escape of the powder gas rearwards, similar to the escapement at the base. It was thought that this would cause the rocket to preserve a more correct balance as well as increase its range, but it has found in practice to have the contrary effect, and the suggested improvement is a failure.